MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 51/1995

Branching Processes

17.-23.12.1995

The conference was organized by P. Ney (Madison) and A. Wakolbinger (Frankfurt); it was the first Oberwolfach meeting on Branching Processes in almost 30 years. Peter Ney and four of the present participants had already contributed to its only predecessor, the illustrious conference of 1967, which was attended by such pioneers in the field like Kolmogorov, Yaglom, Kingman, Harris and Sewastjanov.

The vigorous development of the field in recent years was evident at the conference. Topics of the 50-minute talks which opened each day's session were

- random fields on branching structures
- excursion representation of random trees
- branching particle systems
- random walks on trees
- branching process approach to non-linear equations.

One goal of the conference was to promote communication between various schools . Recently, e.g., there has been much interest in branching populations of particles moving in space, and their scaling limits known as superprocesses. As Anatol Joffe put it, after chairing the Wednesday morning session on spatial models: "The conference has succeeded in breaking down a wall between branching processes and superprocesses."

The increasingly prominent role of random trees in the theory of branching processes showed up clearly at the conference. Originally an offshoot of the theory, random trees, through their combinatorial analysis and representation through excursion, are now having important implications for the theory itself.

Another goal of the conference was a survey of the applications of branching processes outside mathematics. Although late cancellations made it necessary to curtail the planned program somewhat, nevertheless there were interesting talks on applications in population genetics, cell kinetics, on polymerase chain reaction, and image coding.

Once more Oberwolfach has proved its efficacy in attracting first-class mathematicians from the whole world, sometimes even in the face of considerable inconvenience (some of the participants were unable to get flights home before Christmas). The prospects for a third Oberwolfach conference on branching processes appear bright, hopefully this time without the lapse of another 30 years.

VORTRAGSAUSZÜGE

K. ATHREYA:

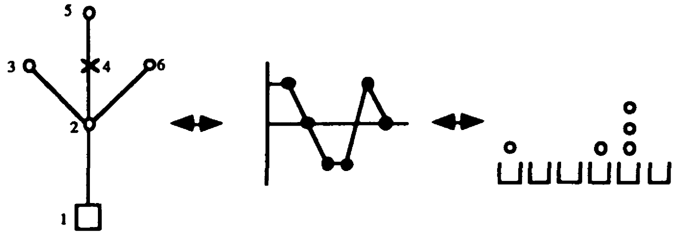
Branching Markov chains, change of measures and the LlogL theorem for branching processes.

For branching Markov chains where the branching is supercritical with no extinction and the Markov motion is ergodic the random probability distribution of the position in the nth generation is shown to converge in probability to the stationary measure of the Markov chain. Under some moment conditions on the offspring distribution and uniform ergodicity on the Markov chain almost sure convergence and large deviation rates are established. Next, following the recent work of Lyons et al, an LlogL theorem is proved for Galton Watson process with arbitrary type space after discussing change of measures for a Markov chain via a nonnegative eigenfunction. As special cases the Bellman Harris and CMJ process are treated.

J. BENNIES:

Some relations between urn models and Galton-Watson trees

A GW-tree with a marked individual can be represented by a random walk bridge and in some cases by an occupancy of cells with balls. Then we identify numbers of children and numbers of balls in cells. To represent a tree with a marked individual we rotate the sequence of occupancy numbers. For the random walk this means that the excursions which represent the tree become bridges.



These relations are for example useful for the following problem:

What is the probability that a uniformly chosen individual has k children, conditioned on the total population-size of the tree?

In the case of the geometric offspring distribution this is equal to the probability of finding k balls in the first cell by distributing $N-1$ balls over N cells according to Bose-Einstein statistics. For the Poisson offspring distribution one has to use the Maxwell-Boltzmann statistics and in the binary case the Fermi-Dirac statistics.

(joint work with G. Kersting)

J. Biggins:

Seneta-Heyde norming in the branching random walk

In the supercritical branching random walk the Laplace transform of the nth generation point process evaluated at ϑ normalized by its expected value forms a martingale, $W_n(\vartheta)$. For ϑ in a certain interval, $(\vartheta_1, \vartheta_2)$, it is known that the martingale converges in L_1 if and only if $EW_1(\vartheta)\log W_1(\vartheta)$ is finite, and its limit is zero otherwise. When $\vartheta = 0$ the martingale becomes the classical Galton-Watson martingale $Z_n/m^n (= W_n(0))$. In that case it is known that there is a sequence of constants (l_n) such that $\{l_n W_n(0)\}$ converges almost surely to a finite limit that is not identically zero. This sequence is called a Seneta-Heyde norming after the result's originators. Similar results have been obtained for other branching models, including for the general (C-M-J) branching process.

For ϑ in $(\vartheta_1, \vartheta_2)$ a Senata-Heyde norming for the martingale $W_n(\vartheta)$ is obtained, so that a sequence $\{l_n\}$ is found such that $l_n W_n(\vartheta)$ converges in probability to a finite limit that is non-zero when the process survives. Though the result is only "convergence in probability" the method used in the proof gives an almost sure Senata-Heyde norming for the general (C-M-J) branching process, which is the "nearest" already known result.

(based on joint work with Andrew Kyprianou)

K. Borovkov:

On distribution tails and expectations of maxima in critical branching processes

Let $\{Z_n\}_{n\geq 0}$ be a critical Galton-Watson process, $Z_0 = 1$, with offspring generating function $f(s)$. Put $M_n = \max_{0\leq k\leq n} Z_k$, $M = \max_{n\geq 0} M_n = \max_{n\geq 0} Z_n$ (the latter is $< \infty$ a.s., for the process dies out w.p. 1). We address the following two problems:

(i) What is the asymptotic behaviour of $P(M > x)$ as $x \to \infty$?

(ii) What is the asymptotic behaviour of $E(M_n)$ as $n \to \infty$?

For the finite-variance case, the answers were known:

$$P(M > x) \sim \frac{1}{x} \quad \text{(Lindvall, 1976)},$$

$$E(M_n) \sim \log n \quad \text{(Athreya, 1988)}.$$

We consider the case when
$(*) f(s) = s + (1-s)^\alpha L(1-s), \alpha \in (1,2], L$ is slowly varying at 0.

To solve (i), we embed our process in an appropriate random walk and establish a universal inequality (valid for any critical GW-process) connecting the distribution of M with that of the maximum of the stopped random walk. The answer is $P(M > x) \sim \frac{\alpha-1}{x}$, if $(*)$ holds.

Then we establish a direct connection between problems (i) and (ii) allowing one to solve (ii) as soon as (i) is solved. The answer to (ii) is: $E(M_n) \sim \log n$, if $(*)$ holds. In conclusion, we discuss another embedding – that of a general branching process into a skip-free random walk. Our results lead then to an asymptotic expression for the non-extinction probabilities of this branching process.

(joint work with V.A. Vatutin, Steklov Math. Inst, Moscow)

H. Cohn:

On the asymptotic patterns of branching processes in varying environment

Let $\{Z_n\}$ be a branching process in varying environment and define $M_n = \max_{i>0} P(Z_n = i)$. A necessary and sufficient condition in terms of the offspring variables $\{X_n\}$ is given for $\lim_{n\to\infty} M_n > 0$. It is also shown that if $W_n = \frac{Z_n}{c_n}$, where $\{c_n\}$ are some norming constants tending to ∞ converges a.s. to a limit W with $P(0 < W < \infty) > 0$, then $\lim_{n\to\infty} M_n = 0$ makes W have a continuous distribution function on $(0,\infty)$. If $\lim_{n\to\infty} M_n > 0$ there exists, up to an equivalence, only one sequence $\{c_n\}$ with a discrete limit distribution for $\{W_n\}$. However, if other growth rates exist in such a case for $\{Z_n\}$ the limits W corresponding to them must have a continuous distribution on $(0,\infty)$.

M. DEKKING:

Mandelbrot percolation and other sets generated by branching processes

We are motivated by the following observation: binary images are in one-to-one correspondence with quad trees. Thus random sets (binary images) correspond to random quad trees. There are many ways to randomize quad trees. We discuss two: tree indexed inhomogeneous Markov chains (with an example of empirical transition probabilities obtained from lung slice data), and fractal percolation with neighbour interaction, which has a structure that we show to be very different from Mandelbrot percolation.

A. ETHERIDGE:

A probabilistic approach to some explosive non-linear heat equations

We are interested in positive solutions to semilinear heat equations of the form $\frac{\partial u}{\partial t} = \Delta u + u^p$ $(*)$ for $p > 1$ and $u(0,x) = \theta\phi(x)$ (with θ a constant). For p an integer this equation can be transformed into one whose solution is expressed in terms of a (time inhomogeneous) branching Brownian motion. By conditioning on the population size we develop a power series in θ. This leads to a new approach to the study of blowup of solutions to the equation $(*)$.

In the special case $p = 2$ we reinterpret the representation in terms of a path-valued (discrete time) Markov process and this leads to a conjecture for the possible forms of the set on which the solution to $(*)$ first becomes unbounded.

S. N. ETHIER:

A measure-valued branching diffusion in population genetics

The usual diffusion approximation of the (measure-valued) Wright-Fisher model by a Fleming-Viot process assumes weak selection. However, a different scaling is possible that permits a diffusion approximation in the presence of strong selection. The limit process is a Girsanov transformation of a very simple

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measure-valued branching diffusion with immigration. As a result, its sample-path properties and unique invariant measure can be determined.

K. FLEISCHMANN:

A continuous super-Brownian motion in a super-Brownian medium

A continuous super-Brownian motion is constructed in which branching occurs only in the presence of some catalysts which evolve themselve as a continuons super-Brownian motion. More precisely, the collision local time, in the sense of Barlow, Evans & Perkins (1991), of a tagged Brownian path with the catalytic mass process governs the branching, in the sense of Dynkin's additive functional approach to superprocesses.

In the one-dimensional case, a new type of limit behavior is encountered: The total mass process converges a.s. to a limit without loss of expected mass (persistence) and with a positive, finite limiting variance given by the expected total Brownian collision local time. On the other hand, starting with a Lebesgue measure, stochastic convergence to that Lebesgue measure holds by some additional law of large number effect.

(joint work with Don Dawson, Carleton Univ. Ottawa)

J. GEIGER:

Decompositions of random trees along distinguished lines of descent

The contour process of a random planar tree is the distance-from-the-root process of the depth-first search. We study a family of random trees including the binary continuous time GW-tree, whose contour processes are strong Markov processes. Expressed in terms of the tree this property states that the subtrees which grow to either side of a fixed path are independent. This allows to give probabilistic constructions of the trees by decomposing them along some distinguished line of descent. To construct trees with stationary contour processes this line of descent is specified by picking a random point uniformly on the tree. If the tree is conditioned on nonextinction up to some time x, then the path to the "left-most" individual alive at x is a suggested candidate to decompose the tree along.

A. GREVEN:

Multiple space-time scale analysis for branching systems

We consider a class of countably many interacting Feller's branching diffusions. The components are labelled by a hierarchical group. The longtime behaviour of this system is analysed by considering space-time renormalized systems in a combination of slow and fast time scales and in the limit of an interaction parameter going to infinity. This leads to a new perspective on the large scale behaviour (in space and time) of critical branching systems in both the persistent and non-persistent cases and including that of the associated historical process. Furthermore we obtain an example for a rigorous renormalization analysis.

In particular we study the family structure of branching systems in equilibrium. The number of distinct families per volume (as the volume tends to infinitiy) is determined as well as the asymptotics of the intensity of a tagged family in large volumes. In addition we relate branching systems and their family structure (i.e. *historical process*) to the genealogical structure arising in Fleming-Viot systems and being described by means of the Poisson-Dirichlet distribution. The Palm-measure arising by picking "a particle" at random is "explicitly" constructed.

B. Griffiths:

Probability distributions on genealogical trees in population genetics

A genealogical tree describes the ancestry of a sample of n genes in a stochastic model of a population. A gene is thought of as an infinitely long sequence of DNA bases, with mutations determining the type of the bases in a stochastic way.

Under this model where mutations can only occur at a base site once, knowledge of the base types in a sample of DNA sequences is equivalent to knowing the genealogical tree. Recent work with Simon Tavaré has been to try and answer ancestral inference questions such as the time to the most recent common ancestor of the sequences, conditional on the DNA observed. The work involves Markov Chain Monte Carlo type techniques simulating back along trees.

A. v. Haeseler:

Modelling the Polymerase Chain Reaction (PCR)

A mathematical model to treat the PCR is introduced. The accumulation of new molecules during a PCR-cycle is regarded as a randomly bifurcating tree. While it is an easy exercise to compute the expected number of molecules it is much harder to obtain the distribution of the number of replications that have occured between a pair of molecules. We develop an approximate formula to do this. The parameters are the efficiency λ of the reaction, the number of initial template molecules and the number of PCR cycles. The goodness of the approximation is tested via simulation. Finally, to model the effect of the intrinsic error rate μ of the polymerase a substitution process is superimposed on the tree. The resulting closed formula for the distribution of pairwise differences of sequences as a function of μ and λ is used to estimate one of them.

(based on joined work with Gunter Weiss)

K. J. Hochberg:

On the long-time behavior of multilevel branching systems

We consider the long-time limiting behavior of the high-density, small mass, short-lifetime diffusion limit of multilevel branching systems with and without spatial motion. Such a two-level system consists of an infinite collection of individuals that are affected by branching at random times, in which groups (families) of "related" individuals undergo a separate, independent branching process. This "upper-level" (family) branching results in a loss of the usual independence assumption for the particle behavior and increases the dependencies in the system.

We find the long-term probability of non-extinction of the resulting measure-valued process and present analogues of Yaglom's conditional limit law for the process conditioned on non-extinction up to a fixed time t and conditioned on non-extinction forever.

If the individual particles also undergo a symmetric stable diffusion, then we show that the resulting "super-2" process (with values on the space of measures on measures on $\mathbb{R}^d$) suffers local extinction in low dimensions and is persistent, in the sense that the initial intensity is preserved in the long run, in high dimensions. This in done by tracing back the genealogy of individuals and studying the resulting evolutionary trees.

(Based on joint work with D.A. Dawson and A. Wakolbinger)

F. den Hollander:

Branching random walk in random environment

At time $t = 0$ place one particle at every site of $\mathbb{Z}^d$. Particles **migrate** according to independent simple random walks jumping at rate 1. Particles **branch** independently at a rate depending on their location, namely, when a particle is at x it: (1) splits into two at rate $\xi(x)$ if $\xi(x) \geq 0$, (2) dies at rate $-\xi(x)$ if $\xi(x) < 0$. Here $\xi = \{\xi(x) : x \in \mathbb{Z}^d\}$ is an i.i.d. $\mathbb{R}$-valued random field, which is kept **fixed** during the evolution.

Let $u(x,t)$ denote the average number of particles at site x at time t. This quantity satisfies the PDE

$$\begin{cases} \frac{\partial}{\partial t}u(x,t) &= \Delta u(x,t) + \xi(x)u(x,t) \\ u(x,0) &\equiv 1, \end{cases}$$

where Δ is the discrete Laplacian. For the special case where the law of $\xi(0)$ is (in the vicinity of) the double exponential $P(\xi(0) > s) = \exp[-e^{s/\rho}]$ $(s \in \mathbb{R})$, with $\rho \in (0, \infty)$ a parameter, we find the asymptotic behavior as $t \to \infty$ of $h_t(x, y)$, the **correlation coefficient** of $u(x, t)$ and $u(y, t)$, as random variables in ξ. The result is

$$\lim_{t \to \infty} h_t(x, y) = \frac{1}{\|w_\rho\|^2} \sum_{z \in \mathbb{Z}^d} w_\rho(x + z) w_\rho(y + z),$$

where $\|\cdot\|$ is the l^2-norm and $w_\rho = (v_\rho)^{\otimes d}$ with v_ρ ground state of the 1-dimensional **nonlinear** difference equation

$$(*) \qquad \begin{cases} \Delta v + 2\rho v \log v = 0 \\ v : \mathbb{Z} \to \mathbb{R}^+. \end{cases}$$

The above result is proved under the condition that the ground state of $(*)$ (i.e., the solution with minimal l^2 norm) is unique modulo translations. We are able to verify this only for ρ sufficiently large. A numerical study of $(*)$ with MAPLE suggests that uniqueness persists for small ρ all the way down to 0.

(joint work with J. Gärtner, TU Berlin, Germany)

P. Jagers:

Coupling and dependence in branching

What happens to a branching population where individual reproduction can be influenced by the population size at the birth of the individual?

Consider such a, population-history dependent, general branching population. Assume reproduction point processes stochastically minorized by those of individuals of the same type but belonging to an imaginary infinite population. Assume that the latter define an a.s. Malthusian (expontially growing etc.) general branching population. Write $m(s)$ for the expected number of children of an s-type individual in the latter, and $m(n, s)$ for the expected offspring size of an s-individual born into an n-size population in the former, population-size dependent case. Then $m(n, s) \geq m(s)$ and if

$$\sum_{n=1}^{\infty} \sup_s \{m(n, s) - m(s)\} < \infty,$$

then the population-size dependent population is also a.s. Malthusian with the same Malthusion growth rate and asymptotic composition as the imaginary limit population.

L^1 convergence may require a little more,

$$\sum_{n=1}^{\infty} n^p \sup_s \{m(n,s) - m(s)\} < \infty$$

for some $p > 2$, is my condition right now.

The method is coupling of individual reproductions, the real reproduction with "what would have been the case if the population were already infinite".

I. Kaj:

Limit processes for age-dependent branching particle systems

We consider systems of spatially distributed branching particles in $\mathbb{R}^d$. The particle lifelengths are of general form, hence the time propagation of the system is typically not Markov. A natural time-space-mass scaling is applied to a sequence of particle systems and we derive limit results for the corresponding sequence of measure-valued processes. The limit is identified as the projection on $\mathbb{R}^d$ of a superprocess in $\mathbb{R}_+ \times \mathbb{R}^d$. The additive functional characterizing the superprocess is the scaling limit of certain point processes, which count generations along a line of descent for the branching particles.

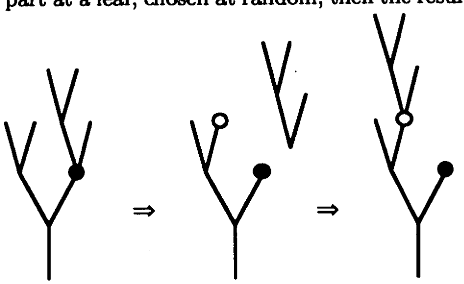
(joint work with S. Sagitov, Almaty, Kazachstan)

G. Kersting:

Surveying the symmetrics of a binary branching tree

A binary branching tree is a rooted tree in the plane, whose edges either end or split into two (both with probability $\frac{1}{2}$) and have i.i.d. exponential lengths.

Thm: If a binary branching tree is cut into pieces at a purely randomly chosen branching point, and if the part without the root is attached to the other part at a leaf, chosen at random, then the resulting tree is again a branching tree.



● = cutting point   ○ = gluing point

The proof uses a representation of the tree via bridges resp. excursions of an exponential random walk (in the spirit of Le Gall).

A similar result holds for subtrees of T, spanned by its root and randomly chosen leaves.

H. KESTEN:

Branching random walk with a critical branching part

We consider limit theorems for the maximal displacemant of a branching random walk when the underlying branching process in critical. In order to get a nontivial limit we have to condition the family tree to have a long lifetime or a large total progeny. Let ζ be the lifetime of a critical Galton-Watson process $\{Z_n\}_{n\geq 0}$ (with $Z_0 = 1$) and ν its total progeny. In addition, let $\{X(v) : v$ a vertex in the family tree of the branching process $\}$ be an i.i.d. family. Denote the root of the family tree by $\langle 0\rangle$ and let $S(v) = \sum X(u)$, with the sum running over all u on the path from $\langle 0\rangle$ to v in the family tree. Finally, $M_n = \max\{S(v) : v$ a vertex in first n generations of the family tree $\}$ and $M_\infty = \max\{S(v) : v$ any vertex in the family tree$\}$. We discuss some limit theorems for M_∞ when we condition on $\{\nu = n\}$ and when $EX(v) \neq 0$ as well as for M_n when we condition on $\{\zeta > \beta n\}$ and when $EX(v) = 0$. In particular, when $EX(v) = 0$, $E|X(v)|^{4+\epsilon} < \infty$, $EZ_1 = 1$, $EZ_1^2 < \infty$, then $\frac{M_n}{\sqrt{n}}$, conditioned on $\{\zeta > \beta n\}$, has a limit distribution.

A. KLENKE:

Multiple scale analysis of clusters of spatial branching models

Cousider $(\Psi_t)_{t\geq 0}$ (critical binary) branching Brownian motion or super-Brownian motion in $\mathbb{R}^2$. It is known that Ψ_t goes to local extinction for initial configuration $H(\rho)$ with intensity $\rho \geq 0$. On the other hand there are relatively small regions (clusters) where the surviving mass piles up. These are known to be "of height $\sim \log t$".

We investigate the profile of clusters more closely. The main concepts are

- Blow-up: $\tilde{\Psi}_t := \frac{8\pi}{\log t}\Psi_t$; $\mathcal{L}(\tilde{\Psi}_0) = \tilde{\mathcal{H}}(t) = \log \mathcal{H}\left(\frac{\log t}{8\pi}\right)$
- Spatial Rescaling: $\tilde{\Psi}_t^\alpha := \mathcal{S}_{\alpha,t}\tilde{\Psi}_t, \alpha \in [0,1]$, where $\mathcal{S}_{\alpha,t}\mu(\cdot) := t^{-\alpha}\mu(t^{\alpha/2})$.

We obtain that $\mathcal{L}^{\tilde{H}(t)}(\tilde{\Psi}_t^\alpha) \Rightarrow \mathcal{L}(Z_{1-\alpha}\cdot\lambda)$, $t \to \infty$, where (Z_s) is Feller's diffusion and λ 2-dim. Lebesgue-measure. In order to get a more refined description we introduce a multiple space scale $A : T \to [0,1]$, which is a decreasing map from a given finite tree T to [0,1]. We observe points $(x_t^e, t \geq 0, e \in T)$ which are scaled according to A:

$$\|x_t^e - x_t^f\| \sim t^{A(e \wedge f)/2}, \quad e \wedge f = \text{ greatest common ancestor}$$

Let $(Z_t^e, e \in T)_{t \geq 0}$ be Feller's diffusion on T, i.e. $Z_t^e = Z_t^f$ for $t < 1 - A(e \wedge f)$ and evolution independent for $t > 1 - A(e \wedge f)$. Denote by $\tau_z \mu(\cdot) = \mu(z + \cdot)$ the shift by $z \in \mathbb{R}^2$. We obtain the following result for (Ψ_t) looked at in the multiple scale A

$$\mathcal{L}^{\tilde{\mathcal{H}}(t)}(S_{A(e),t} \tau_{x_t^e} \tilde{\Psi}_t, e \in T) \overset{t \to \infty}{\Longrightarrow} \mathcal{L}(Z_{1-A(e)}^e \cdot \lambda, e \in T).$$

J. F. Le Gall:

Random trees and Lévy processes

The genealogical structure of a discrete time Galton-Watson branching process is described by a tree, the genealogical tree of the population. Similarly, we introduce the tree structure associated with a general continuous-state branching process. This tree structure is characterized by the so-called exploration process, which represents the motion of a particle that visits all vertices of the tree by moving up and down along its branches. We show that for a general continuous-state branching process, the exploration process is a simple functional of a Lévy process with no negative jump. The knowledge of the exploration process then yields a path-valued process construction of superprocesses with a general branching mechanism.

(joint work with Yves Le Jan)

R. Lyons:

Some results and open questions on random walks on trees

I. We consider simple random walk (SRW) on a Galton-Watson (GW) tree with mean $m > 1$. As first shown by Grimmett & Kesten, this is transient a.s. on nonextinction. This is equivalent to positivity of the effective conductance. If $\mathcal{C}$:= the effective conductance and $\gamma := \frac{\mathcal{C}}{1+\mathcal{C}}$, then the c.d.f. of γ satisfies

$$F_\gamma\left(\frac{s}{1+s}\right) = \sum_{k \geq 0} p_k F_\gamma^{*k}(s).$$

Is the solution absolutely continuous? It appears so from computer calculations.

By using ergodic theory, we show that the speed, or rate of escape, is a strictly positive constant a.s. on nonextinction and give a formula, showing that it is less than $\frac{m-1}{m+1}$. We cannot solve the general multitype case.

II. Now let there be a bias of weight λ towards the root. We still have transience for $0 < \lambda < m$. When $p_0 = 0$, the speed is again a positive a.s. constant, but we have no formula for it. Is it $\leq \frac{(m-\lambda)}{(m+\lambda)}$? The method of proof uses regeneration points and two estimates valid for any tree on which this biased walk is transient, including linear growth of the expected range. However, if $p_0 > 0$, the walk can spend too much time in the bushes, so the speed is positive iff $f'(q) < \lambda < m$. If $p_0 > 0$, is speed monotonic decreasing in λ? How smooth a function of λ is the speed?

We give a couple of explicit calculations for deterministic trees.

(joint work with Robin Pemantle & Yuval Peres)

J. A. López-Mimbela:

A branching processes approach to some systems of nonlinear P.D.E.

Some asymptotics and critical dimensions of systems of nonlinear P.D.E. are investigated by means of a population of individuals undergoing spatial migration and multitype branching. Under mild assumptions on the population model it is shown that the motion parameter of the most mobile type, and the fertility parameter of the most clumping type are responsible for the extinction/persistence of the population in the large time limit. This is used to determine the limit behavior (as $t \to \infty$) of the L^1-norms of the solution components of the LogLaplace evolution system.

In addition, by considering a supercritical multitype branching population, a critical dimension, above which certain systems of semilinear equations (with positive mixed nonlinearities) allow global solutions, is found.

L. Overbeck:

Weakly interacting and non-linear superprocesses

We show that a weak solution to the quasilinear pde with mass creation

$$\frac{\partial}{\partial t}u = \sum_{i,j} a_{ij}(u)\frac{\partial^2}{\partial x_i \partial x_j}u + \sum_k d_k(u)\frac{\partial}{\partial x_k}u + b(u)u$$

can be found by nonlinear superprocesses. In order to show uniqueness of a solution of the above equation, we give a construction of an arbitrary superdiffusion based on the Historical process over an independent product of the Brownian motion and a Poisson process. A nonlinear superprocess can be approximated by a sequence of N-type weakly interacting superprocesses as N tends to infinity. This is a "propagation of chaos" result. We remark on large deviations of this approximation result and give a second type of nonlinear partial differential equation which is solved by the Log-Laplace-Functional of a nonlinear superprocess. An example concerning branching processes conditioned on nonextinction is also given.

A. G. PAKES:

Killing and resurrection of population processes

Questions about island biogeography, and conservation management raise mathematical questions about Markov population processes which allow for exterrial influences, immigration, catastrophe and emigration.

With no immigration, effect of initial population size i on the law of the zero hitting time T is of interest. If $P_i(T < \infty) = 1$ and a limiting conditional law (LCD) exists, then $E_i(e^{\lambda T}) < \infty$ for some $\lambda > 0$ $(*)$. Conversely, if $T \to \infty$ as $\lambda \to \infty$ ("AR-property"), then $(*)$ is sufficient for the LCD to exist.

Examples are mentioned which have an LCD, but lack the AR property. Most comprise some population growth process to a knockout catastrophe.

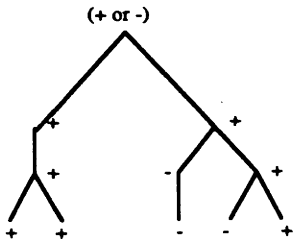
Recent work of others models "fast" immigration from zero using intensities q_{0j} making zero instantaneous. These models admit infinitely many Q-processes, exactly one being honest. An alternative more natural approach is described.

Analysis with "fast" immigration is by lengthy computation with resolvents. The general method has been illustrated for the usual "slow" immigration from zero and with Poisson "knockout" catastrophes. It was shown how exploiting a regeneration at these killing times leads to more general results with a minimum of computation.

Y. PERES:

2-type branching, Ising model and electrical conductance

At the root of a tree network T there is a random (uniform) unknown "spin" $\in \{\pm 1\}$.



Along each edge the spin is flipped with probability ε, retained with probability $1-\varepsilon$. When can the root spin be reconstructed from the nth level spins with probability bounded away from $\frac{1}{2}$? (This is equivalent to non-trivial tail for the Markovian Gibbs state in the Ising Model).

We show reconstruction possible for $1-2\varepsilon >$ br $(T)^{-\frac{1}{2}}$ and not for $1-2\varepsilon <$ br $(T)^{-\frac{1}{2}}$. (Here br (T) is the branching number defined by R. Lyons. br $(T)^{-1}$ is the critical percolation probability).

This extends a recent result of Bleher, Ruiz and Zagrebnov (1995) who treated **regular** trees.

(based on joint work with W. Evans, C. Kenyon, and L. Schulman)

A. ROUAULT:

Branching processes and KPP equations

The distribution function of the rightmost particle in a binary 1-dim branching Brownian motion, as a function of time and space, satisfies the Kolmogorov-Petrowski-Piscounov equation $u_t = \frac{1}{2}u_{xx} + u(1-u)$. Rescaling in the large deviation regime ($t \mapsto \varepsilon^{-2}t, x \mapsto \varepsilon^{-2}x$) the solution has a $0-1$ limit (0 ahead of the front and 1 behind). In the 0 region, u is exponentially going down (Freidlin 1985) and its sharp behaviour is shown (Chauvin-Rouault 1988). The number $Z^\varepsilon(t,x)$ of particles living at $\varepsilon^{-2}t$ ahead of $\varepsilon^{-2}x$ satisfies

$$\varepsilon^{-1}\exp\left(-\frac{V(t,x)}{\varepsilon^2}\right) EZ^\varepsilon(t,x) \to C_1 > 0$$

$$\frac{P(Z^\varepsilon(t,x)\neq 0)}{EZ^\varepsilon(t,x)} \to C_2 > 0$$

where $V(t,x)$ plays the role of a "local reproduction rate", as in the subcritical regime of a branching process. This model can be extended to discrete-time (R 1990) and to inhomogeneous branching diffusion (Ben Arous-R 1993, Rouques 1995).

A. Schied:

Geometric aspects of some variational problems associated with superprocesses

Let $\rho(\nu,\mu) = \sup\{v(\nu) - v(\mu) | \Gamma(v,v) \leq 1\}$ denote the intrinsic distance of two finite measures ν, μ, that is associated with the carré du champs operator Γ of a continuous superprocess. It is shown that ρ coincicdes with twice the usual Kakutani-Hellinger distance. We also give integral representations with the corresponding energy and arclength functionals. The curves minimizing length and energy given its endpoints can be characterized using Radon-Nikodym derivatives. Also we discuss applications in large deviations.

R. Siegmund-Schultze:

Spatial branching processes - some results on equilibria and genealogy

In constrast to the classical Galton-Watson model, spatial branching processes may have non-trivial steady states. Under the assumption of finite intensity, the structure of translation invariant equlibria of critical branching processes in $\mathbb{R}^d$ is well understood (Kallenberg's backward tree method). In the general situation, only partial results are available at this time. Some of them are contra-intuitive, as the fact, that in $\mathbb{R}^1$ there are subcritical branching laws admitting equilibria (of infinite intensity). We give an example: Assume that a mother-particle generates $3^{28\cdot 3^n}$ particles with probability $3^{-82\cdot 3^n}$, placing all of them at the same random position, which has a distance ξ to the mother particle's location being exponentially distibuted with parameter $3^{-2\cdot 3^{n+1}}, n = 1,2,3,\ldots$ With the remaining probability the particle has no descendants.

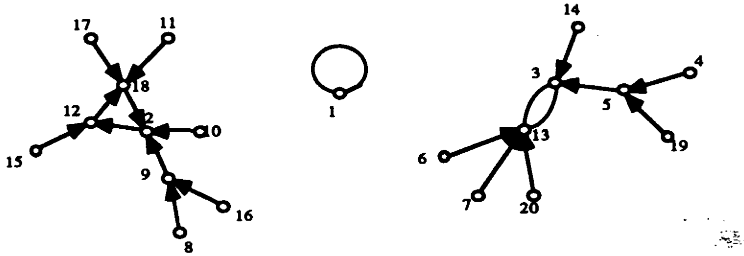
As an example, two results are presented for the case of a general branching model in an arbitrary complete separable space A. These results indicate that also in this general situation the structure of equlibria is closely related to recurrence properties of ancestral lines in the genealogical tree generated by the branching evolution, similar to the situation for finite intensity steady states in $\mathbb{R}^d$.

(joint work with K. Matthes, A. Wakolbinger, K. Nawrotzki)

Y. SUNG:

Random mappings and conditional Galton-Watson forests

A mapping $f : \{1, \dots n\} \to \{1, .., n\}$ can be represented by the directed graph with vertices $1, \dots, n$ and edges $(i, f(i))$.



Erasing the cyclic edges generates a forest where the roots of the trees are the cyclic vertices. If the mapping is chosen uniformly at random then the forest is a randomly labelled conditional Galton-Watson forest where the number of trees has geometric distribution and the individuals have Poisson offspring. By using Burtin's lemma we derive quasi-binomial distributions for certain quantities of conditional Galton-Watson forests.

Z. TAIB:

Branching processes and cell kinetics

A multitype branching process version of the Bell-Anderson cell population model is presented. For this model, we discuss the existence of a stable cell size distribution. We also show that since our process has an explicit genealogy, it is indeed a quite natural setting for studying certain facts known empirically from experimental data and simulation studies. Examples of such facts are the shapes of the so called α-curve and β-curve, as well as certain correlations between close relatives. The case of linear growth is used as an application.

V. VATUTIN:

Branching processes in random environment: the probability of extinction at a given moment

Let $Z(n)$ be the number of particles in a critical branching process in random environment and $T = \min\{n : Z(n) = 0\}$. We show that

$$P\{T = n\} \sim c \cdot n^{-\frac{3}{2}} \quad \text{as } n \to \infty.$$

V. VINGORADOV:

On weak convergence of branching particle systems undergoing spatial motion

We give natural sufficient conditions (in terms of the motion and branching mechanisms) for branching particle systems to belong to domains of attraction of certain superprocesses. The cases of nonhierarchical as well as of hierarchical branching are considered.

(joint with D.A. Dawson, Carleton University, Ottawa and K.J. Hochberg, Bar-Ilan University, Ramat-Gan)

E. WAYMIRE:

Random cascades and positive T-martingales

Random cascades are discussed as a prototypical example within the more general framework of J-P Kahane's notation of a positive T-martingale. For this one has a (loc.) compact metric space T and a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{F}_n\}$ on which are defined positive random functions $Q_n(t), t \in T, n = 1, 2, \ldots$ such that for each fixed $t \in T$, $\{Q_n(t)\}_{n=1}^{\infty}$ is a martingale w.r. to $\{\mathcal{F}_n\}$. In the case of a cascade one has $T = \{0, 1, \ldots, b-1\}^{\mathbb{N}}$ with the usual ultrametric and $Q_n(t) = W_{t|1} \cdot \ldots \cdot W_{t|n}$, for $t|i = (t_1 \ldots t_i)$, $t = (t_1, t_2, \ldots) \in T$, subject to $E(W_{t|n+1}|\mathcal{F}_n) = W_{t|n}$, $\mathcal{F}_n = \sigma(W_\tau : |\tau| \leq n)$; an important case being i.i.d. W_τ's with $EW_\tau = 1$. Other examples include coverage processes, smoothing interacting particle systems, spin-glass and polymer models. Given a positive Radon measure σ on T let $Q_n\sigma(B) = \int_B Q_n(t)\sigma(dt), B \in \mathcal{B}(T)$. Then with probability 1, $Q_n\sigma \underset{(w)}{\Rightarrow} \sigma_\infty$ (essentially by martingale conv. thm).

Write $Q_\infty\sigma = \sigma_\infty$. Then Kahane has shown $Q_n\sigma = Q'_n\sigma + Q''_n\sigma$, where (i) (living part) $EQ'_\infty\sigma(B) = EQ'_1\sigma(B)$, (ii) (dying part) $EQ``_\infty\sigma(B) = 0$. We say **survival** occurs if there is a non-trivial living part. The basic problems concern criteria for survival and computation of fine scale structure of the surviving part. We will show a way to compute survival criteria by SIZE-BIASING and a way to compute fine scale structure from the survival criterion (PERCOLATION METHOD). We then illustrate these methods with an application to cascades on a Galton-Watson tree whose offspring satisfy $EL\log L < \infty$, thereby answering a conjecture from the IMA ('94) branching process conference and extending the results of Peyriere (1977, Duke Math J) to this general setting. The answer is survival if and only if $EW\log W < \log EL$ for i.i.d. generations W, provided $P(L \geq 1) = 1$ and $EL\log L < \infty$.

(based on joint work with Stan Williams)

N. YANEV:

Regeneration and branching

Limit theorems are obtained for a class of two-stage non-negative regenerative processes with infinite means of regenerative cycles. These results are applied to investigate the asymptotic behaviour of critical Bellman-Harris branching processes in the case of finite or infinite offspring variance and finite or infinite immigration mean.

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Branching Processes

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