MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 49/1995

Topologische Methoden in der Gruppentheorie

3. bis 9. Dezember 1995

Die Tagung fand unter der Leitung von Herbert Abels (Bielfeld) und Peter H. Kropholler (Queen Mary and Westfield College, London) statt. 45 Teilnehmer aus Deutschland, Frankreich, Griechenland, Großbritannien, Israel, Rußland, Schweden, der Schweitz und den USA waren der Einladung gefolgt; es wurden insgesamt 31 Vorträge gehalten. Die folgenden Schlagworte geben einen Überblick über die Vielfalt der Themen.

Automorphismen von freien Gruppen
Coxetergruppen und Artingruppen
Endliche Gruppen
Endlichkeitsbedingungen
Geometrische und asymptotische Invarianten
Gruppenoperationen auf topologischen und metrischen Räumen
Kombinatorische Gruppentheorie
Negative und nicht positive Krümmung
Theorie, Berechnungen und Anwendungen von Gruppenkohomologie, z.B.
- Vollständige Kohomologie
- Beschränkte und ℓ^2-Kohomologie

Trennungseigenschaften
Zwillingsgebäude

Die Tagungsleiter bemühten sich, neben den vielen Vorträgen auch Zeit zum freien Gedankenaustausch zu schaffen. Dadurch konnten leider nicht alle Teilnehmer über ihre Arbeit referieren; deshalb sind in diesem Bericht auch Vorträge aufgeführt, die nicht gehalten werden konnten. Für die freundliche und großzügige Atmosphäre des Hauses sei allen Mitarbeitern herzlich gedankt.

Abstracts

Peter Abramenko: Twin buildings and finiteness properties of certain S-arithmetic groups.

Definition. The finiteness length $\varphi(\Gamma)$ of a group Γ is defined by $\varphi(\Gamma) := \sup\{l \in \mathbb{N}_0 \mid \Gamma \text{ is of type } F_l\}$.

Question. Given a reductive group $\mathcal{G}$ over a global function field K and an S-arithmetic subgroup Γ of $\mathcal{G}(K)$, e.g. $\Gamma = \mathcal{G}(\mathcal{O}_S)$, what is $\varphi(\Gamma)$?

This turns out to be a hard problem in general. The following is known:

- a complete classification of all finitely presented Γ's (H. Behr 91),
- $\varphi(\mathrm{SL}_2(\mathcal{O}_S) = \#S - 1$, (U. Stuhler 80)
- $\varphi(\mathrm{SL}_{n+1}(\mathbb{F}_q[t])) = n-1$ if $q \geq \binom{n-1}{[\frac{n-1}{2}]}$ (Abels/A. 87)
- $\varphi(\mathcal{G}(\mathbb{F}_q[t])) = n-1$ and $\varphi(\mathcal{G}(\mathbb{F}_q[t,t^{-1}])) \geq n-1$, provided that $\mathcal{G}$ is an almost simple classical $\mathbb{F}_q$-group of $\mathbb{F}_q$-rank $n \geq 1$ and $q \geq 2^{2n-1}$ (A. 92/95)

The geometric background (action of $\Gamma = \mathcal{G}(\mathbb{F}_q[t])$ on the corresponding Bruhat-Tits building Δ_+) in this last situation is significantly better understood if one considers the action of $G = \mathcal{G}(\mathbb{F}_q[t,t^{-1}])$ on the associated twin building $\Delta = (\Delta_+, \Delta_-)$. Observing that $\Gamma = \mathrm{Stab}_G(O_-)$ for a vertex $O_- \in \Delta_-$, finiteness properties can be deduced from the following general

Theorem. Let G be a group acting "nicely" on a twin building $\Delta = (\Delta_+, \Delta_-)$. Suppose that Δ_+, Δ_- are thick locally finite, n-dimensional buildings and that the (spherical) links occurring in Δ_+, Δ_- are "sufficiently large" and not of exceptional type. Then $\varphi(G) \geq 1$ and $\varphi(\Gamma) = n-1$ for any $\Gamma = \mathrm{Stab}_G(a_-)$, $\varnothing \neq a_- \in \Delta_-$.

The following (very recent) example shows that the condition "sufficiently large" can not be dropped in general:

Counterexample. Let $\mathcal{G}$ be a (minimal, split) Kac-Moody group of rank 3, such that the corresponding Coxeter numbers are $(4,4,4)$. If Γ is a proper parabolic subgroup of $G = \mathcal{G}(\mathbb{F}_2)$, then $\varphi(\Gamma) = 0$.

Alejandro Adem: Automorphisms and cohomology of discrete groups.

Let Γ denote a discrete group of finite cohomological dimension. We outline a method for producing cohomology classes using finite automorphism groups. If $P \subset_\varphi \mathrm{Aut}(\Gamma)$ is a finite p-group, let $\bar{\Gamma} = \Gamma \times_\varphi P$. Let $\bar{P}$ denote a finite subgroup of Γ mapping onto P via standard projection. Then we have (using total dimensions)

Theorem. $\dim H^*(\Gamma, \mathbb{F}_p) \geq \sum_{H^1(P,\Gamma)} \dim H^*(Z_\Gamma(\bar{P}), \mathbb{F}_p)$.

Applications to congruence subgroups illustrate the usefulness of this formula.

Juan Alonso: Homological hyperbolicity.

In joint work with W. Bogley, R. Burton, S. Pride and X. Wang, we have defined, for a group G of type F_{m+1} ($m \geq 1$), homotopical Dehn functions δ_G^t ($1 \leq t \leq m$) (where δ_G^1 is the "classical" Dehn function used by Gromov to define hyperbolic groups) and proved that these are invariant under quasi-isometry and, more generally, under quasi-retractions.

For a group G of type FP_{m+1} ($m \geq 1$) I have developed a corresponding theory of homological Dehn functions Δ_G^t ($1 \leq t \leq G$) which enjoy similar invariance under quasi-isometries and -retractions as the δ_G^t do. The relationship between these Dehn functions is:

Proposition. Suppose that G is a group of type F_{m+1}. Then

(i) $\Delta_G^1 \preceq \bar{\delta}_G^1$, (the "subnegative closure" of δ_G^1)

(ii) $\delta_G^2 \preceq \Delta_G^2$,

(iii) $\delta_G^t = \Delta_G^t$ for all $t \geq 3$.

Definition. A group G of type FP_2 is called homologically hyperbolic if Δ_G^1 is linear.

Theorem. A group is hyperbolic iff it is homologically hyperbolic.

Johathan Alperin: Variations on a theme of Mislin.

Guido Mislin has given a remarkable connection between group structure and cohomology isomorphisms: Let k be a field of prime characteristic p and let H be a subgroup of the finite group G; the restriction map of $H^*(G,k)$ to $H^*(H,k)$ is an isomorphism if, and only if, H contains a Sylow p-subgroup of G and whenever Q is a p-subgroup of H then $N_G(Q)/C_G(Q) \cong N_H(G)/C_H(G)$.

The talk is concerned with analogous results motivated by this theorem and, in particular, establishes necessary and sufficient conditions for the principal blocks of G and H to be stably equivalent via restriction and truncation to the principal block.

Dave Benson: Finite group actions on products of spheres.
(Joint work with A. Adem)

Theorem. Let G be an elementary abelian p group of rank r acting freely on a finite dimensional CW complex $X \simeq (S^n)^t$, in such a way that the basis $u_1, \ldots, u_t$ of $H^n(X; \mathbb{F}_p)$ corresponding to the t spheres is permuted by G. Then the number of orbits of G on $\{u_1, \ldots, u_t\}$ is at least r. Moreover, the mod p cellular chains on X are chain homotopy equivalent to a tensor product of complexes corresponding to the orbits. The complex corresponding to an orbit is tensor induced from a complex for the stabilizer, with the homology of a single n-sphere.

Robert Bieri: On the finiteness length in direct products.

By the finiteness length $\mathrm{fl}\, G$ of a group G I mean the smallest integer m (or ∞) with G not of type F_m. I have reported on recent progress by H. Meinert and R. Gehrke in the problem of computing $\mathrm{fl}\, N$ of a normal subgroup $N \triangleleft G$ with G/N Abelian, when G is a direct product of 3-manifold groups or one-relator groups.
The method is based on computing the BNSR-geometrical invariant $\Sigma^m(G)$. Define a map $\tilde{\mathrm{fl}} : \mathrm{Hom}(G, \mathbb{R}) \to \mathbb{N}_0 \cup \{\infty\}$ by putting $\tilde{\mathrm{fl}}(0) = 0$ and $\tilde{\mathrm{fl}}(\chi) = \inf\{m \mid \chi \notin \Sigma^m(G)\}$. The result is then obtained by a surprising additivity result for $\tilde{\mathrm{fl}}$ in a direct product (which is not available for fl!) and the formula $\mathrm{fl}(N) = \inf\{\tilde{\mathrm{fl}}(\chi) \mid \chi : G \to \mathbb{R},\ \chi(N) = 0\}$.

Martin Bridson: 2-complexes, towers and subgroups of $F \times F$.

One can say considerably more about 2-complexes of non-positive curvature than about arbitrary spaces of non-positive curvature. Moreover, there are many examples of such 2-complexes that are interesting from the point of view of group theory. In this talk I shall discuss some of these examples and then describe a construction peculiar to dimension 2 and use it to give a geometric proof of the Baumslag-Roseblade characterisation of finitely presented subgroups of $F \times F$, where F is a free group of finite rank.

Mike Davis: Bestvina's and Brady's examples of groups of type FP which are not finitely presented.

These examples are of the following form: G is a right angled Artin group and H is the kernel of a homomorphism from G to $\mathbb{Z}$. For appropriate choices of G, H will be of type FP but not finitely presented.

Mike Davis: The cohomology of a Coxeter group with group ring coefficients.

Let (W, S) be a Coxeter system and let $L(W, S)$ be the geometric realization of the poset of those subsets T of S which generate a finite subgroup. There is a formula for $H^*(W; \mathbb{Z}W)$ in terms of the cohomology of $L(W, S)$ and certain of its subcomplexes. It follows from this that W is a virtual Poincaré duality group if and only if W splits as $W = W_0 \times W_1$, where W_1 is finite and $L(W_0, S_0)$ is a homology $(n-1)$-manifold with the same homology as S^{n-1}.

Thomas Delzant: The isomorphism problem for hyperbolic groups.

Let $P := \langle a_1, \ldots, a_r; R_1, \ldots, R_s \rangle$ and $Q := \langle b_1, \ldots, b_n; S_1, \ldots, S_t \rangle$ be two abstract presentations. Suppose one already knows a solution to the word problem for these presentations. It is obvious how to decide whether the two defined groups are isomorphic or not, if one has a priori knowledge of a constant C, such that for some isomorphism $\varphi: P \to Q$ the following holds:

$$|\varphi(a_i)|_B \leq C \quad \text{and} \quad |\varphi^{-1}(b_j)|_A \leq C \quad \text{for all } i \text{ and } j,$$

where $|\ |_A$ is the word metric in the alphabet a_i, and vice versa.

The aim of the talk is to explain how hyperbolic geometry can provide such an estimate. We will discuss in detail the case of a free group. One should also mention that Sela already gave a solution to this problem, at least for hyperbolic groups with finite outer automorphism group; however, the presented solution is stronger, because it provides an effective solution rather than a Turing machine.

Beno Eckmann: Groups of type FP and Euler characteristics.

Let G be a group of type FP, i.e. admitting a resolution P_* over $\mathbb{Z}G$ of finite length n and with all P_i finitely generated projective $\mathbb{Z}G$ modules. Question: can one express the (homological) Euler characteristic $\chi(G) = \sum_0^n (-1)^i \beta_i(G)$, where $\beta_i(G) = \dim_{\mathbb{R}} H_i(G;\mathbb{R})$ is the i-th Betti number of G, by ℓ_2-Betti numbers $\bar\beta_i$ like in the case where the resolution is free over $\mathbb{Z}G$? The answer is "yes" if G has the property

(∗) G fulfills the strong Bass conjecture, or G is residually finite.

Our procedure actually applies, more generally, to any FP-complex P_* over $\mathbb{Z}G$ (and therefore to a space dominated by a finite cell-complex). It is based on the

Theorem. For a finitely generated projective $\mathbb{Z}G$-module P, the Hilbert-G-module $\ell_2 G \otimes_G P$ is, under the condition (∗), isometrically G-isomorphic to $\ell_2 G^{\operatorname{rk} P}$, where $\operatorname{rk} P$ is the "naive" rank $\dim_{\mathbb{R}}(\mathbb{R} \otimes_G P)$ of P.

Thus, under condition (∗), $\ell_2 G \otimes_G P$ is a complex of free Hilbert-G-modules of ranks $\operatorname{rk} P_i$. If $\bar\beta_i$ is the von Neumann dimension $\dim_G \bar H_i$, reduced (co-)homology of that complex, then the standard Euler-Poincaré argument yields

$$\sum_0^n (-1)^i \bar\beta_i = \sum_0^n (-1)^i \operatorname{rk} P_i = \sum_0^n (-1)^i \beta_i = \chi.$$

Ross Geoghegan: Kernels of actions on CAT(0) spaces.
(Joint work with R. Bieri)

We look at exact sequences of groups

$$1 \to N \to G \xrightarrow{p} \operatorname{Isom}(M),$$

where M is a CAT(0) space whose geodesic segments can be prolonged to geodesic lines. We give geometrical conditions, in terms of the action of G on M, necessary and sufficient for N to have the finiteness property F_n, assuming that G has it too. Among other things we show that $N = N(p)$ having type F_n is an open condition on the space of representations p. This generalizes the work of Bieri, Neumann, Strebel and Renz.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Henry Glover: The p-rank one cohomology of $\mathrm{Out}(F_n)$.

We compute the first few cases of the p-rank one Farrell cohomology of $\mathrm{Out}(F_n)$. In particular we show

$$\hat{H}^*(\mathrm{Out}\,F_{p-1};\mathbb{Z})_{(p)} \cong \hat{H}^*(\Sigma_p;\mathbb{Z})_{(p)}$$
$$\hat{H}^*(\mathrm{Out}\,F_p;\mathbb{Z})_{(p)} \cong 2\hat{H}^*(\Sigma_p;\mathbb{Z})_{(p)}$$
$$\hat{H}^*(\mathrm{Out}\,F_{p+1};\mathbb{Z})_{(p)} \cong 3\hat{H}^*(\Sigma_p;\mathbb{Z})_{(p)} \oplus H$$

where H is a sum of $\hat{H}^*(D_{2p};\mathbb{Z})_{(p)}$ and $\hat{H}^*(M_{4p};\mathbb{Z})_{(p)}$, depending on cases mod 20. Here M_{4p} denotes an extension of D_{2p} by $\mathbb{Z}/2$, and $(\ )_{(p)}$ denotes the p-primary component.

Rostislav I. Grigorchuk: Bounded cohomology of discrete groups.

In the first part of the talk we give a survey of results on the bounded cohomology of discrete groups. In particular we explain the rôle of amenable groups and pseudo-characters in the theory. We give a description of the second ℓ^1-homology group $H_2^{\ell^1}(G)$ and the second bounded cohomology group $H_b^2(G)$ in terms of spaces of 2-chains and 2-cochains. This leads to the following

Theorem. If $\varphi : G \to H$ is an epimorphism of groups, then $\hat{\varphi} : H_b^2(H) \to H_b^2(G)$ is injective and $H_2^{\ell^1}(G) \to H_2^{\ell^1}(G)$ is surjective.

Conditions for the nonvanishing of $H_b^2(G)$ are given for groups G which are free products with amalgamation, HNN extensions or one relator groups. We also establish a connection between the vanishing of the averaged width and the vanishing of $H_b^2(G)$

Jens Harlander: Embedding into efficient groups.

A group G is called efficient if it admits a finite presentation $\langle x_1,\dots x_n \mid r_1,\dots r_m\rangle$ with

$$n - m = r(H_1(G)) - d(H_2(G)),$$

where r denotes the torsion free rank and d is the minimal number of generators. We study embedding questions in connection with efficiency. Our main result is the following:

Theorem. Every finitely generated group G can be embedded into some efficient group H. If G is finite, then H can be taken to be finite. If the cohomological dimension of G is 2, one can find H with virtual cohomological dimension 2, otherwise one can arrange $\mathrm{cd}\,H = \mathrm{cd}\,G$.

Hans-Werner Henn: The mod 2 cohomology of $\mathrm{SL}(3,\mathbb{Z}[\frac{1}{2}])$.

We outline a proof of the following result.

Theorem. $H^*(\mathrm{SL}(3,\mathbb{Z}[\frac{1}{2}]);\mathbb{F}_2) \cong \mathbb{F}_2[w_2,w_3] \otimes E(e_3,e_5)$.

(E denotes the exterior algebra, indices give the degrees of the corresponding classes.)

The method of proof is to study $E\Gamma \times_\Gamma (X_\infty \times X_2)$. Here $\Gamma = \mathrm{SL}(3, \mathbb{Z}[\frac{1}{2}])$, X_∞ is the symmetric space $\mathrm{SO}(3)\backslash \mathrm{SL}(3, \mathbb{R})$ and X_2 is the Bruhat-Tits building associated to $\mathrm{SL}(3, \mathbb{Q}_2)$. The "standard approach" of studying the spectral sequence of the map $E\Gamma \times_\Gamma (X_\infty \times X_2) \to (X_\infty \times X_2)/\Gamma$ is too complicated to be carried out. Instead, we use a spectral sequence converging to $H^*_\Gamma((X_\infty \times X_2)_S; \mathbb{F}_2)$, where $(\ )_S$ is the "2-singular part", which has $H^*(\mathrm{GL}(2, \mathbb{Z}[\frac{1}{2}]); \mathbb{F}_2)$ and $H^*(\mathbb{Z}[\frac{1}{2}]^\times \times \mathbb{Z}[\frac{1}{2}]; \mathbb{F}_2)$ as input.

Then we analyze

$$H^*(X_\infty \times X_2, (X_\infty \times X_2)_S; \mathbb{F}_2) \cong H^*((X_\infty \times X_2)/\Gamma, (X_\infty \times X_2)_S/\Gamma; \mathbb{F}_2)$$

and the long exact sequence for the pair $(X_\infty \times X_2, (X_\infty \times X_2)_S)$.

Peter H. Kropholler: Groups with Eilenberg-MacLane complexes of finite type.

Let $\mathrm{H}\mathcal{F}$ be the smallest class of groups containing all finite groups with the property that if G acts on a finite dimensional contractible CW-complex with stabilizers in $\mathrm{H}\mathcal{F}$ then G itself belongs to $\mathrm{H}\mathcal{F}$. In joint work with Guido Mislin, we have shown that an $\mathrm{H}\mathcal{F}$-group G with an Eilenberg-MacLane space of finite type possesses a finite dimensional model for the classifying space $\underline{E}G$; that is, there is a G-CW-complex X with finite isotropy such that X^H is contractible for all finite $H \leq G$.

The construction begins with the poset of non-trivial finite subgroups and depends in a crucial way on homological results proved earlier in joint work with Jonathan Cornick.

Ian Leary: On the cohomology of Coxeter groups.

Using techniques of M. Davis, M. Bestvina was able to exhibit a group Γ such that $\mathrm{cd}_{\mathbb{Z}}\,\Gamma = \mathrm{cd}_{\mathbb{F}_2}\,\Gamma = 3$ and $\mathrm{cd}_{\mathbb{Q}}\,\Gamma = 2$, as a finite-index subgroup of a finitely generated Coxeter group. Warren Dicks and I have considered the non-finitely generated case. We characterise the Coxeter groups that have finite virtual cohomological dimension, and give some information about cohomology of these groups. Examples outlined in this way include

1) A group Γ_1 acting with stabilizers of orders 1, 2 and 4 on a contractible 2-complex, but having no finite-index torsion-free subgroups.
2) A group Γ_2 such that $\mathrm{cd}_{\mathbb{Z}}\,\Gamma_2 = 3$ but $\mathrm{cd}_{\mathbb{F}}\,\Gamma_2 = 2$ for all fields $\mathbb{F}$.

The groups Γ_1 and Γ_2 may be taken to be 2-generator groups by applying the HNN embedding theorem to the non-finitely generated examples occurring as finite-index subgroups of Coxeter groups.

Martin Lustig: The index of a free group automorphism and the Scott conjecture.
(Joint work with D. Gaboriau, A. Jäger, G. Levitt)

Let F_n be a free group of finite rank $n \geq 2$, let $\alpha \in \mathrm{Aut}(F_n)$ and let $\partial\alpha$ be the induced homeomorphism on the Gromov boundary ∂F_n. Let $\mathrm{Fix}(\alpha) := \{w \in F_n \mid \alpha(w) = w\}$ and $\mathrm{Fix}(\partial\alpha) := \{X \in \partial F_n \mid \partial\alpha(X) = X\}$.

Corollary. If $\mathrm{Fix}(\alpha) = \{1\}$, then $\#\,\mathrm{Fix}(\partial\alpha) \leq 4n$.

Let $\mathrm{Fix}_-(\partial\alpha)$ denote the set of all $\partial\alpha$-attractors in $\mathrm{Fix}(\partial\alpha)$. One can show

$$\mathrm{Fix}(\partial\alpha) = \mathrm{Fix}_-(\partial\alpha) \dot{\cup} \mathrm{Fix}_-(\partial\alpha^{-1}) \dot{\cup} \mathrm{Fix}(\partial\alpha|_{\mathrm{Fix}(\alpha)}).$$

Notice that $\mathrm{Fix}(\alpha)$ acts canonically on $\mathrm{Fix}(\partial\alpha)$, preserving the above partition.

Definition. $\mathrm{ind}(\alpha) := \mathrm{rk}(\mathrm{Fix}(\alpha)) + \frac{1}{2}\#(\mathrm{Fix}_-(\partial\alpha)/\,\mathrm{Fix}(\alpha)) - 1$.

Theorem 1. All $\alpha \in \mathrm{Aut}(F_n)$ satisfy $\mathrm{ind}(\alpha) \leq n - 1$.

Notice that this reproves the Scott conjecture $\mathrm{rk}(\mathrm{Fix}(\alpha)) \leq n$, due to Bestvina and Handel.

Definition. Denote by $\hat{\alpha} \in \mathrm{Out}(F_n)$ the canonical image of $\alpha \in \mathrm{Aut}(F_n)$.

$$\mathrm{ind}(\hat{\alpha}) := \max\left\{\sum_{i\in I} \mathrm{ind}(\alpha_i) \mid \hat{\alpha}_i = \hat{\alpha},\ \alpha_i \neq I_w^{-1}\alpha_j I_w\ \forall w \in F_n\right\},$$

where $I_w : v \mapsto w^{-1}vw\ \forall v \in F_n$.

Theorem 1'. $\mathrm{ind}(\hat{\alpha}) \leq n - 1$ for all $\hat{\alpha} \in \mathrm{Out}(F_n)$.

The proof crucially uses (a) the existence of a "good" α-fixed point in the closure of outer space, (b) Gaboriau-Levitt's analysis of the index for free group actions on trees, and a new concept for such actions, called Bounded Back Tracking.

Wolfgang Lück: L^2-invariants and applications to group theory.

We give the basic definition and properties of L^2-Betti numbers of spaces and groups. We use them to prove the following results.

Theorem. Let $1 \to \Delta \to \Gamma \to \Pi \to 1$ be an extension of groups, such that Δ is finitely generated and infinite, Γ is finitely presented and Π contains an element of infinite order. Then

a) $b_1^{(2)}(\Gamma) = 0$.

b) $\mathrm{def}(\Gamma) := \max\{g - r \mid \Gamma \cong \langle s_1, \ldots, s_g \mid R_1, \ldots, R_r\rangle\} \leq 1$.

c) If M is an oriented closed 4-manifold group with $\pi_1(M) \cong \Gamma$ then $|\mathrm{signature}\,(M)| \leq \chi(M)$.

Let F be the Thompson group of orientation preserving PL-automorphisms of $[0,1]$ whose slopes are powers of 2 and whose breaks are contained in $Z[\frac{1}{2}]$. This group is not elementary amenable, does not contain $\mathbb{Z} \times \mathbb{Z}$ as subgroup and BF is of finite type. Hence it is an interesting question whether F is amenable. A necessary, but not sufficient condition is $b_p^{(2)}(F) = 0$ for all p. We show

Theorem. $b_p^{(2)}(F) = 0$ for all $p \geq 0$.

Shahar Mozes: Word metric and Riemannian metric on lattices in semisimple groups. (Joint work with A. Lubotzky, M.S. Raghunathan)

Let G be a semisimple Lie group and $\Gamma < G$ an irreducible lattice. There are two natural classes of metrics on Γ:

d_W - the word metric with respect to some finite set of generators,

d_R - the restriction to Γ of a left invariant metric on G induced from the Riemannian metric on the symmetric space G/K, where $K < G$ is a maximal compact subgroup.

We prove the following theorem conjectured by V. Kazhdan:

Theorem 1. If $\operatorname{rank} G \geq 2$ then the two metrics are equivalent, i.e. there exists a $C > 0$ such that for every $\gamma, \gamma' \in \Gamma$

$$C^{-1}\, d_R(\gamma,\gamma') \leq d_W(\gamma,\gamma') \leq C\, d_R(\gamma,\gamma').$$

Definition. Let Γ be a finitely generated group. An element $\gamma \in \Gamma$ is called a U-element if it is of infinite order and $d_W(\gamma^n, e) = O(\log n)$.

The proof of Theorem 1 is based on

Theorem 2. When G has rank ≥ 2 and $\Gamma < G$ is an irreducible lattice, then an element $\gamma \in \Gamma$ is a U-element if and only if it is virtually unipotent, i.e. γ^n is unipotent for some $n \geq 1$.

Note that Theorem 2 provides group a theoretic criterion for recognizing (virtually) unipotent elements in irreducible lattices in a semisimple Lie group of higher rank.

Gena Noskov: Non-combability of Hilbert modular groups.

The talk is concerned with the problem of describing all S-arithmetic groups which admit a combing in the sense of the book "Word processing and group theory" by Epstein et al. The result of Epstein-Thurston is that $\mathrm{SL}_n(\mathbb{Z})$ is non combable when $n \geq 3$. This is based on the reduction theory for $\mathrm{SL}_n(\mathbb{Z})$ and the higher isoperimetric inequalities for combable groups acting properly and cocompactly on Riemannian manifolds. We show how one can transfer the Epstein-Thurston method to the case of Hilbert modular groups $\mathrm{PSL}_2(\mathcal{O})$, where $\mathcal{O}$ is the ring of integers of a totally real field K of odd degree.

Steve Pride: Second order Dehn functions on groups (and monoids).

Let G be a finitely presented group of type FP_3. Let $\mathcal{P}$ be a finite presentation of G and let A be a finite generating set of $\pi_2(\mathcal{P})$. An element ξ of $\pi_2(\mathcal{P})$ can be represented by a spherical diagram over $\mathcal{P}$, and we let Area(ξ) be the number of regions in a minimal diagram representing ξ. Now ξ has an expression

$$\xi = \sum_{i=1}^{n} \varepsilon_i g_i \xi_i \qquad (\varepsilon_i = \pm 1,\ g_i \in G,\ \xi_i \in A) \tag{$*$}$$

We let $\mathrm{Vol}_A(\xi)$ be the minimal volume of n over the expressions ($*$) for ξ. Then $\delta^2_{\mathcal{P},A}(n) = \max\{\mathrm{Vol}_A(\xi) \mid \mathrm{Area}(\xi) \leq n\}$. Up to equivalence this is independent of the choice of finite representation $\mathcal{P}$ for G and finite generation set A of $\pi_2(\mathcal{P})$. We may therefore refer to δ^2_G — the second order Dehn function for G. This is a quasi-isometry invariant.

In my talk I will describe some results concerning the calculation of this function. Some of this work is contained in a joint paper with J. Alonso, W. Bogley, R. Burton and X. Wang, and some is recent work of my own.

Mark Ronan: Twin trees.

This talk discussed some recent progress by the speaker and J. Tits in the study of twin trees. These objects, which were introduced by Tits and Ronan, arose originally from Kac-Moody groups of rank 2, but it has since become clear that examples arise from other sources.

The talk outlined some of the results in Twin Trees I (Inventiones 1994) by Ronan and Tits, and went on to describe the forthcoming paper Twin Trees II by the same authors. In particular, for any tree T, a graph T^0 was introduced. The vertices are "virtual vertices" in trees twinned with T.

Results showing that any semi-homogeneous tree T admits 2^α non-isomorphic twinnings (α being the cardinality of the vertex set of T) were discussed, and it was described how trees twinned with T could be constructed as subgraphs of T^0 (above).

Jim Roseblade: Finite presentability and wedge products.

Two closely related theorems (proved in collaboration with C.J.B. Brookes and J.S. Wilson) were discussed. The first asserted that a finitely presented Abelian by polycyclic group must have a metanilpotent subgroup of finite index. Since not every finitely generated Abelian by polycyclic group is like this, it follows that the Baumslag-Remeslennikov theorem on embedding finitely generated metabelian groups in finitely presented ones does not extend to Abelian by polycyclic groups. The second theorem concerned a finitely generated kH-module V, where k is a finite field and H is a polycyclic group: If, for some $r \geq 2$, $\bigwedge^r V$ is finitely generated, then $H/C_H(V)$ must be virtually nilpotent. The methods, unfortunately perhaps, were neither topological nor geometrical.

Hamish Short: Brady's example of non-hyperbolic subgroup of a word hyperbolic group.

Gromov's word hyperbolic groups are finitely presented groups, generalizing fundamental groups of compact hyperbolic manifolds. Rips showed that they may have finitely generated subgroups which are not finitely presented (so not word hyperbolic).

Noel Brady has recently constructed an example of a finitely presented subgroup H of a word hyperbolic group G such that H is not word hyperbolic. His construction uses Brady's and Bestvina's recent work on Morse theory for complexes, CAT(0) spaces, cubical complexes, and branched coverings.

I will attempt to describe the principal steps in Brady's construction. If there is enough time, I shall show that any subgroup constructed in this way satisfies a polynomial isoperimetric inequality (a group is word hyperbolic if and only if it satisfies a linear isoperimetric inequality).

Michael Stoll: Rational and transcendental growth series of the higher Heisenberg groups.

It is shown that for any discrete higher Heisenberg group H_n (of length $2n+1$), there is a finite generating set, such that the associated growth series is a rational power series. On the other hand, we prove that for $n \geq 2$, H_n also admits a finite generating set such that the associated power series is transcendental. Specifically, H_2 has a transcendental growth series with respect to its standard (minimal) generating set. Both results hold generally for 2-step nilpotent groups G with $[G,G] \cong \mathbb{Z}$.

Karen Vogtman: The degree of graphs and rational homology of $\mathrm{Aut}(F_n)$.
(Joint work with A. Hatcher)

Inside the space A_n of basepointed marked graphs with fundamental group F_n, we consider the subspace $A_{n,k}$ of graphs of degree at most k, where the degree of a graph is defined to be 2n minus the valence of the basepoint. We show that $A_{n,k}$ is $(k-1)$-connected. Since $\mathrm{Aut}(F_n)$ acts with finite stabilizers, the quotient $A_{n,k}/\mathrm{Aut}(F_n)$ can be used to study the rational homology of $\mathrm{Aut}(F_n)$. This leads to a proof that $H_i(\mathrm{Aut}(F_n);\mathbb{Q})$ is independent of n, for $n > \frac{3}{2}i$, and to computations of $H_i(\mathrm{Aut}(F_n);\mathbb{Q})$ for $i \leq 5$.

Peter Webb: Mackey functors and highest weight categories.

Over a field of characteristic zero, certain generalized Mackey functors form a highest weight category, and in general over a field of characteristic p the projective functors have a filtration by certain precisely described functors which in characteristic zero are the Weyl objects in the highest weight category. The importance of this lies in the fact that there are significant examples of these functors — such as group cohomology with trivial coefficients. The theory also gives information about stable maps $[BG, BH]$, showing that certain algebras constructed from these maps are quasihereditary.

John Wilson: Conjugacy separability of certain Bianchi groups and HNN extensions.

A group G is conjugacy separable if whenever a, b are non-conjugate elements there is a normal subgroup N of finite index such that Na, Nb are non-conjugate in G/N. Some sufficient conditions for amalgamated free products of virtually free groups and HNN extensions of virtually free or virtually polycyclic groups to be conjugacy separable were described, and the significance in the proofs of properties of associated profinite groups and profinite trees were discussed briefly.

Pavel Zalesskii: Conjugacy separability of free products with cyclic amalgamation.

A group is conjugacy separable if for any non-conjugate elements x and y of G there exists some finite quotient of G in which the images of x and y are non-conjugate.

Theorem. (L. Ribes, D. Segal, P. Zalesskii) Let $\mathcal{X}_1$ be the class of all groups that are either free-by-finite or polycyclic-by-finite. For $i > 1$ define recursively the class $\mathcal{X}_i$ to consist of all groups that are free products $G = G_1 *_H G_2$ of groups G_1, G_2 in $\mathcal{X}_{i-1}$ with cyclic amalgamated subgroup H. Then any group in the class $\mathcal{X}' = \bigcup_{i=1}^{\infty} \mathcal{X}_i$ is conjugacy separable.

This theorem answers a question of C.Y. Tang (Problem 8.70 of the Kourova Notebook), whether a free product of polycyclic-by-finite groups with cyclic amalgamation is conjugacy separable.

Further Abstracts

Herbert Abels: Compactness properties of locally compact groups.
(Joint work with A. Tiemeyer)

We introduce a series of compactness properties C_n and CP_n for locally compact groups, which generalize the finiteness properties F_n and FP_n. Here C_1 is compact generation, C_2 is compact presentability. The main result, due to A. Tiemeyer, is that for S-arithmetic groups $\Gamma = G(\mathfrak{o}_S)$, G an *arbitrary* algebraic group defined over a number field k, the group Γ is of type F_n (FP_n) iff $G(k_\mathfrak{p})$ is of type C_n (CP_n) for every local field $k_\mathfrak{p}$, $\mathfrak{p} \in S$. The problem now is to gain a better understanding of the properties C_n and CP_n and to see which (solvable) G have these properties.

Kai-Uwe Bux: Finiteness properties of nice soluble S-arithmetic groups over function fields.

Let $\mathcal{O}_S$ be an S-arithmetic ring in a global function field K. Then there is the following

Example. The group $\Gamma := \mathrm{B}_{n+1}(\mathcal{O}_S)$ of upper triangular matrices in $\mathrm{SL}_{n+1}(K)$ for $(n \geq 1)$ is of type $FP_{|S|-1}$ but not of type $FP_{|S|}$.

The number field case is completely different. Here the group Γ is of type FP_∞. This is implicitly contained in the thesis of A. Tiemeyer; another proof was communicated by P. Abramenko.

Note that in the function field case the finiteness properties do not depend on the rank of B_{n+1}. This is a bit surprising, since in the context of reductive groups one is used to expect that the number of primes as well as the rank of the group are positively related to the finiteness properties of S-arithmetic groups.

To compute the finiteness properties one studies the action of Γ on the product $\overline{X} = \prod_{p\in S} X^{(p)}$ of Euclidean buildings associated to $\mathrm{SL}_{n+1}(K_p)$. One uses the fact that $\mathrm{B}_{n+1}(K_p)$

fixes a chamber at infinity to construct a projection $X^{(p)} \to \Sigma_0^{(p)}$ to the standard apartment. Put these maps together to obtain a $\mathrm{B}_{n+1}(K)$-map $\rho : \overline{X} \to \overline{\Sigma_0} = \prod_{p\in S} \Sigma_0^{(p)}$ where the group acts on the right hand side via projection to the torus. Take H to be a minimal Γ-invariant subspace of $\overline{\Sigma_0}$.

The group Γ acts cocompactly on the preimage $\overline{X}_0 = \rho^{-1}(H)$ and the cell stabilizers are finite. It turns out that $\overline{X}_0$ is $(|S|-2)$-connected. Hence Γ is of type $FP_{|S|-1}$. At the moment this part of the proof makes use of the fact that the buildings $X^{(p)}$ are Moufang. Hence it does not lead to a treatment of the number field case yet.

The fact that Γ is not of type $FP_{|S|}$ is checked for $n = 2$ first. One can do this using Bieri-Strebel invariants. Establishing the same finiteness length for the group $\overline{B}_2(\mathcal{O}_S)$ of all invertible upper triangular matrices over $\mathcal{O}_S$ allows one to reduce the general case to this one by observing that there is a group retract

$$\mathrm{B}_{n+1}(\mathcal{O}_S) \rightleftarrows \overline{B}_2(\mathcal{O}_S).$$

In such a situation the retract is FP_m whenever the left hand group is.

A last remark on using the word "nice" in the title. The proof seems to work in a slightly more general setting. Just start with a Chevalley group over a global function field and then take a Borel subgroup. Their S-arithmetic subgroups are of type $FP_{|S|-1}$ but not of type $FP_{|S|}$.

Olympia Talelli: On cohomological periodicity isomorphisms

A group G is said to have a period q after one step if the functors $H^i(G,-)$ and $H^{i+q}(G,-)$ are naturally equivalent for all $i > 1$. For example free groups, countable locally finite cyclic p-groups have this property. Using the Almost Stability Theorem of Dicks and Dunwoody we show

Theorem. If G has period q after one step then G acts without inversions on a tree T with finite vertex stabilizers.

Corollary. Let G have period q after one step.

α) If G is torsion free, then G is free.

β) If G is torsion then G is a countable locally finite group.

γ) If G is finitely generated then G is free-by-finite.

Moreover, there is an element $g \in H^q(G,\mathbb{Z})$ such that the natural equivalence is induced by the cup product with g, i.e. for any $\mathbb{Z}G$-module A, cup product with g

$$\cup g : H^i(G,A) \to H^{i+q}(G,A)$$

is an isomorphism for $i > 1$ and an epimorphism for $i = 1$.

Note that $g \in H^q(G,\mathbb{Z}) = \mathrm{Ext}^q_{\mathbb{Z}G}(\mathbb{Z},\mathbb{Z})$ is represented by a q-extension

$$0 \to \mathbb{Z} \to A \to P_{q-2} \to \cdots \to P_0 \to \mathbb{Z} \to 0$$

with P_i projective $\mathbb{Z}G$-modules and $\mathrm{pd}_{\mathbb{Z}G} A \leq 1$.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft