MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 40/1995

Nonlinear and Stochastic Systems

24.09. - 30.09.1995

Die Tagung fand unter der Leitung der Herren L. Arnold, Bremen, W. Schiehlen, Stuttgart und W. Wedig, Karlsruhe statt. Von den 35 Teilnehmern kamen 15 aus Deutschland, die übrigen aus England, Frankreich, Italien, Kanada, den Niederlanden, Österreich, Polen, den Russland und aus USA. Wegen dieser starken internationalen Beteiligung wurden alle Tagungsvorträge in englischer Sprache gehalten.

Mit dem Thema "Nonlinear and Stochastic Systems" wurden Vertreter der Mathematik und der Mechanik angesprochen. Von besonderem Interesse waren dabei die nichtlinearen deterministischen Schwingungen und die stochastischen, häufig auch nichtlinearen Systeme. Einen wichtigen Aspekt bildeten die Verzweigungen, welche in der Deterministik und der Stochastik nichtlinearer Systeme auftreten sowie die Ljapunov-Exponenten. Dabei hat sich eine starke Verwandtschaft der mathematischen Methoden gezeigt.

Die einzelnen Vorträge können den folgenden Problemgruppen zugeordnet werden:

- Nichtlineare Dynamik von Satelliten, Strukuren und Verbrennungsprozessen
- Nichtlineare Parameteridentifikation
- Invariante Mannigfaltigkeiten gestörter Dgln.
- Nichtlineare Deskriptorsysteme
- Nichtlineare Mehrkörpersysteme
- Selbsterregte Reibungsschwingungen
- Stochastische Bifurkationen
- Stochastische Dgln. mit Totzeit
- Stochastische Navier-Stokes-Gleichungen
- Statistische Linearisierung
- Numerik stochastischer Dgln.
- Monte-Carlo Simulation
- Lokalisierung ungeordneter Strukturen
- Ljapunovspektrum

- Globale stochastische Stabilität
- Dynamische Zuverlässigkeitstheorie

An jeden Vortrag schloss sich eine lebhafte Diskussion an. Alle Teilnehmer nutzten intensiv die einzigartige Gelegenheit zu einem Gedankenaustausch zwischen verschiedenen Disziplinen.

Vortragsauszüge:

S.T. Ariaratnam, Waterloo, Canada
The Role of Moment Lyapunov Exponents in Stochastic Bifurcations

Consider the deterministic dynamical system described by the ordinary differential equation $\dot{x} = f(x, \alpha), x(0) = x_0 \in R^d, f(0, \alpha) = 0$, where α is a parameter. As α is varied, suppose that the trivial solution $x(t) = 0$ becomes unstable and bifurcates to a new solution at some critical value α. Now suppose that α is perturbed by a random process $\sigma\xi(t)$ so that the new system satisfies the stochastic differential equation $\dot{x} = f(x, \alpha + \sigma\xi(t)), x(0) = x_0 \in R^d$.
The stability of the trivial solution is now examined by considering the linearized system (about $x(t) = 0$): $\dot{v} = A(0, \alpha + \sigma\xi(t)), v(0) = v_0, A = [\partial f_i/\partial x_j]_{x=0}$. The solution $x(t) = 0$ becomes unstable with probability one at a value $\alpha = \alpha_c$ for which the maximum Lyapunov exponent of the linearized system, defined by $\lambda(\alpha) = \max \overline{lim}_{t\to\infty} 1/t \log ||v(t, \alpha, v_0)||$, vanishes. At $\alpha = \alpha_c$ the trivial solution is said to undergo a dynamical or D-bifurcation. The new bifurcating solution is said to undergo a further phenomenological or P-bifurcation at a value $\alpha = \alpha_c^*$ where the probability distribution changes from an unimodal to a bi-modal or multi-modal distribution. According to a result of Baxendale, this occurs when the second zero $\beta = \beta^*$ of the β^{th} moment Lyapunov exponent, defined by $\Lambda(\beta, \alpha) = lim_{t\to\infty} log E||v(t, \alpha, v_0)||^\beta, \beta \geq 0$, is equal to $(-d)$, where d is the dimension of the system.
These concepts are illustrated through one- and two-dimensional examples of systems exhibiting transcritical, pitchfork and Hopf bifurcations.

L. Arnold, Bremen, Germany
Simplicity of Lyapunov Spectrum

We investigate whether simplicity of Lyapunov spectrum is a generic property of products of random matrices (= cocycles) which satisfy the integrability condition of the Multiplicative Ergodic theory. Here are our results:
1. "Simple spectrum" is L^p-dense, $1 \leq p < \infty$, in the set of all cocycles. However

"one-point spectrum" is also L^p-dense, so that "simple spectrum" cannot contain an open set.
2. For products of independent and identically distributed random matrices (with distribution μ), simplicity holds on a set of μ's which contains an open and dense set in both the topology of total variation and the topology of weak convergence, hence is strongly generic in both topologies. (Joint work with N. D. Cong, Hanoi, Vietnam).

P. Bernard, Clermont Ferrard, France

Stochastic Linearization and Large Deviations

Very little is known concerning the quantitative behavior of dynamical systems with random excitation, unless the system is linear. Known techniques imply the resolution of parabolic differential equations (Fokker-Planck Kolmogorov equation), which are degenerated, and in high dimension, for which there is no effective known method of resolution. Therefore, users (physicists, mechanician engineers) concerned with such systems had to design global linearization techniques, known as equivalent linearization. Until now, there was no rigorous justification of these techniques. In this contribution, using Large Deviation Principles, several linearization methods mathematically founded are proposed. These principles are relative entropy and Monsku Vardhan entropy of a Gaussian measure relatively to a Markov kernel. The results are generally different from the numerous practical techniques in the literature, the method of "true linearization" (Kozin) is justified. (Joint work with A. Wu).

J. Brindly, Leeds, U.K.

Nonlinear Dynamics of Ignition and Burning Processes

Chemistry provides us with a rich variety of nonlinear phenomena. All chemical reactions take place at a characteristic rate, k, modelled by the highly nonlinear Arrhenius law $k(\tau) = Aexp(-\epsilon/(RT))$, where T is temperature, which, since all reactions either require energy input (endothermic), or reduce energy (exothermic), leads to strong coupling with an energy equation of "reaction-diffusion" character. Additionally, other features of reactions, for example chain-branching, are also nonlinear.
Typically a chemical reaction involves many, perhaps hundreds, of steps, and depends on the creation, participation and subsequent destruction of radicals. Rational approximation of this finite but hiyh-dimensional system by its essential dynamics on a low-dimensional "slow" manifold is a challenging problem receiving much present attention.

Amongst the more striking and important chemical phenomena are those of ignition and combustion. Ignition is recognizable in mathematical models by bifurcational event or by exitable behavior. Sustained burning, for example of polymers, often depends on a highly nonlinear feedback process between exothermic "flame" and endothermic plastic degraduations for its maintenance. Mathematical modelling of these phenomena has far to go and presents exciting challenges, at least through the introduction of stochasticity.

D. van Campen, Eindhoven, The Netherlands

Nonlinear parametric identification using periodic equilibrium states and chaotic data

The subject of the presentation is the development of a nonlinear parametric identification method using chaotic data. In former research, the main problem in using chaotic data in the parameter estimation appeared to be the numerical computation of chaotic trajectories. This computational problem is due to the highly unstable character of the chaotic orbits. The method presented is based on assumed physical models and has two important components.

Firstly, the chaotic time series is characterized by a "skeleton" of unstable periodic orbits. Secondly, these unstable periodic orbits are used as the input information for a nonlinear parametric identification method using periodic data. As a consequence, problems concerning the numerical computation of chaotic trajectories are avoided. Also, before starting the estimation process, a huge data reduction has been accomplished by extracting the unstable periodic orbits from the long chaotic time series. The method is validated by application to a parametrically excited pendulum, which is an experimental nonlinear dynamical system in which transient chaos occurs.

F.L. Chernousko, Moscow, Russia

On dynamics of retrieval for a space tethered system

The dynamics of space tethered systems have attracted increasing attention in recent years, in view of possible use of such systems in various applications. One of the essential problems that arises here is to construct a rational retrieval process, since the tether may oscillate at increasing amplitudes while being retrieved. We consider nonlinear equations governing the plane (in-orbit) motions of the tether, one end of which is attached to a satellite (probe) of the mass m, while the other is attached to a spaceship of the mass M, $M >> m$. The spaceship moves along a circular orbit, the tether is assumed to be flexible, massless, and inextensible. We propose to control the retrieval process by changing the speed of the retrieval u, and consider two possibilities: a) $u = 0$; b) $u = u_0 = const < 0$. In the case a), the equation is reduced to the nonlinear pendulum equation. In the case b), we have obtained an unique solution that is bounded and vanishes as the length of the tether tends to zero. By combining these regimes, we propose a motion such that, for arbitrary initial conditions (in some region), the tether can be retrieved in finite time, and the angle of deflection is bounded and evaluated. Thus, we have shown that nonlinear retrieval process with bounded angle of the tether deflection is available.

F. Colonius, Augsburg, Germany

Invariant Manifolds for Time Dependent Differential Equations

Families of ordinary differential equations with bounded time-dependent coefficients can be viewed as skew product flows on metric spaces. Using this formulation, one can apply results due to Bronstein / Chernii (1978) in the theory of linear flows on vector bundles in order to construct invariant manifolds from exponentially separated subbundles. If one combines this with the analysis of Lyapunov exponents and Fenichel's uniformization lemma, one obtains stable and unstable manifolds. The result has applications for stability radii of nonlinear systems at a singular point and for time-dependent perturbations of attractors with sensitive dependence on initial conditions. (Joint work with W. Kliemann, Iowa State University, USA.)

H. Crauel, Saarbrücken, Germany

Additive noise destroys a pitchfork bifurcation

We show that if the classical pitchfork bifurcation $\dot{x} = \alpha x - x^3, x, \alpha \in R$ is subjected to additive white noise, then the dynamical bifurcation disappears. That is to say: the nontrivial attractor $A = [-\sqrt{\alpha}, \sqrt{\alpha}]$, which appears after the bifurcation in $\alpha = 0$ in the deterministic system, is replaced by a random attractor $A(\omega)$, which is a one-point set for any $\alpha \in R$. This even holds for arbitrarily small intensity of the additive white noise, i.e., for $dx = (\alpha x - x^3)dt + \epsilon dW_t, \epsilon > 0$, arbitrary. (Joint work with F. Flandoli, Pisa, Italy).

P. Imkeller, Besancon, France

On the smoothness of the laws of the Oseledets spaces of linear random dynamical systems

We study smoothness properties of the laws of the Oseledets spaces belonging to linear random dynamical systems that come from sde's driven by Wiener processes, for example the existence and smoothness of (conditioned) densities with respect to the Riemannian volume on the manifold on which they take their values. Smoothness properties allow a treatment of many problems arising in the study of the asymptotics of rds due to the non-adaptedness of the underlying random invariant measures by the tools of stochastic analysis. The treatment of smoothness properties we propose is based on Malhavin's calculus.

H. G. Davies, New Brunswick, Canada

Nonstationary response of a coupled pitch-roll ship model under modulated excitation

A two degree of freedom system with quadratic coupling is used to describe coupled pitch and roll motion of a ship. We consider periodic excitation with a modulated amplitude. Stability of an exact solution with one mode zero is studied. Loss of stability involves either a rapid transition to an alternate solution branch, or a period doubling. Local stability boundaries are obtained showing the very large effect of modulating frequency. For a reduced approximate problem, global stability can be determined, and the relation between local and global stability discussed. This does not appear possible in the general case, partly because of the coexistence for some parameter values of both periodic and chaotic attractors. A variety of modulated responses, bifurcations and chaotic responses are found.

F. Flandoli, Pisa, Italy

Stochastic Naviar-Stokes equation

We consider a bounded domain $D \subset R^d$, d = 2,3, and a fluid filling D and satisfying the momentum and mass conservation equations $\partial u/\partial t + u\nabla u + \nabla p = \nu\Delta u + f + \partial g/\partial t$, $divu = 0, u|_{\partial D} = 0$ where u = velocity field, p = pressure field, f and $\partial g/\partial t$ are body forces, but g is not differentiable (fast varying impulsive force); for example, $\partial g/\partial t$ is white noise in space-time. Classical energy inequality is meaningless if $g \neq 0$ both mathematically, and physically (g introduces -maybe rarely - large amounts of energy at certain times, so that the kinetic energy $0.5 \int_D |\omega|^2$ does not dissipate to some equilibrium value, but has arbitrary large fluctuations). However, the difference between $u(t, u_0^1)$ and $u(t, u_0^2)$ for different initial velocity fields ω_0^1 and ω_0^2 dissipates; to capture this behavior we introduce a reference trajectory, the solution of the auxiliary Stokes equation (Ornstein-Uhlenbeck equation) $\partial z/\partial t + \nabla q = \nu \Delta z + f + \partial g/\partial t$, $div z = 0, z|_{\partial D} = 0$ and consider the difference $v = u - z$. This dissipates. By this technique (energy inequality for v) one can prove existence of solutions, uniqueness in $d = 2$, some Hausdorff-dimension estimates of the set of possible singularities in $d = 3$, existence of random attractors and invariant measures in $d = 2$, random attractors in the sense of Sebl in $d = 3$, and uniqueness of invariant measure and ergodic theorem in $d = 2$ where the noise is sufficiently uncorrelated. Interesting often problems are the structure of this invariant measure and the zero noise limit.

C. Glocker, München, Germany

Dynamics of Rigid Body Systems with Friction and Impacts

The dynamics of rigid multibody systems with dry friction and impacts is described by different sets of ordinary differential equations with varying dimensions. The conditions of transition between these sets result from contact, impact, and friction laws which are expressed by using multivalued and non-smooth functions. Velocity and acceleration jumps are contained in this formulation. In the planar case the evaluation of the contact laws leads to a Linear Complementarity Problem which can be solved by applying a modified Simplex Algorithm. The known principles of classical dynamics are generalized about these problems and result in variational inequalities and convex programs. As a consequence, certain existence and uniqueness conditions for the dry friction problem are available.

R. Heuer, Wien, Austria

Nonlinear dynamic analysis of shallow shells

Geometrically nonlinear vibrations of shallow shells are treated by applying Berger's approximation to the generalized von Karmans-type plate equations considering hard hinged supports. Shear deformation is considered by means of Mindlin's kinematic hypothesis. Applications of a multimode expansion in the Galerkin procedure to the governing differential equation, where the eigenfunctions of the corresponding linear plate problem are used as space variables, renders a coupled set of ODEs for the generalized coordinates with (mixed) cubic and quadratic nonlinearities. The nonlinear steady-state response of shallow shells subjected to a time-harmonic lateral excitation is investigated and the phenomena of primary, superharmonic, and subharmonic resonances are studied by means of the Perturbation Method of Multiple Scales. Unifying results with respect to the planform of the shell are evaluated. Within the scope of random vibrations, the F.P.K.-equation for the transition probability density of the generalized coordinates and velocities is derived. Its stationary solution gives the probability of eventual snapping after a long time has elapsed. However, the probability of first occurence follows from the (approximate) integration of the nonstationary F.P.K.-equation. The probability of first dynamic snap-through as well as the probability distribution of the asymmetric snap-buckling is derived.

W. Kliemann, Ames, USA

Some Remarks on Dynamic Reliability

We consider a nonlinear dynamical system with random excitation $\dot{x} = X_0(x) + \sum_{i=1}^{m} \xi_i(t) X_i(x)$ on a smooth manifold M. Here $X_0, ..., X_m$ are sufficiently smooth vector fields, and $\xi_i(t)$ is a function of a background Markov diffusion process on a compact manifold. We assume sufficient mixing of $(\xi_i)_{i=1...m}$ and $(X_0, ..., X_m)$. In this context we analyze two concepts of reliability theory: collapse (or sudden failure) described by exiting from some compact set $L \subset M$, and damage accumulation (or aging) described by $\int_0^t f(x(s, x_0, w))ds$, where $x(s, x_0, w)$ denotes the trajectory with initial value $x_0 \in M$, $f : M \rightarrow R^t$ is a damage function. Although the concepts of reliability (failure probabilities, life time, failure location) are related to the transient behavior, quite some insight can be gained from the global qualitative theory of Markov diffusion processes, together with characterizations via nonlinear control theory and global dynamics of skew product flows. We present some results in this direction and analyze the Takens-Bogdanov oscillator as well as a model for ship capsizing in random seas in some details. (Joint work with F. Colonius and G. Häckl, Universität Augsburg).

W. Szemplin'ska-Stupnicka, Warszaw, Poland

On semi-stochastic model of deterministic chaos in a non-linear oscillator

It is observed that the chaotic process (solution of Duffing equation) can be approximated by some single harmonic function with random-like modulated amplitude and amplitude-dependent frequency. This derivation gives size to a hypothesis that the type of chaotic motion can be approximated by a semi-stochastic process. In the paper we propose a model of stochastic approximation of the chaotic process and try to verify it numerically. (Joint work with Z. Kotulski, J. Trebicki.)

E. Kreuzer, Hamburg, Germany

Nonlinear Dynamics in Marine Technology

The evolution of gravity driven water waves interacting with fixed or freely floating objects is an important field of research in marine technology. Many problems are encountered in this field which are nonlinear by nature. Analytical solutions of the fully nonlinear equation have, however, not been found so far. Therefore, it seems that the development of numerical methods is a more promising way towards the solution of these problems.
The international recommendations for the safety against capsiring of seagoing merchant vessels in waves are presented first. These criteria describe static stability requirements only and are based on the assumption that a vessel that is sufficiently stable in calm water will be stable in waves. Obviously the stability parameters at the limit between safe and unsafe depend on the sea state. In the eighties it was tried to take into account the sea state and hull forms by means of a so-called hull from factor. But the real dynamic problem is still not considered. In this presentation the governing equations for the (nonlinear) water-wave problem, including the interaction with fixed objects and floating bodies, and methods for solution are considered.
As an example the model of a crane ship with linear hydrodynamics is considered. A set of 20 differential equations of motion describe the dynamics of the system. Bifurcation analysis and simulations show the full range of nonlinear phenomena from periodic motion to chaotic behavior. (Joint work with R. Kral.)

P.C. Müller, Wuppertal, Germany

Optimal Control of Nonlinear Descriptor Systems

In recent years the modeling of dynamical systems by differential-algebraic equations, i.e. in form of descriptor systems has become more and more important. Although a lot of results has been found still many problems in the analysis and control design of descriptor systems are open. In this contribution some first steps for the optimal control design are presented. Here the notion of "causality" is essential, i.e. it is necessary to distinguish between descriptor systems which solutions depend on the control input only or additionally on its higher order time derivatives. For causal descriptor systems the optimization runs as usual, but for non-causal systems surprising results appear. Then the optimization has to be based on the underlying ODE on a manifold and Pontryagin's Maximum Principle has to be applied to an extended state space system. The results are illustrated by some examples.

N. S. Namachchivaya, Urbana-Champaign, USA
Global and Stochastic Dynamics in Mechanical Systems

In the first part of the paper, we study the discrete nonlinear gyroscopic systems. The aim of our work is to examine the effects of small dissipation, symmetry breaking perturbations and periodic parametric excitations on the bifurcations of gyroscopic systems, when the unperturbed Hamiltonian system with symmetry exhibits double zero resonance instability. It was shown the perturbed autonomous system exhibits a pitchfork-homoclinic bifurcation. These calculations were made using Melnikov functions. Using an interactive algorithm based on Lie transforms, the normal forms for non-autonomous Hamiltonian were derived. Under small perturbations arising from parametric excitations, two mechanisms were identified in such systems that may lead to chaotic dynamics.
In the second part, an approximation for moment Lyapunov exponent, the asymptotic growth rate of the moments of the response of 2-degree of freedom gyroscopic systems driven by real noise was constructed. A perturbation approach was used to obtain explicit expressions for these exponents in the presence of small intensity noise. The usefulness of moment Lyapunov exponent in predicting parameter values at which qualitative changes in probability density function occur was also illustrated. (Joint work with R. McDonald, W. Nagata, H. van Roessel.)

K. Popp, Hannover, Germany
Dynamical behavior of a friction oscillator with simultaneous self and external excitation

Friction induced vibration, also known as stick-slip vibration, occurs in mechanical systems as well as in everyday life. In engineering applications these vibrations are undesired and should be avoided. The robust limit cycle of stick-slip motions can be disturbed by periodically acting forces. As a simple mechanical model of this mechanism a simple-degree-of-freedom friction oscillator with external harmonic excitation is investigated. The resulting system equation is non-smooth due to the non-smooth friction characteristic. Investigating the system behavior, rich bifurcation behavior and different routes to chaos have been observed. In this case, the system dynamics in the three-dimensional state space can be analyzed by a one-dimensional map. Similar to the logistic map, bifurcation and stability analysis can be carried out and Lyapunov exponents can be calculated. The friction characteristic has a significant influence on the dynamical behavior. For example, stick-slip limit cycles require a decreasing friction force characteristic. Experiments show that the friction characteristics found in literature give only a rough approximation. The random components in the measurements due to surface roughness deserves more attention.

H.J. Pradlwarter, Innsbruck, Austria

Advanced Simulation Procedures in Non-linear Stochastic Systems

Experience has shown, that analytical models, while most instrumental and accurate to analyse smaller types of problems, are not necessarily applicable to the class of problems defined in higher dimensions. However, numerical procedures, such as Monte Carlo simulation are free of those restrictions. Though, if they are applied directly, it is almost impossible to describe the structural response in the region of interest, i.e. the tails of the distributions. This information, of course, is a necessary prerequisite for solving exceedance, reliability and other related response problems. For this reason - particularly during the last decade - considerable efforts have been made to adopt variance reduction techniques to problems of stochastic structural dynamics. In this context procedures such as importance, adaptive and directional sampling, etc., were shown to be useful. Other procedures which proved to be must instrumental for analysing stochastic nonlinear problems are the so-called Response Surface Method.
Recently, other types of advanced simulation procedures such as "Double and Clump" and "Russian Roulette and Splitting" have been introduced. These methods are applicable for the case of loading by a random process consisting of a large number of random variables. The method allows to direct Monte Carlo simulation toward the tails of the distribution. These concepts have been discussed in the presentation. Joint work with G.I. Schueller.)

G. Rega, L'Aguila, Italy

Nonlinear dynamics of theoretical and experimental cable models

The elastic suspended cable can be considered a rich model system for nonlinear and chaotic phenomena in structural dynamics. They range from the regular and nonregular responses exhibited by a SDOF model for 2D finite forced dynamics, to the whole complexity of interactions and bifurcations in MDOF models for 3D oscillations. Attractor-basis-manifold phase portraits and global bifurcations mechanisms in SDOF model are highlighted through computational and geometrical techniques. The requirments for a richer mathematical model for actual 3D dynamics are suggested by the results of experimental investigations. Several competing classes of regular response occur for a cable undergoing verticale in-plane support motions under various external and simultaneous internal resonance conditions. First practicable characterization of quasiperiodic and chaotic responses is made by means of reconstructions of the attractors from time series and calculations of dimension in pseudo-phase spaces. A theoretical four d.o.f. model able to match the experimental behavior in different regions of the control parameter space is presented, and asymptotic steady ask amplitude modulated motions are obtained at various resonance conditions.

K.R. Schenk-Heppe, Bremen, Germany

Deterministic and stochastic Duffing-van der Pol oscillators are non-explosive

Do solutions of the D-vdP equation $\ddot{x} = \alpha x + \beta\dot{x} - x^3 - x^2\dot{x} + \sigma_1 x\xi_1(t) + \sigma_2\xi_2(t)$ explode when time passes?
We prove under the assumptions that (read noise case) $\xi_1(t)^4, \xi_2(t)^2$ locally integrable, $P(|\xi_1(t)| + |\xi_2(t)| \leq e_1, \forall t \leq] - c_2, c_2[) > 0$ for some $c_1, c_2 > 0$ the flow $\varphi_t(w) : D_t(w) \rightarrow R_t(w), t \in R$ is global to the forward and local to the backward, i.e. $D_t(w) = n^2, \forall t \geq 0, R_t(w) \neq n^2, \forall t > 0$.
In the white noise case the above result is true without any assumptions. In particular the deterministic equation ($\sigma_1 = \sigma_2 = 0$) is non-explosive forward in time and possesses initial values for which the backward eq. explodes.

M. Scheutzow, Berlin, Germany

Exponential stability of stochastic delay equations

Consider the stochastic delay equation $dX(t) = \sigma X(t-1)dW(t)$, where W is onedimensional Brownian motion and $\sigma \geq 0$ is a parameter. We are interested in stability properties of this equation as a function of σ. We show that for small $\sigma \geq 0$ the exponential growth rate is strictly negative wheras it is strictly positive for large values of σ. More precisely the following is true.
Theorem 1 (stability for small σ): There exists $\sigma_0 > 0$ and a continuous function $\bar{\lambda} : (0, \sigma_0) \rightarrow (-\infty, 0)$ such that for all $f \in C = C([-1, 0], R)$ and all $\sigma \in (0, \sigma_0)$ we have $P(\lim sup 1/t \log |X^f(t)| \leq \bar{\lambda}(\sigma)) = 1$. Here X^f denotes the solution with initial condition $X(s) = f(s), s \in [-1, 0]$. Furthermore $\bar{\lambda}(\sigma) \sim \sigma^2/2$ as $\sigma \rightarrow 0$.
Theorem 2 (instability for large σ): There exists a constant $\kappa \in R$ such that for all $f \in C$ $\{0\}$. We have $P = 1$. Here $||X_t^f||\infty := sup|X^f(t+s)|$. In particular the exponential growth rate is positive for sufficiently large σ.
We prove both results via specially constructed Lyapunov functionals. We also indicate a generalization of Theorem 1 to more general (also certain nonlinear) equations. Further we point out the limitations of classical methods in dealing with stability properties of the equation. (Joint work with S. Mohammed, Carbondale, USA.)

W. Schiehlen, Stuttgart, Germany

Amplitude Bounds of Stochastic Nonlinear Multibody Systems

Dynamical equations describing nonlinear multibody systems are represented in the state space as $\dot{x} = f(x, \xi_t, p), x(t_0) = x_0$ where x is the state vector, x_0 a random initial state vector, ξ_t a stochastic vector process including random parameters and p a deterministic parameter vector. It is assumed $f(0, \xi_t, p) = 0$. An amplitude bound with respect to the initial conditions for a sample trajectory is defined as $B(\xi_t, p, x_0) := \frac{||x(t)||_\infty}{||x_0||}$ on an infinite time interval. For the statistical analysis its inverse $IB(\xi_t, p, x_0)$ is preferable. It yields $B \in [1, \infty]$ and $IB \in [0, 1]$. The amplitude bound has to be computed numerically for multibody systems using Monte-Carlo simulations. Computation results are presented for the Duffing oscillator with positive and negative cubic nonlinearities and excitation by wide-band and narrow band stochastic processes. (Joint work with B. Hu and S. Schaub.)

B. Schmalfuss, Bremen, Germany

Attractors and Random Bifurcations of Stochastic Differential Equations

Let (C_0, B_{C_0}, P) be a canonical Wiener process and $\Theta_L : C_0 \to C_0$ be a family of shift operators such that $\Theta_L\omega = \omega(. + L) - \omega(L), L \in R$ The solution of the Stratonovich equation $dx = (\alpha x - x^3 + g(x))dL + x_0 dw, x(0) = x_0$ generates a cocycle $\varphi(L, \omega, x_0)$. We assume $g(0) = 0$ and some other conditions. $X^s(0) = 0$ is a stationary solution of this equation. We are looking for other stationary solutions if $d > 0$. In a first step we are able to prove the existence of a random attractor that does not contain 0. In a second step we try to prove that the attractor only contains one element. In particular, we can use a random fixed point theorem based on uniform Lyapunov exponents. So we can find a stochastic analogy of pitchfork bifurcations in the sense that new stationary solutions appear. These stationary solutions are given by a random variable $x_f^s(\omega)$ such that $L \to x_f^s(\Theta_L(\omega), d > 0$ solves the above equation. (joint work with L. Arnold.)

K. Sobczyk, Warsaw, Poland

Stochastic Nonlinear Systems and Maximum Entropy Principle

In stochastic dynamics of physical and engineering systems the probability distributions of the response process are seldom achievable. Most often, all what we are able to obtain (from the governing stochastic nonlinear equations) are the moments of the response, or the equations for moments. We use this partial

information about the system response to construct the approximate probabilty distribution; this is made via maximum entropy principle.
If the system of interest is governed by the Ito equation for the vector process $Y(t) = [Y_1(t), ..., Y_n(t)]$, $\gamma \in \Gamma, (\Gamma, F, P)$, $dY(t) = F[Y(t)]dt + \sigma[Y(t)]dW(t,\gamma)$, where $W(t,\gamma) = [W_1(t,\gamma), ..., W_m(t,\gamma)]$ m-dimensional Wiener process, then - under known assumptions - the solution is a Markov diffusion process with drift $A(y)$ and diffusion $B(y) = \sigma(y)\sigma^T(y)$. According to the spirit of the maximum entropy principle the approximate probability density $p(y,t)$ of $y(t)$ is determined as a result of maximization of the entropy functional $H(P) = -\int P(y,t) lup(y,t)dy$ under the constraints 1) normalization condition: $\int P(y,t)dy - 1 = 0$, 2) given moment equations: $\frac{dm_k(t)}{dt} =< G_K(y_k) >_p$, $|k| = 1, 2, ..., K$ where $G_K(y) = \sum_i F_i(y)\frac{\partial h_k(y)}{\partial y_i} + 1/2 \sum_l \sum_{i,j} \sigma_{il}\sigma_{jl}\frac{\partial^2 h_k(y)}{\partial y_i \partial y_j}$. The method is illustrated by the examples. (Joint work with J. Trebicki.)

H. Troger, Wien, Austria

Center Manifold Approach to the Control of a Tethered Satellite System

In station keeping often undesired oscillations of a system of two satellites connected by a thin cable (up to 100 km length) occur. These oscillations should be extinguished by tension control, that is, only by changing the tension in the tether connecting the satellites. However, if the equations of motion are linearized about the radial relative equilibrium the out of plane oscillation decouples from the in plane oscillation and cannot be effected by tension control. Therefore a nonlinear control problem is formulated on the center manifold which is two-dimensional and results from a Hopf bifurcation. By means of computer algebraic programs the whole calculation can be performed analytically. It is shown that the nonlinear analysis results in further restrictions on the control parameters. (Joint work with W. Steiner, A. Stendl.)

W. Szemplin'ska, Warsaw, Poland

Homoclinic bifurcation versus boundary crisis

In the study of global bifurcations in the two-well Duffing oscillator we consider the region of the system parameters when a codimension-two bifurcation part exists: the point where there is an intersection of the local saddle-mode bifurcation of the single-well attractor and the global homoclinic bifurcation of the P-periodic large orbit (cross-well) saddle D_L. Effects of the bifurcations are investigated by numerical calculations and are illustrated by basin of attraction and manifold structure of the saddle D_L, which lies outside the cross-well chaotic

attractor. A particular attention is paid to two different effects of the homoclinic bifurcation g of the saddle D_L:
1) If it occurs in the system parameter region then there exist a cross-well chaotic character attractor, the homoclinic bifurcation destroys its local stability and so destroys its basin of attraction. Therefore it corresponds to the phenomena known as "boundary crisis".
2) Often the homoclinic bifurcation occurs in the region, when two types of periodic attractor exist, it does not affect their stability but destroys their basin boundary. Thus it results in creating a fractal basin boundary and an explosion of the basin of attraction of the large orbit T-periodic cross-well attractor. Joint work with K. Janicki.)
A great contrast between simplicity of the attractor period existing in the region and complexity of the related transient state is also underlined.

W. Wedig, Karlsruhe, Germany
Stability and invariant measures of perturbed dynamical systems

For stability investigations of linear dynamical systems with parametric perturbations we are interested to calculate the top Lyapunov exponents by the invariant measures of the systems appling the Furstenberg-Khasminskii formula. For this purpose we set up the Fokker-Planck equations of the projection angles and solve them by introducing special iterative solution schemes which reduce the parabolic partial differential equations to multi-dimensional limit cycles problems with respect to one rotating phase angle of the linear systems.
The integration into the negative angle direction is performed by Euler forward differences leading to simple explicit schemes in regular cases. Drifts due to numerical dampings are eliminated by global corrections utilizing stationary conditions according to marginal densities in every iteration step.
For oscillators perturbed by bounded Ornstein-Uhlenbeck processes or by harmonic or almost periodic excitations with noise perturbations the iterative schemes are illustrated by demonstration programs realized by a PC-notebook and visualized by an overhead panel on the screen.

V. Wihstutz, North Carolina, USA
Numerics for the Lyapunov exponents of real noise driven linear systems

The results given by Talay and Ground and Talay concerning Lyapunov exponents of stochastic systems with elliptic generator are generalized to linear systems with weaker non-degeneracy properties. An algorithm is represented and the order of convergence is calculated for white or real noise driven linear stochastic differential systems which satisfy Hörmander's hypoellipticity condition.

W.C. Xie, Waterloo, Canada

Mode Localization in Disordered Structures

This presentation extends the classical theory of localization in disordered one-dimensional periodic systems in two different directions: 1) Buckling mode localization in nonhomogeneous beams on elastic foundation; 2) Vibration mode localization in weakly-coupled two-dimensional mass-spring arrays.
In the first part, localization in nonhomogeneous continuous systems is studied. Buckling mode localization in nonhomogeneous beams on elastic foundation is studied. Localization factors, which characterize the average exponential rates of growth or decay of deformation of the beam, are determined using both the method of transfer matrix and the method of Green's function.
In the second part, the method of perturbation is applied to determine the localization factors of disordered weakly-coupled mass-spring array. For one-dimensional disordered mass-spring chain, the first exact analytical results of the loocalization factors are obtained in terms of the system parameters. For two-dimensional disordered mass-spring array, the localization factors are determined in terms of the angle of direction ranging from 0 to 2π.

Berichterstatter: P. Eberhard, Stuttgart, Germany

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