Tagungsbericht 41/1995

Mathematische Methoden der Geodäsie

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The meeting has been organized by Willi Freeden (Geomathematics Group, Kaiserslautern) and Erik W. Grafarend (Geodetic Department, Stuttgart). It brought together researchers from mathematics and geodesy. The main purpose was to find appropriate means of assimilating, assessing and reducing to comprehensible form the readily increasing flow of data from terrestrial and satellite sources and providing an objective basis for scientific interpretation, classification, testing of concepts, and solution of geoproblems. The talks that were delivered reflected well the multiple interest in the audience. There have been lectures on geodetic boundary value problems, inverse problems, differential geometry, statistics, multivariate approximation theory. The themes addressed were given as a block series of talks (indicated below by seperating lines). The talks have been followed by lively discussions and a useful exchange of ideas has taken place.

VORTRAGSAUSZÜGE

G. Anger:

The Efficiency of a Mathematical Model in Natural Sciences and Medicine

Most mathematical problems in science, technology and medicine are inverse problems. In order to solve an inverse problem the following points have to be studied:

- Mastery of the special process both experimentally and theoretically
- Possibility of mathematical modeling of the process
- Mastery of the direct problem both theoretically and numerically
- Studying of the information content of the inverse problem, i. e. , to find out which internal parameters of a system inaccessible to measurement can be determined in a stable and unique manner
- Development of algorithms for the numerical solution of an inverse problem

In general, the solution of the direct can be represented by using integrals (compact operator). If $Af = g$, $R(A)$ infinite dimensional, is a one-to-one mapping, then A^{-1} is discontinuous (F. Riesz 1918). In order to study the efficiency of a mathematical model. We have to include its measurements. In this talk we presented basic principles of inverse theory using different models, especially those which are of interest in geodesy.

M. Bertero:

Linear and Nonlinear Methods for Linear Inverse Problems

Linear inverse problems are, in general, ill-posed and, as a consequence, the corresponding discretized problems are ill-conditioned and affected by numerical instability. Since the solutions (or generalized solutions) of the discretized problems are deprived of physical meaning, one must look for approximate solutions satisfying some *a priori* conditions. This explains the variety of the methods which have been developed for solving this kind of problems. We review linear regularization and filtering methods, linear and non linear iterative methods, constrained iterative methods and probabilistic methods such as maximum likelihood and Wiener filtering. These methods are illustrated with applications to image restoration, seismology and tomography with few data.

C. Jekeli:

Methods to Reduce Aliasing in Spherical Harmonic Analysis of Gravity Anomalies

Despite the sophistication in the construction of todays detailed models of the Earth's gravity field using high-degree spherical harmonic expansions, they generally still process a global set of mean gravity anomalies available (or estimated) on a regular grid of latitude and longitude lines. This transform of discrete data into harmonic coefficients (the spectrum) is a numerical approximation of an integral; and, even if the data themselves have no error, it usually introduces a discretization error, or in the parlance of spectral analysis, an aliasing error – the estimated coefficients are aliased by higher degree gravity field components that cannot be seperated because of the granularity of the data.
The methods of analysis using simple quadratures, least-squares collocation, other modified quadratures, least-squares adjustment, and the newer proposals using planar 2D discrete Fourier transforms all share the same basic structure. For example, for efficient computation, most place considerable restrictions on the allowable statistical error distribution of the data. However, there is significant variation in the aliasing errors of the methods. This paper derives a rigorous formula for the aliasing error in each case, thus yielding a lucid model comparison and indicating ways to ameliorate the error through data smoothing.
Typically, data smoothing is the result of uniformly weighted averaging of gravity anomalies over blocks defined by latitude and longitude lines. The aliasing error formulas clearly show how this procedure fails to attenuate a part of the high-degree spectrum that then aliases the estimated spectrum. Averages over constant spherical caps, though spoiled at higher latitudes by the increasing correlation between neighbours on the latitude/longitude grid, are more definitive in filtering out the high-degree components. Special weighting functions (such as the Gaussian function) can further reduce the aliasing that remains due to the ringing of the uniformly weighted average.

L. E. Sjöberg:

On the Error of Downward Continuation in Physical Geodesy

Analytical continuation of free-air gravity anomalies combined with Stokes's formula is compared with the classical method for geoid determination by Helmert's second condensation method. Assuming that the density distribution of the terrain is correctly known Helmert's method can be regarded as correct, while we show that the analytical continuation method yields an error in geoidal heights that may reach several metres in high mountains. In a similar way we show that a spherical harmonic determination of the geoid based on external gravity data is an error which again, may reach metres over continental areas.

N. Sneeuw:

Spectral Analysis of Single Component Gradiometry

The technical implementation of gravity gradiometry in the form of differential accelerometry over a fixed baseline results inherently in measuring (one or more) single components of the gravity gradient tensor. It is therefore interesting to investigate how the gravity field can be determined from single gradiometry components, and to describe the error characteristics of the field obtained. As has been demonstrated by simulations for the STEP-mission (cross-track in-line component), the resulting spherical harmonic error spectra may be inhomogeneous.
Here the spectral characteristics of single components of the gradient tensor, expressed in a local frame, will be analyzed. The type of representation of error simulation results will be discussed. Moreover optimal gradiometry configurations are proposed in order to obtain homogeneous and isotropic error characteristics.

F. Schneider:

The Solution of Linear Problems in Satellite Geodesy by Means of Spherical Spline Approximation

We consider a certain class of geodetic linear inverse problems $\Lambda F = G$, F: gravitational potential at the surface of the earth and G for example the first radial derivative of the gravitational potential at satellite altitude (linearized satellite-to-satellite tracking) or the second radial derivative of the gravitational potential at satellite altitude (satellite gradiometry). The regularization is done by a refinement of the topologies by considering the inverse problem in a reproducing kernel Hilbert space setting in order to obtain a bounded generalized inverse operator $\Lambda^{\dagger}$. For a numerical realization we assume G to be given at a finite number of discrete points to which we employ a spherical spline interpolation method adapted to the Hilbert spaces. By applying $\Lambda^{\dagger}$ to the obtained spline interpolant we get an approximation of the solution F at the surface of the earth. Finally, some convergence results are formulated if the measurements at satellite altitude increase, where it is important that neither the earth figure nor the shape of the orbit underly severe geometrical restrictions.

P. Maaß:

Wavelet Transforms on Spheres

The aim of this talk is to describe some recent results concerning wavelet transforms on spheres. These transforms are constructed in order to analyze local structures and to efficiently represent functions on spheres.
Motivated by the well established wavelet transform for functions $f \in \mathcal{L}^2(\mathbb{R})$ one seeks a transform of the type

$$L_\psi f(\eta, a) = < f, \psi_{\eta a} >, \quad \eta \in S^2, \quad a \in (0, \infty),$$

where η denotes a translation parameter and a refers to scaling the wavelet ψ. Different approaches are discussed:

- the discrete method by W. Sweldens, based on hierarchically subdividing the sphere into triangles,
- the analytic approach by B. Rubin, which starts from analytic continuation method for approximate convolution identities and
- an algebraic approach (S. Dahlke, P. Maaß) based on group representations acting on functions on the tangent bundle of S^{n-1}.

U. Windheuser:

Spherical Wavelet Transform and Its Discretization

A continuous version of spherical multiresolution is described, starting from continuous wavelet transform on the sphere. Scale discretization enables us to construct spherical counterparts of wavelet packets and scale discrete Daubechies' wavelets. It is shown that singular integral operators forming a semigroup of contraction operators of class (C_0) (like Abel-Poisson or Gauß-Weierstraß operators) lead in canonical way to pyramidal algorithms. Fully discretized (scale and space discrete) wavelet transforms are obtained via approximate integration rules. Shannon's sampling theorem on the sphere is mentioned and exact space discrete wavelet transforms are discussed for band-limited wavelets. Finally applications to geodetic reality are given in more detail.

M. D. Buhmann:

Old and New Results on Radial Functions with Compact Support

In this work, radial basis functions that have compact support and give rise to positive definite interpolation matrices $\{\phi(\|x_j - x_k\|_2)\}_{j,k}$ are considered. Here, ϕ is the radial basis function, $\|\cdot\|_2$ is the Euclidean norm on $\mathbb{R}^n$ and $x_j \in \mathbb{R}^n$ are prescribed, distinct centres. The aim of the interpolation method is to approximate given data $\{f(x_j)\}_j$ by

$$s(x) = \sum_j \lambda_j \, \phi(\|x - x_j\|_2), \qquad x \in \mathbb{R}^n,$$

where the λ_j are to be defined through the interpolation conditions $s(x_j) = f(x_j)$ for all j. If the number of data is very large, it can be advantageous to have a ϕ of compact support. Several earlier approaches are reviewed in this talk and a new class of radial function is introduced and analyzed. Moreover, the differences to the well-established globally supported ones are discussed, including their approximational behaviour.

M. Schreiner:

Radial Basis Functions on the Sphere

Radial basis functions on the sphere are an appropriate tool for many approximation methods on the sphere (splines, finite elements, wavelets) and play an important role also for non-spherical approximation. Besides their structural simplicity the main advantage in the use of these functions is that they are strongly related to invariant pseudodifferential operators on the sphere. Thus, they are likely to play an essential role in future geodetic applications.
After a general introduction in radial basis functions on the sphere the talk concentrates on new results concerning locally supported spherical radial basis functions. By using these functions, the numerical effort of any of the mentioned approximation techniques can be significantly reduced.

W. R. Madych:

Recovery of Band Limited Functions via Generalized Splines

It is known that univariate band limited functions can be recovered from their values on certain discrete subsets, $\{x_n\}$, as the limits of piecewise polynomial spline interpolants when the order of the splines goes to infinity. See (1) Schoenberg, J. Analyse Math. , XXVII (1974), 205-229 for the case when $\{x_n\}$ is the integer lattice and (2) Lyubarskii and Madych, Jour. Funct. Anal. , Vol. 125, No. 1, Oct 1994, 201-222 for the significantly more general case when $\{x_n\}$ is such that the collection of functions in ξ $\{e^{-ix_n\xi}\}$ is a Riesz basis for $\mathcal{L}^2([-\pi,\pi])$, among other examples this includes the integer lattice and any sufficiently small perturbation. The goal of this talk is to indicate that the methods introduced in (2) can be modified to obtain meaningful extensions:

- Replace piecewise polynomial splines by more general families of interpolators of the form $\sum_{x_n} c_n\varphi_k(x-x_n)$ and still have the same recovery result as $k\to\infty$. For example, our results include the cases $\varphi_k(x) = 1/(k^2+x^2)$ and φ_k =Daubechies scaling function with scaling filter of length $2k$ and maximum sampling moments convolved with its symmetry.
- Recovery results for multivariate band limited functions from their values of irregularly distributed sampling sets using polyharmonic splines and selected interpolation methods.

O. Kounchev:

Splines which are Piecewise Solutions of Elliptic Equations on Manifolds

It is our purpose to establish a new paradigm in the area of smoothing of multivariate scattered data, which provides a natural generalization of the univariate spline theory. So far the results obtained for multivariate polynomial splines and in the theory of radial basis functions have not succeeded to establish a satisfactory paradigm in the multivariate world, in the sense of natural extension of the beautiful spline theory in one dimension.

K. -R. Koch:

Bayesian Statistics with Applications

Starting from Bayes' theorem the essence of the Bayesian approach is outlined. The choices of prior probability density functions are discussed starting from noninformative prior densities. Prior densities based on maximum entropy are introduced. It leads to the normal distribution in case of a given expected value and a variance. Conjugate priors are mentioned which are used in linear models. The posterior density of the unknown parameters resulting from Bayes' theorem contains all the information needed for statistical inference. For the point estimation the expected value or the value for which the posterior density becomes maximal are taken. Confidence regions are obtained by highest posterior (HPD) regions. Hypothesis tests are solved by confidence regions or simply by computing posterior odds. The results of the parameter estimation in linear models using noninformative or informative density functions are given. Finally as an extension of Bayesian statistics Bayes nets for reasoning under uncertainty are presented. The computation of

the joint and the marginal density functions and the propagation of probabilities in these nets are outlined.
As examples for the application of Bayesian statistics two procedures of an automatic interpretation of digital images are presented. Both methods introduce labels for the objects to be identified. In the first procedure the labels are defined as random variables of a Markov random field and in the second one as random variables of the nodes of a Bayes net. The labels are estimated by maximizing their density functions.

W.-D. Schuh:

Computational Geodesy as a Tool for Solving Problems within Satellite Gravity Field Missions

In spite of new developments in computer design, the standard least squares procedure for the estimation of spherical harmonic coefficients of the gravitational potential from orbit and satellite gravity gradiometer data for high degree gravitational fields exceeds present capabilities. Therefore, special techniques have to be developed to accomplish this task. With simulated satellite gravity gradiometry (SGG) and satellite-to-satellite tracking (SST) data sets the numerical behaviour of the normal equiations are analyzed. A white noise stochastic model but also band-limited and coloured noise models are employed. With the help of orthogonality considerations and special numbering schemes tailored iterative procedures are developed. Special investigations allow to use parallel resources and also robust statistical methods within these iterative procedures. Tailored pre-conditioning strategies improve the convergence rate and decrease the number of iterations drastically.

W. Pachelski:

Barycentric Coordinates: an Introduction to Geodetic Applications

Apart from global reference frames geodesy requires also local reference frames to locally describe detailed structure of the Earth surface, monitor local geodynamic phenomena, fulfill functions of a Land Information System, as well as to perform corresponding geodetic operations. Presumably, such frames should be as much as possible autonomous, i. e. once they are properly defined they should refer as little as possible to any external reference frame.
As a possible tool for such frames there are considered *Moebius* barycentric coordinates, which can be defined through any simplex in an n-dimensional space, e. g. by means of triangular or tetrahedral bases in 2-D or 3-D, respectively. Their theoretical background is reviewed, as well as their relation to oblique coordinates and use in some geodetic applications such as the *Ansermet's* resection problem, a photogrammetric positioning

problem, and shape functions in the method of finite elements.
Main properties of barycentric coordinates consist in their invariance with respect to linear transformations, as well as in a seperation of nonmetric and metric relations between geometric constructs.

L. Meister:

Estimation of Rotating Body Attitude by Quaternion Filter

A problem of optimal estimation of a rotating body attitude with both differential equation of motion and observation of stars is considered. The problem is formulated as a conditional extremum problem and using the language of quaternions an analytic solution to the problem is obtained. It is shown that the mutual position of the observed stars influences on the number of possible solutions, and the optimal estimation of the attitude has to be selected from the different classes of functions. The case of only one observed star serves to illustrate the feasibility of the presented method.

U. C. Herzfeld:

Geostatistical Methods for Interpolation and Classification of Remote-Sensing Data

Along three examples, geostatistical methods for interpolation and classification of remote-sensing data are developed and demonstrated.
The first method is kriging, a family of interpolation and extrapolation techniques mathematically related to least-squares prediction. Application of ordinary kriging to satellite radar altimeter data from Lambert Glacier/Amery Ice Shelf, Antarctica, yields maps/grids with a 3km resolution. Construction of a time series of such grids from altimeter data of SEASAT (1978), GEOSAT GM (1985-1986), GEOSAT ERM (1987-1989), and ERS-1 (since 1992) facilitates analyses of changes in the Antarctic ice stream/ice shelf system, such as elevation changes and changes in the position of the grounding line.
While interpolation utilizes the primary information in the data, a newly developed geostatistical classification method is geared at deriving secondary information from elevation data or backscatter data. Based on statistical properties, elements of surface structures are used for automated geologic/geomorphologic mapping. This is applied in a geologic segmentation of the western Mid-Atlantic Ridge flank near 26 deg North.
A combination of high-resolution and low-resolution techniques is attempted in a study of the 1993-1995 surge of Bering Glacier, Chugach Mountains, Alaska. GPS-located video data collected from small aircraft are used in combination with ERS-1 SAR images to (a) help understand the relationships between ice velocity, surface stress patterns, crevassing and iceberg calving during the surge, and (b) provide a technique for surface classification

based on SAR data in general.

L. Svensson:

Quality of Methods in Physical Geodesy

Different aspects of the problems of physical geodesy are obtained from the point of view of functional analysis and from the point of view of the corresponding discretized problems, i. e. regarding the problems of determining, e. g. by least squares methods, a finite number of coefficients from a finite number of observations. The first point of the present talk is that functional analysis results are the asymptotic versions of results for corresponding discretized problems when the net of observations is successively refined ad infinitum.
The second point is that it is essential, in dealing with the discretized problems, to make a proper discussion of the errors, seperating systematic errors, which also include the approximation errors, from random errors and their propagation from observations to results (gross errors being neglected here).
The third point is the derivation of asymptotic formulae for propagated errors in some important cases, including problems of heterogeneous sets of data such as the altimetry-gravimetry problem. Methods of integral formulae, collocation, least squares and Galerkin type solutions are discussed. Explicit formulae are given in some idealized cases, still sufficiently general to be useful as rules of thumb.

J. Otero:

The Simple Problem of Molodensky

The simple problem of Molodensky is the following: *given a function g defined in ω, find a function u such that*

$$\Delta u = 0 \text{ in } \Omega, \qquad < \nabla u, x > +2u = g \text{ on } \omega, \qquad u(x) \to 0 \text{ as } x \to \infty \tag{1}$$

where ω is a closed surface in $\mathbb{R}^3$, and Ω is the domain exterior to ω. This boundary problem appears several times in Geodesy with u being the *dirturbing potential*, ω being a *telluroid* and g being related to observed quantities like the *gravity*. By inversion relative to a sphere and Kelvin transform, this problem is equivalent to

$$\Delta v = 0 \text{ in } \Omega', \qquad < \nabla v, x > -v = g' \text{ on } \omega'. \tag{2}$$

This boundary problem (2) also arises in the gravity space approach to the vectorial Molodensky problem.
In the first part of this talk, we shall present some open problems about the uniqueness

and existence of solutions of the general linear (vectorial and scalar) Molodensky problems. We shall also take advantage of this atmosphere to present a uniqueness theorem for a Robin boundary value problem arising in the classical gravimetric determination of the Geoid.
In the second part, we shall show how the solution of the linearized vectorial Molodensky problem may be reduced to a sequence of simple problems of Molodensky. At this point it will be necessary to give some uniqueness, existence and regularity of solutions theorems for the simple problem of Molodensky.
Summing up, we shall try to make clear the important role that boundary problems like (1) and (2) play in Physical Geodesy.

E. Groten:

Quasigeoid Heights and Deflections of the Vertical for Germany:
A New Solution at IPGD

Utilizing new gravity material for Europe and mainly for East part of Germany it was possible to evaluate a new quasigeoid model for Germany, based on a remote zone model, using Rapp's OSU 91A and IGM-3, and a superimposed terrestrial (point values) set of Δg which is extremely dense in Eastern Germany, enabling to model the geoid in steps of 3'. The terrestrial set is supplemented by a reasonable (5 by 5 km) terrain model in order to get the various refining corrections to conventional spherical approximations. Belikov's well known "scaling" approach (non-harmonic supplement) gives way to a representation which basically corresponds, in well surveyed areas such as East Germany, to a spherical harmonics expansion of truncation degree $n = 3600$. Even though a more detailed terrain model of 20 by 20 meters will soon be available a substantial improvement for geoid heights cannot be expected. The deflection of the vertical given by us may take more profit out of such more detailed terrain models but the use of precise deflections is decreasing in modern geodesy so our approximation appears adequate. The accuracy of the geoid was found to be of the order of a few centimeter.

M. Belikov:

Local Geoid Determination with Very High Resolution

The local gravity field modelling is considered in the framework of Cauchy-Kowalevskaya problem. The iterative regional harmonic analysis is applied in order to stabilize the solution. The high resolution of the solution is provided by the implementation of pseudo-harmonics associated with solid spherical functions and scaled harmonic analysis technique. In the elaborated method the original data distribution is used without need to transform data to a regular grid. This method has been applied for determination of gravimetric geoid for Germany. The resolution of the regional geoid is shown to be 5km with the accuracy of several cm over some hundred km, in comparison with GPS and levelling.

W. Light:

Variational Approaches to Approximation on Spheres

Radial Basis Functions on $\mathbb{R}^d$ can be viewed as coming from a variational theory. It is now well-known that such a theory is possible on spheres S^{d-1}. Such a theory leads to approximations of the form

$$v(x) = \sum_{i=1}^{m} \lambda_i \phi(x a_i) + n(x)$$

where $a_1, \ldots, a_m$ are fixed points in S^{d-1}, $\lambda_i \in \mathbb{R}$ and n is a spherical harmonic polynomial (for practical purposes having low degree). The aim is to solve the interpolation problem $v(a_j) = c_j$, $j = 1, \ldots, m$. Again, this theory (and its applications to geodesy) is well understood. We will elaborate on the known results and in particular provide error estimates.

R. Haagmans:

Accurate Radon Domain Based Interpolation for Satellite Tracks (ARTIST): A Solution for interpolation of Trace and Missing Trace Data?

ARTIST has been developed for interpolation of densely measured data along more or less parallel, but widely spaced traces. One can think of satellite altimeter data, marine gravity data, aero-magnetic or aero-gravity data, and seismic data. Now, if a small scale lineament is supported by several neighbouring traces, most "standard" interpolation techniques used within geodesy fail to interpolate this structure. The Radon domain, being the mapping of the integral along a line onto one point, allows us to detect significant line-like structures, and subsequently to interpolate one-dimensionally the structure along the corresponding lines in space domain. These interpolated "control" points are merged with the original set, and are finally two-dimensionally interpolated with minimum curvature splines. The procedure is applied to a synthetic gravity data set and real-life aero-magnetic data. The interpolated results are superior to the "classical" results, showing a reduced bias and a 32% improvement in the rms of the difference in case of the synthetic set. Next, the procedure is applied to a synthetic seismic data set for interpolation of missing traces. Also here a significant improvement over classical procedures is obtained. The goal is to extend the procedure for more general data like from satellite altimetry. Therefore, the detection procedure is to be improved by means of nonstationary filtering with wavelets, and an iterative search in the Radon domain.

J. D. Ward:

Nonstationary Wavelets on the m-Sphere, Localization and Uncertainty

We discuss classes of nonstationary wavelets generated by spherical basis functions, which comprise a subclass of Schoenberg's positive definite functions on the m-sphere. The wavelets are intrinsically defined on the m-sphere, are independent of the choice of coordinate system and may be easily orthogonalized. Decomposition, reconstruction and localization for these wavelets will be discussed. In the special case of the 2-sphere, we derive an uncertainty principle that expresses the trade-off between localization and the presence of high harmonics – or high frequencies – in expansions in spherical harmonics. We discuss the application of this principle to the wavelets that we construct.

F. N. Narcovich:

Spherical Basis Functions, Hermite Interpolation, and Intrinsic Wavelets on the Sphere

We discussed interpolating data generated via integrating a distribution against a C^∞ function on a closed, compact Riemannian manifold $|M|^m$ via C^∞ strictly positive definite kernels on $|M|^m$. We gave a number of examples of such kernels for the m-sphere and the m-torus. Spherical basis functions (n spherical splines) are special cases of such kernels on S^m; they have the form

$$G(\vec{p}\cdot\vec{q}),$$

where $P_l(m+1;t)$ is the l^{th} degree Legendre polynomial in $m+1$ dimensions.
Define the convolution as "star" product via

$$G*H=\sum_{l=0}^{\infty}a_l b_l\frac{\omega_m}{d_{m,l}}P_l(m+1;\vec{p}\cdot\vec{q}),$$

where G is as above, and H has a similar expansion with the coefficients in the expansion than being the b_l's. We construct sampling spaces of the form

$$V=\operatorname{span}\{G(\vec{p}\cdot\vec{\xi_1}),G(\vec{p}\cdot\vec{\xi_2}),\ldots,G(\vec{p}\cdot\vec{\xi_N})\},$$

and show that the inner product

$$<f,g>=\int_{S^m}f(\vec{p})\overline{g(\vec{p})}\,d\omega(\vec{p}),$$

where $f=\sum f_j G(\vec{p}\cdot\vec{\xi_j}), g=\sum g_k G(\vec{p}\cdot\vec{\xi_k})$, has the form

$$<f,g>=\underline{g}^*A_*\underline{f},$$

where $\underline{f}^T = (f_1\, f_2 \cdots f_N)$ and $\underline{g}^T = (g_1\, g_2 \cdots g_N)$; the matrix A_* has entries $G * G(\vec{\xi_j} \cdot \vec{\xi_k})$. This matrix is always strictly positive definite, and may be explicitly computed in many cases. Similar results apply for the case of $|M|^m$.

A. De Santis:

New Regional Models for Geodesy

In the last ten years, some new techniques have been introduced and developed for modelling the magnetic field in restricted regions (the so-called regional magnetic fields). This note will describe the basis of the spherical cap models as: Spherical Cap Harmonic Analysis (SCHA), Translated Origin Spherical Cap Analysis (TOSCA), and Adjusted Spherical Harmonic Analysis (ASHA).
Above methods are exact (except ASHA which is an approximation of SCHA) solutions of Laplace's equation in a cap-like region.
After showing the main characteristics of the spherical cap methods in the frame of geomagnetism, it is given also an example of application for local gravity field representation. The expressions of: disturbing field potential, anomaly gravity, geoid undulations and deflection of the vertical are shown.

B. Heck:

Integral Equation Methods in Physical Geodesy

Starting from one and the same geodetic boundary value problem several forms of integral equations can be constructed, based on various representations of the potential function. We applied single and double layer as well as Brovar's generalized single layer and volume potentials to construct integral equations with strongly different types of singularities; as an example the "simple" Molodensky problem is chosen. The properties of the boundary integral operators can easily be studied in the case of the corresponding spherical integral equations, which are expanded in series with respect to a shrinking parameter ε. Finally the numerical properties of the integrals related to Boundary Element Methods are studied. It is shown that the strong singular integrals can be transformed into weak singular and regular ones; the numerical effort for evaluating these integrals is estimated.

R. Klees:

A Wavelet Galerkin Approach to Gravity Field Determination

We treat equivalent integral equation formulations of Geodetic Boundary Value Problems which aim at recovering the Earth's gravitational field from functionals given on the Earth's surface. The corresponding integral operators are linear bounded operators from $H^r \to H^{r-\tau}$ with integer orders $\tau \in \{-1, 0, 1\}$. Galerkin methods and finite elements on the boundary lead to dense linear spectras of algebraic equations with $\sim 10^5$ unknowns. We show how compactly supported wavelets can be used to approximate the dense matrix by a sparse one. We discuss how the compression rate depends on the order of the integral operator and the number of vanishing moments of the wavelet basis functions. Finally we point out some numerical problems which still have to be solved before the method can successfully applied to global gravity field determination.

P. Holota:

Coerciveness of the Linear Gravimetric Boundary-Value Problem and a Geometrical Interpretation

In this paper the linear gravimetric boundary-value problem is discussed in the sense of the so-called weak solution. For this purpose a Sobolev-weight space was constructed for an unbounded domain representing the exterior of the Earth and quantitative estimates were deduced for the trace theorem and equivalent norms. In the generalized formulation of the problem a special decomposition of the Laplace operator was used to express the oblique derivative in the boundary condition which has to be met by the solution. The relation to the classical formulation was also shown. The main result concerns the coerciveness (ellipticity) of a bilinear form associated with the problem under consideration. The Lax-Milgram theorem was used to decide about the existence, uniqueness and stability of the weak solution of the problem. Finally, a clear geometrical interpretation was found for a constant in the coerciveness inequality and the convergence of approximation solutions constructed by means of the Galerkin method was proved.

F. Sacerdote:

Stochastic BVP in Physical Geodesy

Many fundamental problems of physical geodesy are formulated as BVP's; the input data however are measurements affected by observational noise. Therefore the problem arises on how this noise propagates to the solutions; this calls first of all for an understanding of what is a solution with noise at the boundary, i. e. what is a solution of a stochastic BVP.

In literature such problems have been studied by a number of mathematicians, among which we mention Rozanov and its Russian school. Yet their theoretical results do not cover the simplest but most important case; namely when the boundary noise is a white noise. This case has been recently attached and some new significant results are obtained.

W. Keller:

Applications of Overdetermined Problems

The problem of overdetermined problems in function-spaces was considered. In order to estimate a solution the well known BLUE-estimation principle was generalized.
The generalized BLUE principle was applied to satellite gradiometry leading to inversion-free solution algorithms.

P. J. G. Teunissen:

Integer Least Squares Processing of GPS Phase Observations; Theory and Results

One of the major problems in processing GPS phase observations is estimating the double-difference (DD) ambiguities as integers. Based on carrier phase data only, short observational time spans result in strongly correlated ambiguities and in very elongated ambiguity confidence ellipsoids. As a result the estimation of the integer least-squares ambiguities becomes an extremely time consuming task, when traditional search-methods are applied. In this contribution, it will be shown both analytically as well as numerically, that this can be explained by the distinctive discontinuity that is present in the spectrum of conditional variances of the DD-ambiguities. The least-squares Ambiguity Decorrelation Adjustment method allows an efficient estimation of the integer ambiguities over short observational time spans. This method removes the discontinuity from the spectrum, thereby returning transformed ambiguities that are much less correlated and that show a dramatic improvement in precision. In this contribution the theory en performance of the method will be discussed.

H. van Gysen:

Estimability Analysis of Satellite Altimetry

An estimability analysis is undertaken for the standard estimation problems of satellite altimetry, i. e. for the collinear, and local and global crossover adjustment problems. For each problem we consider the estimability of mean sea surface heights, residual radial orbit error and residual tidal signals, and present explicit characterisations of the nullspace of the estimation problem and develop a variety of partial minimum-norm solutions. Analytical solutions are given (on the assumption that the data are in the form exact repeats of the altimeter measurement cycle, with no missing data). We look also at the mixed-mission crossover adjustment problem involving two (or more) altimeter satellites, and at various augmentations of the basic estimation problem (e. g. , allowing for the simultaneous evaluation of empirical sea state bias parameters). The analysis makes use of generalised inverses of partitioned matrices, and of Kronecker matrix products. Finally, we point to ways in which the analysis procedure we do here can be applied to other geodetic estimation problems.

A. Dermanis:

The non-linear and space-time geodetic problem:

An attempt is made to generalize the geometric theory of generalized inverses of linear operators to the non-linear finite dimensional case by replacing quotient spaces generated by linear subspaces with fiberings. The geodetic datum problem arising from the use of coordinates as unknowns while observations are invariant under rigid or similarity transformations (injectivity defect) is investigated without resorting to linearization. The non-linear Baarda transformation is derived which maps a given solution into the one of the same solution subspace (fiber) closest to a fixed element. The differential geometry of the solution fibers is investigated.
The space-time geodetic datum problem, where continuous (adjusted) observations are available for a time interval, is investigated by studying the geometry of the space-time solution manifold. Several solutions (called motions) are presented in the form of a transformation from a given reference motion into one with prescribed optimality or selection criteria. In particular motions which are everywhere orthogonal to the local single epoch fiber are introduced as generalizations of Meissl's inner solution and the differential equations of inner constrains are derived. It is proved that in the case where the observations are invariant under rigid transformations the orthogonal motions are geodesics and furthermore the ones of shortest length among all those joining the two boundary fibers corresponding to the initial and final observation epoches.

D. Lelgemann:

On the Generation of the Geodetic Datum of Terrestrial Land Surveying Systems

The concepts of geodetic analysis techniques are governed by the problem to investi-

gate the information content of data (observations). Therefore, stability investigations, error investigations as well as the interpretation of results with respect to cause / effects govern geodetic analysis techniques.
Those ideas, in particular the interpretation problem, is explained in some detail in the frame of the determination of the so-called geodetic datum (translation of origin, rotation of the axis, scale factor) of the Deutsches Hauptdreiecksnetz (DHDN), the reference of geodetic coordinates and maps in Germany, with respect to the worldwide GPS reference system.
It was found, that the transformation parameters are
Translationvector $[\Delta x_1, \Delta x_2, \Delta x_3]^t = [638\text{m}, 24\text{m}, 444\text{m}]^t$
Rotationvector $[\varepsilon_1, \varepsilon_2, \varepsilon_3]^t = [-0.''97, -0.''45, -1.''60]^t$
scale factor $m = 0$.
By a parametertransformation (choice of a local basis in the fundamental point Rauenberg in Berlin of the DHDN) it could be shown that the translation vector was generated due to
- neglection of the height anomaly and the deflection of the vertical at the fundamental point Rauenberg and
- the use of the Bessel ellipsoid as reference surface.
The rotation of the DHDN is due to small observation errors of the astronomical latitude ($d\phi = -0.''2$) and longitude ($d\Lambda = 0.''2$), but in particular due to a large azimut error ($dA = -1.''7$) at the fundamental point Berlin-Rauenberg of the DHDN in the course of the establishment of the DHDN in the middle of the last century.

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