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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

# Tagungsbericht 44/1995

# Empirical Processes: Theory and Applications
## 22.10. - 28.10.1995

Die Tagung fand unter der Leitung von P.L. Davies (Essen), P. Gaenssler (München) und W. Stute (Giessen) statt.

Eine Grundidee vieler statistischer Verfahren besteht darin, durch zufällige Fehler entstandene Streuungen durch Mittelung der Daten zu reduzieren. Durch Einführen des zugehörigen Zählprozesses — des Empirischen Prozesses — ist man in der Lage, die Analyse unterschiedlichster Fragestellungen auf ein Studium der zugrundeliegenden Prozesse zurückzuführen. In den letzten beiden Jahrzehnten ist ein umfangreiches Arsenal an Methoden entwickelt worden, welches heute erfolgreich in so verschiedenen Gebieten wie der Zeitreihenanalyse, der Survival Analysis oder bei Kurvenschätzungen Anwendung findet.

Ziel der Tagung war es, auf unterschiedlichen Teilgebieten arbeitende Experten zu einem gemeinsamen Gedankenaustausch zusammenzuführen. Schwerpunkte des Tagungsprogramms und der Diskussion waren:

- Allgemeine Empirische Prozesse
- Empirische Prozesse in der Zeitreihenanalyse
- Kurvenschätzprobleme
- Changepointproblematik
- Multivariate Verfahren
- Goodness-of-fit Probleme
- Semi- und Nichtparametrische Verfahren
- Bootstrap
- Analyse unvollständiger Daten

Die Tagung wurde von 49 Teilnehmern besucht. Es wurden insgesamt 36 Vorträge gehalten, davon 7 Haupt- bzw. Übersichtsvorträge. Die fruchtbaren und intensiven Diskussionen haben den Teilnehmern neue wertvolle Impulse für ihre zukünftige Forschung vermitteln können.



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M. Arcones

**Weak convergence of triangular arrays of empirical processes**

We study the weak convergence of a general triangular array of empirical processes having a general infinitely divisible distribution. Let $\{X_{nj} : 1 \leq j \leq K_n\}$ be $S_{nj}$-valued independent random variables, and let $f_{nj}(\circ, t) : S_{nj} \longrightarrow \mathbb{R}$ be real-valued functions. For some functions $c_n(t)$ we then consider the processes $Z_n(t) = \left(\sum_{j=1}^{K_n} f_{nj}(X_{nj}, t)\right) - c_n(t)$. First we study necessary and sufficient finite-dimensional approximation conditions for the weak convergence of $\{Z_n(t) : t \in T\}$. Then we investigate convergence in specific cases: convexity, V-C (maximal entropy) conditions, bracketing conditions with majorizing measures.

P.K. Bhattacharya

**Rank-cusum charts for sequential detection of change in distribution**

The problem of sequential detection is considered in three modes, viz., location change in a symmetric distribution, and location change or scale change in an arbitrary distribution. Nonparametric counterparts of the Page-cusum chart based on ranks and signs are constructed in these models. The resulting rank cusum charts are described by doubly-indexed processes of linear rank statistics whose weak limits are the same as the weak limits of the processes which describe the corresponding parametric cusum charts.

M.V. Boldin

**On residual empirical processes and rank estimates in an autoregressive model with infinite variance**

Consider the autoregressive model

$$u_i = \beta u_{i-1} + \varepsilon_i, \qquad i \in \mathbb{Z},$$

where $\beta$ is an unknown parameter, $|\beta| < 1$, and $\{\varepsilon_i\}$ are i.i.d. random variables with unknown d.f. $G(n)$ and possibly infinite variance.
Let $u_0, \ldots, u_n$ be observations, $\varepsilon_k(\theta) = u_k - \theta u_{k-1}$, and let $\rho_k(\theta)$ denote the rank of $\varepsilon_k(\theta)$. Put

$$l_n(\theta) = \sum_{t=1}^{n-1} \theta^{t-1} \sum_{k=t+1}^{n} \psi_1\left(\frac{\rho_{k-t}(\theta)}{n+1}\right) \psi_2\left(\frac{\rho_k(\theta)}{n+1}\right), \qquad |\theta| < 1.$$

The rank estimate $\beta_{n\rho}^*$ of $\beta$ is the solution of the equation $l_n(\theta) = 0$. It is proved that $\beta_{n\rho}^*$ is $\sqrt{n}$-asymptotically normal and has some other attractive properties. The method is based on an analysis of the residual empirical process.



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R. Cao Abad

**Applications of almost sure representations for censored/truncated data to the bandwidth selection problem**

The strong representation of the survival function estimator for truncated and censored data given by Gijbels and Wang (1993) is used to derive an asymptotic representation of the mean integrated squared error (MISE) for the problem of density estimation under truncation and censoring. An asymptotic expression for the bootstrap MISE is derived by the same method. Rates of convergence to the minimizer of MISE as well as the limit distribution can be obtained for the bandwidth selector that results from minimizing the asymptotic bootstrap MISE.

S. Csörgő

**Principles of curve estimation under long-range dependence**

Three basic problems of large-sample non-parametric curve estimation theory will be discussed. One is estimating the marginal density function, or its derivatives, of a stationary long-range dependent sequence under instantaneous Gaussian subordination. The second is estimating a function in a fixed-design regression model when the errors form a long-range dependent sequence under instantaneous Gaussian subordination. The third is estimating a regression function with a random design and long-range dependent errors. In all three cases, the asymptotic distributions of the usual kernel estimators are studied. The phenomena disclosed are unconventional; they are surprising in themselves and relative to each other.

V. de la Peña

**Wald's equations: Decoupling inequalities**

In this talk we give a survey of decoupling inequalities. The theory is motivated by a re-formulation of Wald's equation for randomly stopped sums of independent random variables. This re-formulation allows for the viewing of the stopping time as independent from the sequence. We continue by introducing a decoupling inequality for the tail probabilities of U-statistics comparing it to that of a sum of conditionally independent random variables. A unification of the above examples is proved by a presentation of the theory of decoupling inequalities which works in general for making comparisons between the sums of dependent random variables and sums of conditionally independent (decoupled) random variables. We also show how decoupling inequalities can be used to improve on the approximations provided by the use of square function inequalities for martingales. We finish by developing new decoupling inequalities which are used to settle the 49 year old problem of extending Wald's equation to the case of U-statistics.

R.M. Dudley

**Consistency of M-estimators and 1-sided bracketing**

Let $(X, \mathcal{A}, P)$ be a probability space and $(\Theta, d)$ a metric space. Let $\rho : X \times \Theta \longmapsto [-\infty, \infty]$, where $\forall \theta \in \Theta, \rho(\cdot, \theta)$ is measurable on $X$. Let $X_1, \ldots, X_n, \ldots$ be strictly i.i.d. $(P)$. Then $\hat{\theta}_n$ are *approximate M-estimators* if $P_n\rho(\cdot, \hat{\theta}_n(X_1, \ldots, X_n)) - \inf_{\theta\in\Theta} P_n\rho(\cdot, \theta) \longrightarrow 0$ in



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outer probability (i.o.p.) as $n \to \infty$. Let $\theta_0$ be such that for some measurable $a(\cdot)$ on $X$, and $h(x,\theta) := \rho(x,\theta) - a(x), +\infty > \inf_{\theta\in\Theta} Ph(\cdot,\theta) = Ph(\cdot,\theta_0) > -\infty$. Then consistency means $\hat{\theta}_n \longrightarrow \theta_0$ i.o.p. We seek to infer it from Glivenko-Cantelli conditions $\|P_n - P\|_{\mathcal{F}} \longrightarrow 0$ for suitable $\mathcal{F}$. Beginning with $\mathcal{F}_1 := \{\rho(\cdot,\theta) : \theta \in \Theta\}$, improvements due to Huber (1967) are: $\mathcal{F}_2 = \{f - a(\cdot) : f \in \mathcal{F}_1\}, \mathcal{F}_3 = \{f/b(\theta) : f \in \mathcal{F}_2\}, b(\theta) > 0$, and $\mathcal{F}_4 : \forall f \in \mathcal{F}_3 \exists g \in \mathcal{F}_4 b \geq g$ (one-sided bracketing). Beyond Huber, one drops local compactness of $\Theta$ and following van de Geer (Ann. Stat. '93) takes $\mathcal{F}_3' := \{g \circ f : f \in \mathcal{F}_3\}$ for suitable $g$ and applies VC '81, GZ '84, Talagrand '87 Glivenko-Cantelli work.

L. Duembgen

## Modulation estimators: Selection plus shrinkage

It is shown that a nonparametric extension of the classical James-Stein estimator for a multivariate normal mean has many desirable properties such as asymptotic Bayes-optimality uniformly over large classes of priors. The estimators are of the form

$$\hat{\xi} = \left(\hat{f}(X)_i X_i\right)_{1\leq i\leq n},$$

where

$$X \sim \mathcal{N}(\xi, \sigma^2 I_n)$$

is the observed random vector, and $\hat{f}(\cdot) \in \mathcal{F}$ belongs to a class $\mathcal{F} \subset [0,1]^n$ of weight functions. Examples include the class $\mathcal{F}$ of nonincreasing functions $f : \{1,2,\ldots,\} \longrightarrow [0,1]$ or the class of $f$ with total variation not greater than some constant $M$. The analysis rests on empirical process methodology.

J.H.J. Einmahl

## The two-sample problem in $\mathbb{R}^m$ and measure-valued martingales

The two-sample problem is one of the classical problems in mathematical statistics. It is well-known that in dimension one the two-sample Smirnov test possesses two basic properties: it is distribution free under the null hypothesis and it is sensitive to all alternatives. In the multidimensional case, i.e., when the observations in the two samples are random vectors in $\mathbb{R}^m, m \geq 2$, the Smirnov test loses its first basic property.
In correspondence with the above, we define a solution of the two-sample problem to be a stochastic process, based on the two-samples, which is (a) asymptotically distribution free under the null hypothesis, and which is, intuitively speaking, (b) as sensitive as possible to all alternatives. Despite the fact that the two-sample problem has a long and very diverse history, starting with some famous papers in the thirties, the problem is essentially still open for samples in $\mathbb{R}^m, m \geq 2$.
In this paper we present an approach based on measure-valued martingales and we will show that the stochastic process obtained with this approach is a solution to the two-sample problem, i.e., it has both the properties (a) and (b), for any $m \in \mathbb{N}$.



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M. Falk

**On MAD and comedians**

A popular robust measure of dispersion of a random variable (rv) $X$ is the *median absolute deviation from the median* med(|X − med(X)|), MAD for short, which is based on the median med(X) of $X$. By choosing $Y = X$, the MAD turns out to be a special case of the *comedian* med((X − med(X))(Y − med(Y))), which is a robust measure of correlation between rvs $X$ and $Y$.
We investigate the comedian in detail, in particular in the normal case, and establish strong consistency and asymptotic normality of empirical counterparts.

D. Ferger

**The two-sample problem in general measurable spaces**

We propose a test for the two-sample problem in the case of $S$-valued observations, where $S$ is an arbitrary measurable space. The test is shown to be an exact level-alpha test, which is consistent against general alternatives. Moreover, we determine the local power of the test under several types of contiguous alternatives. In the special case of location or scale alternatives a comparison with rank-tests and Kolmogorov-Smirnov tests is made. Weight-functions are used to improve the local power, if the two sample-sizes are unbalanced.

U. Gather

**Estimation in censoring models**

A so-called hospital model of combining informative and noninformative censoring is considered, where -- different from a simple proportional hazards model of random censoring -- the observed survival times and the indicator variables are not independent. In such models we discuss the estimation of the survival time distribution and show connections to other models used in such situations.

I. Gijbels

**Local likelihood in hazard regression**

We consider a proportional hazards regression model with nonparametric risk factor. We discuss estimation of the risk factor function and its derivatives in two cases: when the baseline hazard function is modeled parametrically and when it is modeled nonparametrically. We establish the asymptotic normality of the estimators, and compare their asymptotic efficiencies.

R.D. Gill

**Nonparametric estimation under censoring and passive registration**

The classical censorship model assumes that we follow an individual continuously up to the time of failure or censoring, so observing this time as well as the indicator of its type. Under passive registration we only get information on the state of the individual at random observation times. In this paper we assume that these observation times are



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the times of events in an independent Poisson process, stopped at failure or censoring; the time of failure is also observed if not censored. The model is shown to be related to the problem of estimating a density known to be monotone. This leads to an explicit description of the nonparametric maximum likelihood estimator of the survival function (based on i.i.d. observations from this model) and to an analysis of its large sample properties.

E. Giné

**On the LIL for degenerate U-statistics and U-processes**

The LIL for U-statistics is not yet totally solved. A result possible from the view point of sufficient integrability conditions is:

$$Eh^2 < \infty \Rightarrow \frac{1}{(n \log \log n)^{m/2}} \sum_{i_1 \neq \ldots \neq i_m \leq n} h(X_{i_1}, \ldots, X_{i_m})$$
$$\longrightarrow \{Eh(X_1, \ldots, X_m) \cdot g(X_1) \ldots g(X_m) : Eg^2(X_1) \leq 1\}$$

($h$ symmetric, P-degenerate, $X_i$ i.i.d. (P)), recently proved by M. Arcones and E. Giné. Some sharp necessary conditions (obtained by Zhang and Giné) will also be presented.

W. González Manteiga

**Goodness of fit test based on the bootstrap of the empirical regression process**

In this work three bootstrap procedures for the so-called empirical regression process are studied: the wild, naive and smooth bootstrap. The first one is proved to be consistent while the others are not. Applications to goodness of fit tests are developed, and some simulations show the good behaviour of the wild bootstrap with respect to the other competitors.

P. Groeneboom

**Nonparametric estimation of a convex density**

The behavior of a sieved nonparametric maximum likelihood estimator of a decreasing convex density on $[0, \infty)$ is discussed. It is shown how this estimator can be defined implicitly as a functional of the empirical quantile process by using the Fenchel duality theorem. The local limiting behavior of the corresponding "Fenchel process" is also discussed and the limiting behavior of the sieved NPMLE is characterized as an (again) implicitly defined functional of integrated Brownian motion $+t^4$.

J. Hoffmann-Jørgensen

**The asymptotic behavior of maximum estimators**

In the study of the asymptotic behavior of $M$-estimators and similar estimators (e.g., $Z$-estimators) one is naturally presented with the problem of convergence in law of set valued random elements. However, it turns out that the natural topology for the problem is the upper Fell topology which has the unpleasant property that all real valued continuous



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functions are constant. This means that the usual notion of convergence in law trivializes (anything converges in law to anything). This calls for a revised definition of convergence in law which is done by replacing continuous functions by upper semicontinuous functions. With this new concept all the usual rules of convergence in law prevail (under appropriate topological conditions). Applying the calculus of the revised notion of convergence in law to estimators we obtain extensions and a unification of a large number of asymptotics for $M$ or $Z$-estimators including the classical asymptotic normality, stochastic differentiability and the cube root asymptotics.

A. Janssen

### A permutation test for the generalized Behrens-Fisher problem

The classical Behrens-Fisher problem is a two sample testing problem for the hypothesis of equality of means under normally distributed errors with unequal variances.
In practice the assumption of normally distributed error variables is not always realistic. What can be done in that case? In the nonparametric setting we propose an extension of Pitman's permutation test. Actually we show that the permutation version of the Welch test works well. It is an asymptotic level $\alpha$ test for the extended nullhypothesis.
The result is a special case of a general theorem about permutation statistics. It also gives us the asymptotic power function under local or fixed alternatives. The consistency and efficiency of the underlying tests can be discussed.

E. Khmaladze

### "Chimeric" alternatives and related goodness of fit theory

Intuitively, "chimeric" alternatives form the subclass of contiguous alternatives that remain distant from the hypothesis, but which are impossible to detect with any of the classical goodness of fit tests. In other words, the classical empirical process

$$\nu_n = \{\nu_n(f) = \sqrt{n} \int f(d\mathbb{P}_n - d\mathbb{P}), f \in C\}$$

has, under chimeric alternatives, the same distribution as under the null distribution $\mathbb{P}$, for any compact subset $C$ of $L_2(\mathbb{P})$.
It is suggested to replace $\nu_n$ by processes $w_{nN}$ of the form

$$w_{nN} = \{\nu_n(K_N f), f \in C\}$$

where $\{K_N\}_1^\infty$ is a sequence of norm preserving operators converging weakly to 0. It is shown that for the subfamily of chimeric alternatives corresponding to the choice of $\{K_N\}_1^\infty$, $w_{nN}$ plays the same role for goodness of fit theory as $\nu_n$ does for compact contiguous alternatives.
Examples of chimeric alternatives are discussed.



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C. Klüppelberg

## Gaussian limit fields for the integrated periodogram

Functionals of the two-parameter process

$$K_n(x,\lambda) = \int_{-\pi}^{\lambda} \frac{1}{n} \left| \sum_{t=1}^{n} X_t e^{-iyt} \right|^2 f(y)dy, \quad 0 \leq x \leq 1, \quad -\pi \leq \lambda \leq \pi,$$

(for a smooth function $f$) of a sample $X_1, \ldots, X_n$ from a linear process $(X_t)_{t\in\mathbb{Z}}$ have been used to detect a changepoint of the spectral distribution. These results are based on FCLT's for $K_n(x,\lambda)$ with a Gaussian limit field. We prove FCLT's for $K_n(x,\lambda)$ under the optimal moment condition of a fourth finite moment. Our approach is via an approximation of the integrated periodogram by a finite linear combination of sample autocovariances. This yields a representation of the Gaussian limit field as a (rescaled) Kiefer process.

R.Y. Liu

## Multivariate ordering based on data depth: concept and applications

For a multivariate random sample, a *data depth* can be used to measure the depth (or the centrality) of a given sample point with respect to the whole sample. Consequently, it provides a center outward ordering of the sample points, and a new notion of location and scale parameters for the underlying distribution. We will discuss some applications of such an ordering to nonparametric multivariate inference in the areas of rank tests, the construction of confidence regions, and the finding of P-values in testing hypotheses. In the context of quality control, we will also present some control charts for multivariate processes which can be visualized and interpreted easily.

D.M. Mason

## The fractal nature of empirical increments

Let $W$ be a standard Brownian motion on $[0,1]$. Orey and Taylor (1974) proved the following version of the Lévy modulus of continuity theorem for $W$: For any $0 \leq \Lambda \leq 1$, set

$$D^{\pm}(\Lambda) = \{t \in [0,1) : \limsup_{h\downarrow 0} \pm \frac{(W(t+h) - W(t))}{\sqrt{2h\log(1/h)}} \geq \Lambda\}.$$

$D^{\pm}(\Lambda)$ is a random fractal with Hausdorff dimension

$$\mathrm{dimD}^{\pm}(\Lambda) = 1 - \Lambda^2 \quad \text{a.s.}$$

We establish a functional analogue of this result for a general class of processes which includes the uniform empirical process, Brownian motion and the superposition of certain stationary independent increment processes. One such functional result is the following: For any absolutely continuous function $f$ on $[0,1]$ with $f(0) = 0$ such that $\lambda^2 := \int_0^1 f^2(u)du \leq 1$, let

$$S(f) = \left\{ t \in [0,1) : \liminf_{h\downarrow 0} \sup_{0\leq s\leq 1} \left| \frac{W(t+sh) - W(t)}{\sqrt{2h\log(1/h)}} - f(s) \right| = 0 \right\}.$$



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One has $\dim S(f) = 1 - \lambda^2$ a.s.

I.W. McKeague

**Outperforming the Gibbs sampler empirical estimator for nearest neighbor random fields**

Given a Markov chain sampling scheme, does the standard empirical estimator make best use of the data? We show that this is not so and construct better estimators. We restrict attention to nearest neighbor random fields and to Gibbs samplers with deterministic sweep. The structure of the transition distribution of the sampler is exploited to construct further empirical estimators which are combined with the standard empirical estimator to reduce asymptotic variance. The extra computational cost is negligible. When the random field is spatially homogeneous, symmetrizations of our estimator lead to further variance reduction. The performance of the estimators is evaluated in a simulation study of the Ising model.

H.-G. Müller

**Discontinuous versus smooth regression**

In regression applications, for instance in human growth studies, sometimes the question arises whether the underlying regression function is smooth or, alternatively, has jump discontinuities. To address this problem, we look at squared differences of the observations with varying spans between 1 and $L = L(n)$. It is shown that these statistics follow an asymptotic linear model with the error variance and the sum of squared jump sizes as well as a quantity depending on the smooth part of the regression function as parameters. We exploit this to derive consistency mean squared error and asymptotic normality for the estimate of the sum of the squared jump sizes, which then provides an asymptotic test for the presence of discontinuities.

A. Nikabadze

**Empirical martingales – censored observations**

Let $\{X_i\}_1^n$ be i.i.d. random variables with unknown d.f. $F$. Let $\mathcal{F} = \{F_\theta, \theta \in R^k\}$ be a parametric family of d.f.'s depending on a finite-dimensional parameter $\theta$. Consider testing the hypothesis that $F \in \mathcal{F}$. Assume that instead of $\{X_i\}_1^n$ one is only able to observe $Z_i = X_i \wedge Y_i$ and $\delta_i = I\{X_i \leq Y_i\}$, where $Y_1, \ldots, Y_n$ are i.i.d. random variables with d.f. $G$, also independent from $\{X_i\}_1^n$. Let $\hat{F}_n$ be the Kaplan-Meier estimator of the unknown d.f. $F$. Let $\tilde{\alpha}_n = \sqrt{n}(\hat{F}_n - F_{\theta_n})$, where $\theta_n$ is a $\sqrt{n}$ consistent estimator of $\theta$. We construct a transformation of $\tilde{\alpha}_n$, $T\tilde{\alpha}_n$, which under the hypothesis weakly converges to the Brownian Motion. Then, as a goodness of fit statistic, we may use any proper continuous functional of $T\tilde{\alpha}_n$, rather than $\tilde{\alpha}_n$, to obtain distribution freeness.



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Y. Nikitin

**Asymptotic efficiency of nonparametric statistics based on transformations of empirical process**

In order to find new goodness-of-fit statistics with simple limit distributions and high efficiency we consider functionals like $\|\cdot\|_p (1 \leq p \leq \infty)$ of transformed empirical processes. Transformation may mean centering (Watson and Darling), extracting the martingale part (Khmaladze and Aki), special weighting (Cabaña), etc. We find large deviation results of new statistics and their exact (local) Bahadur efficiency. We investigate also when these statistics attain maximal efficiency in the case of location and Lehmann alternatives.

D. Nolan

**The infinite-order U-process**

The infinite-order U-statistic is a U-statistic where the degree of the kernel increases to infinity with the sample size. Frees (1986 Annals of Statistics; 1989 Scandinavian Journal of Statistics) first considered this kind of U-statistic.
He showed that the Nelson-Aalen estimator of the cumulative hazard in survival analysis and a nonparametric renewal function estimator are examples of infinite-order U-statistics. We consider the statistic indexed by a collection of kernel functions, the infinite-order U-process. Through the Hoeffding decomposition and the U-process rates of convergence developed by Arcones and Giné, conditions can be found on the order of the kernel and the metric entropy of the index functions to yield a functional limit theory for the infinite-order U-process.

J. Praestgaard

**Survival analysis under order restrictions**

The talk will consider problems in estimating survival curves from right censored data when order restrictions are present. These may, for instance, be of the form $\bar{F}_1(t) \geq \bar{F}_2(t)$ if cohorts of males and females are under study, and it is known that female survival is more likely than male. Asymptotic distributions under such restrictions will be presented. They are related to concave majorants of Brownian Motion.

M. Talagrand

**An inequality for the supremum of the empirical process**

The supremum of the empirical process over a bounded class of functions satisfies a Bennett-type deviation inequality.

S. van de Geer

**Quasi-likelihood estimation**

Consider a response variable $Y$, with conditional expectation $\mathbb{E}(Y|T=t) = \mu_0(t)$, where $T = (X, Z)$ is a 2-dimensional covariate. Suppose that

$$\mu_0(t) = F(\theta_0 x + m_0(z)), \quad t = (x, z),$$



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with $F$ a given link function, $\theta_0 \in \mathbb{R}$ an unknown parameter, and $m_0$ an unknown function in the Sobolev class $\{m : \int (m^{(k)}(z))^2 dz < \infty\}$. Given a variance function $V(\mu)$, the quasi-(log-)likelihood is

$$Q(y;\mu) = \int_y^\mu \frac{y-s}{v(s)} ds.$$

The penalized quasi-likelihood estimator $(\hat{\theta}_n, \hat{m}_n)$, based on i.i.d. copies $(Y_1, T_1), \ldots, (Y_n, T_n)$ of $(Y, T)$, is obtained by maximizing

$$\sum_{i=1}^n Q(Y_i; F(\theta X_i + m(Z_i))) - \lambda_n^2 \int (m^{(k)}(z))^2 dz.$$

If $F$ is the identity link, and $V \equiv 1$, this estimator is known as a partial smoothing spline. We prove asymptotic normality of $\hat{\theta}_n$. Our main assumption is that a conditional expectation given $Z = z$ is smooth enough, as a function of $z$. For example, in the partial smoothing spline case, we assume that $\int (h^{(k)}(z))^2 dz < \infty$, with $h(z) = E(X|Z = z)$. The smoothness assumption ensures that $(\hat{\theta}_n, \hat{m}_n)$ is an interior point of the hardest 1-dimensional submodel. Therefore, the derivative in the worst possible subdirection, of the penalized quasi-likelihood, is zero. We use asymptotic equicontinuity of empirical processes, to show that remainder terms appearing in this derivative are negligible.

A. van der Vaart

### Semi-parametric likelihood theory

By way of examples we 'define' the notion of a semi-parametric likelihood, which is a stochastic process indexed by the parameter(s) of a statistical model. We study three objects defined relative to the likelihood process: the maximum likelihood estimator (point of maximum), the likelihood ratio statistic for testing 'Euclidean hypotheses' and the observed information (the second derivative of the profile likelihood). The desired results are: asymptotic normality, asymptotic chi-squaredity and consistency for the efficient information. We discuss a method of proof and the cases in which this method is successful. Empirical processes are a necessary, but not sufficient tool.

J.L. Wang

### Dimension reduction methods for censored regression data

Without parametric assumptions, high-dimensional regression analysis is already complex. This is made even more complex when data are subject to the presence of censoring. The approach taken in this article is to seek ways of reducing the dimensionality of the regressor before applying nonparametric smoothing techniques. In particular, we show how to extend the methodology of Sliced Inverse Regression (SIR) to incorporate the censoring indicator in addition to the observed response variable. A double-slicing scheme is proposed and a two-stage procedure is described to incorporate randomly censored response variables.



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J.A. Wellner

**Empirical processes: Progress and problems**

Empirical process theory has developed rapidly over the last 20 - 25 years, with key ideas coming from developments in weak convergence theory, the theory of Gaussian processes, probability in Banach spaces, and the recent development of isoperimetric inequalities and methods. The resulting set of tools and methods have proved to be very useful for a wide range of problems in statistics. In my talk, I briefly surveyed some of these developments with examples of the gains in empirical process theory, and applications of the theory to statistics: bootstrap methods, rates of convergence of nonparametric estimators, study of nonlinear functionals of the empirical measure $\mathbb{P}_n$, and recent progress in $M$- and $Z$-estimation theorems for semiparametric models. Several problems for future research were also discussed briefly.

J. Yukich

**Approximation of functions using probabilistic methods**

We consider the problem of approximating a smooth target function and its derivatives by superpositions and translations of a fixed activation function. The approach involves probabilistic methods based on central limit theorems for empirical processes indexed by classes of functions.

L.-X. Zhu

**A stopping time concerning sphere data with applications**

Denote by $A(x) = \{a : |a^T x| \leq h\}$ a circle zone on a sphere surface, for each given $h > 0$. For a given integer $m$, we investigate how many zones chosen randomly at least are needed for containing one point on the surface $m$ times. As an application, the life of a sphere roller is investigated. We present the empirical formulas for mean, standard deviation and distribution of the life of a sphere roller. Moreover, the limit behaviour of the above stopping time is obtained.

Berichterstatter: Winfried Stute



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