MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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ALLGEMEINE UNGLEICHUNGEN

12.11. bis 18.11.1995

The seventh International Conference on General Inequalties was held from 12 to 18 November 1995, at the Mathematisches Forschungsinstitut Oberwolfach (Black Forest, Germany). The organizing committee was composed of C. Bandle (Basel), W.N. Everitt (Birmingham), L. Losonczi (Debrecen and Kuwait) and W. Walter (Karlsruhe). Wolfgang Reichel (Karlsruhe) served cheerfully and efficiently as secretary of the meeting. The meeting was attended by 50 participants from 19 countries.

The opening remarks were made by W. Walter who gave a brief history of the General Inequalties meetings held at Oberwolfach. He remarked that six members of the first meeting held in 1969 were also at this, the seventh meeting.

This was the first General Inequalties meeting held at the Institute since the appointment of the new Director, Professor Matthias Kreck. Professor Kreck had written, in December 1994, to all members of organizing committees for meetings held at the Institute. In that letter he reported that his overall impression of the meetings is very positive but he considers that most meetings are overloaded with too many talks; he makes the pertinent and significant point that Oberwolfach is not a conference centre but a research institute.

The organizing committee for General Inequalties 7 took due note of these remarks of Professor Kreck and responded by a sensible and structured reduction in the number of programmed lectures, but at the same time invited all the participants to consider organizing informal discussion groups outside the formal programme. This led to a very successful overall programme for the meeting and the organizing committee takes this opportunity to thank all the participants for their help and cooperation in making arrangements for the lectures, and for the excellence of their contributions.

The organizing committee thanks Professor Kreck for the opportunity to meet with him during the meeting. The committee pointed out that by the diverse nature of inequalities in mathematics the subject lent itself to reports from a wide basis of areas and applications, and this by itself suggested that a larger than usual number of lectures is required.

At the end of the week the organizing committee considered that the meeting had met with the ideas of the Director, and that it had been successful in arranging for the report of new results in inequalties and in stimulating new developments in this wide and diverse field of mathematics.

After discussion with Professor Kreck, responsibility for the continuation of the General Inequalties meetings at the Institute now passes to a new organizing committee with membership C. Bandle (Chairperson, Basel), L. Losonczi (Debrecen and Kuwait) and M. Plum (Karlsruhe).

A reception was held on the Thursday evening in honour of Wolfgang Walter, who had attended all seven of the General Inequalties meetings and served as chairman for all but the first meeting. A special mention was made of his distinguished contribution to the subject of mathematical inequalities in carrying editorial responsibility for the published proceedings of all seven of the meetings.

Catherine Bandle offered closing comments on the last day of the meeting; she paid tribute to the high standards set and maintained, firstly by all the participants in contributing to the programme of the meeting, and then to the staff of the Institute for their help and cooperation.

C. Bandle W.N. Everitt L. Losonczi W. Walter

Vortragsauszüge

J. ACZEL:

Role of inequalities in characterizing selection probabilities with aggregation (Joint work with G. Maksa)

We suppose that selection probabilities are functions of scale values of the options from which we select. On one hand the aggregated probabilities of each option should be functions of the individual scale values. We deal with ratio scales (which are transformed componentwise by linear transformations). These premisses yield a system of functional equations and inequalities. For disproving a conjecture of A.A.J. Marley about the uniqueness of a frequently used solution, surprisingly the inequalities keeping the aggregated probabilities between 0 and 1 proved to be most difficult to satisfy. When we got a handle of it, we soon got not only a counter-example but also the general solution. Monotonicity (increasing in one variable, decreasing in the others), which is a natural requirement, leads to further inequalities.

R.P. AGARWAL and P.Y.H. PANG:

Opial-type inequalities involving higher order partial derivatives of two functions

In this paper we offer very general Opial-type inequalities involving higher order partial derivatives of two functions of two independent variables. From these inequalities we then deduce extended and improved versions of several recent results.

C.D. AHLBRANDT:

Connections between discrete variational inequalities, continued fractions, and discrete Riccati equations

Results relating convergence of continued fractions to convergence of an associated series go back to Euler's work in 1762. Relationships between convergence of continued fractions and existence of recessive solutions of linear recurrences were given by Pincherle in 1894. In 1956 W.T. Reid provided what has been termed a "Roundabout Theorem" for the continuous case which related a number of topics including Jacobi Conditions of the Calculus of Variations, existence of solutions of matrix Riccati equations, and positive definiteness of quadratic functionals. Because of the work of Kalman, matrix Riccati equations are important to filtering and control. In Kalman's original work, which was developed for control problems arising in the design of the space shuttle, the Riccati equations are discrete. Discrete problems are important because of their role in computer solution of filtering and control problems with discrete measurements of variables. A discrete version of Reid's Roundabout Theorem was given in joint work of Ahlbrandt and Hooker. A great deal of research continues to be done on the questions of existence and construction of stabilizing solutions of discrete matrix Riccati equations. In discrete variational theory, quadratic inequalities are related to "disconjugacy" in a sense of absense of generalized zeros. The purpose of this paper is to show in increasing generality that disconjugacy (in the discrete case) is a sufficient condition for existence of recessive and dominant solutions and hence provides information about convergence of matrix continued fractions and about the solution space of discrete matrix Riccati equations.

M.S. ASHBAUGH:

Inequalities for the first eigenvalue of the clamped plate and buckling problems

in 1877 Rayleigh conjectured that the lowest frequency of vibration of a clamped plate of given area occurs when the plate is circular. This conjecture was finally proved by Nadirashvili in 1992 (with important earlier contributions due to Szegő and Talenti). An analogous conjecture concerning the critical buckling load of a clamped plate was made by Pólya and Szegő around 1950. This conjecture remains open. The talk surveys the state of our knowledge of these and related problems, including their n-dimensional generalizations. In particular, we discuss our recent work with R. Benguria proving Rayleigh's clamped plate conjecture for dimension 3 (and 2) and with R. Laugesen proving inequalities for the clamped plate problem (for $n \geq 4$) and for the buckling problem. In the latter cases, the inequalities have the form of the conjectured lower bounds but contain an unwanted factor slightly less than 1. Our bounds for the clamped plate problem follow from detailed analysis involving Bessel functions, and compare favorably to earlier bounds of a similar form due to Talenti. Our bounds for the buckling problem follow from earlier bounds due to Payne and Krahn, as first noted by Bramble and Payne.

M.S. ASHBAUGH and R. BENGURIA:

Proof of the Payne-Pólya-Weinberger conjecture in spaces of constant sectional curvature

We consider the Dirichlet problem for the Laplace-Beltrami operator defined on a domain Ω of S^n. Denote by $\lambda_1(\Omega)$ and $\lambda_2(\Omega)$ the two lowest eigenvalues. Let B_1 be a geodesic ball in S^n having the same lowest eigenvalue as Ω, i.e., $\lambda_1(B_1) = \lambda_1(\Omega)$. If the domain Ω is contained in a hemisphere of S^n we prove that $\lambda_2(\Omega) \leq \lambda_2(B_1)$ with equality if and only if Ω is a geodesic ball. We also prove that the ratio of the lowest two eigenvalues of a geodesic ball is an increasing function of the radius of the ball. These two results imply the PPW conjecture for domains contained in a hemisphere of S^n, i.e., $\lambda_2(\Omega)/\lambda_1(\Omega) \leq \lambda_2(\Omega^*)/\lambda_1(\Omega^*)$, where Ω^* is a geodesic ball having the same volume as Ω. Again, equality is attained if and only if Ω is a geodesic ball.

R: BADORA:

On the stability and separation theorems for n-additive functions

A function from a product $S_1 \times S_2 \times \cdots \times S_n$ of semigroups S_i, $i = 1, 2, \ldots, n$, into a linear lattice Y is called *n-additive* [*n-subadditive*] {*n-superadditive*} if and only if it is additive [subadditive] {superadditive} with respect to each variable.

At first we generalize Albert-Baker's theorem on Hyers-Ulam stability for n-additive functions to the class of amenable semigroups. Using this result we prove that a pair of functions n-subadditive P and n-superadditive Q defined on a product $S_1 \times S_2 \times \cdots \times S_n$ of amenable semigroups S_i, $i = 1, 2, \ldots, n$, with values in a complete Archimedean linear lattice Y satisfies

$$Q(x_1, x_2, \ldots, x_n) \leq P(x_1, x_2, \ldots, x_n), \tag{1}$$

$(x_1, x_2, \ldots, x_n) \in S_1 \times S_2 \times \cdots \times S_n$, and

$$\sup\{P(x_1, \ldots, x_n) - Q(x_1, \ldots, x_n) : (x_1, \ldots, x_n) \in S_1 \times \cdots \times S_n\} \in Y \tag{2}$$

is separated by an n-additive function $A : S_1 \times S_2 \times \cdots S_n \to Y$, i.e.,

$$Q(x_1, x_2, \ldots, x_n) \leq A(x_1, x_2, \ldots, x_n) \leq P(x_1, x_2, \ldots, x_n), \tag{3}$$

for all $(x_1, x_2, \ldots, x_n) \in S_1 \times S_2 \times \cdots \times S_n$.

Next we show that, in Abelian case, hypothesis (2) can be replaced by a weaker one.

C. BENNEWITZ:

On a general version of the HELP inequality

The inequality $\{\int_0^\infty |u'|^2\}^2 \le 4\int_0^\infty |u|^2 \int_0^\infty |u''|^2$ was given by Hardy and Littlewood in 1932. In 1971 Everitt considered the following generalization:

Is there a finite constant K such that

$$\left\{\int_0^\infty (p|u'|^2 + q|u|^2\right\}^2 \le K^2 \int_0^\infty |u|^2 \int_0^\infty |-(pu')' + qu|^2$$

for functions that make the right hand side finite, under some technical assumptions on the coefficients p and q? The inequality is now known as the HELP inequality. Everitt was able to show that the value of K is completely determined by the behaviour of the so-called Neumann m-function for the equation $-(pu')' + qu = \lambda u$ on $[0,\infty)$. This was later used by Everitt and several collaborators to determine the constant K in all those cases where the m-function can be explicitly determined. Everitt also showed that the existence of a finite K (as distinguished from the best value of K) depends only on the behaviour of the m-function in a neighbourhood of 0 and ∞.

The author considered, in 1984, the following generalization of the HELP inequality. Let S and T be ordinary, formally symmetric differential expressions on a real interval I, and let $(\cdot,\cdot)_S$, $(\cdot,\cdot)_T > 0$ be corresponding Hermitean symmetric Dirichlet integrals. Consider the possible existence of a finite constant K such that

$$|(u,u)_S|^2 \le K^2(u,u)_T(v,v)_T$$

for all u, v satisfying $Su = Tv$ for which the right hand side is finite. A characterization was given of the constant K which reduces to that of Everitt in his case, although the m-function was not explicitly mentioned. Here we show that in cases when an m-function (now matrix-valued) exists, the condition given is of the same form as that of Everitt. A class of examples where a finite constant exists is given. We also show that using recent asymptotic results for the m-matrix by Bennewitz and Wood one can give conditions that guarantee the existence or non-existence of a finite constant, in terms of the behaviour of the leading coefficients of S and T, and of the spectral character of the origin with respect to a natural boundary condition associated with the forms $(\cdot,\cdot)_S$, $(\cdot,\cdot)_T$.

M. BOHNER:

On positive definiteness of discrete quadratic functionals

We consider a linear Hamiltonian difference system for the so-called singular case so that discrete Sturm-Liouville equations are included in our theory. By introducing focal points of matrix-valued and generalized zeros of vector-valued solutions we define disconjugacy of the system and show that the system being disconjugate, i.e., the discrete version of the strengthened Jacobi condition, is equivalent to positive definiteness of a certain discrete quadratic functional. This Reid Roundabout Theorem gives some more conditions equivalent to the strengthened Jacobi condition; among them solvability of a certain Riccati matrix difference equation. As an application of our theory, we treat Sturm-Liouville equations as special Hamiltonian systems and achieve a Reid Roundabout Theorem for those important functionals also.

A. BRILLARD:

Subdifferential inequalities and asymptotic analysis of elliptic partial differential equations

It is known that the solution u of an elliptic partial differential equation, written in divergence form, is also in many cases the solution of a minimization problem: $F(u) = \min_{v \in K} F(v)$, where K is a closed subset of a functional space (generally a Sobolev space).

Many situations have been studied in the last years, where the solution depends on a small parameter ε: flows of viscous fluids in periodic porous media [3], adhesive thin films bonding elastic materials [5], elliptic equations involving critical Sobolev exponents [2],...

In order to describe the mathematical behavior of $(u_\varepsilon)_\varepsilon$, when ε goes to 0, it can be useful to prove the convergence of the associated functionals $(F_\varepsilon)_\varepsilon$, in a sense which implies the convergence of the minimizers: the so-called epi-convergence [1] or Γ-convergence [4].

This epi-convergence is established using subdifferential inequalities and appropriate test-functions, in order to pass to the limit in these inequalities.

Various examples will illustrate this method.

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F. BROCK:

An approach to rearrangement via polarization
(Joint work with A.Yu. Solynin)

Many known symmetrizations – including some kind of continuous (k,n)-Steiner symmetrization – can be approximated in L^p by a consecutive sequence of polarizations. A lot of important functional inequalities (including integral inequalities of Dirichlet-type and for convolutions, and also comparison theorems for partial differential equations) can be proved very easily, if the symmetrization in them is replaced by a polarization. The interaction of these two principles yields new elementary proofs of the inequalities for the symmetrization.

B.M. BROWN:

HELP inequalities for $2n$-th order differential equations

This talk discusses the possibility of a HELP-type inequality for real even order symmetric differential expressions. It will be shown that Everitt's condition for the existence of the classical HELP inequality has a matrix analogue for inequalities associated with higher order differential expressions. A number of examples using both analytic and numerical methods wil be presented.

R. BROWN, D. EDMUNDS, and J. RÁKOSNÍK:

Some remarks on higher-order Poincaré inequalities

Let Ω be a (bounded or unbounded) domain in $\mathbb{R}^n$ and let $W \equiv W(X,Y)$ be an abstract m-th order Sobolev space determined by the Banach function spaces X, Y defined on Ω. Let F be functional on W with the properties:

(F1) F is continuous;

(F2) $F(\lambda u) = \lambda F(u)$ for all $\lambda > 0$;

(F3) $F(u) = 0 \Longrightarrow u = 0$ if u belongs to $\mathcal{P}_{m-1} \cap W$, where $\mathcal{P}_{m-1}$ is the class of polynomials in $\mathbb{R}^N$ of degree at most $m-1$.

Edmunds, Opic, and Rákosník using an approach pioneered by Amick in 1978 have shown that the Poincaré inequality

$$\|u\|_X \leq K\{|F(u)| + \|u\|_Y\} \tag{1}$$

holds if and only if $A \in [0,1)$, where A is the ball measure of noncompactness of the embedding $W \hookrightarrow X$. (1) or equivalent inequalities have applications in the spectral theory of partial differential equations. However, A is difficult to calculate, and in most known examples of (1) with $n > 1$ the embedding is compact; i.e., $A = 0$. Here we develop an approach which is independent of A and prove:

THEOREM. *Assume that F satisfies* (F1), (F2), *and* (F3). *Suppose also that there exists a Banach function space Z such that $W \hookrightarrow\hookrightarrow Z$ and a (possibly nonlinear) functional G defined on W, continuous at 0 with respect to the norm*

$$\|u\|_{Z,Y} := \|u\|_Z + \|\nabla^m u\|_Y$$

such that $G(0) = 0$. Then a sufficient condition for the Poincaré inequality (1) to hold is that the inequality

$$\|u\|_X \leq K_1\{\|u\|_{Z,Y} + |G(u)|\}$$

holds for all $u \in W$.

This theorem gives new examples of (1) for which $0 < A < 1$; as a byproduct (1) also yields new types of weighted embeddings of the form $W \hookrightarrow X$ as well as weighted Friedrichs inequalities.

W. EICHHORN:

Inequalities in the theory of the price index

We introduce and compare several axiomatic approaches for the definition of price indices with emphasis on inequalities.

M. ESSÉN, D.F. SHEA, and C.S. STANTON:

Some best constant inequalities of $L(\log L)^\alpha$-type

Let $F = f + i\tilde{f}$ be analytic in the unit disc with $\tilde{f}(0) = 0$. M. Riesz proved in 1924 that there is a constant C_p such that

$$\|\tilde{f}\|_p \leq C_p \|f\|_p, \quad 1 < p < \infty. \tag{1}$$

The best constant was found by Pichorides in 1972. We consider also Zygmund's inequality from 1929:

$$\|\tilde{f}\|_1 \leq A \int_0^{2\pi} |f| \log^+ |f| \frac{d\theta}{2\pi} + B. \tag{2}$$

Also in 1979, Pichorides proved that for every $A > 2/\pi$, there is a constant B such that (2) holds.

We give a general method based on Cole's theorem which can be used to deduce these and other inequalities of this type with best constants. In the case of (2), we prove that there are constants A and B such that

$$\|\tilde{f}\|_1 \leq \frac{2}{\pi} \int_0^{2\pi} |f| \log^+ |f| + A \int_0^{2\pi} |f| \log^+ \log^+ |f| + B. \tag{3}$$

Here $2/\pi$ is best possible.

This is a special case of the following inequality which holds for $0 < \alpha < \infty$,

$$\begin{aligned} \int_0^{2\pi} |\tilde{f}|(\log(1 + |\tilde{f}|))^{\alpha-1} \quad &\leq \quad \frac{2}{\pi\alpha} \int_0^{2\pi} |f|(\log^+ |f|)^\alpha \\ &+ A \int_0^{2\pi} |f|(\log^+ |f|)^\beta \log^+ \log^+ |f| + B. \end{aligned}$$

Here $2/\pi\alpha$ ist best possible, $\beta = (\alpha - 1)^+$, and A and B are constants.

J. FLECKINGER:

Antimaximum principle for some "demilinear" equation
(Joint work with J.P.Gossez, P.Takac, and F. de Thélin)

I- RECALLS

Let Ω be a regular and bounded domain in $\mathbb{R}^n$; we consider the following classical equation

$$(1) \qquad -\Delta u = au + f \text{ in } \Omega; \quad u = 0 \text{ on } \partial\Omega.$$

Here $f \in L^2(\Omega)$ is given; a is a real parameter. Denote by λ_1 the principal eigenvalue of the Dirichlet Laplacian and by φ the associated eigenfunction such that:

$$-\Delta\varphi = \lambda_1\varphi \text{ in } \Omega; \quad \varphi = 0 \text{ on } \partial\Omega.$$

If we impose $||\varphi||_{L^2} = 1$ and $\varphi > 0$, then φ is unique.

It is well known that:

If $a < \lambda_1$, then there exists a unique solution u; moreover if $f \geq 0$, $f \not\equiv 0$, then $u \geq 0$. We say that (1) satisfies the maximum principle if for $f \geq 0$, $f \not\equiv 0$, any solution u is also nonnegative. The maximum principle holds for (1) if and only if $a < \lambda_1$.

If $a = \lambda_1$, the Fredholm alternative holds; (1) has a solution if and only if $\int \varphi f = 0$. This implies immediately that if $a = \lambda_1$ and if $f \geq 0$, $f \not\equiv 0$, then (1) has no solution.

It has been proved by Clément-Peletier in 1979 that

Proposition: (Antimaximum Principle). *If $f \in L^2(\Omega)$, and $f \geq 0$, $f \not\equiv 0$, then there exists $\delta(f) > 0$ such that if $\lambda_1 < a < \lambda_1 + \delta(f)$, any solution to (1) satifies $(*)$:*

$$(*) \qquad u < 0 \text{ in } \Omega; \quad \partial u/\partial n|_{\partial\Omega} > 0$$

where $\partial/\partial n$ denotes the exterior normal derivative.

II- THE p-LAPLACIAN

We give a new proof of the antimaximum principle which can be adapted to the "p-Laplacian":

$$(2) \qquad -\Delta_p u = a|u|^{p-2}u + f \text{ in } \Omega; \quad u = 0 \text{ on } \partial\Omega.$$

If $f \in L^\infty$ and smooth, we show that:

- Maximum principle holds for (2) if and only if $a < \lambda_1$, where λ_1 denotes in that case the principal eigenvalue of the Dirichlet p-Laplacian.

- If $f \geq 0$, $f \not\equiv 0$, and if $a = \lambda_1$, then (2) has no solution in $W_0^{1,p}$.

- There exists $\delta(f) > 0$ such that if $\lambda_1 < a < \lambda_1 + \delta(f)$ then the antimaximum principle holds for (2), i.e. u statisfies $(*)$.

III- THE "DEMILINEAR" EQUATION:

Consider now

$$(3) \qquad -\Delta u = au^+ - bu^- + f \ \text{ in } \ \Omega; \quad u = 0 \ \text{ on } \ \partial\Omega,$$

where as above $f \in L^\infty$; here $u^+ = max(u, 0)$ and $u = u^+ - u^-$.

Theorem 1. *The maximum principle holds for* (3) *if and only if* $b < \lambda_1$.

Theorem 2. *Assume that* $f \geq 0$; $f \not\equiv 0$, *and let* $B \leq 0$. *Then there exists* $\delta(f, B) > 0$, *such that for* $\lambda_1 < a < \lambda_1 + \delta(f, B)$, *for* $B < b < \lambda_1 + \delta(f, B)$, *the antimaximum principle holds, i.e., any solution* u *satisfies* $(*)$.

Corollary 1. *Let* f *be as in Theorem* 2 *and let* $B \leq 0$. *Then there exists* $\delta(f, B) > 0$ *such that* (3) *has no solution when* $\lambda_1 < a < \lambda_1 + \delta(f, B)$ *and* $B < b < \lambda_1$.

Corollary 2. *Let* f *be as above. There exists* $\delta(f) > 0$ *such that* (3) *has at least 2 solutions when* $a < \lambda_1$ *and* $\lambda_1 < b < \lambda_1 + \delta(f)$.

M. FLUCHER:

Sobolev inequality for general functionals and low energy limits

In collaboration with S. Müller [1] we study variational problems of the form

$$\sup \left\{ \int_\Omega f(u) : \ u \in H_0^1(\Omega), \ \int_\Omega |\nabla u|^2 \leq \varepsilon^2 \right\}$$

with small ε. The corresponding Euler equation is the semilinear Dirichlet problem

$$\begin{aligned} -\Delta u_\varepsilon &= \lambda_\varepsilon f'(u_\varepsilon) \ \text{ in } \ \Omega, \\ u_\varepsilon &= 0 \ \text{ on } \ \partial\Omega \end{aligned}$$

with large λ_ε.

Our goal is to obtain qualitative information on the extremals u_ε as $\varepsilon \to 0$. First we derive a generalized Sobolev inequality. It covers Sobolev's and Poincaré's inequality for capacity as special cases. Then we show that maximizers concentrate at a single point in the sense of the concentration-compactness alternative of P.L. Lions. In a second paper we identify the concentration point. It turns out to be a harmonic center of Ω [2]. Finally we show that the extremals suitably rescaled in x tend to an entire solution of the variational problem. The nonlinearity f is allowed to be discontinuous. Thus the results apply to certain free-boundary problems as well [3].

References

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T. FURUTA:

$A \geq B \geq 0$ ensures $(B^r A^p B^r)^{1/q} \geq (B^r B^p B^r)^{1/q}$ for $r \geq 0$, $p \geq 0$, $q \geq 1$ with $(1+2r)q \geq p+2r$ and its applications

In what follows, a capital letter means a bounded linear operator on a complex Hilbert space H. An operator T is said to be positive (in symbol: $T \geq 0$) if $(Tx, x) \geq 0$ for all $x \in H$. As an extension of the Löwner-Heinz theorem, we establish the following "order preserving operator inequalities".

Theorem A (1987). *If $A \geq B \geq 0$, then for each $r \geq 0$*

(i) $$(B^r A^p B^r)^{1/q} \geq (B^r B^p B^r)^{1/q}$$

and

(ii) $$(A^r A^p A^r)^{1/q} \geq (A^r B^p A^r)^{1/q}$$

hold for p and q such that $p \geq 0$ and $q \geq 1$ with $(1+2r)q \geq p+2r$.

Theorem A yields the following famous Löwner-Heinz inequality when we put $r = 0$ in (i) or (ii) of Theorem A: $A \geq B \geq 0$ ensures $A^{\alpha} \geq B^{\alpha}$ for any $\alpha \in [0, 1]$.

Several applications of Theorem A are discussed.

R. GER:

Delta-exponential mappings in Banach algebras

An intriguing interplay between the theory of delta-convex mappings (in the sense of Veselý and Zajiček) and the Hyers-Ulam stability problems is developed by studying a functional inequality

$$(*) \qquad \|F(x+y) - F(x)F(y)\| \leq f(x)f(y) - f(x+y).$$

This is an "exponential version" of the inequality

$$\|F(x+y) - F(x) - F(y)\| \leq \|x\| + \|y\| - \|x+y\|,$$

proposed first by D. Yost and then generalized to

$$\|F(x+y) - F(x) - F(y)\| \leq f(x) - f(y) - f(x+y).$$

Superstability phenomenon in connection with $(*)$ is examined and reported on.

M. GOLDBERG:

Homotonic mappings

Let $\mathbf{V}$ be a complex linear space of bounded complex-valued functions defined on an arbitrary set $\mathbf{T}$. A functional $\varphi : \mathbf{V} \to \mathbf{C}$ will be called *homotonic* if

$$|f| \leq g \text{ implies } |\varphi(f)| \leq \varphi(g), \quad f, g \in \mathbf{V}.$$

The same will hold for a mapping $\Phi : \mathbf{V} \to \mathbf{V}$ from $\mathbf{V}$ into itself.

In the first part we obtain bounds for homotonic functionals, by means of the usual sup norms,

$$\|f\|_{\infty} \equiv \sup_{t \in \mathbf{T}} |f(t)|, \quad f \in \mathbf{V}.$$

We provide several examples regarding well known functionals on matrices, such as the spectral radius, the numerical radius, and two families of l_p norms.

The second part is devoted to homotonic mappings and to bounds obtained by weighted sup norms of the form

$$\|f\|_{w,\infty} \equiv \sup_{t\in\mathbf{T}} |w(t)f(t)|, \quad f \in \mathbf{V},$$

where w is a positive member of $\mathbf{V}$, bounded away from zero. Much of the discussion addresses the case where $\mathbf{V}$ is an associative algebra, and $\times$, the multiplication in $\mathbf{V}$, is *homotonic*, i.e.,

$$|f_1| \leq g_1, \ |f_2| \leq g_2 \ \text{implies} \ |f_1 \times f_2| \leq g_1 \times g_2, \quad f_1, f_2, g_1, g_2 \in \mathbf{V}.$$

We give simple conditions on the weight function w that assure power boundedness for $\|\cdot\|_{w,\infty}$. Our main result proves that for $\|\cdot\|_{w,\infty}$, multiplicativity, strong stability, and quadrativity, are each equivalent to the condition $|w^{-2}| \leq w^{-1}$.

W.K. HAYMAN:

A uniqueness theorem for the Nevanlinna class $\mathcal{N}$

Suppose that $f(z) \in \mathcal{N}$, i.e., $f(z)$ is meromorphic of bounded characteristic in $D : |z| < 1$ (so that $f(z) = f_1(z)/f_2(z)$, where $|f_j(z)| < 1$ in D).

Let z_n be a sequence in D such that, as $n \to \infty$,

$$|z_n| \to 1 \ \text{and} \ \sum(1 - |z_n|) = \infty.$$

We define

$$\eta_n = \prod_k |z_k - z_n|,$$

where the product is taken over all k, such that $0 < |z_k - z_n| < \frac{1}{2}(1 - |z_n|)$.

Then if, as $n \to \infty$,

$$(1 - |z_n|) \log (|f(z_n)/\eta_n|) \to -\infty, \ \text{we conclude} \ f(z) \equiv 0.$$

This result refines a theorem of Danikas (1994) and also generalises a theorem of Khavinson (1963). A proof of the theorem will be sketched and some consequences and examples discussed.

D. HINTON:

Some extremal properties of eigenvalue problems
(Joint work with R.C. Brown and S. Schwabik)

We derive upper and lower bounds for the eigenvalues of second order eigenvalue problems by applying a one-dimensional Sobolev inequality and Prüfer transformations. The bounds are in terms of integrals of the coefficients. Applications are given to counting eigenvalues in intervals $(-\infty, \lambda]$, estimating eigenvalue gaps, Liapunov inequalities, and de La Valeé-Poussin type inequalities. For the operator $L(y) = -y'' + q(x)y$ with Dirichlet boundary conditions, a new proof is given for maximizing eigenvalues subject to an L_1 norm constraint on q.

B. KAWOHL:

On the minimal deformation of a membrane under rearrangement of a given load

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain and let $0 \leq f_0 \in L^q(\Omega)$ with $q > n/2$ be given. Consider the Poisson problem $-\Delta u = f_0$ in Ω, $u = 0$ on $\partial\Omega$. Clearly u depends on f_0. Now vary the right hand side in the differential equation over the set $K(f_0) := \{f \in L^q(\Omega) | \ f$ is equimeasurable to $f_0\}$.

Under which rearrangement of f_0 does the L_p-norm of u become minimal? Does the answer depend on p?

Intuitively one will redistribute f_0 in such a way that the maximal load is close to the boundary $\partial\Omega$. In the lecture this heuristic reasoning is replaced by a proof, in the course of which the class of admissible loads f is first enlarged to the convex hull $C(f_0)$ of $K(f_0)$. Then it is shown that minimizing loads are actually located in the extremal points of $C(f_0)$, i.e., in $K(f_0)$.

The problem was discovered by my daughter Anne while we were hanging up laundry. However, the proofs were found in cooperation with V. Ferone, A. Alvino, and G. Trombetti from Naples.

K. KARTÁK:

Generalized integrations and inqualities

Solutions of ordinary differential equations $x' = f(t, x)$ with Carathéodory f are considered, using an axiomatized integration. Special paradoxical results will be mentioned (nonexistence or nonuniqueness of solutions in the linear case).

Some questions concerning multipliers of $f \in P$ (= Perron integrable) are discussed. An estimate for

$$\int_a^b fg, \quad f \in P, \quad g \in BV,$$

is given.

H. KÖNIG:

Construction of measures, and a general inequality for non-additive set functions

For some time the author works on the aim to restructure those parts of measure theory which deal with the construction of measures from more primitive data, as circumscribed by the extension theorem of Carathéodory and the representation theorems of Daniell-Stone and Riesz. The basic idea is to replace the traditional outer measure due to Carathéodory by some new outer and inner envelopes for set functions. This idea led to substantial extensions of the above theorems and to a unification of the entire discipline. At the same time the new formations turned out to be useful in the domain of the so-called non-additive set functions. The lecture presents a general inequality on isotone set functions which is a far extension and simplification of the famous capacitability theorem due to Choquet 1959.

Q. KONG and A. ZETTL:

Inequalities for eigenvalues of Sturm-Liouville problems

The prevailing method for studying the dependence of the eigenvalues on the problem data is based on the variational characterization, in particular the min-max theorem. Here we present an entirely different approach. This consists simply of computing the derivatives and is carried out for: the end-points, the boundary conditions, the coefficients and the weight function. In the latter cases the derivative is in the sense of Fréchet in the Banach space L^1 which, we believe, is the natural space in which to study the regular Sturm-Liouville problems.

A. KUFNER:

Interpolation inequalities in sum and product form in weighted spaces

We deal with inequalities of the type

$$\|\nabla^j u\|_X \le c\|\nabla^m u\|_Y^{\alpha}\|u\|_Z^{1-\alpha}, \quad 0 < \alpha < 1,$$

and

$$\|\nabla^j u\|_X \le c(\|\nabla^m u\|_X + \|u\|_Z),$$

with non-negative integers j, m, $0 \le j \le m$. Here, ∇^i is the gradient of order i and X, Y, Z are weighted Lebesgue or Hölder spaces. Conditions on the weights will be given which guarantee the validity of the inequalities mentioned. Also fractional order derivatives will be considered.

L. LOSONCZI:

Minkowski's inequality for two variable homogeneous mean values
(Joint work with Zs. Páles)

We study Minkowski's inequality

$$(1) \qquad M(x_1 + x_2, y_1 + y_2) \le M(x_1, y_1) + M(x_2, y_2)$$

and its reverse

$$(2) \qquad M(x_1 + x_2, y_1 + y_2) \ge M(x_1, y_1) + M(x_2, y_2) \quad (x_1, x_2, y_1, y_2 > 0),$$

where M is either the "sum mean"

$$S_{a,b}(x,y) = \left(\frac{x^a + y^a}{x^b + y^b}\right)^{1/(a-b)} \qquad (x, y > 0,\ a \ne 0)$$

or the "difference mean"

$$D_{a,b}(x,y) = \left(\frac{x^a - y^a}{a}\frac{b}{x^b - y^b}\right)^{1/(a-b)} \qquad (x, y > 0,\ x \ne y,\ ab(a-b) \ne 0),$$

a, b being (real) parameters.

In both cases we determine the exact domain of the parameters (a, b) such that (1) or (2) holds.

E.R. LOVE:

A conjectured inequality of T.J. Lyons

The conjectured inequality is

$$\alpha \sum_{j=0}^{n} \binom{\alpha n}{\alpha j} x^{\alpha j}(1-x)^{\alpha(n-j)} \leq 1,$$

where $\binom{\alpha n}{\alpha j} = \dfrac{(\alpha n)!}{(\alpha j)!(\alpha(n-j))!}$, $0 \leq \alpha \leq 1$ and $0 \leq x \leq 1$.

We have proofs of this for $\alpha(n+1) \leq 1$ and a little more. We are fairly confident that for fixed n the left side is concave in x, and is an increasing function of α; but this confidence is based partly on numerical calculation of particular cases. Some weaker inequalities have been proved, but this one still awaits proof or disproof.

M. MARCUS:

An inequality in Sobolev spaces

Consider the energy integral,

$$(1) \qquad I_\Omega(u) = \int_\Omega \left(\sum_{|\alpha|=m} |D^\alpha u|^q + |u|^p \right) dx, \quad u \in W_{m,q}(\Omega),$$

where Ω is a domain in $\mathbb{R}^n$ satisfying the cone condition, $1 \leq q < p$, $n < mq$. To simplify some expressions we assume also that Ω has unit volume. We are interested in determining the rate of increase of $I_\Omega(u)$ as $\|u\|_{L_\infty} \to \infty$ while $\|u\|_{L_1}$ remains bounded or at least satisfies the inequality

$$(2) \qquad O(u) = \|u\|_{L_\infty} / \|u\|_{L_1} \geq k,$$

where k is "large". This question comes up in the investigation of a variational problem with small parameters. To answer this question we establish the following result:

There exist positive constants c, k such that

$$(3) \qquad I_\Omega(u) \geq c\|u\|_{L_\infty}^{p-n\beta} + \left(\int_\Omega |u| \right)^p, \quad \beta = \frac{p-q}{mq},$$

for all u satisfying (2). We note that $p - n\beta > q$. The exponent $(p - n\beta)$ is optimal, i.e., the inequality does not hold (in general) for larger exponents.

Inequality (3) can be reformulated in homogeneous form,

$$\|u\|_{L_\infty} \le c' |u|^{\sigma}_{m,q} |u|^{*(1-\sigma)}_{p}, \tag{4}$$

where $|u|_{m,q} = \left(\int_\Omega \sum_{|\alpha|=m} |D^\alpha u|^q\right)^{1/q}$, $|u|^*_p = \left(\int_\Omega |u|^p - (\int_\Omega |u|)^p\right)^{1/p}$ and $\sigma = n/[m(p - n\beta)]$. This inequality is similar to the corresponding case of the Gagliardo-Nirenberg inequality:

$$\|u\|_{L_\infty} \le c'' \|u\|^{\sigma}_{W_{m,q}} \|u\|^{1-\sigma}_{L_p}. \tag{5}$$

Clearly (4) implies (5) for those functions $u \in W_{m,q}(\Omega)$ which satisfy (2). It can be shown that (5) does not imply (4), no matter how large is k.

G.V. MILANOVIĆ:

Integral inequalities for algebraic polynomials

Let $\mathcal{P}_n$ be the class of all algebraic polynomials $P(x) = \sum_{k=0}^{n} a_k x^k$ of degree at most n and $\|P\|_{d\sigma} = \left(\int_{\mathbb{R}} |P(x)|^2 d\sigma(x)\right)^{1/2}$, where $d\sigma(x)$ is a non-negative measure on $\mathbb{R}$. We determine the best constant in the inequality $|a_k| \le C_{n,k}(d\sigma)\|P\|_{d\sigma}$, for $k = 0, 1, \ldots, n$, when $P \in \mathcal{P}_n$ and such that $P(\xi_k) = 0$, $k = 1, \ldots, m$. The cases $C_{n,n}(d\sigma)$ and $C_{n,n-1}(d\sigma)$ were studied by Milanović and Guessab [1], and only for the Legendre measure by Tariq [3]. In particular, we consider the cases when the measure $d\sigma(x)$ corresponds to some classes of orthogonal polynomials on the real line. For many details in this subject we refer to our monograph [2].

Let $m \in \mathbb{N}$, $a_s > 0$, $s = 0, 1, \ldots, 2m - 1$ and $\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_{2m-1}$ be $(2m)$-th roots of unity, i.e., $\varepsilon_s = \exp(i\pi s/m)$, $s = 0, 1, \ldots, 2m - 1$. We introduce orthogonal polynomials relative to the inner product

$$(f, g) = \sum_{s=0}^{2m-1} \varepsilon_s^{-1} \int_{l_s} f(z)\overline{g(z)}|w(z)|\, dz,$$

where l_s are the radial rays in the complex plane which connect the origin $z = 0$ and the points $a_s\varepsilon_s$, $s = 0, 1, \ldots, 2m - 1$, and $z \mapsto w(z)$ is a suitable complex (weight) function, and then we investigate some inequalities and extremal problems in L^2-norm for polynomials $P \in \mathcal{P}$.

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R.N. MOHAPATRA:

Bernstein tpye inequalities for rational functions

Bernstein type inequalities for polynomials provide estimates for the derivatives of a polynomial in terms of the norm of the polynomial. The extremal results show connection between the location of zeros of the polynomial and these estimates are known. We consider several recent extensions to rational functions when we have information about zeros of the rational function and when we know about information on the poles. We shall also indicate new directions of research associated with it.

R.J: NESSEL:

A resonance principle with rates in connection with pointwise estimates for the approximation by interpolation processes
(Joint work with L. Imhof)

Recently there have been established various results concerned with rates of convergence for a number of important interpolation processes. It is the purpose of this paper to discuss some of the relevant pointwise estimates in the light of our previous general resonance principle with rates. This principle indeed delivers a condensation of singularities on the limes superior of arbitrary, not necessarily countable index sets. More specifically, with the aid of the Borel-Cantelli lemma it is shown that a direct theorem of P. Vértesi (1971) and J. Szabados (1986) for a trigonometric $(0, M)$-interpolation process (M odd) is sharp on a set of full measure. The same holds true in connection with an equivalence assertion obtained by X.L. Zhou (1993) for the approximation by the classical Fejér-Hermite polynomials, the existence of a counterexample is established, simultaneously delivering the sharpness of the Jackson as well as of the Bernstein estimate on sets of full measure. Based on the resonance principle mentioned and on a convergent interpolatory process constructed by Szabados-Vértesi it is even possible to show that the Jackson estimate of Zhou is sharp for all points.

Zs. PÁLES:

Separation with bilinear and quadratic functions and inequalities for second order derivatives of nonsmooth functions

Assume that X is a locally convex linear space and that $P : X \times X \to \mathbb{R}$ and $R : X \times X \to [-\infty, \infty[$ are positively bihomogeneous functions

$$\begin{aligned} P(\lambda x, y) = P(x, \lambda y) = \lambda P(x,y) \\ R(\lambda x, y) = R(x, \lambda y) = \lambda R(x,y) \end{aligned} \qquad \text{for } \lambda > 0,\ x, y \in X.$$

Consider the following three problems:

I. Find necessary and sufficient conditions for the existence of a bilinear function $Q : X \times X \to \mathbb{R}$ such that

$$R \le Q \le P \quad \text{on} \quad X \times X.$$

II. Find necessary and sufficient conditions for the existence of a bilinear function $Q : X \times X \to \mathbb{R}$ such that $Q \le P$ on $X \times X$.

III. Characterize those functions P which are pointwise suprema of families of bilinear functions, i.e.,

$$P(x,y) = \sup\{Q(x,y) : Q \text{ is bilinear and } Q \le P\}.$$

These questions are motivated by searching for first and second order derivatives for nonsmooth functions.

All the above three problems are solved, and also analogous problems with symmetric bilinear functions and positive semidefinite symmetric bilinear functions are discussed.

J. PEČARIĆ:

Arithmetic mean - geometric mean and related matrix inequalities

Arithmetic mean - geometric mean inequality for two positive definite Hermitian matrices is a well known result. Recently, M. Sagae and K. Tanabe (Lin. Multilin. Alg. 37 (1994), 279–282) proved a weighted generealization of this result in a case of n positive definite Hermitian matrices. These and other recent results are explained.

C.E.M. PEARCE:

Inequalities of Gauß type

Gauß' inequality

$$\upsilon_2^2 \leq \frac{5}{9}\upsilon_4$$

connecting the second and fourth absolute moments of a distribution with nonnegative support and nonincreasing density function has been refined and extended in a number of ways. We review a rich body of recent work, much not yet published or still in progress, which has developed from two extensions by Pólya. Versions now exist incorporating a number of functions, discrete distributions and derivatives of first and higher order.

M. PLUM:

Numerical existence proofs for weak solutions of nonlinear elliptic boundary value problems

Consider the nonlinear elliptic boundary value problem

$$(1) \qquad u \in H_1^0(\Omega), \quad \int_\Omega [\nabla u \cdot \nabla \varphi + F(x,u)\varphi]\,dx = 0 \quad (\varphi \in H_1^0(\Omega)),$$

where $\Omega \subset \mathbb{R}^n$ is some bounded Lipschitz domain, and $F : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a C_1-smooth nonlinearity with subcritical growth at ∞. In the lecture, a computer-assisted method for proving the existence of a solution of problem (1) within an explicit $H_1^0(\Omega)$-neigborhood of some approximate solution $\omega \in H_1^0(\Omega)$, will be presented. It is based on the computation of a bound for the defect $-\Delta\omega + F(\cdot,\omega)$ in the norm of the dual space $H_{-1}(\Omega)$, of a $(H_{-1} \to H_1^0)$-norm bound for the inverse of the linear operator $L : H_1^0(\Omega) \to H_{-1}(\Omega)$

obtained by linearization of problem (1) at ω, and of the constants in some Sobolev inequalities. Numerical examples will illustrate the method.

Th.M. RASSIAS:

On a problem of S.M. Ulam and the asymptotic stability of the Cauchy functional equation with applications

In 1940 S.M. Ulam proposed the following problem. Given a group G_1, a metric group G_2 with metric $d(\cdot,\cdot)$ and a positive number ε, does there exist a $\delta > 0$ such that if $f : G_1 \to G_2$ satisfies

$$d(f(xy), f(x)f(y)) < \delta \text{ for all } x, y \in G_1,$$

then a homomorphism $h : G_1 \to G_2$ exists with

$$d(f(x), h(x)) \leq \varepsilon \text{ for all } x \in G_1\,?$$

Some new results for Ulam's problem are proved. It is also explained how the Hyers-Ulam stability theory can be used to study the asymptotic derivative of some nonlinear operators especially in fixed point theory.

J. RÄTZ:

Convexity of power functions with respect to symmetric homogeneous means

If $M : \mathbb{R}_+^* \times \mathbb{R}_+^* \to \mathbb{R}_+^*$ is an $\mathbb{R}_+^*$-homogeneous symmetric mean, briefly $M \in \mathfrak{M}$, then a function $\varphi : \mathbb{R}_+^* \to \mathbb{R}_+^*$ is called

$$\begin{array}{ll} \text{strictly } M\text{-convex} & \text{if } \varphi[M(x,y)] < M[\varphi(x), \varphi(y)], \\ M\text{-affine} & \text{if } \varphi[M(x,y)] = M[\varphi(x), \varphi(y)], \\ \text{strictly } M\text{-concave} & \text{if } \varphi[M(x,y)] > M[\varphi(x), \varphi(y)] \end{array}$$

(for all $x, y \in \mathbb{R}_+^*$, $x \neq y$).

It is clear that the function $\varphi_p : x \mapsto x^p$ ($p \in \mathbb{R}$) is σ-affine (σ the geometric mean) for every $p \in \mathbb{R}$ and

$$\begin{array}{ll} \text{strictly } M_c\text{-convex} & \text{if } c(p^2 - p) > 0, \\ \text{strictly } M_c\text{-concave} & \text{if } c(p^2 - p) < 0, \end{array}$$

where $M_c(x,y) := \left(\frac{x^c+y^c}{2}\right)^{1/c}$ $(c \neq 0)$.

This last result is extended to two 2-parameter families $(E_{r,s})_{r,s\in\mathbb{R}}$ (Stolarski means) and $(M_{r,s})_{r,s\in\mathbb{R}}$ (Gini means) containing all power means M_c and σ. Instead of $c(p^2-p) >, =$ or < 0 we may have there $(r+s)(p^2-p) >, =$ or < 0.

Cases of $M \in \mathfrak{M}$ are shown where the situation for the power functions is completely different.

J. RÄTZ:

Convex functions with respect to the logarithmic mean
(Joint work with J. Matkowski)

If $L : (0,\infty) \times (0,\infty) \to (0,\infty)$ denotes the logarithmic mean and I an open subinterval of $(0,\infty)$, then a function $\varphi : I \to (0,\infty)$ is called convex with respect to L (briefly L-convex) if

$$\varphi[L(x,y)] \leq L[\varphi(x),\varphi(y)] \text{ for all } x,y \in I.$$

Strict L-convexity as well as (strict) L-concavity are defined analogously.

The power functions $x \mapsto x^p$ ($x \in (0,\infty)$, $p \in \mathbb{R}$) are classified with respect to L-convexity and L-concavity, and the inequality between L and the arithmetic and the geometric means is obtained as a corollary.

The method of proof leads to more general considerations and to a characterization of the weighted geometric mean.

W. REICHEL:

A strong comparison principle for the radial p-Laplacian

For the p-Laplacian operator $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$, $p>1$, the *strong comparison principle* (SCP) states, that for two functions v, w with

$$\begin{aligned} \Delta_p w + f(x,w) &\leq \Delta_p v + f(x,v) \text{ in a domain } \Omega \\ w &\geq v \text{ on } \partial\Omega \end{aligned}$$

the alternative $w \equiv v$ or $w > v$ in Ω holds. Here $f(x,u)$ is a locally lipschitz and decreasing function of u.

(SCP) holds under the requirements $v, w \in C^2(\Omega)\cap C(\overline{\Omega})$ and $\nabla v, \nabla w \neq 0$ in Ω. By an example of Walter, it is known that (SCP) can fail if $\nabla v, \nabla w \neq 0$ is violated.

The operator Δ_p, acting on functions u depending only on $r = |x|$, can be viewed as an ordinary differential operator of second order

$$L_p^{N-1}u = r^{1-N}(r^{N-1}|u'|^{p-2}u').$$

We give smoothness conditions on $f(r,u)$, such that (SCP) holds under the modified assumption

$$\begin{aligned} 0 \leq L_p^{N-1}w + f(r,w) &\leq L_p^{N-1}v + f(r,v) \text{ in } [a,b] \\ w &\geq v \text{ at } r = a, b. \end{aligned}$$

The main tool to prove (SCP) in the radial case is the study of the initial value problem for the operator L_p^{N-1}.

S. SAITOH:

Natural norm inequalities in nonlinear transforms

Let E be an arbitrary nonvoid abstract set and let $H_K(E)$ be a Hilbert (possibly finite-dimensional) space admitting a reproducing kernel $K(p,q)$ on E. Then, the Hilbert space $H_K(E)$ is composed of complex-valued functions $f(p)$ on E such that

(1) $$K(\cdot,q) \in H_K(E) \text{ for any fixed } q \in E$$

and, for any member f of $H_K(E)$ and for any fixed point q of E,

(2) $$(f(\cdot), K(\cdot,q))_{H_K} = f(q).$$

In general, a reproducing kernel $K(p,q)$ on E satisfying (1) and (2) is uniquely determined by the Hilbert space $H_K(E)$ and is a positive matrix in the sense that for any points $\{p_j\}_j$ of E and for any complex numbers $\{C_j\}_j$

(3) $$\sum_{j,j'} C_j\overline{C_{j'}}K(p_{j'},p_j) \geq 0.$$

Conversely, a positive matrix $K(p,q)$ on E satisfying (3) determines uniquely a functional Hilbert space (reproducing kernel Hilbert space $\equiv$ RKHS) $H_K(E)$ satisfying (1) and (2).

We shall consider RKHS $H_K(E)$ as an input function space of the following nonlinear transform

$$\varphi : f \in H_K(E) \longrightarrow \sum_{n=0}^{\infty} d_n(p) f(p)^n, \tag{4}$$

where $\{d_n(p)\}$ are nonvanishing functions on E. We shall see that the nonvanishing assumption on the functions $\{d_n(p)\}$ is not essential, in our arguments.

In this nonlinear transform φ, we shall show that the images $\varphi(f)$, $f \in H_K(E)$, belong to a Hilbert space **H** which is naturally determined by the nonlinear transform φ, and there exists a natural norm inequality between the two norms $\|\varphi(f)\|_H$ and $\|f\|_{H_K}$.

Some general theorems for these facts and their miscellaneous applications with concrete norm inequalities are given.

G. TALENTI:

An inequality by L.E. Fraenkel

We present an approach to the following inequality:

$$\int_{-\infty}^{+\infty} dx \int_{0}^{+\infty} \varphi^p y^{-2-p/2} dy \le \text{const.} \left\{ \int_{-\infty}^{+\infty} dx \int_{0}^{+\infty} (\varphi_x^2 + \varphi_y^2) \frac{dy}{y} \right\}^{p/2}.$$

considered by L.E. Fraenkel. Such an approach is based on the isoperimetric inequality for the hyperbolic plane.

P. VOLKMANN:

Die Funktionalgleichung $f(x) + \max\{f(y), f(-y)\} = \max\{f(x+y), f(x-y)\}$

(Gemeinsame Arbeit mit R.M. Redheffer)

Es sei G eine abelsche Gruppe. Eine Funktion $f : G \to \mathbb{R}$ löst die Funktionalgleichung genau dann, wenn sie additiv ist oder wenn sie folgende Form besitzt: $f(x) = |a(x) + p| - p$ mit $p \geq 0$ und einer additiven Funktion $a : G \to \mathbb{R}$.

W. WALTER:

A new method in the theory of differential inequalities

A method based on monotonicity arguments is presented, which turns out to be simple and effective when dealing with comparison theorems and maximum principles for ordinary differential equations. The following problems are discussed.

1. A generalization of M. Hirsch's theorem on strictly monotone flows generated by a system $y'(t) = f(t, y)$, where f satisfies only Carathéodory hypotheses and is locally Lipschitz continuous and quasimonotone increasing in y.

2. Minimum and comparison theorems for second order differential equations of the form $(\varphi(x, u'))' + g(x, u) = 0$, where $\varphi(x, s)$ is defined in $J \times \mathbb{R}$, $J = [a, b]$, and increasing in s. In particular, the radial Δ and Δ_p operator in $\mathbb{R}^n$ and the Monge-Ampère operator $\det(D^2u)$ for radial solutions are covered.

3. Comparison theorems for initial value problems $(\varphi(x, u'))' = g(x, u)$, $u(x_0) = u_0$, $u'(x_0) = u_1$, and applications to existence and uniqueness for radial solutions of blow-up problems for the equation $\Delta_p u = g(r, u)$ are treated.

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