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"Function Spaces"

17.-23.09.1995

The international conference on "Function Spaces" was held from September 17 to September 23, 1995 in Oberwolfach. The organizing committee consisted of A. Frölicher (Genève), H. Herrlich (Bremen) and G. Preuß (Berlin). The meeting was opened by G. Preuß who welcomed 23 participants from 10 countries and proposed one minute of silence for the late colleagues V. Koutnik (Czech Republic) and K. Morita (Japan). Simultaneously, a conference on category theory took place in Oberwolfach. On account of this reason there was ample time for fruitful discussions between and after the presented lectures. The special flavour of this meeting resulted from the fact that scientists working in many different areas of function spaces came together. Thus, e.g. pure topologists joined workers in the field of applications in analysis.

The talks and discussions dealt mainly with the theory of function spaces in the following contexts:

I) Axiomatic Set Theory (Herrlich)

II) General Topology:
(a) topological spaces (Hušek, Schröder)
(b) generalized topological spaces [pretopological spaces; convergence spaces; and convergence approach spaces] (Lowen-Colebunders, Richter; Giuli, Vainio; Kent)
(c) sequential spaces (Frič)
(d) proximity spaces (Bentley)
(e) semiuniform convergence spaces (Preuß)

III) Category Theory (Gähler, Nguyen, Porst, Strecker)

IV) Applications:
(a) logic (Kleisli)
(b) topological algebra (Frič)
(c) analysis (Adam, Frölicher, Kriegl)
(d) duality theories (Erné)

Besides other ones the following results are very remarkable:

1. H. Herrlich has shown earlier that in ZF set theory compactness splits into various forms (A–, B–, C–, and D–compactness) that behave quite differently. Here he demonstrated the surprising fact that the Ascoli Theorem holds in ZF
   (a) for A–compactness (same for B–compactness resp. C–compactness) iff the Boolean prime ideal theorem holds,
   (b) for D–compactness iff the axiom of choice holds.

2. M. Hušek (jointly with S. Watson) solved Herrlich's 1991–problem whether metrizable spaces are almost coreflective in Top in the following surprising way: the answer is 'no'; however it is 'yes' provided Top is replaced by the category of T_0–spaces. [Equivalently (in the terminology of function spaces): a topological space X is a T_0–space if and only if the function space functor $C(-,X)$ from the category of metrizable spaces to Top is a weak retract of the function space functor $C(-,M_X)$ for some metrizable topological space M_X.]

3. E. Lowen–Colebunders has demonstrated earlier that (a) in the categroy Pr Top of pretopological spaces the exponential objects are precisely the finitely generated ones. Here she demonstrated that, moreover, (b) the finitely generated spaces form the largest cartesian closed coreflective subcategory of Pr Top – a result that contrasts sharply with the situation in Top (though smaller than Pr Top, Top has larger cartesian closed coreflective subcategories than the one consisting of finitely generated topological spaces). Using different methods, G. Richter demonstrated that (c) the result (a) above extends to T_0–, T_1–, and T_2–pretopological spaces.

4. H.L. Bentley (jointly with M. Hušek) presented a Stone–Weierstraß theorem in the setting of proximity spaces.

5. G. Preuß introduced earlier semiuniform convergence spaces (s.u.c.sp.) as a convenient setting for studying topological problems. Here he demonstrated that the locally compact s.u.c.sp. form a cartesian closed coreflective subcategory of the category SU Conv of s.u.c.sp. whose exponential objects can be easily described. Furthermore, the concepts "locally compact" and "compactly generated" coincide in contrast to the situation in Top . Note that the Hausdorff axiom is not needed whereas in Top the (compact Hausdorff)–generated spaces form a cartesian closed subcategory but the (compact)–generated spaces do not form a cartesian closed subcategory.

6. Using a suitable symmetric monoidal closed category $\underline{B}$ and a suitable cartesian closed category $\underline{C}$, H. Kleisli constructed an adjoint situation $\underline{B} \underset{F}{\overset{U}{\rightleftarrows}} \underline{C}$ that can serve as a model for linear logic.

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7. The uniform boundedness principle is usually formulated for locally convex spaces. A. Frölicher (joint work with Cl.-A. Faure) demonstrated convincingly that by replacing the space of linear continuous functions by the one of linear bornological functions, i.e. by working in the category of bornological spaces (instead of locally convex ones) one obtains much nicer results. Moreover, the classical theorems can be recovered from the new results in an elegant manner.

8. A. Kriegl reported on joint work with P. Michor concerning infinite dimensional Lie groups. The most spectacular result: every Lie algebra homomorphism integrates to a Lie group homomorphism, provided that the source group is simply connected and the image group is regular. This has many applications, since it was shown also that the important function spaces of diffeomorphisms are regular Lie groups. Among the many modern tools which are used one finds the calculus for convenient vector spaces and latest results of E. Adam on the approximation property.

9. M. Erné developed a symmetric duality theory covering about a dozen familiar dualities between certain categories of ordered sets on one hand and certain categories of topological algebras on the other hand as special cases. The duality functors in both directions are suitable function–space functors.

At the end of the conference several of the participants expressed the hope that a similar conference could be held again in some years in the inspiring atmosphere of the Mathematical Research Institute of Oberwolfach.

A. Frölicher H. Herrlich G. Preuß

ABSTRACTS

E. Adam

Approximation properties

Classically, a locally convex space E is said to have the approximation property if the space $E' \otimes E$ of operators on E of finite rank is dense in the space $\mathcal{L}(E, E)$ of continuous linear operators on E with respect to the trace of the topology of uniform convergence on relatively compact subsets of the completion $\widehat{E}$ on $\mathcal{L}(\widehat{E}, \widehat{E})$. In the setting of the infinite dimensional calculus due to Frölicher and Kriegl, the objects of which are the socalled convenient vector spaces forming a category that admits several isomorphic descriptions, it turns out to be fruitful to consider other structures than locally convex ones, in particular bornological ones. The question, whether the operational tangent space in a point of a manifold modelled on convenient vector spaces coincides with the kinematic one, leads us to formulate approximation property in terms of convenient vector space structure: which conditions ensure denseness of $E' \otimes E$ in $L(E, E)$ for E a convenient vector space, with respect to several more or less natural topologies on the space of morphisms? We give some stability properties for three different kinds of approximation property and show that many of the naturally appearing spaces of smooth functions have the strongest among these.

H.L. Bentley (joint work with M. Hušek)

A Stone–Weierstrass Theorem for Proximity Spaces

As a consequence of the version of such a theorem which appears in the book "Topological Spaces" by E. Čech, we prove:
($*$) *Theorem:* Let X be a proximity space, $P^*(X)$ the algebra over $I\!R$ of all proximally continuous $f : X \longrightarrow I\!R$, and A a subalgebra of $P^*(X)$ containing the real constants. On $P^*(X)$, place the sup norm topology. Then the following are equivalent for any $g \in P^*(X)$:
(1) $g \in Cl\,A$.
(2) $g\mathcal{H}$ converges for any filter $\mathcal{H}$ such that $f\mathcal{H}$ converges for every $f \in A$.
This theorem has formed the basis for establishing a Katětov dimension theory for proximity spaces.

M. Erné

A symmetric generalization of Stone–type dualities

There are at least a dozen of more or less known dualities between certain categories or *ordered* sets, algebras or (semi)lattices and certain categories of *topological* spaces (algebras, ordered sets etc.), for example, between

- spatial frames and sober spaces,
- bounded distributive (semi)lattices and Stone spaces,
- superalgebraic lattices and A–discrete T_0–spaces,
- semilattices and compact totally disconnected topological semilattices (alias algebraic lattices),

etc. Some other more algebraically flavored dualities, where a topological approach is not so evident, turn out to be similarly structured.
We develop a general "symmetric duality theory", covering these and many other dualities (also some new ones), based on so–called R–invariant subset selections, assigning to each ordered set a certain collection of subsets. Given two such selections $\underline{X}, \underline{Z}$, we establish a duality between the categories $\underline{ZSX}$ and $\underline{XSZ}$, where the objects of $\underline{ZSX}$ are $\underline{Z}$–sober spaces with an open base of $\underline{X}$–compact sets, and morphisms are $\underline{X}$–proper maps between them (such that the inverse image maps preserve $\underline{X}$–compact open sets). In many cases (but not always), the involved duality functors in both directions are obtainable as spaces of functions into the components of a "schizophrenic object" $(S_{\underline{Z}}, S_{\underline{X}})$, where $S_{\underline{Z}}$ is the Sierpinski space S or obtained from S by dropping the empty closed set. More involved is the problem of determining products, function spaces etc. in the general categories $\underline{ZSX}$ (if such constructions exist at all).

R. Frič

Sequentially continuous functions and sequential envelopes

The ring of all sequentially continuous (real–valued) functions can be studied as an OBJECT, but as well as a TOOL to study the underlying space (carrying a seq. convergence).
1. OBJECT. It is known that the ring $B(R)$ of all Baire functions carrying the pointwise convergence yields a sequential completion of the ring $C(R)$ of all continuous functions. We investigate various sequential convergences related to the pointwise convergence and the process of completion of $C(R)$. In particular, we prove that the pointwise convergence fails to be strict (the proof is due to Ján Borsík) and prove the existence of the categorical ring completion of $C(R)$ which differs from $B(R)$. (Remember, not all seq. conv. rings do have a completion. In fact we prove that the subcategory of all seq. conv. complete rings is epireflective in the category of all seq. conv. rings having a completion.)

2. TOOL. The sequential envelope is the sequential analogue of the Čech–Stone compactification or the Hewitt real compactification. We give a short survey of what is known about the sequential envelopes (it is about the extension of seq. continuous functions) and put forward some questions related to the preservation of sequential convergence of a given type when passing from sequentially continuous functions to their continuous extensions.

A. Fröhlicher (joint work with Cl.–A. Faure)

On the Uniform Boundedness Principle (U.B.P.)

The classical version of the U.B.P. uses locally convex spaces E, F and states that, under certain conditions (e.g. if E is barelled or if E is locally complete) two bornologies on the function space $L(E, F)$ of linear continuous maps $E \longrightarrow F$ are identical. The two respective bornologies on $L(E, F)$ are described by means of the bornologies on E, F which are determined by the locally convex topologies. This indicates that one should consider bornological vector spaces E, F instead of locally convex ones and study the function space $L(E, F)$ formed by the linear bornological maps $E \longrightarrow F$. One obtains in fact nicer results in this situation. We shall say: the U.B.P. holds for $L(E, F)$ if for $B \leq L(E, F)$ one has $B(a) \subseteq F$ bounded for all $a \in E \iff B(A) \subseteq F$ bounded for all bounded $A \subseteq E$. In the following F is supposed to be a "linear bornological space", i.e. that one has for $A \subseteq F$: $A \subseteq F$ bounded $\iff \ell(A) \subseteq \mathbb{R}$ bounded for all linear bornological $\ell : F \longrightarrow \mathbb{R}$. This condition holds iff there exists a locally convex topology on F inducing the bornology.

Theorem. For a bornological vector space cond. 1), 2), 3), 4) are equivalent and implied by 5):
1) The U.B.P. holds for $L(E, \mathbb{R})$;
2) The U.B.P. holds for $L(E, F)$ for any linear bornological space F;
3) For $b : E \times G \longrightarrow F$ bilinear, G any bornological vector space, F as above, one has b partially bornological $\Longrightarrow b$ bornological;
4) Every barrel of E is bornivorous (i.e. E is "barrelled");
5) E is Mackey-complete.

This leads to consider the class of Mackey-complete linear bornological spaces. This class has excellent properties since both conditions can be shown to carry over form F to $L(E, F)$. By imposing in addition a separation condition one obtains the so-called convenient vector spaces which are the natural objects in general differentiation theory if one wants to get, together with the smooth maps as morphisms, a cartesian closed category. For this, the U.B.P. plays a fundamental role. This was pointed out by A. Kriegl who proved in particular that 5) $\Longrightarrow$ 2). The consequence of the U.B.P. for convenient vector spaces is the following: a map $f : E \longrightarrow F$ is smooth if

$\ell \circ f : E \longrightarrow I\!R$ is smooth for all $\ell \in L(F, I\!R)$.
For the proofs one could use the classical theorem. But it is shorter to begin from the scratch by means of the following lemma: Let A, B be sets, $A \times B \longrightarrow I\!R$ a map noted $(a, b) \longmapsto a * b$. If $A * B \subseteq I\!R$ is unbounded, there exist sequences $a_i \in A$, $b_j \in B$ and $t \in \ell^1$ s.t. for $n \in I\!N$ one has $|\sum_{i=1}^{\infty} t_i \cdot a_i * b_n| \geq n$.

W. Gähler

Monadic topologies and representation

The notion of monadic topology provides an interesting extension of the classical topology and even of the fuzzy topology. This notion depends on a fixed partially ordered monad $(\varphi, \leq, \eta, \mu)$ consisting of a covariant functor $(\varphi, \leq)$ of SET to the category of almost complete sup–semilattices (in which all non–empty suprema exist) and two natural transformation η and μ such that (φ, η, μ) is a monad over SET. Examples are the partially ordered filter monad, more general, the partial ordered fuzzy filter monad, and the partially ordered stack monad.
A monadic topology on a set X is a mapping $p : X \longrightarrow \varphi X$ such that $\eta_X(x) \leq p(x)$ holds for all $x \in X$ and $p = nb \circ p$, where $nb : \varphi X \longrightarrow \varphi X$ is the associated neighbourhood operator $\mu_X \circ \varphi p$ of p. p is uniquely given by means of nb and, extending φX by an "improper" object O_X to $\tilde{\varphi} X$, also by the related interior operator int : $\tilde{\varphi} X \longrightarrow \tilde{\varphi} X$. Moreover, p is uniquely given by the set T of all open φ–objects M, which are defined by $nb\, M = M$ or, equivalently, by int $M = M$. T also determines p uniquely. We have that a subset T of φX consists of all open φ–objects with respect to a monadic topology if and only it it is closed with repect to all non–empty suprema and with respect to all infima, so far as they exist.
We obtain a more well-known situation defining principal φ–objects $P_{a,\alpha}$ with $a \in \psi X$ and $\alpha \in L$ by means of a contravariant functor $(\psi, \leq)$ of SET to the category of all complete inf–semilattices and a special complete lattice L, called a valuation lattice. Then under some assumptions the monadic topologies can be characterized by the related open ψ–objects $a \in \psi X$. In the fuzzy filter case we have that a subset τ of ψX consists of the open ψ–objects with respect to a monadic topology if and only if τ is closed with respect to all suprema and all finite infima. Moreover, in the fuzzy stack case we have that a subset τ of ψX consists of the open ψ–objects with respect to a monadic topology if and only if τ is closed with respect to all suprema.

E. Giuli (Joint work with M.M. Clementino and W. Tholen)

Closure operators

To develop topology in an arbitrary category $\underline{X}$, a subobject structure and a closure operator w.r.t. this subobject strutcure are needed. A subobject structure is provided by fixing an $(\underline{E}, \underline{M})$-factorization structure for morphisms which extends to a factorization structure for sinks in $\underline{X}$. The images $f(m)$ are given by the second factor of the $(\underline{E}, \underline{M})$-factorization of $f \circ m$, and inverse images $f^{-1}(n)$ are given by pullback. Denoting by $subX$ the complete lattice of all subobjects of an object X, a closure operator of $\underline{X}$ w.r.t. $\underline{M}$ is defined as a family

$$e = \{e_X : subX \to subX\}_{X \in \underline{X}}$$

satisfying the properties $(m, n \in subX)$: 1. $m \leq e_X(m)$; 2. $m \leq n \Rightarrow e_X(m) \leq e_X(n)$; 3. For $f : X \to Y, f(e_X(m)) \leq e_Y(f(m))$.
Let $\underline{X}$ be a category with subobjects, with finite products and with a closure operator e. An object X is called Hausdorff if the diagonal $\delta_X : X \longrightarrow X \times X$ is closed, and it is called connected if δ_X is dense. X is called compact if, for every object Y, the projection $p : X \times Y \longrightarrow Y$ satisfies $p(e_X(m)) = e_Y(p(m))$ for every $m \varepsilon$ sub X.
By adding conditions on e (such as idempotency, (weak) hereditariness, etc.) and (or) on $(\underline{E}, \underline{M})$ (such as Frobenius Reciprocity Law, the Beek–Chevalley Property, etc.) many classical topological results related to separation axioms, compactness and to connectedness can be obtained in the general context of a category with closure.

H. Herrlich

Remarks on Ascoli's Theorem

Consider the following version of Ascoli's Theorem:
(AT) Let X be a locally compact T_2–space, let Y be a metric space, and let $F \subset C(X, Y)$. Then the following are equivalent.

(1) F is compact in the compact–open topology,

(2) (a) F is closed in Y^X w.r.t. the product topology,
(b) $\forall x \in X \quad \pi_x[F]$ is closed in Y, and
(c) F is equicontinuous.

Does (AT) hold in ZF? No. The precise status of (AT) in ZF depends on the definition of compactness. Consider the following non–equivalent versions:

(1) X is A–compact provided X has the Heine–Borel property.

(2) X is B–compact provided in X every ultrafilter converges.

(3) X is C–compact provided X is completely regular and in the ring $C^*(X)$ every maximal ideal is fixed.

(4) X is D–compact provided every infinite subset of X has a complete accumulation point in X.

Theorem 1. Equivalent are:
(a) (AT) holds for A–compactness,
(b) (AT) holds for B–compactness,
(c) (AT) holds for C–compactness (in the completely regular version),
(d) the Boolean prime ideal theorem (PIT) holds.

Theorem 2. Equivalent are:
(a) (AT) holds for D–compactness,
(b) the axiom of choice (AC) holds.

M. Hušek (joint work with S. Watson)

Function spaces on metrizable spaces

H. Herrlich asked a nice question in 1991 that we reformulate and modify as follows: Describe those topological spaces X for which there exists a metrizable space M_X such that the function space functor $C(-,X)$ from metrizable spaces is a weak retract of the function space functor $C(-,M_X)$. (The original formulation is in terms of almost coreflections: Is the class of metrizable spaces almost coreflective in Top?)
We proved, jointly with S. Watson, that exactly T_0–spaces have the above property (or all spaces have the property if one considers pseudometrizable spaces instead of metrizable ones - an answer to the question of L. Bentley).
The method of the proof shows that if one restricts the functors $C(-,X)$ to nonarchimedean metrizable spaces, then one may find M_X to have the same property. The procedure generalizes to higher cardinals κ in the sense that one can consider spaces having κ–discrete open (or clopen) bases or even κ–disjoint open bases for the domain of function spaces and for M_X.
It is still an open problem (posed by Bentley and Hušek, and being a subproblem of another Herrlich's question concerning projectivity classes) whether one may take (perfectly) normal spaces, instead of metrizable ones, in answering the above question.

D.C. Kent (joint work with P. Brock)

Diagonal Axioms for Convergence Approach Space

In 1967, C.H. Cook and H.R. Fischer defined the following dual axioms for a convergence space (X, q).

F: Let J be a non–empty set, $\psi : J \to X$, and let $\sigma : J \to \mathbb{F}(X)$ have the property $\sigma(y) \xrightarrow{q} \psi(y)$, for all $y \in J$. Let $\mathcal{F} \in \mathbb{F}(J)$. Then: $\psi(\mathcal{F}) \xrightarrow{q} x \Rightarrow \kappa\sigma\mathcal{F} \xrightarrow{q} x$.

R: Let J be a non–empty set, $\psi : J \to X$, and let $\sigma : J \to \mathbb{F}(X)$ have the property $\sigma(y) \xrightarrow{q} \psi(y)$, for all $y \in J$. Let $\mathcal{F} \in F(J)$. Then: $\kappa\sigma\mathcal{F} \xrightarrow{q} x \Rightarrow \psi(F) \xrightarrow{q} x$.

In these axioms, $\kappa\sigma\mathcal{F} = \bigcup_{F\in\mathcal{F}} \bigcap_{y\in F} \sigma(y)$. It is known that a convergence space is topological iff F holds and regular iff R holds. In the category CAP of convergence approach spaces (introduced by E. Lowen and R. Lowen in 1991), one may show that the above axioms translate to the following:

(F): If J is a non–empty set, $\psi : J \to X, \sigma : J \to F(X)$ and $\mathcal{F} \in F(J)$, then $\lambda(\kappa\sigma\mathcal{F}) \leq \lambda(\psi\mathcal{F}) + \sup_{y\in J} \lambda(\sigma(y))(\psi(y))$;

(R): If J is a non–empty set, $\psi : J \to X, \sigma : J \to F(X)$ and $\mathcal{F} \in F(J)$, then $\lambda(\psi\mathcal{F}) \leq \lambda(\kappa\sigma\mathcal{F}) + \sup_{y\in J} \lambda(\sigma(y))(\psi(y))$.

Here $(X, \lambda) \in |CAP|$. The approach spaces (Lowen '89) are precisely those objects in CAP satisfying (F); the regular objects in CAP are those which satisfy (R). When we consider $pqs - Met^{\infty}$ as a subcategory of CAP (the former being function $d : X \times X \to [0, \infty]$ satisfying only $d(x, x) = 0$), (F) is equivalent to the triangle inequality, while (R) is equivalent to $d(x, y) \leq d(x, z) + d(y, z), \forall x, y, z \in X$. From this we deduce that an object in $pq - Met^{\infty}$ is regular in CAP iff it is pseudo–metrizable.

H. Kleisli

How to topologize the space of continuous morphisms between topological Banach balls?

The following problem of linar logic is presented: Find models for the full ILL, e.g. pairs of functors $B \underset{F}{\overset{U}{\rightleftarrows}} C$ where i) $(B, \otimes, \multimap, T)$ is a SMCC, ii) $(C, \times, [\ \ ], I)$ is a CCC and iii) $F \dashv U$ is a monoidal adjunction. (See: G.M. Bierman, Univ. Cambridge Comp. Lab. TR No 333 (1994)). The same for the classical LL, where in addition the existence of a dualizing object $\perp$ is required.
A model for classical LL is described in the hope that the presentation might convince members of the categorical topology community to produce further models. Let Barr denote the following category introduced by M. Barr 20 years ago. The objects are unit balls of Banach spaces carrying a convenient kind of locally convex topology. The morphisms are continuous maps which are compatible with the absolute convex

structure of the unit balls. Let Brown be the category of Hausdorff spaces and k-continuous maps which R. Brown showed to be cartesian closed 30 years ago. Finally let U : Barr $\longrightarrow$ Brown be the forgetful functor. The problem consists in modifying the topology of compact convergence on the space $[B,C]$ of morphisms $B \longrightarrow C$ in order to obtain an internal hom–object $B \multimap C$ satisfying property i) and such that the dualizing object is the unit ball $O\mathbb{C}$ of $\mathbb{C}$. That has been achieved by M. Barr in 1976 (CTGD, Vol. XVII–4). A proof of property iii) has been presented last year by Rosicky, Künzi and Kleisli at the Workshop on Categorical Topology in L'Aquila (Sept. 1994).

A. Kriegl (with P. Michor)

Regular Infinite Dimensional Lie Groups

The notion of 'regular Fréchet in group' has been introduced by Omori, Mäeda and Yoshioka in an attempt to find conditions which ensure the existence of exponential mappings. Later it has been weakened by Milnor who required that smooth curves in the Lie algebra integrate to smooth curves in the group in a smooth way. We use this later concept, but for general Lie groups modelled on convenient vector spaces. Up to now nobody found a non–regular Lie group and in particular the important function spaces of diffeomorphisms form regular Lie groups. We were able to show that regular Lie groups allow to push surprisingly for the geometry of principal fibre bundles: parallel transport exists and flat connection integrate to horizontal foliation as in finite dimensions. As consequences we obtain that Lie algebra homomorphisms integrate to Lie group homomorphisms, if the source group is simply connected and the image group is regular. Furthermore we gave stability results for short exact sequence of Lie groups and used this to give a new proof for the formula of the derivative of the exponential mapping.

E. Lowen Colebunders (joint work with G. Sonck)

On the largest coreflective Cartesian closed subconstruct of Prtop

We show that the subconstruct Fing of Prtop, consisting of all finitely generated pretopological spaces, is the largest Cartesian closed coreflective subconstruct of Prtop. This implies that in any coreflective subconstruct of Prtop, exponential objects are finitely generated. Moreover, in any finitely productive, coreflective subconstruct, exponential objects are precisely those objects of the subconstruct that are finitely generated. In particular for Prtop itself, this means that the class of exponential objects coincides with Fing. We give a counterexample showing that without finite productivity the class of exponential objects in a coreflective subconstruct C of Prtop can be strictly smaller than $\mathcal{C} \cap \underline{\text{Fing}}$.

T. Nguyen

Characterizations of S– and SH–classes

Given an $\mathcal{E}$–co–wellpowered $(\mathcal{E}, \mathcal{M})$ –cat. $\mathcal{C}$.
Theorem: A full and iso-closed subcat. $\mathcal{B}$ of $\mathcal{C}$ is weakly multi–$\mathcal{E}$–reflective if and only if $\mathcal{B}$ is closed in $\mathcal{C}$ under the formation of $\mathcal{M}$–subobjects.
If there ex. a class $\mathbb{P}$ of $\mathcal{C}$–objects such that $\mathcal{C}$ has enough $\mathbb{P}$–projectives and for any two $\mathcal{C}$–objects C_1, C_2 there exists a $P \in \mathbb{P}$ and $\mathcal{E}$–morphisms $P \xrightarrow{e_1} C_1$ and $P \xrightarrow{e_2} C_2$, then one has
Theorem: $\mathcal{B}$ is closed under the formation of $\mathcal{M}$–subobjects and $\mathcal{E}$–quotients if and only if $\mathcal{B} = \boldsymbol{M}$–Inj for some $\boldsymbol{M} \subseteq \boldsymbol{M}_{\mathbb{P}\mathcal{E}}$, where $\boldsymbol{M}$–Inj is the full subcat. of $\mathcal{C}$ consisting of all $\boldsymbol{M}$–injective objects and $\boldsymbol{M}_{\mathbb{P}\mathcal{E}}$ is the conglomerate of all small sources $(P \xrightarrow{e_i} C_i)_I$ with $P \in \mathbb{P}$ and $e_i \in \mathcal{E}$ for each $i \in I$.

H.-E. Porst

Motions and cartesian closedness

The conceptual necessity of using cartesian closed categories in topology and analysis in motivated through modelling the motion of a (non point-like) body in space by "continuous" maps in different ways – (1) path (of a particle of the body) as a function of particles, (2) position (of the body) as a function of time, (3) position (of a particle) as a function of time and particle – which all ought to be equivalent.

G. Preuß

Local compactness in semiuniform convergence spaces

Recently semiuniform convergence spaces have been studied by the author as a common generalization of (symmetric) limit spaces (and thus of symmetric topological spaces) as well as of uniform limit spaces (and thus of uniform spaces) with many convenient properties such as cartesian closedness, hereditariness and the fact that products of quotients are quotients (published in Math. Japonica **41**, 465–491). They form the suitable framework for studying continuity, Cauchy continuity and uniform continuity as well as convergence structures in function spaces, namely simple convergence, continuous convergence and uniform convergence.
In this talk the localization of compactness has been introduced in the realm of semi-

uniform convergence structures. It turned out that the categroy LC-SUConv of locally compact semiuniform convergence spaces (and uniformly continouous maps) is a bicoreflective cartesian closed subcategory of the category SUConv of semiuniform convergence spaces (and uniformly continuous maps) containing all compact semiuniform convergence spaces. Furthermore, LC-SUConv is the bicoreflective hull of all compact semiuniform convergence spaces (formed in SUConv), in other words: a semiuniform convergence space is locally compact iff it is compactly generated. As is well-known this statement is not true in the realm of topological spaces.
Though the category CGHaus of compactly generated Hausdorff topological spaces (= k-spaces) has been used by several authors as a nice framework for homotopy theory, topological algebra and duality theory because of its cartesian closedness, it has the disadvantage that its objects have not been described by means of suitable axioms in constrast to the situation for semiuniform convergence spaces (or for limit spaces). In our context the kelleyfication (= compactly generated modification) kX of a Hausdorff topological space X is obtained by forming the underlying topological space of the locally compact modification (= bicoreflective modification of X with respect to LC-SUConv) of X (considered as a semiuniform convergence space).

G. Richter

Exponentials in the category of Hausdorff pretopological spaces

In 1993 E. Lowen-Colebunders and G. Sonk discovered that the exponential objects in the category of all pretopological spaces are just the finitely generated ones. This quite unexpected internal characterization requires essentially the initially dense (non Hausdorff) space B and the respective external description of exponentials by F. Schwarz, 1983. This approach fails badly to apply to the Hausdorff case. But there is a completely different, purely pretopological, and very elementary method which reproves the above mentioned result and works for Hausdorff spaces as well as for T_0- and T_1-spaces. In any case, the exponentials are just the finitely generated ones, hence discrete for T_1 and T_2.

J. Schröder

Survey on Urysohn Spaces

A topological space is called Urysohn, if any two distinct points can be separated by disjoint closed neighbourhoods. The difference between Hausdorff- (H) and Urysohn-spaces (U), though small, has important consequences in many areas of General Topology. Consider the following cardinal functions:

H1) $|X| \leq 2^{L(X)\chi(X)}$, Archangelski 1969
U1) $|X| \leq 2^{aL(X)\chi(X)}$, Bella & Cammaroto 1988
H2) $|X| \leq 2^{c(X)\chi(X)}$, Hajnal & Juhasz 1967
U2) $|X| \leq 2^{U_c(X)\chi(X)}$, Schröder 1992
H3) $|X| \leq 2^{s(X)\psi(X)}$, Hajnal & Juhasz 1967, X T_1
U3) $|X| \leq 2^{U_s(X)\psi_c(X)}$, Schröder 1992, X T_2

The general question, whether the separation axiom can be weakened in H1 and U1 is unsolved. It can be weakened to T_1 in H1 in the case of compact spaces [Gryzlow] and to T_2 in U1 in the case of absolutely closed spaces [Dow & Porter].
Regular λ–compact spaces with large pseudocharacter $\psi(x, X) \geq \lambda$ for every $x \in X$ have at least 2^λ points [Alas1992]. This is wrong for T_2 spaces. Is it true for Urysohn spaces? The answer is yes for countable compact spaces. The general case is unsolved.
Quasi hereditarily Lindelöf (QHL) T_2 spaces may have arbitrary cardinality, QHL Urysohn spaces have cardinality $\leq 2^{\aleph_0}$ [Hajnal & Szentmiklossy 1982], see also [Schröder 1992].

G.E. Strecker

Flows and Wolfs

In 1994 Herrlich and Meyer introduced the concept of flow as a common generalization of the concepts of sink and of set–indexed sources, and they investigated categories that have $(\underline{E}, \underline{M})$-factorization structures for flows (i.e. $(\underline{E}, \underline{M})$-categories for flows). They characterized several types of completeness properties for categories in terms of factorization structures for flows. We approach this topic from the standpoint of a functor $\underline{A} \xrightarrow{F} \underline{B}$, define F–flows and obtain results concerning functors that have $(\underline{E}, \underline{M})$-factorization structures for flows (i.e. $(\underline{E}, \underline{M})$-functors for flows). E.g., each such functor is coadjoint. Since a category $\underline{A}$ is an $(\underline{E}, \underline{M})$–category for flows iff $id_{\underline{A}}$ is an $(\underline{E}, \underline{M})$–functor for flows, our results turn out to be extensions of those of Herrlich and Meyer.
Thm. Given functors $\underline{A} \xrightarrow{F} \underline{B} \xrightarrow{G} \underline{C}$ with one coadjoint and the other $\underline{I}$–complete for some category $\underline{I}$, then $G \circ F$ is $\underline{I}$–complete as well.
Cor. Functors with flow factorization structure are compositive.
Cor. If $\underline{A} \xrightarrow{F} \underline{B}$ is coadjoint and either $\underline{A}$ or $\underline{B}$ has products, then F has a flow-factorization structure.
We propose the name "wolf" for the concept dual to that of "flow".

R. Vainio

A different approach to connectedness and disconnectedness

This approach is applicable whenever function spaces satisfy the exponential laws. It is presented for limit spaces and continuous maps. Classes of limit spaces are always assumed to be homeomorphism–closed. Let Θ and Σ be such classes, and let $C_c(\Sigma, \Theta)$ denote the class generated by all spaces $C_c(S,T), S \in \Sigma$, $T \in \Theta$. Finally asume Σ contains all singleton spaces. We say Θ is a fix–class for Σ, if $C_c(\Sigma, \Theta) = \Theta$. Every Σ has a smallest and a largest fix–class, namely the class of singleton spaces and the class of all limit spaces. Intermediate fix–classes are given by, for instance, the totally disconnected spaces and the separation axioms T_0, T_1 and T_2.
Let $c\Theta$ denote the largest class, for which Θ is a fix–class, and $d_{\Theta_0}\Sigma$ the largest fix–class for Σ contained in Θ_0. For a suitable choice of initial classes, the related classes $c\Theta$ and $d_{\Theta_0}\Sigma$ define natural concepts of connectedness and disconnectedness, respectively. Operators c, d_{Θ_0} define a Galois connection between classes that are closed under formation of subspaces and refinement of structure, and classes that are closed under formation of finite products and continuous images. Hence, the operators reverse inclusion, and the composed operators cd_{Θ_0} and $d_{\Theta_0}c$ are closure operators.

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