MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 26/1996

Variationsrechnung

7.-13.07.1996

Die Tagung fand unter Leitung von L. Ambrosio (Pavia), F. Hélein (Cachan) und S. Müller (Zürich) statt. Die Teilnehmer repräsentierten ein breites Spektrum aktueller Entwicklungen in der Variationsrechnung, und die Tagung führte zu einem intensiven Austausch zwischen unterschiedlichen Teilgebieten. Im Mittelpunkt standen Variationsprobleme aus Geometrie und Physik, Gradientenflüsse, Variationsansätze in der Bildverarbeitung, sowie allgemeine Fragen der Variationsrechnung (Unterhalbstetigkeit, Regularität und Γ-Konvergenz).

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

Giovanni Alberti

Asymptotic behaviour of a non-local model in phase transitions

We study the asymptotic behaviour as $\epsilon \to 0$ of the functional

$$F^\epsilon(u) := \frac{1}{\epsilon}\Big[\int_x\int_{x'} J_\epsilon(x'-x)(u(x')-u(x))^2 dx'\,dx + \int_{x\in\Omega} W(u(x))dx\Big]$$

where Ω is a regular domain in $\mathbb{R}^N$, $u \in L^1(\Omega, \mathbb{R})$, W is a double well potential which vanishes in ± 1, $J : \mathbb{R}^N \to \mathbb{R}$ is an interaction potential with

1) J is even (i.e., $J(x) = J(-x)$), possibly anisotropic,
2) J is nonnegative,
3) $\int_{\mathbb{R}^N} J(x)dx$, $\int_{\mathbb{R}^N} J(x)|x|dx < \infty$.

Under these hypothesis we prove the following result:
Theorem 1: There exists a function $\sigma : S^{N-1} \to \mathbb{R}$ which is strictly positive and bounded, such that $x \mapsto |x|\sigma(x/|x|)$ is convex on $\mathbb{R}^N$ and the sequence F^ϵ is equicoercive and converges (in the sense of Γ-convergence in $L^1(\Omega)$) to

$$F^0(u) := \begin{cases} \displaystyle\int_{S_u} \sigma(\nu_u) d\mathcal{H}^{N-1} & \text{if } u \in \mathrm{BV}(\Omega, \pm 1) \\ +\infty & \text{elsewhere.} \end{cases}$$

Here $\mathrm{BV}(\Omega, \pm 1)$ is the class of BV functions on Ω which take values ± 1 only, if $u \in \mathrm{BV}(\Omega, \pm 1)$ S_u is the jump set of u (that is, the reduced boundary of the finite perimeter set $\{u(x) = 1\}$) and ν_u its measure theoretic normal.

The functional F^ϵ represents the free energy of a continuous limit of spin lattices in the Ising model. In this context u can be viewed as a magnetization density.

The result in Theorem 1 is achieved proving the existence of an increasing optimal profile for the 1-dimensional transition and then showing that the optimal profile in the N-dimensional case for a planar transition with normal direction e exists and in fact depends only on the direction e. The key result of this approach is the possibility of finding explicit solutions of the following 1-dimensional problem: given

$$F(u) := \int_{x\in\mathbb{R}}\int_{x'\in\mathbb{R}} J_\epsilon(x'-x)(u(x')-u(x))^2 dx'\,dx + \int_{x\in\mathbb{R}} W(u(x))dx$$

where J is fixed, find W such that the minimum of F on the class $X = \{u : \mathbb{R} \to [-1,1]$ such that $u(\pm\infty) = \pm 1\}$ is a prescribed increasing function.

François Alouges
Computation of liquid crystals and micromagnetic equilibrium configurations

The problem of numerically finding liquid crystals equilibrium configurations, when working with the Oseen-Frank model for nematics, is of a non-convex optimization type. When further assumptions on physical constants are enforced, it reduces to finding minimizing harmonic maps from the domain where the crystal is embedded in S^2 the unit sphere on $\mathbb{R}^2$. We present, first in this case, and then on the former more general case a preconditioned projected gradient algorithm that produces an energy decreasing sequence of configurations. Results of a 2D finite element code are shown describing in a simple framework the "escape in the third dimension" phenomenon and the force the crystal exerts on immersed bodies. In 3D we check the minimality of the map $x/|x|$ with respect to the relative values of the physical constants.

The micromagnetic model describes the distribution of magnetization inside a ferromagnetic body at mesoscopic scale. As the magnetization minimizes an energy and is of constant norm inside the body, it appears to be a natural extension of the liquid crystal problem. However a lot of new difficulties occur in that case. Namely a non local term gives rise to an exterior problem and physical experiments show the natural formation of domains, walls, lines and singular points. We present the flower-vortex transition, which is one of the preliminary results of a code currently under development jointly with S. Labbé, P.Y. Bertin, L. Halpen and F. Rogier.

Bernard Dacorogna, Paolo Marcellini
On the Cauchy-Dirichlet problem for first order nonlinear systems

The following theorem holds:
Theorem: Let Ω be an open set of $\mathbb{R}^n$. Let E be a compact set of $\mathbb{R}^{m\times n}$, such that

$$PcoE = RcoE,$$

where $PcoE$ and $RcoE$ are, respectively, the polyconvex and the rank-one convex envelope of E. We also assume two technical properties that we call segment property and extreme points property.

Finally, let $\varphi \in C^1(\bar{\Omega}; \mathbb{R}^m)$ (or piecewise C^1) such that

$$D\varphi(x) \in E \cup \operatorname{int} RcoE, \quad x \in \Omega.$$

Then there exists (a dense set of) $u \in W^{1,\infty}(\Omega; \mathbb{R}^m)$ such that

$$\begin{cases} Du(x) \in E & q.e.\ x \in \Omega \\ u(x) = \varphi(x) & x \in \partial\Omega. \end{cases}$$

Gianni DalMaso
Special functions with bounded deformation

In some recent papers in collaboration with L. Ambrosio, G. Bellettini and A. Coscia we introduced, for every open set $\Omega \subset \mathbb{R}^n$, the space SBD($\Omega$) of special functions with bounded deformation. It is defined as the set of all functions $u \in L^1(\Omega, \mathbb{R}^n)$ whose distributional gradient Du has a symmetric part $Eu = \frac{1}{2}(Du^T + Du)$ that can be represented by a bounded measure with singular part concentrated on the jump set J_u of u.

We proved a compactness theorem for certain subsets of SBD(Ω) which allows to solve minimum problems of the form

$$\min_{u \in \mathrm{SBD}(\Omega)} \left\{ \int_{\Omega \setminus J_u} f(\mathcal{E}u)dx + \mathcal{H}^{n-1}(J_u) + \text{ lower order terms} \right\},$$

where $\mathcal{E}u$ is the density of the absolutely continuous part of Eu with respect to the Lebesgue measure and $\mathcal{H}^{n-1}$ is the $(n-1)$-dimensional Hausdorff measure. These problems can be considered as a weak formulation of some variational problems which arise in fracture mechanics for linearly elastic-perfectly brittle materials.

Guy David

Uniform rectifiability of quasiminimal hypersurfaces in $\mathbb{R}^n$

(This is joint work with S. Semmes)

Let a (fixed) annulus $A = B_1 \setminus B_0 \subset \mathbb{R}^n$ be given, and let $\mathcal{F}$ be the set of compact sets $E \subset A$ with finite Hausdorff measure $H^{n-1}(E)$ and that separate ∂B_0 from ∂B_1. We call quasiminimal surfaces sets $E \in \mathcal{F}$ such that

$$H^{n-1}(E \setminus F) \leq M H^{n-1}(F \setminus E)$$

for all competitors $F \in \mathcal{F}$, and some constant $M \geq 1$. Then E is the union of a set of measure zero and an irreducible quasiminimizer E^* such that E^* is Ahlfors-regular (i.e., with "density" bounded and bounded below), and $\mathbb{R}^n \setminus E^*$ is composed of exactly two connected components that are John domains. In particular, E^* is uniformly rectifiable.

Frank Duzaar

Minimization of conformally invariant energies in homotopy classes

(This is joint work with E. Kuwert)

Suppose $X \subset \mathbb{R}^N$ is compact and $\alpha \in \pi_n(X)$ is a given homotopy class of maps. The purpose of the talk was to analyze the behaviour of minimizing sequences $u_k \in \alpha$ for a conformally invariant functional of the form

$$F(u) = \int_{S^n} f(u(\xi), du(\xi))d\xi.$$

The minimizing sequence may not converge strongly due to the phenomenon of separation of n-spheres. This was first described in the work of Sacks and Uhlenbeck. We say that a family $(u^i)_{i \in I}$ of nonconstant maps is a weak limit sets of the sequence $(u_k)_{k\in\mathbb{N}}$ if there exist pairwise unbounded (as $k \to \infty$) conformal automorphisms h_k^i of the sphere such that $u_k \circ h_k^i \to u^i$ weakly for any $i \in I$. If in addition the sum of the energies of the maps u^i is equal to the limit of the energies of the sequences u_k we then say that the convergence is in energy.

Theorem: If u_k is a minimizing sequence for F and $\alpha \in \pi_n(X)$ then there exists a subsequence and a weak limit set $(u^i)_{i\in I}$ such that u_k converges in energy to $(u^i)_{i\in I}$. Moreover, any u^i is F minimizing in its own homotopy class $\{u^i\} \in \pi_n(X)$ and the family $(\{u^i\})_{i\in I}$ is a decomposition of the given homotopy class α.

Maria J. Esteban

Existence of stationary solutions for the Maxwell-Dirac equation
(This is joint work with V. Greorgiev and E. Séré)

The Maxwell-Dirac equation models the interaction of a (relativistic) electron with its own electromagnetic field. It can be viewed as a "semiclassical" model, because the electromagnetic field is classical (is not quantized).

There exist stationary solutions of the nonlinear (nonlocal) equation for phases $\omega \in (-m, m)$ where m is the mass of the electron. This can be proved by looking for critical points of the corresponding action functional. Actually, by using a nonlinear family of ad hoc ansatzes (indexed by the angular momentum), one can prove the existence of an infinity of distinct solutions for $\omega \in (0, m)$.

The variational procedure one has to use is a kind of modified linking. Concentration/compactness plays an important part both in the obtention of a priori bounds and in the passing to the limit in the nonlinear terms.

Irene Fonseca

Recent developments in the calculus of variations: Relaxation results

In the case where lower semicontinuity of the energy

$$u \in W^{1,p} \mapsto \int_\Omega f(\nabla u)dx$$

fails, one of the approaches adopted to identify the effective energy of a given final state $u \in W^{1,p}(\Omega; \mathbb{R}^d)$, where $\Omega \subset \mathbb{R}^N$ is an open, bounded domain, reduces to considering the relaxed energy

$$\mathcal{F}^{q,p}(u;\Omega) := \inf_{\{u_n\}} \Big\{ \liminf_{n\to\infty} \int_\Omega f(\nabla u_n)dx : \quad u_n \in W^{1,q}_{loc}(\Omega;\mathbb{R}^N) \cap W^{1,p}(\Omega;\mathbb{R}^d), \\ u_n \rightharpoonup u \text{ in } W^{1,p}(\Omega;\mathbb{R}^N) \Big\}.$$

Here $p \le q$, $0 \le f(\xi) \le c(1+|\xi|^q)$ for all $\xi \in M^{d\times N}$, and one seeks to identify an integral representation for $\mathcal{F}^{q,p}(u;\Omega)$, i.e.,

$$\mathcal{F}^{q,p}(u;\Omega) = \int_\Omega \bar{f}(\nabla u)dx + \mu_s(\Omega),$$

where $\bar{f}$ is now the effective bulk energy density, and μ_s is a Radon measure, singular with respect to the N-dimensional Lebesgue measure $\mathcal{L}^N$.

If $p = q$ it is well known that

$$\mathcal{F}^{q,p}(u;\Omega) = \int_\Omega Qf(\nabla u)dx,$$

where the quasiconvexification of f is defined as

$$Qf(\xi) = \inf \Big\{ \int_{(0,1)^N} f(\xi + \nabla\varphi(x))dx : \varphi \in W_0^{1,\infty}((0,1)^N;\mathbb{R}^d) \Big\}.$$

In this talk we consider the case $p < q$, a prototype example relevant in the study of cavitation being given by $f(\xi) = |\xi|^2 + |\det \xi|$, $\xi \in M^{3\times 3}$. Using a global method of relaxation introduced in collaboration with G. Bouchitté and L. Mascarenhas, in joint work with G. Bouchitté and J. Malý we prove that for $p > q - \frac{1}{N}$ one has

$$\mathcal{F}^{q,p}(u;\Omega) = \int_\Omega Qf(\nabla u)dx + \mu_s(\Omega).$$

Gero Friesecke
Variational methods and quantum many-body systems

The many-body energy functional of quantum mechanics has received little attention from workers in the Calculus of Variations, but for its minimization or approximation (e.g.: by nonquadratic functionals in fewer variables!) a number of structurally very interesting ideas have been suggested by physicists such as Thomas, Fermi, Dirac, Bloch, Hohenberg, Kohn.

I will discuss some rigorous results and open problems, and explain hidden and previously unnoticed connections to other areas of Analysis, such as: the study of cancellation effects in exponential sums via the method of stationary phase; or the development of new tools to quantify the failure of strong convergence in weakly convergent sequences (Tartar; Gérard).

Nicola Fusco
A partial regularity result for a class of free discontinuity problems

We study the regularity of the free discontinuity set K minimizing the Mumford-Shah functional

$$(1) \qquad \mathcal{F}(u,K) = \int_{\Omega\setminus K} [|\nabla u|^2 + \alpha(u-g)^2]\,dx + \beta\mathcal{H}^{n-1}(\Omega\cap K)\ ,$$

where Ω is a bounded open set in $\mathbb{R}^n$, α, β are positive scaling factors, $g \in L^\infty(\Omega)$, K is a compact subset of $\mathbb{R}^n$ and $u \in C^1(\Omega\setminus K)$.
Using a lower semicontinuity result of Ambrosio, existence of minimizers for the functional (1) has been proved by De Giorgi, Carriero and Leaci. More precisely they show that there exists a function $u \in BV(\Omega)$ such that the pair (u, K) minimizes (1), where K is the closure of the discontinuity set of u.
It is also easy to show that u is a solution of the Neumann problem

$$\begin{cases} -\Delta u = \alpha(u-g) & \text{in } \Omega \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } K \end{cases}$$

while the set K, which is $(n-1)$-countably rectifiable, satisfies the first variation equation

$$\int_{\Omega\setminus K}[|\nabla u|^2\mathrm{div}\eta+\alpha(u-g)^2\mathrm{div}\eta-2<\nabla u,\nabla u\cdot\nabla\eta>]\,dx+\int_K \mathrm{div}^K\eta\, d\mathcal{H}^{n-1}=0$$

for any $\eta\in C_0^1(\Omega,\mathrm{I\!R}^n)$, where $\mathrm{div}^K\eta$ is the tangential divergence along K. In this talk we present a recent result, due to Ambrosio, Fusco and Pallara, concerning the partial regularity of the discontinuity set K. We show that K outside a closed set of $\mathcal{H}^{n-1}$ zero measure is indeed a $C^{1,\alpha}$ hypersurface.

Karsten Große-Brauckmann
Moduli spaces of constant mean curvature surfaces with few ends

The ends of embedded complete constant mean curvature surfaces with finite topology are asymptotically Delaunay. For genus 0 and 3 of 4 ends we give necessary conditions for the set of Delaunay surfaces which is attained asymptotically. We do this under a symmetry assumption, namely we assume three orthogonal planes for surfaces with 4 ends (they are either + or X-shaped) and two orthogonal planes for Y-shaped surfaces with 3 ends.

Gerhard Huisken
Dynamical behaviour of stable constant mean curvature surfaces in Lorentzian manifolds
(This is joint work with S.T. Yau, Harvard)

Consider an asymptotically flat initial data set (M^3,g,K), where g and K correspond to the induced metric and second fundamental form, respectively, of an isometric embedding of (M^3,g,K) in a Lorentzian manifold (L^4,h) satisfying the Einstein-Vaccuum equations ${}^L\mathrm{Ric}\equiv 0$. In the simplest case g and K satisfy

$${}^MR-|K_{ij}|^2=0,\quad \mathrm{tr}(K_{ij})=0,\quad {}^3\nabla_iK_{ij}=0,$$

where ${}^MR$ is the scalar curvature and "maximal slicing" is assumed. Near infinity g and K satisfy

$$g_{ij} \;=\; \delta_{ij}\Big(1+\frac{2m}{r}\Big)+\mathcal{O}\Big(\frac{1}{r^2}\Big),$$

$$K_{ij} = \frac{3}{2r^2}\Big\{P_i r_j + P_j r_i - \langle P, \nabla r\rangle(\delta_{ij} - r_i r_j)\Big\} + \mathcal{O}\Big(\frac{1}{r^3}\Big),$$

where $(m, \vec{P})$ is the invariant ADM 4-momentum, and $r_i = x_i/r$ is the radial direction.

The "centre of mass" is then geometrically defined by a unique stable constant mean curvature foliation $\{{}^2\Sigma_r\}$ near infinity. When (M^3, g, K) is propagated through (L^4, h) according to the Einstein-Vaccuum equations ${}^L\mathrm{Ric} \equiv 0$, it is shown that the 2-dimensional constant mean curvature spheres ${}^2\Sigma_r$ of the radial foliation move with an asymptotic speed $\vec{v}$ in agreement with the special relativity: $\vec{P} = m\vec{v}$.

Tom Ilmanen

A strong maximum principle for singular minimal hypersurfaces

Let M, N be stationary, connected n-dimensional hypersurfaces in a Riemannian domain $\Omega \subseteq P^{n+1}$, possibly with singularities. We prove a strong maximum principle as follows:

Theorem 1: If

(a) $M = \partial E \cap \Omega$, $N = \partial F \cap \Omega$, $E \subseteq F \subseteq \Omega$,

(b) $\mathcal{H}^{n-2}(\mathrm{sing}M) = 0$ (N can be any stationary varifold),

then either $M = N$ or $M \cap N = \emptyset$.

(Alternatively, we can replace (a) and (b) by the hypothesis $\mathcal{H}^{n-2}(M \cap N) =$ 0.

Corollary 2: If M is connected, stationary, and $\mathcal{H}^{n-2}(\mathrm{sing}M) = 0$ then $\mathrm{reg}M$ is connected.

The proof is similar to L. Simon's and Moschen's analogous result for area-minimizers, with two new ingredients

(a) Reduction to the case of stable minimal hypersurfaces via interposition.

(b) Replacement of the intrinsic Harnack inequality of Bombieri, DeGiorgi and Giusti by a more elementary and robust mean value inequality.

As a byproduct we have

Theorem 3: Every tangent cone of a stable minimal hypersurface (with

$\mathcal{H}^{n-2}(\text{sing}M) = 0)$ has multiplicity one.
Corollary 4: (Nonlinear extrinsic Harnack inequality) If disjoint stationary hypersurfaces approach closely somewhere, they approach closely everywhere.

Jürgen Jost
The theory of harmonic maps

The theory of harmonic maps started with the results of Eells-Sampson, Al'ber, Hartman, and Hamilton about the existence and uniqueness of harmonic maps with values in Riemannian manifolds of nonpositive curvature. These results employed the heat-flow method.

In the work of Hildebrandt-Kaul-Widman, it was then shown that variational methods and techniques from the regularity theory of semilinear elliptic systems could be used to solve the Dirichlet problem for harmonic maps with values in simple convex geodesic balls with possibly positive curvature. Also, they found the prototype of an example for a singular weakly harmonic map.

More recently, the theory of harmonic maps has been extended to maps between metric spaces, motivated by certain questions about lattices in algebraic groups. The talk then sketched a general framework for the regularity theory for generalized harmonic maps by extending and employing notions from the theory of Dirichlet forms of Beurling-Deny. It is shown that under quite general conditions, minimizers of generalized Dirichlet forms with values in a metric space with nonpositive curvature are Hölder continuous. The proof uses a Harnack inequality of Biroli-Mosco and Sturm, a telescoping lemma of Giaquinta-Giusti-Hildebrandt as well as arguments of Meier developed originally in the context of the approach of Hildebrandt et al.

At the end of the lecture, a volume with contributions by students, colleagues, and friends was presented to Stefan Hildebrandt on the occasion of his sixtieth birthday, in recognition of both his fundamental mathematical contributions and his founding and formative influence on the Oberwolfach meeting on the "Calculus of Variations".

Richard D. James

Thin films

(This is joint work with K. Bhattacharya)

Thin films (of thickness $h > 0$) provide an interesting area for future mathematical research. There are opportunities for taking standard theories (including, say, bulk and surface energy) and studying Γ-convergence as $h \to 0$, and also for studying new effects that occur at small scales, such as spontaneous vibration of beams, unusual surface and multilayer effects, GMR, etc. In this talk I studied Γ-convergence of

$$E^{(h)}(y) = \frac{1}{h}\int_{\Omega_h} \alpha|\nabla^2 y|^2 + W(\nabla y(x);\theta)dx$$

where $y : \Omega_h \to \mathbb{R}^3$, $\Omega_h = S \times (0,h) \subset \mathbb{R}^3$ is the thin film, and W is a stored energy for martensitic materials (e.g. it has "wells"). As $h \to 0$, minimizers of $E^{(h)}$ satisfy $y^{(h)} \to \bar{y}$ in $W^{2,2}(\Omega_1, \mathbb{R}^3)$, $\frac{1}{h}y_{,3} \to \bar{b}$ in $W^{1,2}(\Omega_1,\mathbb{R}^3)$, where we have rescaled to Ω_1, and $(\bar{y},\bar{b})$ minimize

$$\int_S \alpha\Big[|\nabla_p y|^2 + 2|\nabla_p b|\Big] + W(y_{,1}, y_{,2}, b;\theta)dz_1dz_2.$$

For specific martensitic materials, this leads to rather different behavior of thin films as compared to bulk materials.

Paolo Marcellini

Elliptic versus parabolic regularization for the equation of prescribed mean curvature

(This is joint work with Keith Miller, Berkeley)

We consider the equation of prescribed mean curvature

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\Big(\frac{u_{x_i}}{(1+|Du|^2)^{1/2}}\Big) + h(x) = 0 \quad x \in \Omega,$$

with the Dirichlet boundary condition $u = 0$ on $\partial\Omega$.

If $h(x)$ is "too large", precisely if there exists a subset $G \subset \Omega$ such that

$$\Big|\int_G h(x)dx\Big| > P(G),$$

where $P(G)$ is the perimeter of the set G, then the equation has no solution. On the contrary, the regularized elliptic problem (with $\epsilon\Delta u$ added in the equation) and the regularized parabolic problem (with u_t added), have solutions.

We describe the behaviour of the elliptic regularized solution $u_\epsilon(x)$ as $\epsilon \to 0$, and of the parabolic regularized solution $u(x,t)$ as $t \to \infty$.

Alexander Mielke
On quasiconvexity at the boundary
(This is joint work with Pius Sprenger)

For elasticity problems having boundary points with Neumann conditions there is up to now no good stability theory. The notion of quasiconvexity at the boundary by Ball and Marsden [1984] supplies a necessary condition, however easily checkable sufficient conditions other than full convexity are missing. We propose the notion of *polyconvexity at the boundary* which involves Null-Lagrangians N having the additional property that $\int_\Omega N(\nabla\varphi)dy = 0$ even if φ does not vanish along the flat part of $\partial\Omega$. It is shown that spatially localized perturbations at a boundary point x_0 can not lower the energy if the density is polyconvex at the boundary in $\nabla y(x_0)$ in the direction $\nu(x_0)$.

For linearized isotropic elasticity polyconvexity at the boundary holds for $7\lambda + 6\mu \geq 0$ and $\mu \geq 0$ while quasiconvexity at the boundary is valid for $\lambda + \mu \geq 0$ and $\mu \geq 0$.

Felix Otto
Dynamics of labyrinthine pattern formation in magnetic fluids: a mean–field theory

We are interested in the flow of a droplet of viscous ferrofluid in the Hele–Shaw cell under a transverse magnetic field. The (two–dimensional) phase configuration is observed to evolve into a labyrinthine pattern. We show that the conventional model for this flow has the form of a gradient flux w.r.t. an energy functional, which is the sum of magnetic and surface energy. In particular, we are interested in the behaviour of this flow problem in the regime of large magnetization $M^2 \gg 1$. In this regime, the details of the pattern evolution are observed to be highly sensitive to changes in the initial configuration. This is reflected in the linear stability analysis of the circular phase configuration, a stationary point of the dynamics which is more and more unstable in the limit $M^2 \uparrow \infty$. In order to capture the "generic" behaviour of the dynamical system in this regime, we need a selection principle which dismisses those non–generic solutions. We propose a selection principle for the limit $M^2 \uparrow \infty$ which is based on the natural implicit discretization in time of our gradient flux formulation. We prove that this approach leads (in an appropriate scaling) to the equation

$$\partial_t s - \Delta s^2 = 0$$

for $s(t,x) \in [0,1]$, the local spatial average of the phase configuration $\chi(t,x) \in \{0,1\}$ ($\chi(t,x) = 1$ if x lies in the two–dimensional cross–section of the fluid at time t, $\chi(t,x) = 0$ else). Thus this quantity, which contains information on the "microstructured zone", evolves deterministically, although χ is essentially unpredictable.

Tristan Rivière

Minimizing p-harmonic maps in homotopy classes from S^3 into S^2

We prove that contrary to the case of maps from S^3 into S^3, there exist infinitely many homotopy classes from S^3 into S^2 having a minimizing 3-harmonic map.

We prove that the first eigenforms of the linear operator $\Delta^{1/2} = d*$ on $\mathrm{Ker}\, d \cap \Lambda^{2n} S^{4n-1}$ are stable for the associated conformal invariant non-linear variational problem and we deduce, in particular, that the Hopf map from S^3 into S^2 minimizes the p-energy in its homotopy class for $p \geq 4$ and it remains true locally for $3 \leq p < 4$. Finally, we prove that the Hopf map minimizes the p-energy for $p \geq 3$ among a class of symmetric fibrations from S^3 into S^2.

Friedrich Sauvigny

Uniqueness of Plateau's problem for certain contours with a one-to-one, non-convex projection onto a plane

Let $\Omega \subset \mathbb{R}^2$ be a bounded C^2-domain satisfying a uniform concavity condition. Then we give explicit conditions on the maximum norm and the Lipschitz constant of the boundary distribution $g : \partial\Omega \to \mathbb{R}$ which guarantee that each solution of Plateau's problem for the contour

$$\Gamma := \Big\{(x, y, z) \in \mathbb{R}^3 : \ (x, y) \in \partial\Omega, \ z = g(x, y)\Big\}$$

is necessarily a graph above the x, y-plane. Since the nonparametric problem has at most one solution, we obtain a uniqueness result for the parametric problem. Scherk's minimal surfaces appear as comparision surfaces replacing the planes in the arguments of H. Kneser and T. Rado.

Scott Spector

Cavitation in Elastic Solids

After presenting photographs of certain experiments on elastomers and optical fibers, the speaker discussed John Ball's approach to the formation of holes in such materials. Some of the results obtained by Müller and Spector in their existence theory were then outlined. The main emphasis was on the significance of surface energy, invertibility condition (INV), and their useful consequences, i.e., continuity outside a set of small Hausdorff dimension, Lusin's (N) condition, and a simple representation for the distributional Jacobian. Condition (INV) is (essentially) the requirement that a mapping be monotone in the sense of Lebesgue and that a hole created in one part of the body not be filled by material from elsewhere.

Open problems in this field were then presented and a brief discussion of the 3 well-established competing theories of cavitation – that of Gent, Ball, and Marcellini – was attempted. A partial list of open problems follows.

- The main open problem of this area: Are the radial minimizers in fact global minimizers when the body is the unit ball and the loading is radially symmetric?
- There are conceptual difficulties with Müller and Spector's definition of surface energy. What can be done to improve this?

- Condition INV is not closed under left composition by diffeomorphisms. What additional hypotheses might remedy this problem?
- The inverse deformation – what is its domain and what kind of differentiability properties does it enjoy?

Klaus Steffen

New results on the existence of parametric H-surfaces in Riemannian 3-manifolds

In this joint work with Frank Duzaar we prove new existence theorems for disc-type 2-dimensional parametric surfaces with prescribed (continuous, bounded) mean curvature $H : N \to \mathbb{R}$ and given (oriented, rectifiable, contractible, closed) boundary curve Γ in a Riemannian 3-manifold N (connected, oriented, complete, with $\partial N = \emptyset$, compact or noncompact and homogeneously regular). Contrary to earlier work on this problem by Hildebrandt and Kaul and by Gulliver we do not assume that Γ is contained in a normal coordinate domain on N. For example, we obtain the following version of a classical result proved by Wente in $\mathbb{R}^3$:
Theorem: There exist constants $a(N)$, $h(N) > 0$ such that one has existence for all Γ bounding a disc-type surface of area $A \leq a(N)$ and all H with $\sup_N |H| < h(N)A^{-1/2}$.
Here, $a(N)$ and $h(N)$ can be computed in terms of certain optimal isoperimetric constants of N. For instance, if $N = S^3$ then we obtain existence for all Γ spanned by a disc in S^3 of area $A < \frac{1}{2}|S^2|$ and all H with $\sup_{S^3} |H| < 2|S^3|^{-1}(|S^2| - 2A)$. On the other hand, Gulliver has proved that a great circle in S^3 does not bound any surface of constant mean curvature $H \neq 0$ in S^3. Thus the condition imposed on Γ cannot be improved. Our variational method also allows to prove existence under an integral condition of type $\int_N |H|^3 < c(N)$ (with no condition on Γ) or under a bound $\sup_D |H| \leq h$ if Γ is contractible in a subdomain $D \subset N$ with boundary mean curvature $\geq h$.

Michael Struwe
Wave maps

For any compact "target" manifold $N \subset \mathbb{R}^n$ with $\partial N = \emptyset$ and finite energy data $(u_0, u_1) : \mathbb{R}^2 \to TN$ in joint work with S. Müller the existence of a global weak solution $u : \mathbb{R} \times \mathbb{R}^2 \to N \subset \mathbb{R}^n$ of the Cauchy problem for wave maps

(1) $\Box u = u_{tt} - \Delta u = A(u)(Du, Du) \perp T_u N$

(2) $u|_{t=o} = u_0,\ u_t|_{t=0} = u_1$

is established.

The proof uses a viscous approximation to (1) and a weak compactness result of Freire-Müller-Struwe, based on the determinant structure exhibited by (1) in a suitable "gauge" and $\mathcal{H}^1$ - BMO duality.

The result makes contact with work of Christodoulou-Tahvildar-Zadeh, Yi-Zhou, Hélein, Evans, and Bethuel.

Luc Tartar
H-measures

I have introduced H-measures first for homogenization questions, then used them for propagation results. Patrick GERARD introduced independently the same object for other purposes.

If u_n satisfies $(u_n)_{tt} - \Delta u = 0$ with $u_n(\cdot, 0) = v_n \rightharpoonup 0$ in H^1 weak and $u_{nt}(\cdot, 0) = w_n \rightharpoonup 0$ in L^2 weak, then $u_{nt}, u_{nx_j} \rightharpoonup 0$ in $L^\infty(0, \infty, L^2)$ weak-$*$, but one cannot predict the limit of the density of energy $e_n = \frac{1}{2}u_{nt}^2 + \frac{1}{2}|\mathrm{grad} u_n|^2$ (the Div-Curl lemma implies $u_{nt}^2 + |\mathrm{grad} u_n|^2 \rightharpoonup 0$). One needs more information on initial data (some H-measures) and one proves then that an H-measure associated to u_n satisfies a p.d.e. in (x, ξ) corresponding to a phase-free version of geometrical optics. Taking in account the initial data has been done by Gilles FRANCFORT and François MURAT (with hints from Patrick GERARD).

H-measures are constructed to compute limits of $L_1\varphi_n \overline{L_2\psi_n}$ where φ_n, ψ_n converge to 0 in L^2 weak and L_1, L_2 are "pseudodifferential" operators of order 0.

H-measures use no characteristic length, but H-measures can be generalized in order to include one characteristic length, as done by P. GERARD (semiclassical measures). I described them with a variant that I introduced, which corrects a mistake in the derivation of the same objects by another method by Pierre-Louis LIONS and Thierry PAUL, based on WIGNER transform. I described a simple approach obtained with P. GERARD based on 2-points correlations and BOCHNER-SCHWARTZ theorem. I showed why this approach is not adapted to some situations where many small scales are present and interact.

Peter Topping
The harmonic map heat flow from surfaces

We consider the heat flow between 2-spheres - i.e., the solution $u : S^2 \times [0, \infty) \to S^2 \hookrightarrow \mathbb{R}^3$ of

$$\frac{\partial u}{\partial t} = \tau := (\Delta u)^{\perp}.$$

Asymptotic studies show that the flow converges to a bubble tree at a sequence of times $t_i \to \infty$, though in general this convergence is not uniform in time.

In the case that the components of the bubble tree all show the same orientation, we provide uniform-in-time convergence results in all reasonable senses.

Rugang Ye
The Chern-Simons functional and Seiberg-Witten Floer homology

Consider a rational homology sphere Y of dimension 3 and a spin^C-structure on it, with the associated spinor bundle S and line bundle L. Let $\varphi \in \Gamma(S)$, $a \in \mathcal{A} = \{\text{unitary connections on } L\}$. The Seiberg-Witten equation is

$$\begin{cases} *F_a + \langle e_i\varphi, \varphi\rangle e^i = 0 \\ \mathcal{D}_a\varphi = 0, \end{cases}$$

where F_a is the curvature of a and $\mathcal{D}$ the Dirac operator of a. The Chern-Simons functional is

$$\mathrm{cs}(a,\varphi) = \frac{1}{2}\int_Y (a-a_0)\wedge(F_a + F_{a_0}) - \frac{1}{2}\int_Y \langle \varphi, \mathcal{D}_a\varphi\rangle.$$

Using a joint work with G. Wang we establish
Theorem: The Bott-type Seiberg-Witten homology is well-defined and a diffeomorphism invariant.

This homology is constructed in terms of the solutions of the Seiberg-Witten equation.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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