MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 48/1995

New Trends in the Teaching and Learning of Mathematics

26.11.-2.12.1995

The conference was organized by David Bressoud (St. Paul, USA), Urs Kirchgraber (Zurich, Switzerland), and Ed Packel (Lake Forest, USA).

In view of
- the recent progress in our understanding of the processes connected with the learning of mathematics,
- the enormous curricular changes (in particular in the US), and
- the growing awareness of the importance of high quality mathematics teaching on all levels,

it seemed appropriate to organize an Oberwolfach meeting in order to exchange results, new ideas, and discuss some of the more controversial issues.

One of the key goals of the conference was to enhance the interaction between experts from various countries. Of the 24 participants 9 came from the US, 8 from Germany, 3 from Switzerland and 1 from each of the following countries: Australia, Austria, France and the UK.

Subsequently, we give an overview of the talks presented at the meeting. It goes without saying that the informal conversations and discussions were at least as important as the talks themselves.

Various presentations dealt with – roughly speaking – *curricular questions* with the main emphasis on real-world applications of mathematics: be it that new contents are taught within genuine rather than superficial problem situations (e.g. dynamical systems, cf. Barker, Hughes-Hallet, Kirchgraber, Reichel), or be it that traditional topics such as changes of rate are effectively used to provide orientation in complex aspects of reality (Henn, Schmidt). Stillwell, too, deals with curricular questions but stays inside pure mathematics where his approach to tying together fundamental ideas of algebra and geometry is solely based on linear and quadratic equations.

New approaches to known pieces of knowledge, made possible through *technology*, showed up in the presentations by Hartmann, Packel and Smith. For one thing, the computer is used to (dynamically) visualize ideas of Calculus, for another, technology allows a more 'natural' inductive approach to mathematical concepts and problems. The influence of the computer on the teaching of geometry was dealt with only by Europeans (Hölzl, Klemenz, Laborde) – apparently because of a strong Euclidean tradition that is still prevalent in some European countries.

Schwank, Tall and Wollring's cognitive-oriented *research contexts* provided a counter-balance, as it were, to pure curricular questions. However, as could be seen in Dubinsky's talk, both branches need not necessarily exclude each other. In their respective talks a changing paradigm of work and methodology is discernible which is reflected in the broader mathematics educators community: a shift from experiments identifying children's errors and their possible remediation to case studies within a more holistic framework (taking into account the social settings while incorporating a more conscious problematising of the nature of mathematics itself).

Jahnke and Rickey showed that the *historic dimension* of mathematics is well received by students provided the teacher who uses history in his or her classes has an authentic relationship to it. Mere interludes providing dates and facts are certainly not enough. The question of authentic teaching styles is central to Hefendehl-Hebeker: her various aspects of *explaining mathematics* are informed by the maxime that the talk in the classroom should be as lucid as possible. Finally, a survey of the progress and problems of American *school mathematics reforms* were given by Kennedy and Solow.

The conference started with a talk by Bressoud and ended with the one by Kramer, both of which dealt with *aspects of proofs* in highly advanced mathematics.

In response to the participant's presentations, of which the abstracts are listed below, some questions arose relating to research, curriculum and technology issues. David Bressoud recorded and edited a list of those questions and made it available on his World Wide Web page (http://www.math.macalstr.edu/~bressoud/). I include his list after the abstracts.

Reinhard Hölzl

ABSTRACTS

W. BARKER:

The Case for Radical Reform of Calculus

Two of the major problems with students who complete the traditional American Calculus sequence are their (1) lack of understanding of the basic concepts of the course and (2) lack of knowledge and appreciation of the important applications of the subject. This talk describes and illustrates some steps that can be taken to address these problems. A logistic model for Maine lobsters will provide a "provocative" conclusion for the presentation.

D. BRESSOUD:

A Search for Proof

Before we decide how and when to teach proofs, we must understand what we mean by proof and how we use it. It is often viewed as a means of convincing someone of the veracity of a mathematical statement, but in fact is more often a tool for testing our own understanding and refining our concepts.

This talk looks at yet another aspect of proof, as exploration of mathematics with its own surprises and insights and unexpected revelations. This exploration is illustrated by the search for a proof that the number of Alternating Sign Matrices of size $n \times n$ is given by $\prod_{j=0}^{n-1} \frac{(3j+1)!}{(n+j)!}$. In seeking this proof, Mills, Robbins and Rumsey were led instead to a proof of a seemingly unrelated result: Macdonald's conjectured generating function for the number of cyclically symmetric plane partitions.

E. DUBINSKY:

Using Theoretical and Empirical Research into Learning as a Basis for the Design and Implementation of Instruction in College Level Mathematics Courses

My talk describes and situates a general theory of learning mathematics. This theory focuses on what might be happening (or not happening) in the minds of college students as they struggle to understand mathematical concepts. In particular it proposes mental constructions that students might make in order to succeed in these struggles.

The design and implementation of instruction is based on an attempt to help students make the constructions proposed by the theory. The instruction is evaluated on the basis of students' apparent success in making the desired mental constructions. The theoretical analysis is evaluated by how well the students appear to be learning mathematics.

This approach has been used for topics in Discrete Mathematics, Precalculus, Calculus and Abstract Algebra. Examples and results are provided.

W. HARTMANN:

A New, Old Approach to Calculus

The impact of new technology resulting in the "Rule of Three", which emphasizes the graphical, numerical and analytical aspects of calculus, is generally accepted today as an important concept in the teaching of precalculus and calculus. Many new textbooks have added exercises and new material, which assume that students have graphing calculators. But these additions tend to be local and fragmentary. Only a few reform projects have resulted in more global changes which call for a complete redesign of undergraduate mathematics education.

The chance for new methodological approaches, made possible by today's technology, is outlined and illustrated by the min-max-problem for one variable functions. Some traditional (school-) methods are outmoded. Current and future teachers will have to be introduced to methods which are new to them. Central topics in the math curriculum may have to be moved into another context. Beginners courses would have to adopt a functional approach rather than continuing within the traditional axiomatic and structural format. A functional approach should start with everyday problems. This also takes into account the way human beings think, learn and create. Thus the new approaches needed in mathematics teaching turn out to be nothing but the obvious way people deal with a problem the first time they are confronted with it.

L. HEFENDEHL-HEBEKER:

Aspects of Explaining Mathematics

Mathematics is not self-explanatory nor is there a guarantee that a neat explanation automatically causes understanding. Nevertheless guidelines for explaining mathematics and for a lucid teaching behaviour can be formulated. Among these are: to reveal the point of view, to establish central ideas as a focus of attention, to become and make aware of the employed strategies, to mark out translations between levels of representation, to give orientations by looking forward and looking back. On the basis of our own teaching experience we put together some aspects of this kind.

W. HENN:

Mathematics as an Orientation in a Complex World

Besides the acquisition of elementary cultural techniques, the most important goal of education in mathematics is to make students value mathematical methods for handling future problems from various parts of life. My talk gives a brief survey of the present situation in Germany (especially the state of Baden-Württemberg) as to the teaching of modelling and real-world applications at the secondary level. Some units I have worked out are intended to illustrate how mathematics can improve orientation in a complex world: The notion of "convexity" explains why a married couple possibly has to pay more tax in Germany than an unmarried one. Insight into how to analyze data is helpful in many fields: An example is a simple model for the consumption of fuel. Other examples concern the Dow-Jones-Index and the reconstruction of distorted data. The Riemann integral is needed to define the HIC (Head Injury Criterion) which quantifies the results of crash tests. Those who understand what local rates of change are will understand why a rainbow appears, etc.

R. HÖLZL:

The Role of Computers in the Teaching and Learning of Geometry

We report and reflect on the outcomes of interpretative case studies that were conducted to investigate the role of computers in the teaching and learning of geometry. In doing so I focus on student's individual problem solving strategies without teacher intervention as well as teacher-centred whole-class settings. Either way, Dynamical Geometry

Environments are an integral part of the teaching and learning arrangements.

D. HUGHES-HALLETT:

Harnessing Student's Interests

This talk starts with a survey of the changes that are taking place in school and university mathematics education in the US and around the world. The reasons for these changes – concern over equity and access, student performance, and advances in technology – led to the observation that the students themselves are changing. The new students are demanding more in the way of understanding and visualization, and more of a sense of the powerful ways in which mathematics is used.

To teach these students, we must harness their interests and harness the technology. Doing so requires changing the attitudes of some students (one of whom, for example, believed that using trial and error to solve a problem is "cheating"). The Calculus Consortium Based at Harvard uses "The Rule of Three" (representing ideas graphically, numerically, and symbolically wherever possible) as well as practical problems to get students actively involved in learning mathematics, and in thinking about the practical meaning of what they learn.

The examples given are largely from differential equations – a topic which deserves a much larger place than it had in the traditional curriculum – and included examples of the probing questions asked by students in such a course.

H. N. JAHNKE:

The Historical Dimension of Mathematical Understanding. A Teaching Experience with Johann Bernoulli's Textbook on Differential Calculus

J. Bernoulli wrote his textbook on the Differential Calculus in 1691/2 when teaching the new infinitesimal methods to the Marquis de l'Hospital. The manuscript was lost for a long time until in 1922 a copy of it was found at the university library of Basel. In summer 1994, some parts of this book were read with students (grade 11) of a Gymnasium near Bielefeld. This teaching experience showed how students, confronted with the ideas of an important mathematician, are motivated to think about their own ideas on Calculus.

D. KENNEDY:

Changing the Rules in American Schools: Fast Food and Slow Reforms

Despite much lively conversation among a few basic coteries of educational theorists over the past century, the instructional model in the American secondary school system has remained remarkably unchanged. One powerful reason for this is that the system has grown so quickly, while the individual schools have adapted to remain faithful to a primitive tribal model, wherein a few tribal sages are entrusted with passing on the lore of the tribe to future generations. Viewed in this anthropological context, the success of the current mathematical reform movement seems to be more than unprecedented; it actually seems to be unreasonable.

A comparison to the success of McDonalds' restaurants (unreasonable for similar anthropological reasons) suggests that the current mathematics reforms are succeeding because they have appeared at a singularly opportune moment in the evolution of our tribal culture. By implication, the changes we see happening now could actually redefine education just as McDonald's has redefined lunch.

U. KIRCHGRABER:

Teaching Dynamical Systems at the Gymnasium

A short overview is given of the Swiss school system and the Mathematics curriculum of the upper Gymnasium of which a relatively broad introduction to Calculus is an essential section. In spite of the new technology that has moved into classroom, curve sketching is still the clue and predominant topic there. In the talk I propose a modest but (hopefully) balanced syllabus on differential equations instead. Some of the issues are;

- Modelling with differential equations based on first principles (conserved quantities, the theory of motion: how to introduce Newton's fundamental laws of Mechanics?)
- The philosophical nature of differential equations
- The geometrical picture
- Solution formula, the value of numerical methods, what is a qualitative result?
- The genuine Swiss contribution: Control of a chain of River Power Plants
- Putting into perspective: understanding the principle of Weather Forecasting, the Moon and the origin of Mankind.

H. KLEMENZ:

Computer Aided 3D-Geometry at the Gymnasium

Mathematics teaching has been enriched for the last few years through use of interactive software tools. However, the tools available are mostly directed at plane geometry and offer little if any help for construction in space. Unlike 2D-tools "3D-Geometer" was developed for spatial constructions, that is, spatial objects can be created by the user without forcing him to do preparatory constructions in the plane first. Display of objects in 3D-Geometer is either graphical or symbolic; solutions are automatically recorded.

My talk first illustrates the interactive user interface of 3D-Geometer by means of simple examples. Choosing some tasks from everyday lessons, I then give a survey of the possibilities of such a software tool.

J. KRAMER:

Spectacular Results in Mathematics - Teaching of Mathematics

In recent years proofs of long-standing problems such as the Mordell Conjecture or Fermat's Last Theorem have been provided. We are concerned to present such interesting new mathematical developments to students on all levels by showing them the ideas and - as far as possible - the methods used to solve these problems. On the one hand this increases the motivation of the students, on the other hand this illustrates the power of modern mathematics. By discussing one or two examples we show in our talk how such courses could be modelled.

C. LABORDE:

Teaching and Learning Geometry in Computer Based Environments

The dynamical graphical possibilities of computers may deeply change the kind of geometry which is taught and learned. The talk presents new kinds of geometric problems which can be given to learners and discusses the role of the computers in the solving process of the students. The talk is mainly illustrated by examples with the software Cabri géomètre (now available on a pocket calculator).

E. PACKEL:

Mathematica as a Tool for Student Exploration and Discover: Examples and Issues

A variety of examples, mostly from calculus and differential equations, are presented to show how a computer algebra system (in this case *Mathematica*) can be used to engage and challenge students working in a laboratory environment. The presentation is accompanied by a discussion of some of the pedagogical issues raised by the use of such technology.

H.-Ch. REICHEL:

New Trends and Concepts for the Teaching of Mathematics in Austria

The new trends referred to in the title concern two items: (1) Organization and (2) Contents.

Ad 1: parallel to the common math-lectures, and during the final three years, Austria's gymnasia offer free-choice courses (2 hours per week) on various topics (Wahlpflichtfächer). We present a list of 26 topics and discuss the main aims and features of our suggestions for these mini courses which, in most cases, end with a modest "research" paper that is done by the students and is part of the final exam, "Matura" ("Abitur", in Germany.) - More details in (Reichel et al.: Fachbereichsarbeiten und Projekte im Mathematikunterricht, Verlag Hölder-Pichler-Tempsky, Wien).
Ad 2: Discussion of concrete examples: (1) a mini course on problem solving that involves empirical studies concerning student's attitudes. (2) A new chapter recently included in vol. 7 of Reichel-Müller-Hanisch: Mathematisches Arbeitsbuch, Verlag Hölder-Pichler-Tempsky, Wien, (the most frequently used school-book in Austria) concerning Discrete Dynamical Systems, i. e. Difference-Equations. We discuss the new material and give a didactic reflection on why this subject should be taught. (3) New trends in using computers at school: we present a simple discrete model for medical scanning and image processing in the context of computer tomography together with a technique for solving larger systems of linear equations

V. F. RICKEY:

The Importance of Using History in Teaching Mathematics

Mathematics has a rich and interesting history that teachers must utilize to improve student motivation and understanding. This position could be argued in the abstract, but examples of what has worked for me will be more interesting than philosophy. Here are some examples that could be presented: What to say on the first day of calculus class.

What to say on November 27. Gregory of Saint Vincent and the logarithm. Euler and the trigonometric functions. Perrault and the tractrix. Participants and the meeting are able to select examples for presentation form a longer list.

W. SCHMIDT:

Efficient Teaching of Mathematics: Experience from a 10-Year-Project on a German Gymnasium

Learning mathematics is only possible if pupils pay attention and are motivated. One way to ensure this may be to let pupils apply what they are learning. This kind of "applied mathematics" differs from the usual sense.

To understand why that works, some recent results of the theory of neuronal processes, especially the function of the hippo campus with respect to "explicit learning", are helpful.

I. SCHWANK:

Predicative vs. Functional Modes of Cognitive Organization

In cognitive science different theories concerning the construction of mental models and the representation of knowledge in memory have been established. In mathematics education the interplay between external representations and the construction of mental models is an important matter of research concerning the process of understanding mathematical concepts. In the process of learning and understanding mathematical concepts suitable microworlds play the role of an operating system by which new concepts are embedded into the so far represented knowledge and the use of them is managed. Understanding occurs when an new idea can be fitted into a larger framework of previously-assembled ideas, which have been viable during the past acting in the microworld.

Since 15 years we are doing research on the mathematical thinking of 12-14 years old students of different intellectual level in different countries (Germany, China, Indonesia). Constructive teaching experiments have been done with single students constructing and analyzing algorithms in special microworlds. The qualitative analysis of the videotapes enables to reconstruct the process of mathematical concept formation. It can be shown that great individual differences in the students methods of mental analysis do exist. We distinguish between predicative and functional cognitive structures. Predicative thinking emphasizes the preference for thinking in terms of relations and judgements; functional thinking emphasizes the preference for thinking in terms of courses and modes of action. There is a resonance effect between external representation in the microworld and the construction of the mental model.

D. A. SMITH

Calculus: Concepts, Computation, Composition, Cooperation

A general survey of the award-winning reform calculus course developed at Duke University by Project CALC: Calculus As a Laboratory Course. The key features of the course are real-world problems, hands-on activities, discovery learning, writing and revision of writing, teamwork, intelligent use of available tools, and high expectations of students.

A. E. SOLOW:

The ART of Combining Assessment, Reform, and Technology

In this talk, we first review the current literature on assessment in mathematics, including the assessment framework developed by Judith Garfield. Then we look at the relationship between assessment and reform and the difficulty of assessing what we value in a time when the definitions of what we value is changing. To this mix, we then add technology and consider several examples that deal with the interconnectedness of these three ideas.

J. STILLWELL:

Building mathematics on the foundations of number theory and geometry

Many students arriving at university today have forgotten the ideas from algebra and geometry that are essential for an understanding of calculus (and other advanced branches of mathematics). I believe that these ideas fall into disuse because the school curriculum is mathematically disorganised – it does not have a clear idea of foundations, how to build on them, and how to reinforce the superstructure by links between its different parts.

It is argued in this talk that number theory and geometry are the most suitable foundations, and that a very large superstructure can be tied together by the ideas of linear and quadratic equations. This claim will be illustrated by examples linking the number concept to geometry, trigonometry and calculus.

D. TALL:

Mathematical Growth

This presentation discusses research that has studied the growth of knowledge in the individual and the roles of visualisation and symbolisation. Building from enaction with the environment, two modes of development occur, one is visual (or figural) in which objects such as geometric figures are given meaning through construction, discussion and definition leading on to Euclidean proof, the other uses symbolism in arithmetic, algebra, calculus etc. which has the power to represent both processes to do mathematics and concepts to think about. The student developing the latter cognitive structure has a more powerful and simpler way of solving complex problems than the student who copes with separate procedures for each task. The difficulties encountered in different types of symbolism and the curriculum implications for different students are discussed.

B. WOLLRING:

Qualitative empirical analyses on the conceptions of probability of preparatory- and elementary-school children

In a research project we analyze preparatory- and elementary-school children's conceptions within stochastic situations, making use of the procedure of interpretative interaction analysis. The disposition is competence-oriented and not deficit-oriented. On the basis of documented results on children's and adult's stochastic conceptions, especially the analysis of heuristics (KAHNEMAN and TVERSKY), intuitive conceptions (FISCHBEIN), control illusions (LANGER) and animistic conceptions (WOLLRING), as well as classical field-studies with standardised problem-settings (FISCHBEIN, FALK), we use video-documented laboratory interviews, namely "game-interviews" as experiment design, in which children have to make decisions and comment on those within familiar risk situations. The stochastic situations are represented in various ways, based on the assumption that stochastic conceptions are being built up within "subjective domains of experience" (BAUERSFELD). As far as the evaluation is concerned, the video-tapes and specially developed types of transcripts - of varying documental density - are used. The interviews are being evaluated with the help of consentaneous interpretative validation of suitable transcripts by the team, focused by didactic, psychological and sociological categories, which themselves influence the organisation of the documents. The following results were found: Preparatory- and elementary-school children's stochastic conceptions are, when provoked within risk situations, strongly determined by subjective experiences and they are interspersed with control illusions and animistic conceptions. As a rule, a priori-problems, due to the lack of a repertoire of arithmetic strategies, are dealt with by making use of compensational strategies; this confirms known results. Remarkable competencies can be found in the frequentative field: Within their own limited range of articulation children show seeds of argumentation in order to estimate parameter and in order to put hypotheses to a test.

LIST OF QUESTIONS

The following research, curricular, and pedagogical questions arose in response to the presentations given at this conference. They represent some of the important issues and problems that the participants jointly agree should be studied. The list is far from complete and should not be interpreted as an attempt to put forth a research program or agenda. Neither is it claimed that these questions are original or of equal importance. We only hope that they will serve to stimulate workers in the field to obtain new results and to improve the learning of mathematics by students throughout the world.

1. What are appropriate methodologies for answering curricular and pedagogical questions?

2. Are learning theories transferable across cultural and subject matter boundaries? Can they be applied to different topics and different groups of students in different countries?

3. What are the different learning styles for mathematics that are prevalent among post-secondary students? How do these learning styles relate to various theories of learning? How immutable is the learning style of an individual student?

4. What are the differences between how mathematics is learned by experts and by novices of different kinds?

5. What do faculty and students mean by the word "understanding"? What is meant by "clarity"? What is the relationship between clarity and precision in the minds of students and faculty?

6. Do the tools of technology change students' understanding of mathematics, and if so how? For example: some people argue that

learning geometry with a software package does not promote the same understanding of geometry as learning in a paper and pencil environment. How can we transform this claim into a research question and what methodology can be developed to investigate this question?

7. What are the student conceptions of the different notions of equality and approximate equality? How are these conceptions affected by technology?

8. What are the difficulties that students have with formal mathematical language such as the use of "for all," "there exists," two-level quantifiers, and negation, and with the relationship of formal mathematical language to everyday language?

9. Why is the concept of a solution to a differential equation difficult? What is the nature of that difficulty? In particular, do students find it difficult to understand-symbolically, graphically, and visually-what it means to be a solution to a differential equation or initial value problem?

10. What pedagogical strategies can be effective in helping students understand the systematic development of mathematical theories?

11. How can we most effectively teach students to use definitions as a mathematician does, and in particular to turn a definition into "an operative form"?

12. What is the relationship between time spent on mathematics outside of class and the level of student understanding? What pedagogical strategies are most effective in improving the quantity and quality of the time students spend on mathematics?

13. What course designs and pedagogical strategies are most effective in taking into account the wide range of abilities and backgrounds of the students?

14. What are the pedagogical advantages and disadvantages of the different ways in which technology can be used? Among these are visualization, the use of built-in mathematical tools, and programming.

15. How does class size affect learning? How is this affected by technology and cooperative learning? What group sizes in cooperative learning best support learning?

16. What are the advantages and disadvantages of using applications from both inside and outside mathematics and of using history? Do they improve the students' retention of the mathematics and/or the retention of the students in mathematics? What is their effect on understanding, and the appreciation of mathematics both for its internal beauty and its usefulness?

17. What form or forms of proof are appropriate in different contexts for student learning and how should they be dealt with pedagogically?

18. What algebra is appropriate as preparation for post-secondary work? How is the answer affected by subject? How is it affected by technology?

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