MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Differential-Algebraic Equations, Related Fields of Theory and Applications

29.10. – 4.11.1995

The conference was organized by Roswitha März (Berlin) and Linda R. Petzold (Minneapolis).
Seventeen comprehensive and eleven shorter lectures as well as intensive topical afternoon discussions, which were continued until late at night sometimes, revealed the impressive variety of the problems investigated and of the results obtained. Furthermore, important and interesting relations were shown and still open resp. not sufficiently solved problems became obvious.
The increasing importance of differential-algebraic equations as adequate, natural models in various fields of application (*circuit simulation, multibody system dynamics and ecology etc.*) was pointed out once again.
In particular, a considerable gain in knowledge became obvious in the understanding and treatment of index-2 systems as well as with respect to applications e.g. in circuit simulation.
On the other side, it turned out that practicable notions of stability and stability analyses for exact solutions and their numerical counterparts were still an open problem and, thus, at the agenda with top priority.

The meeting brought together researchers from 14 countries, many of them just starting their careers.
What was remarkable about this meeting was the strong presence and great activity of female mathematicians, namely, there were 24 female and 21 male participants. The program of lectures, too, was slightly dominated by the female mathematicians.
The high scientific level and the outstanding working atmosphere Oberwolfach is know for was emphasized, characterized and complemented by female intelligence, beauty and kindness.

Submitted Abstracts

Global error estimators for the numerical treatment of differential-algebraic equations

S. Adam

Technische Universität Dresden

To estimate the error of the produced numerical solution one mostly investigates the local discretization error using embedding techniques. However, from a more practical point of view it would be desirable to estimate the global error with an acceptable amount of work, too.

In my talk I will present estimators for the global error of special methods for the numerical treatment of semi-explicit differential-algebraic equations of index 1. Runge-Kutta methods, Rosenbrock methods and BDF methods are analyzed. The basis of the error estimators are asymptotic expansions of the global error of the considered methods. The functions, contained in the main error term, satisfy a linear, semi-explicit differential-algebraic equation of index 1, denoted as error DAE.

Using a condition on the principal error function (with respect to the differential part of the solution of the original problem), one can solve this error-DAE exactly. By means of these functions and the above mentioned asymptotic expansions we derive our error estimators.

Furthermore we explain in detail how to use these estimators in practice in an efficient way. Finally we present numerical results underlining the quality of our special estimators.

Optimized β-blocking for nonlinear semiexplicit index 2 DAEs

C. Arévalo

Universidad Simon Bolivar

(in collaboration with *C. Führer* and *G. Söderlind*)

β-blocking may be used to stabilize non-stiff implicit linear multistep methods for DAEs of index greater than 1. This technique was developed for Euler-Lagrange equations of index 2, in which the algebraic variable appears linearly. The technique can be extended to general semiexplicit index 2 DAEs by introducing the β-blocking operator in a "one-leg" fashion, in order to apply the stabilization only to the algebraic variable.

The Adams-Moulton methods up to order 3 can be stabilized in this way. Other methods of higher order are studied. In particular we focus on difference-corrected methods based on the BDF ρ polynomials. Adams-Moulton methods fall in this category too provided that higher order difference corrections are used.

The stabilizing difference operator is chosen optimally to get the smallest possible root for the characteristic polynomial which determines the stability of the resulting method. The class of higher order difference-corrected BDF methods is searched for maximal stability and minimal error constants. Practical aspects of solving the implicit difference equations are discussed. Numerical experiments corroborate these results.

Uniform perturbation estimates – two case studies

M. ARNOLD

Universität Rostock

For a given differential-algebraic equation (DAE) the perturbation index gives a measure for the sensitivity of a solution w.r.t. small perturbations. If we consider, however, *classes* of DAEs (e.g. all DAEs that arise as semi-discretizations of a given partial DAE by the method of lines) then these error bounds might be too optimistic. We illustrate this fact by 2 examples and define as alternative the *uniform* perturbation index that gives simultaneously error bounds for *all* DAEs of a given class. We prove that in one example each individual DAE has perturbation index 1 but the uniform perturbation index is 2. Another example illustrates that the class of all finite difference semi-discretizations may even have *no* uniform perturbation index if the given partial DAE has perturbation index 2.

Forward dynamics, elimination methods and formulation stiffness in robot simulation

U.M. ASCHER

The University of British Columbia – Vancouver

(in collaboration with *K. Pai* and *B.P. Cloutier*)

The numerical simulation problem of tree-structured multibody systems, such as robot manipulators, is usually treated as two separate problems:

(i) the forward dynamics problem for computing system accelerations, and

(ii) the numerical integration problem for advancing the state in time.

The interaction of these two problems can be important and has led to new conclusions about the overall efficiency of multibody simulation algorithms. In particular, the fastest forward dynamics methods are not necessarily the most numerically stable, and in ill-conditioned cases may slow down popular adaptive step-size integration methods. This phenomenon is called "formulation stiffness."

In this talk, we first unify the derivation of both the composite rigid body method and the articulated-body method as two elimination methods to solve the same linear system, with the articulated body method taking advantage of sparsity. Then the numerical instability phenomenon for the composite rigid body method is explained as a cancellation error that can be avoided, or at least minimized, when using the articulated body method. Specifically, we show that the articulated-body method is better suited to deal with certain types of ill-conditioning than the composite rigid body method.

Steady state and periodic solutions in parameter-dependent DAEs

YIN BAI

Philipps–Universität Marburg

We discuss singularity and periodic solution in parameter dependent semi-explicit DAEs. We will apply classical bifurcation theory to algebraic part to detect solution branches bifurcating from bifurcation point, and Hopf bifurcation theory to differential part to study existence of periodic solution, as well as their intersection.

On linear subspaces of solutions to linear DAEs of index 1. An approach to boundary value problems

K. BALLA

Hungarian Academy of Sciences – Budapest

Shooting methods for solution of linear boundary value problems use a fixed basis in the solution space that may vary very fast while the specific subspace in question varies slowly. For the index-1 tractable DAEs we show that, in contrary, the solutions of the adjoint equation are related to the complementary subspace and give to rise Lagrangian transfer of linear relations prescribed at a given (arbitrary) point. We derive this result in two different ways, directly and by means of a reduction theorem. Backdraws concerning (in)stability of Lagrangian transfer are eliminated if one applies the ideas of Abramov's transfer exactly as it takes places at ODEs.

A direct transcription tool for trajectory optimal control problems

K. BRENAN

The Aerospace Corperation - Los Angeles

In this talk, the current status of the "large-scale" flight optimization and sizing code FONSIZE, will be described. FONSIZE utilizes the direct transcription method to solve trajectory optimal control problems with path constraints. In the direct transcription approach, the optimal control problem is discretized by a collocation formula into a sparse, large-scale parameter optimization problem. Then a nonlinear programming algorithm is used to determine the optimal discretized solution which included numerical approximations for the state, the control and the adjoint variables at a set of mesh points. Recently, the FONSIZE program has been interfaced with a generalized reduced gradient algorithm specifically designed to exploit the sparsity structure of the Hermite-Simpson collocation equations. A summary of numerical tests performed to examine the efficiency of the new algorithm will be presented for a set of trajectory optimal control problems. The importance of understanding the theory and numerical solution of differential-algebraic equations during the problem formulation and subsequent solution process will be discussed. Finally, current strategies used in direct transcription software for controlling the accurace of the direct transcription numerical solution will be summarized.

Numerical methods for the Toda Lattice Equations

M.P. CALVO

Universidad de Valladolid

Numerical one-step methods are considered for the integration of the Toda Lattice Equations. A comparison between symplectic and isospectral integrators will be presented and the experimental findings will be backed by theoretical analysis.

Jacobian Reuse in explicit integrators for higher index DAEs

ST.L. CAMPBELL

North Carolina State University - Raleigh

Systems $F(y', y, t) = 0$ with $F_{y'}$ identically singular are known as differential algebraic equations (DAEs) and occur in a variety of applications. Most numerical methods for DAEs either require special structure or low index. Two alternative approaches (explicit

integration (EI) and implicit coordinate partitioning (ICP)) have been proposed for numerically integrating more general higher index DAEs. This paper examines some of the mathematical issues involved in the efficient implementation of the EI method especially in regards to the Newton iteration where most of the computational effort occurs. It is seen that Jacobian reuse with EI integrators will often lead to discontinuous vector fields. It is shown analytically how these vector fields can still be integrated. Numerical tests are given which support the theory.

Stabilization of numerical solutions of boundary value problems exploiting invariants

E. Eich

Fachhochschule München

Solving boundary value problems (BVPs) numerically is an important task when dealing with problems of optimal control and parameter identification of multibody systems. In this paper the numerical solution of BVPs for differential algebraic equations (DAEs) is discussed. The method of choice is multiple shooting.

Multibody systems are described by index 3 DAEs. Optimal control problems are higher index DAEs in the case of singular controls or state constraints. The common procedure of solving higher index DAEs is to reduce the index by differentiating the algebraic equations until index 1 DAEs or ordinary differential equations (ODEs) are obtained which can be treated directly. Unfortunately, the numerical solution of the index reduced problems often suffers from instabilities introducing a drift from the original algebraic conditions. We interprete the higher index constraints as invariants of the ODE and exploit these invariants in order to improve accuracy, stability and efficiency by a new projection technique. Obviously, conservation properties e.g. for energy or momentum can be used in this sense, but the symplectic structure of Pontryagin's Maximum principle allows for deriving further invariants.

Solving the shooting equations by Newton's method requires the computation of sensitivity matrices. This is performed by solving the initial value problems for the variational ODEs together with their invariants or by differentiation of the discretization scheme. The effort of this step can be significantly reduced due to the invariants.

The techniques are demonstrated on the example of a flight path optimization problem.

Index reduction

E. GRIEPENTROG

Fachhochschule für Wirtschaft und Technik – Berlin

The reduction concept developed by Y. Boyarincev for linear DAEs with constant coefficients has been extended to quasilinear problems by E.-M. Reich. The reduction process ends in any case after a finite number k of steps; then a criterion decides whether an underlying ODE exists or not. If an underlying ODE exists the quasilinear problem is called k-reducible. By the author it was proved that k-reducibility in the linear (time-variable) case is equivalent to the index-k-property, where k means the differentiation index. The proof is based on the fact that each reduction step really reduces the differentiation index by 1.

A ROW type approach for integral form DAEs arising in charge oriented network equations

M. GÜNTHER

Technische Hochschule Darmstadt

Considering integrated digital circuits based on MOS-technology, capacitance-oriented reciprocal models are inappropriate for time domain analysis. Only a charge-oriented approach succeeds in correctly reflecting the effects in the transistor according to intrinsic charge flows.

Using modified nodal analysis, this ansatz yields differential-algebraic equations in integral form of the type

$$A \cdot \frac{d}{dt}(q(x(t))) - f(x) - s(t) = 0.$$

The direct approach used in most simulation packages is based on the integration of the charge flow $\dot{q}$ by backward differentiation formulas. However, the physical quantities the user is really interested in – x, the node potentials and branch currents through voltage sources – appear only in algebraic form.

We introduce a ROW type scheme that offers a step size prediction and error control directly based on node potentials and currents. This ansatz exploits the integral form structure of the network equations and solves the most important systems arising in digital circuit simulation up to index 2. Its construction is carefully discussed, as well as convergence analysis, derivation of order conditions, impact of inconsistent initial values and implementation aspects. The numerical simulation of industry relevant test circuits confirmed the theoretical results.

Asymptotic properties of solutions of differential-algebraic equations and asymptotic stability of numerical integration methods

M. HANKE

Humboldt–Universität zu Berlin

A number of stability notions for numerical integration methods for ODEs is based on a simple scalar test equation (e.g. A-stability, L-stability etc.). The justification of this approach is given by LYAPUNOV's theory: The linearization of an autonomous ODE at a statinary point gives criteria for the asymptotic stability of that point. A number of similar results for index-1, -2, and -3 DAEs is discussed in the talk. Following the approach of th ODE case we are led to linear constant coefficient DAEs. It is shown that properties like, e.g., A-stability are preserved if integration methods are applied to such systems. Then, contractivity for index–1 and (linear) index–2 DAEs is defined. Criteria are given which ensure that numerical methods which are B-stable for ODEs remain B-stable for DAEs, too. In doing so, the importance of certain geometric properties of DAEs is demonstrated.

Asymptotic expansions of RK methods for regularized index 2 DAEs

I. HIGUERAS–SANZ

Universidad Publica de Navarra - Pamplona

Regularized index 2 differential algebraic equations

$$\begin{aligned} u' &= f(u,v) \\ 0 &= g(u+\varepsilon u') \qquad t \in [0,T] \end{aligned}$$

are considered. The solution of these systems has an asymptotic ε-expansion with two kind of terms; one of them is only significant near the boundary layer.

For this index 1 system Runge-Kutta methods can be used. We are interested on the error between the solution of the original index-2 system $(u_0(t), v_0(t))$ and the numerical solution $(u_n^\varepsilon, v_n^\varepsilon)$, thus the dependence of the numerical solution on the parameter ε is studied.

Differential-algebraic equations of the optimal synthesis problem

F.M. Kirillova

Academy of Sciences of Belarus – Minsk

The paper deals with the classical optimal feedback control problem for linear systems under perturbations. A new statement of the problem is given to construct positional solutions corrresponding to real motions. The method under consideration is based on introducing and solving defining (algebraic) equations with changing number of variables and inhomogeneous differential equations. As application, the problem of stabilization by bounded controls is considered.

Singular Cauchy problems for systems of nonlinear functional-differential equations

N.B. Konyukhova

Russian Academy of Sciences – Moscow

The system of n nonlinear functional-differential equations (FDE) is considered on a semi-infinite interval $T \leq t < \infty$. The operator in the right side does not need to be of the Volterra type. As $t \to \infty$ either the limit value of the desired function or the boundedness condition of a solution is given (the case that a condition is imposed at a finite singular point can be reduced to the above ones by a change of the variable t). The local and nonlocal theorems on existence and uniqueness of solution of such singular Cauchy problems are stated. The sufficient conditions of nonuniqueness of a solution are formulated, i.e., the existence of n-parameter (or k-parameter, $0 < k < n$) family of solutions satisfying given conditions at infinity. As particular cases of the considering systems we refer to systems of ordinary differential equations (ODE) with nonsummable singularities at a finite or infinite initial points (including differential-delay equations), systems of integro-differential equations and so on. The direction developed has been stimulated, in particular, by the problems arising in a study of stable manifolds of solutions as a whole for degenerating systems of ODE and FDE. It is important to specify these manifolds when setting and investigating singular boundary value problems for these systems, and also in order to verify the methods of boundary value transfer for numerical solution of them. Singular Cauchy problems themselves are of special interest.

On the numerical treatment of linear and nonlinear differential-algebraic equations

P. KUNKEL

Universität Oldenburg

Starting with the definition of local and global characteristic values for linear time-varying differential-algebraic equations we first develop a normal form and an existence and uniqueness result for solutions on which we can base a numerical procedure. These characteristic values are then related to the differentiation index and the perturbation index. Finally we extend these results to the treatment of nonlinear differential-algebraic equations.

Embedded Runge-Kutta methods for the solution of stiff ODEs and DAEs

A. KVÆRNØ

University of Trondheim

First some arguments will be given for why there might still be a need for more Runge-Kutta methods. When this need is justified, a set of embedded Runge-Kutta methods of order 3(2), 4(3) and 5(4) will be presented. These methods are all of DIRK type and are constructed for solving stiff ODEs and DAEs of index 1. Emphasis has been put on the construction of the error estimating methods. Testing indicates that these methods are both robust and effective, in particular for the solution of index 1 DAEs.

Two-point-boundary value problems in DAEs and nonlinear oscillation phenomena

R. Lamour

Humboldt–Universität zu Berlin

The generalization of ODE results to the world of DAEs is considered from the analytical point of view. For boundary value problems in ODEs the nonsingularity of the so-called *shooting matrix* is responsible for the (local) uniqueness of a solution. In the DAE case under natural assumptions the determination of consistent initial values combined with the shooting equation forms an operator with a nonsingular Jacobian matrix, which means also locally uniqueness of the solution. In the special case of periodic solutions the generalization to the index 1 case of the Theorems of Floquet, Lyapunov and Andronow-Witt is given.

Numerical results of the shooting code MSHDAE (Multiple SHooting for DAEs) based on the described generalization are presented.

On numerical solution of several multi-parameter spectral problems for ODE systems by the modified phase-function method

T.V. Levitina

Russian Academy of Sciences – Moscow

If a Dirichlet or Neumann condition is imposed on the surface of the ellipsoid, the variables are separated in the scalar wave equation in ellipsoidal coordinates, and the problem in hand is reduced to a system of three identical ordinary differential equations, each being defined on a separate interval and subject to its own boundary conditions. Thus, the three-parameter self-adjoint Sturm-Liouville problem arises: the equations are coupled by two separation constants and the eigen frequency of the ellipsoid, i.e., the spectral parameters, which must be so chosen that all the equations of the system have simultaneously nontrivial solutions, each satisfying the corresponding boundary conditions. The problem obtained is *right definite* , therefore the existence of the eigenvalue of a given multi-index is known. The effective globally converging numerical algorithm is proposed for calculating eigen frequencies and separation constants. It exploits the ideas stated by Abramov et al. , and is based on the modified Prüfer angle concept. The merit of the method is illustrated on the example of several calculations of the sound field and caustic surfaces in an ellipsoid.

The life-span of backward error analysis for numerical integrators

CH. LUBICH

Universität Tübingen

(in collaboration with *E. Hairer*)

Backward error analysis is a useful tool for the study of numerical approximations to ordinary differential equations and differential-algebraic equations. The numerical solution is formally interpreted as the exact solution of a perturbed differential equation, given as a formal series in powers of the step size (usually divergent). For a rigorous analysis, this series has to be truncated.

In this talk we study the influence of this truncation to the difference between the numerical solution and the exact solution of the perturbed differential equation. Results on the *long-time behaviour* of numerical solutions are obtained in this way. We present applications to the approximation of periodic orbits and *Hopf bifurcation*, and to energy conservation and approximation of invariant tori in *Hamiltonian systems*.

On ill-conditioning for index-1 DAE

R.M.M. MATTHEIJ

Eindhoven University of Technology

As is well-known, solutions of DAE depend in a rather complex way on their parameters. For linear problems this means that the algebraic variable may depend on (higher) derivatives of the source term.

We investigate ill conditioning of index-one problems, i.e. DAE which are in some sense close to higher index problems and indicate a suitable conditioning constant concept.

Control problems for linear differential-algebraic equations

V. MEHRMANN

Technisch Universität Chemnitz–Zwickau

We discuss control problems

$$\text{Min} \int_{t_0}^{t_f} (x^T Q(t)x + u(t)^r Ru(t))dt \text{ subject to } E(t)\dot{x} = A(t)x + B(t)u,\ x(t_0) = x^0.$$

An optimal solution is obtained via the solution of a differential–algebraic Riccati equation, which does not have a unique solution. If this equation is solved in a least squares sense, then this turns out to be again an optimal control problem of the same type but for

$$\dot{x} = \tilde{A}(t)x + \tilde{B}(t)u + f.$$

Regularization of time-varying differential-algebraic control systems by output feedback

N.K. Nichols

University of Reading

Smooth singular value decompositions are used to reduce a time-varying descriptor system, governed by the equations

$$E(t)\,dx/dt = A(t)x + B(t)u, \qquad y = C(t)x,$$

to a condensed form that reveals controllability and observability properties. The matrix E may be singular for all t, and may lose rank for isolated values of t. Derivative and proportional output feedback controls of the form

$$u = Fy - G\,dy/dt$$

are constructed to ensure that the system is index one and pointwise regular. Results show that under certain conditions the feedback can be selected to ensure that the closed loop matrix $E + BGC$ has constant rank over the interval of interest and is 'near optimally' conditioned. This property ensures that a lower bound on the pointwise 'distance to instability' of the closed loop system is maximised and that the differential and algebraic parts of the system are well-defined and can be decoupled in a numerically stable way.

The smooth singular value decompositions are determined from the solutions to a system of ordinary differential matrix equations. A novel 'orthogonality – preserving' numerical integration scheme is used to compute the right and left singular factors. A form of deferred correction leads to results of unusually high accuracy.

The inverted n-bar model

P. RENTROP

Technische Hochschule Darmstadt

The planar inverted n-bar model and its multibody formulation are presented. The descriptor formulation, which is based on a set of redundant coordinates, results in a differential-algebraic (DAE) system of index 3. A minimum set of coordinates characterizes the state space formulation, which corresponds to an ordinary differential equation (ODE) system. The regular structure of the descriptor form allows a complete formulation of the equations of motion. On this base, by induction arguments the state space form can be derived analytically. We present these equations, since the inverted n-bar model serves as an instructive example for the drift phenomena in numerical simulation, for the design of a controller and for the balancing problem in the context of neural networks.

Cheap enhancement of symplectic integrators

I.M. SANZ-SERNA

Universidad de Valladolid

(in collaboration with *M.A. López-Marcos, R.D. Skeel*)

We show how to increase the accuracy of symplectic integrators by postprocessing the numerical solution Y_n to obtain an improved approximation $\pi(Y_n) =: y_n$. The mapping π can be inexpensively approximated by forming differences of the sequence $\{y_n\}$.

Oscillating integrated circuits
Numerical stability analysis for ODEs and DAEs

P. SELTING

Technische Universität München

A special topic in circuit simulation is the computation of stable limit cycles of oscillating electric circuits. Self–excited oscillating circuits transfer a constant input signal into an oscillating periodic output signal. The frequency is determined by the basic electronic elements, which are affected by production tolerances. In stability analysis the effects of these production tolerances are investigated, because they can cause a loss of stability, i. e. a bifurcation behaviour. Bifurcation is one of the main reasons for the birth of an irregular behaviour of a system.

A charge oriented approach for the investigation of local stability of systems of ordinary differential equations is presented. For that purpose the monodromy matrix is computed. Its dominant eigenvalues help to characterize the reliability of the electronic circuit.

For the investigation of industrial relevant circuits with a commercial simulator it is necessary to adapt that approach to differential algebraic equations.

Mixed DAEs and PDEs in flexible multibody systems

B. SIMEON

Technische Hochschule Darmstadt

The multibody system approach provides a basic methodology in modelling various mechanical systems like vehicles, robots, air- and spacecrafts. In case of rigid bodies, the mathematical modelling leads either to second order ordinary differential equations (ODEs) in minimum coordinates, the state space form, or, more generally, to differential-algebraic equations (DAEs) of index 3 in redundant coordinates, the descriptor form. Due to the research activity of the last decade, both approaches and their relation are now well understood. Advanced modelling and simulation software is available.

On a more detailed modelling level, however, not all bodies can be assumed to remain rigid and elasticity effects need to be taken into account in terms of the partial differential equations (PDEs) of elastomechanics. Important applications in this field are, e.g., vehicles with elastic chassis for optimal ride quality or aircrafts with elastic body-wing structure and landing gears described as multibody systems.

The talk is concerned with the modelling and numerical treatment of such *flexible multibody systems* composed of rigid and elastic bodies. Using the instructive example of a flexible slider crank mechanism, the presentation concentrates on a descriptor form approach which consists essentially of a mixed system of DAEs and PDEs. A partitioning into rigid motion entries and additional flexible motion entries is applied which allows an interpretation of the equations as a singularly perturbed system. Numerical results both in the case of linear and nonlinear elasticity behavior illustrate the problems encountered in the simulation of flexible multibody systems.

Stability and dissipativity for differential-algebraic equations

G. SÖDERLIND

Royal Institute of Technology

Stability for DAEs is largely an open problem. So far, attempts have been made to adapt the usual stability notions for ODEs, such as Lyapunov stability and monotonicity conditions, to the DAE case. These standard stability notions must be applied to an ODE on a manifold, the constraint mainfold $\mathcal{M}$. When the DAE is discretized the numerical solution will normally not stay on this manifold. It is therefore of interest to study also the stability of solutions *outside* $\mathcal{M}$. We show by simple examples that stability *on* $\mathcal{M}$ does not imply the stability *of* $\mathcal{M}$ in the nonlinear case. In other words, it is generally not sufficient to consider the conventional approach of perturbing initial values as no solutions to the DAE exist outside $\mathcal{M}$. Rather than perturbing initial values, we consider the effects of perturbing the equations, while the initial values are kept the "same" and consistent. Thus, given a DAE

$$F(t, x, \dot{x}) = 0, \qquad x(0) = x_0 \quad \text{consistent},$$

we consider the perturbed problem

$$F(t, x + \delta x, \dot{x} + \delta\dot{x}) = r; \qquad \delta x(0) = 0,$$

and ask that δx remain uniformly bounded for $t \geq 0$ for certain classes of perturbations, e.g. $r \in L^p$. More precisely, we say that the system is stable if

$$\forall \varepsilon > 0\, \exists \delta > 0 : \ \|r\|_p \leq \delta \Rightarrow \|\delta x\|_\infty \leq \varepsilon,$$

where the norms are defined in the usual way. This stability concept is directly related to the notion of perturbation index. We also discuss dissipativity by considering special classes of perturbations, viz. impulse and step responses. This leads to a modification (depending on the constraint) of the usual logarithmic norm.

The numerical solution of differential algebraic-integral equations arising in the modelling of catalytic combustion with radiation

A. Spence

Bath University

Modelling catalytic combustion in monolith reactors produces systems of differential algebraic equations (DAE's) coupled with Fredholm Integral Equations. This coupling of an initial value problem with a boundary value problem presents numerical difficulties. The talk is concerned with the efficient solution of these systems. Specifically, the talk is as follows:

- The physical problem is described and equations are derived.
- The numerical solution of the DAEs is discussed with specific attention paid to the equations arising in catalytic combustion.
- An algorithm for DAE-integral equations is outlined.
- Numerical results on a model problem and some comparison with actual experiment are given.

Solving index-2 differential algebraic equations in circuit simulation

C. TISCHENDORF

Humboldt–Universität zu Berlin

The talk deals with index-2 differential algebraic equations arising from a modern modelling technique in circuit simulation, the modified nodal analysis (MNA). The approaches, the classical one and the charge oriented one, are analyzed. We present a class of circuits, for which both approaches lead to the same index in the lower index case (≤ 2). Further, a careful numerical analysis provides sharp estimations for solutions of perturbed index-2 initial value problems. The BDF method is shown to be feasible under natural assumptions to the errors in the initial values and to the defects arising from solving the nonlinear equations in each step. The sensitive defects can be kept small using a defect correction what is a generalization of the projection idea of Ascher and Petzold for Runge-Kutta methods. Finally, we present our numerical results for two examples, the NAND-gate model and the model of a ring modulator.

Boundary function method for singularly perturbed problems

A.B. VASILIEVA

Moscow State University

We consider the ODEs containing small parameters at the highest derivatives – so-called singularly perturbed differential equations (SPDE).

The solution of some boundary value problem for SPDE has in generally two *boundary layers* (in the neighbourhood of boundary points). The asymptotic expansion for such solutions can be constructed by using the so-called *boundary function method* (BFM). The boundary value problem might have also the solutions with *interior layers*. Such solutions are called contrast structures (CS). We can apply the BFM to investigate different CSs and their stability.

Issues in the numerical solution of vehicle models

R. von Schwerin

Universität Heidelberg

Vehicle system dynamics is a challenging area of application of DAEs in technical simulation. The governing equations are derived from hydraulics, electronics, control theory, mechanics and other fields that each have their own approach to modelling and solving the arising equations. The interaction of various components in a car, however, necessitiate a combined interdisciplinary solution of the dynamical system represented by the vehicle.

Special topics relevant to an efficient treatment of such mechatronical systems will be addressed and exemplified in this talk.

Asymptotic error expansions for singular BVPs

E. Weinmüller

Technische Universität Wien

We discuss the existence of an asymptotic expansion for the global error of a simple finite difference scheme for the numerical solution of boundary value problems for ODEs with a singularity of the first kind. Such problems occur when certain classes of DAEs are transformed into a system of ODEs and a system of algebraic equations and there is a rank change of the leading coefficient matrix. An effective technique for the numerical solution of ODEs is the Iterated Defect Correction Method, IDeC, originally used as a method for the estimation of the global error. We use IDeC to solve the problem

$$y''(t) = f\Big(t, \frac{y(t)}{t^2}, \frac{y'(t)}{t}\Big), \quad 0 < t \leq 1; \quad y(0) = \alpha,\ y(1) = \beta$$

and investigate the structure of the global error of the scheme for certain classes of linear singular problems. It turns out that the error expansion is no longer as perfect as in the classical situation. This is caused by unbounded inhomogeneities in variational equations due to singularity. We discuss typical situations and explain how the features of the problem and its solution influence the form of the asymptotic error expansions and their final length.

Partitioned W-methods for multibody systems in descriptor form

J. Wensch

Martin–Luther–Universität Halle–Wittenberg

W-methods as a special formulation of linear implicit methods have been used to integrate stiff ODEs. They avoid the solution of nonlinear systems and allow a partition of the system in stiff and nonstiff components.

We use W-methods to integrate the equations of motion of a multibody system for the same reasons. The partitioned Jacobian includes only these blocks that are necessary for the solution of the constraint equations. Further, the coefficients of these methods are partitioned. A convergence theorem is presented. Methods of order 2, 3, 4 with 3, 5, 11 stages have been constructed.

DAE's with impasse points and relaxation oscillations

R. Winkler

Humboldt–Universität zu Berlin

First, we deal with simple impasse points of quasilinear DAEs of differential index 1. We are able to handle them by transforming the independent variable t. This way we obtain an augmented system which remains a regular DAE even in the impasse point. We compute the solution together with t which depends on the new variable s and determine the impasse point by solving the scalar equation $t'(s) = 0$.

For semiexplicit DAEs originating as degenerate equations of singularly perturbed systems we show how to continue the solution by jumps. Finally, the algorithm is illustrated by results of an implementation based on a BDF-Code.

Bifurcations of spiral waves in reaction-diffusion-systems

C. Wulff

Konrad–Zuse–Zentrum für Informationstechnik – Berlin

We consider a reaction-diffusion system on the whole plane

$$u_1 = \Delta u + f(u,v),\ v_1 = \delta \Delta v + g(u,v),\ u,v : R^2 \to R,\ \delta \geq 0.$$

An example is the Oregonator model of the Belousov-Zhabotinski-reaction. This system is equivariant under the Euclidean symmetry. We examine the transition from rigidly rotating spiral waves to meandering spiral waves. This is a Hopf-bifurcation in the corotating frame. Thereby the infinitely extended domain causes difficulties since there the angle derivative $\frac{\partial}{\partial\phi}$ is not bounded w.r.t. the Laplacian. In the resonance case when the rotation frequency is a multiple of the Hopf frequency we prove the bifurcation of drifting spiral waves. Analogous phenomena occur in the case of periodic forcing of rigidly rotating spiral waves. Our results give a rigorous mathematical explanation of experiments on spiral wave dynamics from chemistry.

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