MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 37/1995

Knotentheorie

10.09. – 16.09.1995

The meeting has been organized by Joan Birman (New York) and Maxim Kontsevich (Berkeley). Some of the subjects treated in the talks are following:

a) Vassilev invariants of knots and links

b) Finite type invariants of 3-manifolds

c) Groups of 3-manifolds

d) Topological quantum field theory

e) Invariants of plane curves and Legendian knots

f) Knizhnik–Zamolodchikov equations and link invariants.

The interest of the audience showed up in lively discussions after the talks and in the evenings after dinners.

VORTRAGSAUSZÜGE

P. CARTIER:

Infinitesimal deformations of categories

The monodromy of Knizhnik–Zamolodchikov equations can be viewed as a way of making deformations of a tensor category. This method can be applied to produce deformations of representations of braid groups and a combinatorial description of the Vassiliev–Kontsevich invariants of knots and links.

T. COCHRAN:

Stability of lower central series of compact 3-manifold groups

If G is a group, then its lower central series is defined by $G_1 \equiv G$, $G_{\alpha+1} \equiv [G, G_\alpha]$ and $G_\alpha = \bigcap_{\beta<\alpha} G_\beta$ if α is a limit ordinal.

The *length of* G is the least ordinal α such that $G_\alpha = G_{\alpha+1} = G_{\alpha+2} = \ldots$. Perfect groups have length 1, abelian groups have length 2, non-abelian free groups and surface groups have length ω (the first infinite ordinal $G_\omega = \bigcap_{n=1}^{\infty} G_n$). In fact, J. Levin only recently exhibited the first example of a finitely presented group with length greater than ω! His example was of length $\omega + 1$. We ask what lengths are possible for compact 3-manifold groups. This seems to be strongly related to the Parafree Conjecture and seems to have some relationship with 4-dimensional topological surgery. In particular, the Parafree Conjecture, a long-standing question of G. Baumslag, is equivalent to the statement that all groups of ribbon link exteriors have length ω.

As our main results we show that, while Fuchsian groups and Seifert fibered 3-manifold groups generally have length ω, there *do exist* closed *hyperbolic* 3-manifolds whose groups have extremely long length (at least 2ω at present). It should be possible (but we have not done it) to find a link exterior with these properties.

P. DELIGNE:

Vassiliev invariants for tangles in $\sigma \times [0,1]$, σ is a Riemann surface

Let us consider tangles in $\mathbf{C} \times [0,1]$, with n strands, each going from a point $(s_i, 0)$ in $\mathbf{C} \times \{0\}$ to $(s_i, 1)$ in $\mathbf{C} \times \{1\}$. Put $s = (s_1, \ldots, s_n) \in X = \mathbf{C}^n \setminus \{\text{diagonals}\}$. By Artin,

$$\pi_1(X, s) = \{\text{colored braid group}\} \hookrightarrow \mathcal{T}(X, s) = \{\text{set of such tangles, mod isotopy}\}.$$

Looking at Vassiliev invariants is akin to looking at the unipotent completion of π_1. More precisely, the vector space dual to that of Vassiliev invariants of order $< n$ is akin to $\mathbf{Q}[\pi_1]/I^n$, I is the augmentation ideal. This quotient of the group algebra of π_1 is motivic — for instance carries a mixed Hodge structure. Philosophy:

(a) this "motivic" structure formally extends to Vassiliev invariants

(b) exactness of the weight filtration explains why Vassiliev invariants of singular knots can be "integrated" to Vassiliev invariants of knots

(c) C can be replaced by any non singular algebraic curve.

M. GOUSSAROV:

How to build all knotted graphs with the same invariants of degree $\leq n$

I will describe geometric operations preserving knotted graph invariants of degree $\leq n$. If a knotted graph cannot be obtained from another one by these operations then there exists an invariant of degree $\leq n$ which takes different values on the graphs. In cases more complicated than knots and string links, I include in the class of finite degree invariants partially defined invariants like Milnor's invariants of links.

T. KOHNO:

Elliptic KZ system and associated invariants

We start with describing Vassiliev invariants for pure braids from de Rham homotopical viewpoint. It is shown that 0-th cohomology of the bar complex of the logarithmic forms on the configuration space is isomorphic to the space of Vassiliev invariants for pure braids. using this technique, we show the converse to the statement due to Stanford that if $\gamma \in P_n$ lies in the k-th stage of the lower central series, any order k Vassiliev invariant for $\gamma - 1$ is zero. In the second part, we study a generalization of the KZ equation, a system associated with elliptic solution of the classical Yang–Baxter equation. With the aid of a work due to Etingof and others, we give an integral representation for a Vassiliev invariant for a link in the torus times unit interval associated with a certain weight system related to $sl(n, \mathbf{C})$.

X.-S. LIN:

On Ohtsuki's invariants of integral homology 3-spheres

An attempt is made to conceptualize the derivation as well as to facilitate the computation of Ohtsuki's rational invariants λ_n of integral homology 3-spheres extracted from Reshetikhin–Turaev $SU(2)$ quantum invariants. Many interesting consequences will follow from our computation of λ_2. One of them says that λ_2 is always an integer divisible by 3. It seems interesting to compare this result with the fact shown by Murakami that λ_1 is 6 times the Casson invariant. Other consequences include some general criteria for distinguishing homology 3-spheres obtained from surgery on knots by using the Jones polynomial.

W.LÜCK:

L^2-torsion and 3-manifolds

We introduce for a finite CW-complex whose L^2-Betti numbers are all trivial and whose Novikov-Shubin invariants are all positive a positive real number called *combinatorial L^2-torsion.* It behaves like a "multiplicative Euler characteristic", namely, it behaves like an Euler characteristic for push outs and fibrations of certain type and is a homotopy invariant. Tools for the computations of L^2-Betti numbers, Novikov-Shubin invariants and combinatorial L^2-torsion are given. For example combinatorial L^2-torsion can be computed for an irreducible Haken 3-manifold from a presentation of the fundamental group without using further topological information. Examples are knot complements. There is the conjecture that the difference of combinatorial L^2-torsion and analytic L^2-torsion for compact manifolds is $\ln(2)/2 \cdot \chi(\partial M)$ which has been recently verified in the closed case. This implies for a prime Haken 3-manifold whose boundary is empty or a disjoint union of incompressible tori that the combinatorial L^2-torsion is up to a multiplicative non-zero constant Gromov's simplicial volume and up to a non-zero multiplicative constant the sum of the volumes of the hyperbolic pieces in the Jaco-Shalen-Johannson-Thurston splitting by incompressible tori.

G. MASBAUM:

A simple proof of integrability of quantum invariants at roots of unity of prime order

In his recent work relating quantum invariants to the Casson invariant, H. Murakami has used the fact that the Reshetikhin–Turaev invariants of homology 3-spheres at roots of unity of prime order are algebraic integers. Unfortunately, his proof of this fact is by a very complicated computation. In this talk, I want to present a simple skein-theoretical argument found in joint work with Justin Roberts (Berkeley). It shows rather easily that the Turaev–Viro invariant is integral; the same result for the Reshetikhin–Turaev invariant follows as a consequence. Our argument also works for manifolds wich aren't homology spheres, as well as for the extension of the invariant for manifold-link pairs.

M. POLYAK:

Invariants of plane curves and Legendrian knots

Recently V.Arnold introduced in axiomatic form three basic invariants J^+, J^- and St o plane curves and generalized later the invariants $J^\pm$ to the case of fronts of Legendrian knot (i.e. roughly speaking, to cooriented curves with cusps).

We present an elementary combinatorial formulas for the invariants $J^\pm$, St in terms of a Gauss diagram of a curve. Similar formulas for the invariants $J^\pm$ of Legendrian fronts ar provided and an invariant St' of Legendrian fronts extending the invariant St of plane curves is introduced.

A relation of Vassiliev knot invariants to plane curve invariants of finite degrees is discussed.

L. ROZANSKY:

Derivative of the Jones polynomial and finite type invariants of rational homology spheres

We derive a formula for the colored Jones polynomial of a link in the limit of large K. The Jones polynomial is presented as an itegral over the product of coadjoint orbits corresponding to the representations attached to the link components. The integrand is a product of an exponential whose exponent is proportional to K and a preexponential factor which is a series in K^{-1}. This presentation allows us to define an infinite sequence of (perturbative) invariants of rational homology spheres by formally substituting integrals instead of sums in the Reshetikhin-Turaev surgery formula and calculating these integrals by stationary phase approximation. The invariants can be expressed in terms of derivatives of the colored Jones polynomial of a link on which the surgery is performed.

The integral formula for the Jones polynomial implies certain bounds on the powers of colors coming with particular powers of K^{-1} in the expansion of teh Jones polynomial of algebraically split links. By using the bounds we prove that perturbative invariants are of finite type with respect to the definitions given by T. Ohtsuki and S. Garoufalidis. We relate the weights of these invariants to closed trivalent graphs with edges corresponding to link components and vertices corresponding to the triple Milnor linking numbers.

We also use the bounds on the expansion of the Jones polynomial in order to provide a simple rederivation of the results of H. Murakami and T. Ohtsuki about the properties of the Reshetikhin-Turaev invariants of rational homology spheres at prime values of K. We derive an explicit surgery formula for Ohtsuki's invariants and show that it coincides with our formula for perturbative invariants. Thus we establish a direct relation between both types of invariants.

T. STANFORD:

Milnor invariants of singular knots

Milnor's μ invariants may be defined for singular knots with much less indeterminacy than in the case of links. I am investigating the possibility that some of these μ invariants of singular knots descend to Vassiliev invariants of (non singular) knots. The easy case is the order 2 invariant (of knots). There is olso an order 4 invariant, and probably more. The hope is that invariants can be obtained which are not in the Alexander polynomial, providing a connection between quantum / Vassiliev invariants and the topology of the complement of a singular knot.

V. TURAEV:

A Shadow viewpoint on Vassiliev invariants

Theorem. *All Vassiliev invariants of knots in S^3 canonically extend to shadows on S^2.*

The proof uses the Hopf fibration $S^3 \to S^2$ and Goussarov theorem about Vassiliev invariants. The result allows to relate the second order Vassiliev invariant of knots with Arnold's strangeness of loops in $\mathbf{R}^2$.

A.VAINTROB:

Algebraic srtuctures related to Vassiliev invariants

We show that the theory of Vassiliev invariants of links is intrinsically related with various algebraic structures of Lie type, such as Lie superalgebras, Yang–Baxter Lie algebras or Leibniz algebras.

These structures can be used to produce new invariants or to explain relations between old ones. Lie superalgebras, in particular, explain coinsidence of weight systems corresponding to Lie algebras of series O and Sp and the supersymetry of the Alexander–Conway polynomial.

O. VIRO:

Finite degree invariants of algebraic curves

Real algebraic curves in plane and 3-space have invariants similar to Vassiliev invariants of knots and Arnold's invariants of immersions $S^1 \to \mathbf{R}^2$.

I mean to discuss these invariants and their relations with their topological counter-parts.

P. VOGEL:

Semi-simple Lie superalgebras are insufficient

There exists a graded $\mathbf{Z}$-algebra Λ acting in a natural way on many modules of 3-valent diagrams. Every simple Lie superalgebra with a non-trivial invariant bilinear form induces a character on Λ. Classical and exceptional Lie algebras and the Lie superalgebra $D(2,1,\alpha)$ produce eight distinct characters on Λ and eight distinct families of weight systems on chord diagrams. As a consequence we prove that weight functions coming from semisimple Lie superalgebras do not detect every element in the module $\mathcal{A}$ of chord diagrams.

S.-W. YANG:

Kontsevich integral and higher degree Vassiliev invariants

Could you believe that the finite type Vassiliev invariant theory is a cohomology theory on a complicated space i.e. Vassiliev complex $\mathcal{V}_n$, $n = 1, 2, \ldots$ (thus, the higher degree theory exists automatically), and the generalized (higher degree) Kontsevich integral is just a special de Rham theory on the terrible non-manifold space (Vassiliev complex). The Arnold relation of closed 1-forms on the configuration space of complex plane makes everything work well. A reduction property in the homology theory of $\mathcal{V}_n$ is established to prove the finite integration of higher degree Kontsevich integral. This reduction is a generalization of "isolated chord axiom" in the algebra of chord diagram.

Main porposes of my talk:

(1) Construct the principal part P_n of Vassiliev complex $\mathcal{V}_n$ (P_n is a $2n$-dimensional manifold) and give a cell decomposition of P_n. The cells correspond to diagrams.

(2) Describe the reduction theorems and chain complexes $\mathcal{D}(n)$ associated with $\mathcal{V}_n$.

(3) For each knot K, there is a cannonical special $2n$-form $\omega(K)$ on P_n (with some local coefficients). When the knots K_n deform on a parameter space U, we get a $2n$-form on $P_n \times U$. By slant operation, we have the Kontsevich integral.

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