MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

# Tagungsbericht 27/1996

# Singularitäten

14.07. – 20.07.1996

The conference was organized by V.I. Arnold (Moskau), G.–M. Greuel (Kaiserslautern) and J.H.M. Steenbrink (Nijmegen) and attended by about 50 participants from Europe, North America and Japan. There were 20 talks, ranging from new developments in the theory of real and complex curves, 2 and 3–dimensional singularities, complete intersections, up to applications of singularity theory to theoretical physics. A special highlight of this conference was the "Brieskorn Day" with talks by Brieskorn's teacher F. Hirzebruch and some of his students in honour of Brieskorn's 60th birthday. A talk on minerals of the "Grube Clara" by Mr. Günter, the head of "Mineralien Museum" at Oberwolfach together with a visit of that museum contributed to the exiting atmosphere during the conference.

# Vortragsauszüge

VLADIMIR I. ARNOLD

## Singularities of plane curves and of wave fronts

1. General nonsense: symplectization, contactization, complexification of all mathematical theories as the method of conjectures discovery.
2. Möbius theorem on the 3 inflection points of noncontractible curves in $\mathbb{R}P^2$ and the principle of the economy of the algebraic realizations of topological phenomena.
3. Wave front reversal and the conjecture on the four cusps on the front as the generalization of the last geometrical theorem of Jacobi and of the four vertices theorem.
4. Sturm theory as the generalization of Morse theory to higher derivatives; and its conjectural extensions to functions of more than one variable.
5. Local additive invariants of immersed plane curves and of wave fronts — theories by V.I. Arnold, Viro, Shumakovich, Aicardi, Polyak, Lin, Wang, Goryunov, Burry, Chernov, Tabachnikov, Chmutov, Vassiliev, Merkov and others, on the $J^+$, $J^-$, $St$ invariants, their relations to algebraic geometry, quantum fields, statistical sums, Vassiliev invariants of knots, Kontsevich integrals and new invariants of Legendrian and framed knots in the solid torus.

MICHAEL POLYAK

## On finite–type invariants of curves and fronts

In 1993 V.I. Arnold studied the discriminant of singular (i.e. non–generic) immersions of an oriented circle into the plane in the space of all immersions. The consideration of three different strata of this discriminant enabled Arnold to introduce three new basic invariants ($J^+$, $J^-$ and $St$) of generic plane curves. $J^\pm$ and $St$ were defined axiomatically via their "jumps" under different deformations of a curve and additivity under connected summation. A similar $St$–type theory of multi–component curves was introduced earlier by Vassiliev, Fenn–Taylor, etc.

We propose an elementary combinatorial way to produce and study different numerical finite–type invariants of curves on surfaces in a systematical and unified manner. Our technique is well–suited for both, one– and multi–component curves, and allows a parallel treatment of these cases. The invariants are defined in terms of a simple combinatorical invariant of a curve — its Gauss diagram. Gauss diagrams present a natural rich source of invariants, as it is a complete invariant of spherical curves (and, together with the index, of planar ones) and encodes all the information about a curve in an elementary fashion. In our approach we count, with some signs and coefficients, the number of different subdiagrams of the Gauss diagram. We prove that any such invariant is of finite type (with degree less or equal half of the maximal number of chords in the corresponding subdiagrams). In particular, the order 1 invariants $J^\pm$, $St$ (and the index $i_{123}$ of multi–component curves) are obtained by counting appropriate 2–chord subdiagrams. Our technique easily generalizes to Legendrian fronts.

Motivated by the similarity of our formulas to Gauss diagram expressions for low–degree Vassiliev knot invariants, we establish various relations of finite–type invariants of curves to the Vassiliev knot invariants and discuss simple examples of such relations.

JIM DAMON

## On the legacy of free divisors

Several of the main directions in Brieskorn's work have involved the Milnor fibres of isolated singularities, discriminants of versal unfoldings, the interplay of deformation theory and

topology, and the topology of the complements of Coxeter arrangements. A common element in these topics is the occurrence of free divisors introduced by K. Saito. For example, Arnold's and Brieskorn's factorisation theorems for the Poincaré polynomials of complements of Coxeter arrangements were shown by Terao to be a consequence of their freeness. This talk explains how the role of freeness interweaves the topology of these objects and provides a legacy of its freeness in several ways.
The rigid free divisors yield rich classes of "almost free divisors and complete intersections" via pull-back and transverse intersection. This leads to a singular version of the Milnor fibre which has the correct homotopy properties: A joint theorem with D. Mond allows the associated "singular Milnor number" to be computed as the length of a determinantal module. This leads to a proof of the factorisation theorem as a direct consequence of the singular Milnor fibre properties.
A second legacy of freeness results from the associated deformation theory for studying the pull-back varieties. Provided the free divisor $V$ gives rise to the appropriate version of Morse-type singularities, then the associated "discriminant" for this theory is a free divisor. This partially explains the abundance of free divisors but also shows that a result of Bruce and Terao on the freeness of bifurcation sets extends to maps into higher dimensional spaces. It also explains the failure of freeness of the Manin-Schechtman discriminantal arrangements due to their being only parts of the "true discriminants".

KYOJI SAITO

## Duality of weight systems and positivity of Fourier coefficients of $\eta$-products associated to the weight system

In the Nice congress (1970), E. Brieskorn established the relationship between simple singularities and simple Lie groups (algebras). In attempt to establish further relationships between singularities (simply elliptic, 14 exceptional unimodular, etc.) and posibly infinite dimensional Lie algebras (which are still to be found and to be constructed), we introduce the $\eta$-product (as a possible building block for the algebra) as follows:

$$\eta_W(\tau) := \prod_{i\in M_W} \eta(i\tau)^{e_W(i)}.$$

Here i) $W = (a, b, c\colon h)$, called a regular system of weights, is a system of 4 positive integers $0 \le a, b, c < h$, such that

$$\chi_W(T) := T^{-h}\frac{(T^h - T^a)(T^h - T^b)(T^h - T^c)}{(T^a - 1)(T^b - 1)(T^c - 1)}$$

is regular except at $T = 0$. Then one has a development $\chi_W(T) = T^{m_1} + \ldots + T^{m_\mu}$,
ii) the index set $M_W$ and the exponents $e_W(i)$ (for $i \in M_W$) are defined by the relation

$$\prod_{i=1}^{\mu} (\lambda - \exp(2\pi\sqrt{-1}\, m_i/h)) = \prod_{i\in M_W} (\lambda^i - 1)^{e_W(i)},$$

iii) $\eta(\tau) := \exp(2\pi\sqrt{-1}\,\tau/24)\prod_{n=1}^{\infty}(1 - q^n)$ $(q = \exp(2\pi\sqrt{-1}\,\tau))$ is the Dedekind $\eta$-function.
$\eta_W$ is an almost holomorphic automorphic form of weight $2a_0 := \sum e_W(i)$, whose congruence group and character are easy to determine. $\eta_W$ is cuspidal (holomorphic) if and only if $\nu_W := -\sum_{i\in M_W}(h/i)e_W(i)$ is negative (resp. non-positive).

**Conjecture:** All the Fourier coefficients of the development $\eta_W = \sum_{n\in\mathbb{Q}_{>0}} c_n q^n$ are non-negative, if and only if $\nu_W \ge 0$.

The conjecture is proved for a wide class of weight systems $W$ (including the cases $\widetilde{E}_6$ $(1,1,1{:}3)$, $\widetilde{E}_7$ $(1,2,2{:}4)$, $\widetilde{E}_8$ $(1,2,3{:}6)$ when $\nu_W = 0$), using Eisenstein series, which describe the Hecke eigen forms of the same type as the $\eta$-products.

## “Brieskorn Day” — Tuesday 16.07.1996

| | |
|---|---|
| $9^{00} - 9^{30}$ | *G.-M. Greuel: Some aspects of Brieskorn's mathematical work* |
| $9^{45} - 10^{45}$ | *Friedrich Hirzebruch: Singularities and exotic spheres* |
| $11^{00} - 12^{00}$ | *Peter Slodowy: A new identification of subregular singularities in simple Lie algebras* |
| $15^{30} - 16^{15}$ | *Wolfgang Ebeling: Milnor lattices and monodromy groups* |
| $16^{30} - 17^{15}$ | *Claus Hertling: Applications of the Brieskorn lattice $H_0''$* |
| $17^{30} - 18^{15}$ | *Egbert Brieskorn: Singularities and Polyhedra* |
| $20^{00}$ | *Mrs. H. Brieskorn, M. Kreck, J. Steenbrink: Music* |

GERT-MARTIN GREUEL

### Some aspects of Brieskorn's mathematical work

The first talk of this day which was organized in honour of Brieskorn's 60th birthday was devoted to a short overview of Brieskorn's mathematical work. In this work we see clearly Brieskorn's idea of unity of mathematics and the success in relating different mathematical structures:

- Topological — differential — analytic (discovery of exotic spheres as neighbourhood boundary of singularities),
- Resolution — deformation (simultaneous resolution of ADE–singularities)
- Lie groups — equations (construction of ADE–singularities from the corresponding simple Lie groups)
- Transcendental — algebraic (construction of local Gauß–Manin connection)
- Continuous — discrete ((generalized) braid groups, Milnor lattices and Dynkin diagrams)

In the subsequent talks details and further developments of some of these topics are explained by Brieskorn's teacher Prof. F. Hirzebruch and some of Brieskorn's students (W. Ebeling, P. Slodowy, C. Hertling).

FRIEDRICH HIRZEBRUCH

### Singularities and exotic spheres

Bericht über das akademische Jahr 1965/66. Brieskorn ist C.L.E. Moore Instructor am M.I.T., Jänich ist an der Cornell University, dann am IAS in Princeton. Ich bin in Bonn. Es gibt ausgedehnte Korrespondenz. Vom 30.9.–7.10.1965 bin ich bei einer Konferenz in Rom (Bericht über Brieskorn's simultane Auflösungen). Dort erreicht mich Brieskorn's Brief vom 28.9.1965: *“Ich habe in den letzten Tagen die etwas verwirrende Entdeckung gemacht, daß es vielleicht 3–dimensionale normale Singularitäten gibt, die topologisch trivial sind. Ich habe das Beispiel heute nachmittag mit Mumford diskutiert, und er hatte bis heute abend noch keinen Fehler gefunden; hier ist es:* $X = \left\{x \in \mathbb{C}^4 \mid x_1^2 + x_2^2 + x_3^2 + x_4^3 = 0\right\}$.” Beweis durch Auflösung und Berechnung aller Invarianten des Umgebungsrandes. In den Proc. Nat. Acad. Sci. USA erscheint allgemeiner das Beispiel $x_1^2 + \ldots + x_k^2 + x_{k+1}^3 = 0$ ($k$ ungerade).
Bericht über die umfangreiche Korrespondenz, die sich anschließt, über Brieskorn's Entdeckung der Arbeit von Pham, die es ihm ermöglicht, Milnor's assertion in einem Brief an Nash zu beweisen — Milnor an Nash am 13.4.1966: *“The Brieskorn example is fascinating. After staring at it for a while, I think I know which manifolds of this type are spheres but the statement is complicated and a proof does not exist. Let $\Sigma(p_1, \ldots, p_n)$ be*

*the locus* $z_1^{p_1} + \ldots + z_n^{p_n} = 0$, $|z_1|^2 + \ldots + |z_n|^2 = 1$, *where* $p_j \geq 2$*....*" Dann gibt Milnor die Bedingung a) oder b) für die Exponenten an. — Allmählich wird es allen Beteiligten klar, daß zur Bestimmung der differenzierbaren Struktur die Berechnung der Signatur von $z_1^{p_1} + \ldots + z_n^{p_n} = 1$ $(n \geq 3,\ n$ ungerade) erforderlich ist. Darüber gibt es manche Briefe von Brieskorn an mich und umgekehrt. Brieskorn schreibt seine Arbeit für die Inventiones Bd. 2 (1966). In diesem Zusammenhang hat er auch $\Sigma(2,3,5,30)$, $30 =$ Coxeterzahl von $E_8$, studiert und schließlich die kleinen Auflösungen dieser Singularität in Kurven gemäß $E_8$–Baum und damit die simultane Auflösung der Flächenfamilien $x_1^2 + x_2^3 + x_3^5 + t^{30} = 0$ (Parameter $t$) bewerkstelligt und den übriggebliebenen Fall seiner Math. Ann. Arbeit von 1966 (über die ich in Rom berichtete) erledigt. Verständnis im Rahmen der Wurzelsysteme und der Weylschen Gruppe wurde erzielt (Brieskorn's Brief an Frau Tjurina vom 13.9.1966) — Jänich hatte $O(n)$–Mannigfaltigkeiten $W^{2n-1}(d)$ studiert (zwei Orbittypen mit Isotropiegruppen $O(n-2)$, $O(n-1)$ und Orbitraum $D^2$, $S^1$) und diese klassifiziert, sowie Knotenmannigfaltigkeiten $M^{2n+1}(k)$, auf denen $O(n)$ operiert (drei Orbittypen $O(n-2)$, $O(n-1)$, $O(n)$ mit Orbitraum $D^4$, $S^3 - k$, $k$ ($k$ der Knoten)). Ich bringe die beiden in USA befindlichen zusammen durch Bericht vom März 1966, z.B. $W^{2n-1}(d)$ ist $\Sigma(2,\ldots,2,d)$ und $M^{2n+1}$(Torusknoten 3,5) ist $\Sigma(2,\ldots,2,3,5)$. Brieskorn schreibt am 29.3.1966: *"Klaus Jänich und ich hatten von diesem Zusammenhang unserer Arbeiten nichts gemerkt, und ich war vor Freude ganz außer mir, wie Sie nun die Dinge zusammengebracht haben."*

Ich hatte hier in Oberwolfach die gleiche Freude, darüber erzählen zu können.

PETER SLODOWY

## A new identification of subregular singularities in simple Lie algebras

This talk is a report and an improvement on recent work of V. Hinich (Israel J. Math. 76 (1991)) which provides a new, conceptual proof of (parts of) Brieskorn's theorem (ICM, Nice 1970) identifying the singularity along a subregular nilpotent orbit in a simple Lie algebra $\mathfrak{g}$ with a Kleinian singularity (RDP, simple sing.) of type $A$, $D$, $E$ (for $\mathfrak{g}$ of type $A$, $D$, $E$; for $\mathfrak{g}$ of type $B_n$, $C_n$, $F_4$, $G_2$ one has $A_{2n-1}$, $D_{n+1}$, $E_6$, $D_4$). The main idea of Hinich is to exploit the geometry of generalized flag manifolds $G/B$ (lines and line bundles) in connection with Springer's resolution of the nilpotent variety of $\mathfrak{g}$ to derive the intersection matrix of the prospective minimal resolution of the singularity in question. In our presentation, we strongly exploit transversality properties of slices to further simplify Hinich's arguments.

WOLFGANG EBELING

## Milnor lattices and monodromy groups

Twenty years ago, Brieskorn started to work on the deformation theory of the exceptional unimodular singularities. Brieskorn was very much interested in the question: To which extend is the subtle geometry of a singularity and its deformations reflected in the invariants?

The invariants which he considered include the Milnor lattice $L$, the monodromy group $\Gamma$, the set $\Delta$ of vanishing cycles and the set $B^*$ of all (strongly) distinguished bases.

We report on Brieskorn's work on the deformation theory of the exceptional unimodular singularities and on these invariants. The relative strength of these invariants with respect to geometrical problems is discussed. We give a survey on related work of Brieskorn's students Knörrer, Balkenborg, Bauer, Bilitewski, Voigt, Kluitmann, Krüger, Dörner, Balke, Kaenders and myself and we mention some further developments.

CLAUS HERTLING

## Applications of the Brieskorn lattice $H_0''$

1970, in his paper on the complex monodromy of isolated hypersurface singularities, Brieskorn laid the foundations for the study of the local Gauß–Manin connection for hypersurface singularities. At the center of the algebraic description of the Gauß–Manin connection stands one object, which is nowadays called the Brieskorn lattice. If $f : (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ is a holomorphic function germ with an isolated singularity at 0, this is

$$H_0'' = \Omega^{n+1}_{\mathbb{C}^{n+1},0} / df \wedge d\Omega^{n-1}_{\mathbb{C}^{n+1},0} .$$

In this talk the definition and basic properties of the Gauß–Manin connection and the Brieskorn lattice are resumed. An overview is given of many applications and results, which have been obtained in the last 26 years:

Brieskorn's proof of the monodromy theorem, statements concerning the Jordan blocks of the monodromy, Malgrange's description of the Bernstein polynomial with $H_0''$ and applications of M. Saito and Stahlke, Varchenko's mixed Hodge structure on the cohomology of the Milnor fibre, spectral numbers and their upper semicontinuity for deformation (Steenbrink), applications to the $\mu$–constant stratum, classifying spaces for the polarized mixed Hodge structure on the cohomology of the Milnor fibre and for the Brieskorn lattice (Hertling), M. Saito's infinitesimal Torelli theorem for $H_0''$, global Torelli theorems with respect to $H_0''$ for several families of singularities by Hertling.

EGBERT BRIESKORN

## Singularities and Polyhedra

I reported about work of my students Thomas Fischer, Alexandra Kaess, Ute Neuschäfer, Frank Rothenhäusler and Stefan Scheidt. This work describes the neighbourhood boundaries of quasi–homogeneous surface singularities in a new way. It is known that these neighbourhood boundaries are quotients $G/\Gamma$ of a 3–dimensional Lie group $G$ and a discrete subgroup $\Gamma$. For example, for the quotient singularities $\mathbb{C}^2/\Gamma$ the group $G$ is $\mathrm{Spin}(3) = S^3$, the group of unit quaternions, and $\Gamma$ could for example be one of the three binary polyhedral groups (binary tetrahedral $\mathbb{T}$, binary octahedral $\mathbb{O}$, binary icosahedral $\mathbb{I}$). This gives the three singularities $E_6$, $E_7$, $E_8$. For the next set of examples, the simply–elliptic singularities $\widetilde{E}_6$, $\widetilde{E}_7$, $\widetilde{E}_8$, the group $G$ is the Heisenberg group, and $\Gamma$ is a congruence subgroup of the lattice of its integral matrices. In most cases however, $G$ is $\mathrm{SU}(1,1)$ or some covering of it, and $\Gamma$ comes from a Fuchsian group $\overline{\Gamma} \subset \mathrm{PSU}(1,1)$ acting on the hyperbolic plane $\mathbb{H} = \{z \in \mathbb{C} \mid |z| < 1\}$. All of this is well known.

Now I describe a very original construction discovered by Thomas Fischer in his 1992 PhD–thesis:

Let $\overline{\Gamma} \subset \mathrm{PSU}(1,1)$ be discrete with compact quotient $\mathbb{H}/\overline{\Gamma}$. Assume that $\overline{\Gamma}$ has at least one point in $\mathbb{H}$ with nontrivial isotropy subgroup. *Choose* such a point $o \in \mathbb{H}$. Let $p$ be the order of its isotropy group $\{\overline{\gamma} \in \overline{\Gamma} \mid \overline{\gamma}(o) = o\}$. Let $\Gamma \subset \mathrm{SU}(1,1)$ be the inverse image of $\overline{\Gamma}$. For many singularities, the neighbourhood boundary is of the form $\mathrm{SU}(1,1)/\Gamma$ with a suitable $\overline{\Gamma}$. For example, for the 14 quasihomogeneous exceptional 1–modular singularities $E_{12}$, $E_{13}$, $E_{14}$, $Z_{11}$, $Z_{12}$, $Z_{13}$, $Q_{10}$, $Q_{11}$, $Q_{12}$, $W_{12}$, $W_{13}$, $S_{11}$, $S_{12}$, $U_{12}$ the group $\Gamma$ is the group of orientation–preserving automorphisms of $\mathbb{H}$ in the group $\Sigma(p,q,r)$ generated by the reflections in the sides of a hyperbolic triangle with angles $\pi/p$, $\pi/q$, $\pi/r$. In this case, the choice of $o \in \mathbb{H}$ amounts to choosing one of the integers in the so called Dolgachev triple $(p,q,r)$. We shall indicate this by underlining this number, e.g. $(2,3,\underline{7})$. Fischer's construction:

$$\mathrm{SU}(1,1) = \left\{ \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix} \,\middle|\, a\bar{a} - b\bar{b} = 1 \right\} = \left\{ x \in \mathbb{R}^4 \,\middle|\, x_0^2 + x_1^2 - x_3^2 - x_4^2 = 1 \right\} =: \mathbb{S}$$

is a 3–dimensional pseudosphere with Minkowski–metric with signature $(+,-,-)$. Up to a factor $-1/8$, this agrees with the Killing metric. The construction will be done in $\mathbb{R}^4$

with $\langle x,x\rangle = x_0^2 + x_1^2 - x_3^2 - x_4^2$. Let $C^+$ be the positive cone $C^+ = \{x \in \mathbb{R}^4 \mid \langle x,x\rangle > 0\}$ and $\pi : C^+ \to \mathbb{S}$ be the retraction by central projection $\pi(x) := x/\sqrt{\langle x,x\rangle}$. For any $g \in \mathbb{S}$, let $H_g$ be the halfspace $H_g := \{x \in \mathbb{R}^4 \mid \langle x,g\rangle \leq 1\}$. Its boundary $\partial H_g$ is the affine tangent space $\partial H_g = T_g(\mathbb{S})$. For any $z \in \overline{\Gamma}(o)$ in the chosen special orbit $\overline{\Gamma}(o) \subset \mathbb{H}$, let $L_z$ be the coset $L_z = \{\gamma \in \Gamma \mid \gamma(o) = z\}$. It has the cardinality $2p$. Let $Q_z \subset \mathbb{R}^4$ be defined by

$$Q_z := \bigcap_{g \in L_z} H_g.$$

$Q_z$ is a 4–dimensional prism, the product of $\mathbb{R}^2$ with a plane $2p$–gon. Consider

$$P := \bigcup_{z \in \overline{\Gamma}(o)} Q_z$$

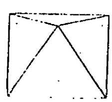
and $\partial_+ P := \partial P \cap C^+$.
$\partial_+ P$ is the support of a 3–dimensional polyhedral complex and $\pi : \partial_+ P \to \mathbb{S}$ is a homeomorphism, which transfers the polyhedral structure to $\mathbb{S}$. The following definition and theorem of Fischer analyzes this structure:

**Definition:** $F_g = C^+ \cap \partial H_g \cap (Q_{g(o)} \setminus \bigcup_{\substack{z \in \Gamma(o) \\ z \neq g(o)}} Q_z)$

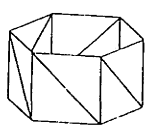
**Theorem:**

1. *$F_g$ is a compact polyhedron in the Minkowski–3–space $\partial H_g$*
2. *$\{F_g\}_{g\in\Gamma}$ is the set of 3–dimensional faces of a 3–dimensional polyhedral complex with support $\partial_+ P$.*
3. *$\Gamma$ operates simply transitively on $\{F_g \mid g \in \Gamma\}$.*
4. *$\{\pi(F_g)\}$ is a tesselation of $\mathbb{S}$ by totally geodesic polyhedra in this Minkowski–pseudosphere. $\Gamma$ acts simply transitively on the set of these $\pi(F_g)$, so each of them can serve as a fundamental domain.*
5. *Hence $\mathbb{S}/\Gamma$ is obtained from $F_G$ by pairing faces and identifying them in a specified way given by $\Gamma$ and the construction.*

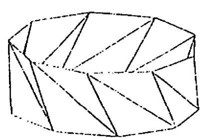
Fischer calculated the examples $(2,3,\underline{7})$, $(2,3,\underline{8})$, $(2,3,\underline{9})$. These fit in very well with the classical cases $E_6 = (2,3,\underline{3})$, $E_7 = (2,3,\underline{4})$ and $E_8 = (2,3,\underline{5})$. I myself added an analysis of the cases $\tilde{E}_6$, $\tilde{E}_7$, $\tilde{E}_8$. The following pictures show the resulting 9 fundamental domains:



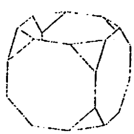
(2,3,3)



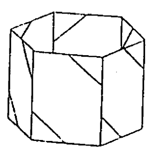
$\tilde{E}_6$



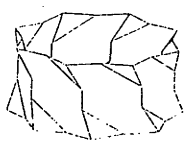
(2,3,9)



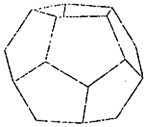
(2,3,4)



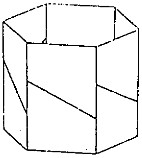
$\tilde{E}_7$



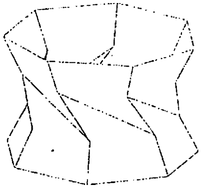
(2,3,8)



(2,3,5)

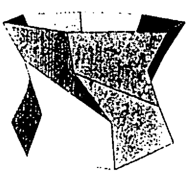


$\tilde{E}_8$



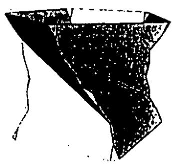
(2,3,7)

The other four students worked out all 14 exceptional $(p,q,\underline{r})$ with the exception of $r = 2$. As a result, a pattern seems to emerge. The following shows a sample of their pictures:



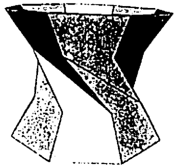
$F_4$ für $\Gamma(2,3,\underline{7})$

$E_{12}$



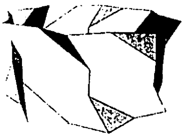
$F_4$ für $\Gamma(2,4,\underline{5})$

$E_{13}$



$F_4$ für $\Gamma(3,3,\underline{4})$

$E_{14}$

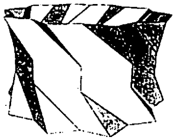


$F_4$ für $\Gamma(2,3,\underline{8})$

$Z_{11}$

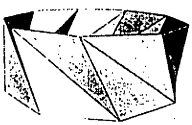
$F_4$ für $\Gamma(2,4,\underline{6})$

$Z_{12}$



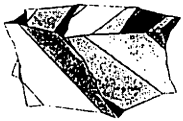
$F_4$ für $\Gamma(3,3,\underline{5})$

$Z_{13}$



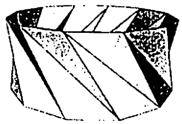
$F_4$ für $\Gamma(2,3,\underline{9})$

$Q_{10}$



$F_4$ für $\Gamma(2,4,\underline{7})$

$Q_{11}$



$F_4$ für $\Gamma(3,3,\underline{6})$

$Q_{12}$

I presented some conjectures on the series–patterns. Work in progress by Ludwig Balke may lead to a new and original way of looking at symmetry–breaking.



Gefördert durch DFG Deutsche Forschungsgemeinschaft

Eugenii Shustin

## Equiclassical deformations of plane curves

Let $\Delta$ be the discriminant in the space $\mathbb{P}^{d(d+3)/2}$ of plane curves of degree $d$ over an algebraically closed field of characteristic 0. Let $\Delta_{int} \subset \Delta$ be the set of irreducible curves. We consider the equiclassical stratification:

$$\Delta_{int} = \bigcup_{g,c} V_{d,g,c},$$

where $V_{d,g,c}$ is the set of curves of (geometric) genus $g$ and class $c$ (degree of the dual curve). The question on the geometry of equiclassical strata was posed by W. Fulton.

**Theorem:** *If $c \geq 2g - d + 2$, then a generic member $C$ of any component of $V_{d,g,c}$ is a curve with ordinary nodes and cusps.*

This improves the previously known result by Diaz and Harris (1989): $c \geq 2g - 1$. Moreover, under condition $c \geq 2g - d + 2$, the dimension of any component of $V_{d,g,c}$ is $c - g + d + 1$, and it is non–singular at any "nodal–cuspidal" point $C$. But $V_{d,g,c}$ may be reducible even under the condition $c \geq 2g - 1$, what happens, for instance, in Zariski's example $V_{6,4,12}$.

**Corollary:** *Any curve of degree $d \leq 10$ can be deformed into a curve with nodes and cusps, having the same degree, genus and class.*

Christoph Lossen

## Geometry of families of nodal curves on the blown–up projective plane

Joint work with Gert–Martin Greuel and Eugenii Shustin

We deal with the following general problem: given a smooth rational surface $S$ and a divisor $D$ on $S$, when is the variety $V_{irr}(D,k)$ of nodal irreducible curves in the complete linear system $|D|$ with a fixed number $k$ of nodes non–empty, when non–singular of the expected codimension ("T–variety") and when irreducible?
For $S = \mathbb{P}^2$, these questions are completely answered by the classical result of F. Severi (1921) stating, that the variety $V_{irr}(dH,k)$ of irreducible curves of degree $d$ having $k$ nodes is a non–empty T–variety if and only if $0 \leq k \leq (d-1)(d-2)/2$, and the result of J. Harris (1985) stating that it is always irreducible. A modification of Severi's method did lead to a sufficient (smoothness–)criterion for general rational surfaces $S$, namely $K_S \cdot C < 0$ for each $C \in V_{irr}(D,k)$.
We concentrate on the case $S = \mathbb{P}^2_r \xrightarrow{\pi} \mathbb{P}^2$, the projective plane blown–up at $r$ generic points $p_1, \ldots, p_r$. Let $E_0$ denote the strict transform of a generic straight line on $\mathbb{P}^2$ and $E_i := \pi^{-1}(p_i)$. Then a curve $C \in V_{irr}(dE_0 - \sum d_i E_i, k)$ corresponds to a plane curve of degree $d$ having (not necessarily ordinary) $d_i$–fold points at $p_i$ $(1 \leq i \leq r)$ and $k' \leq k$ nodes outside. In this situation, the above general criterion reads as $3d > \sum_{i=1}^r d_i$ (which is always fulfilled for $r \leq 8$.
We obtain asymptotically improved (that is, quadratic in $d$, $d_i$) sufficient conditions for the T–variety property of $V_{irr}(dE_0 - \sum d_i E_i, k)$, which give an improvement for $r, d \gg 0$. Moreover, we obtain sufficient conditions for the non–emptiness and the irreducibility of the same type. For the proof, we use the deformation theory to deduce the smoothness and the irreducibility from the vanishing of some $H^1$'s. To show the existence of irreducible curves with a given number of nodes, we combine a modification of a proof by A. Hirschowitz and the smoothing of nodes.

Andrew du Plessis

## Versal deformations in families of projective hypersurfaces

The following was proved:

**Theorem:** *Let $\Gamma$ be a hypersurface of degree $d$ in $\mathbb{P}^r(\mathbb{C})$ with isolated singularities only. Let*

$$\tau^* = \begin{cases} 4(d-1) & \text{if } d \geq 5 \\ 18 & \text{if } d = 4 \\ 16 & \text{if } d = 3 \end{cases}, \quad \tau^{**} = \begin{cases} 3(d-1) & \text{if } d \geq 5 \\ 9 & \text{if } d = 4 \\ 8 & \text{if } d = 3 \end{cases}$$

(i) *If the sum $\tau(\Gamma)$ of the Tjurina numbers of the singularities of $\Gamma$ is less than $\tau^*$, then the family of all hypersurfaces of degree $d$ induces simultaneous versal unfoldings of all the singularities of $\Gamma$.* (The case $r = 2$ was previously obtained by E. Shustin.)

(ii) *If $\tau(\Gamma) < \tau^{**}$, and $H$ is any hyperplane transverse to $\Gamma$, then the family of all hypersurfaces of degree $d$ agreeing with $\Gamma$ on $H$ induces simultaneous versal unfoldings of all the singularities of $\Gamma$.*

Moreover, the bounds are best possible, in that there exist hypersurfaces of degrees $\tau^*$, $\tau^{**}$ (whenever such hypersurfaces can exist, so $\tau^*, \tau^{**} \leq (d-1)^r$) not versally unfolded as in (i), (ii), respectively.
The methods used make extensive use of the properties of discriminant matrices of hypersurface singularities, and in particular of their relation to the instability modules of unfoldings of (weighted–)homogeneous function–germs versal only up to a certain weight.

Charles T. C. Wall

## Application of the discriminant matrix to highly singular plane curves

Let $\Gamma$ be a reduced (but perhaps reducible) curve in the projective plane $\mathbb{P}^2\mathbb{C}$, defined by an equation $F(x, y, z) = 0$ of degree $d$. We say the vector field $a\partial/\partial x + b\partial/\partial y + c\partial/\partial z = \xi$ has degree $r$ if $a, b, c$ are homogeneous of degree $r$, and that $\xi \in \operatorname{Ann} F$ if $\xi F = 0$.

**Theorem:** *Let $r$ be the least degree of any $0 \neq \xi \in \operatorname{Ann} F$. Then $0 \leq r \leq d-1$ and we have $(d-1)(d-r-1) \leq \tau \leq (d-1)(d-r-1) + r^2$.* ($\tau$ denotes the sum of the Tjurina numbers at the singular points of $\Gamma$.) *If $2r + 1 > d$, then $\tau \leq \frac{3}{4}d^2 - \frac{3}{2}d$.*

**Notes:** The theorem only gives effective information when $\tau > \frac{3}{4}d^2 - \frac{3}{2}d$, the "highly singular" case.
If $r = 0$, $\Gamma$ consists of concurrent lines and $\tau = (d-1)^2$.
If $r = 1$, $\Gamma$ has a 1–parameter symmetry group $\subset PGL_3$.
For $d = 12$ we have $\tau = 121$ if $r = 0$, $\tau = 110$ or $111$ if $r = 1$, $99 \leq \tau \leq 103$ if $r = 2$ and $\tau \leq 97$ otherwise.

**Ideas of proof:** Regard $f(x, y) = F(x, y, 1)$ as a deformation of $f_0(x, y) = F(x, y, 0)$.
The theory of the discriminant matrix leads to $(d-1)^2 - \tau = \dim(\mathbb{C}[x, y]/K)$, where $K = \{\overline{\psi} \in \mathbb{C}[x, y] \mid \overline{\psi} f \in \langle \partial f/\partial x, \partial f/\partial y \rangle\}$. Set $I = \{\psi \in \mathbb{C}[x, y, z] \mid \psi f \in \langle \partial F/\partial x, \partial F/\partial y \rangle\}$.
One shows that, provided $z = 0$ is transverse to $\Gamma$, $\mathbb{C}[x, y, z]/I$ is a free $\mathbb{C}[z]$–module, so its rank $((d-1)^2 - \tau)$ can be computed by substituting $z = 0$.
Next, $\gamma \in I$ if and only if there exists a vectorfield $\xi = a\partial/\partial x + b\partial/\partial y + c\partial/\partial z \in \operatorname{Ann} F$.
The bound essentially comes from the observation that $\xi, \eta \in \operatorname{Ann} F$ implies that $\xi \wedge \eta$ is a multiple of $\nabla F$, which leads to

$$0 \longrightarrow \mathbb{C}[x, y, z] \longrightarrow \operatorname{Ann} F \xrightarrow{\Delta} \mathbb{C}[x, y, z],$$

where the first map is multiplication by $\xi$ of minimal degree $r$, and $\Delta\eta = \psi$ if $\xi \wedge \eta = \psi \nabla F$. This leads easily to $0 \to \mathbb{C}[x, y, z] \to I \to \mathbb{C}[x, y, z]/\langle\gamma\rangle$ and, with care, to $0 \to \mathbb{C}[x, y] \to \overline{I} \to \mathbb{C}[x, y]/\langle\overline{\gamma}\rangle$ , and the result follows easily.

Victor A. Vassiliev

## Algebraicity of surface potentials and the monodromy of complete intersections

By a famous theorem of Newton, the standard charge distributed on a sphere in $\mathbb{R}^n$ does not attract interior particles. Ivory (1809) extended this theorem to the attraction by ellipsoids, and Arnold (1982) to the attraction by arbitrary hyperbolic surfaces: such a surface does not intersect the particles inside the hyperbolicity domain. We investigate qualitative properties of the attraction force in arbitrary components of the complement of the attracting surface, in particular (following one other famous theory of Newton) the question if it is algebraic or not.
The ramification of this force is controlled by a monodromy group; we identify it with a proper subgroup of the local monodromy group of a complete intersection of codimension 2 in $\mathbb{C}^n$: it is generated by reflections in vanishing cycles spanning a sublattice of corank 1 in the vanishing homology group (with twisted coefficients if $n$ is odd).
Studying this monodromy group we prove, in particular, that the attraction force of a hypersurface of degree $d$ in $\mathbb{R}^n$ coincides with algebraic vector functions everywhere in $\mathbb{R}^n$ outside the attracting surface if $n = 2$ or $d = 2$, and is non–algebraic in all domains other than the hyperbolicity domain if the surface is generic and $d > 2$, $n > 2$ and $n + d > 7$.

Sabir M. Gusein–Zade

## Invariants of generic plane curves and of singularities

V. Arnold has constructed three basic invariants of the first order (in the sense of V. Vassiliev) for plane curves — $J^+$, $J^-$ and $St$. The first two of them can be defined for irreducible real plane curve singularities. There is the following statement relating $J^-$ with a "convensional" singularity invariant: $J^-/2 \bmod 2 = \mathrm{Arf}$, where Arf is the arf–invariant of the singularity — the arf–invariant of its intersection form mod 2 and the quadratic function which is equal to 1 for each vanishing cycle.

Victor Goryunov

## Polynomial invariants of plane curves and Bennequin numbers

The aim of the talk is to survey recent results by S. Chmutov, H. Mutakami and the speaker on the restrictions of the classical polynomial link invariants to Legendrian curves in the standard contact solid torus and 3–space. We point out the sets of the rules which completely define these restrictions in terms of the underlying plane fronts. Unlike the case of arbitrary framed links, when the framed versions of the polynomials are Laurent in the framing variable $x$, the polynomials of Legendrian links do not contain any negative powers of $x$. We give a series of estimates of the Bennequin–Tabachnikov numbers implied by this basic property.

Klaus Altmann

## 3–dimensional non–isolated Gorenstein singularities

Let $\sigma \subset \mathbb{R}^n$ be a polyhedral cone and denote by $Y_\sigma := \operatorname{Spec} \mathbb{C}[\sigma^\vee \cap \mathbb{Z}^n]$ the corresponding toric variety. There are three main questions concerning the deformation theory of $Y_\sigma$:

1) The vector space $T^1$ (describing the infinitesimal deformations) is $\mathbb{Z}^n$–graded. For which $R \in \mathbb{Z}^n$ do we have $T^1(-R) \neq 0$?

2) Find genuine deformations (e.g. flat $k$-parameter families) containing $Y_\sigma$ as special fibre.

3) Describe the semi-universal deformation of $Y_\sigma$.

Question 1) can be answered for any $\sigma$ and $R$: $T^1(-R)$ may be directly given in terms of the polyhedron $Q(R) := \sigma \cap [R = 1]$. Question 3) can be completely answered for isolated toric Gorenstein singularities. Finally, we approach Question 2) for 3-dimensional (non-isolated) toric Gorenstein singularities, i.e. when $\sigma$ equals the cone over a lattice polygon in $\mathbb{Z}^2$.

THEO DE JONG

## Quasi-determinantal singularities

Consider a normal surface singularity $(X,x)$, and let $\pi : (\widetilde{X}, E) \to (X,x)$ be a resolution. Artin defined $(X,x)$ to be rational if and only if $R^1\pi_*\mathcal{O}_{\widetilde{X},x} = 0$. If in addition $(X,x)$ is a hypersurface singularity, then $(X,x)$ belongs to the A-D-E list (DuVal). So in this case $(X,x)$ can be described explicitely. Our question is: How to get equations for rational surface singularities?
Some known results:

i) $\mathrm{mult}(X,x) = 2 \Rightarrow$ A-D-E
ii) $\mathrm{mult}(X,x) = 3$ (Tjurina)
iii) quotient singularities (Riemenschneider)
iv) minimal number of equations $= \binom{\mathrm{mult}(X,x)}{2}$ (Wahl)
v) reduced fundamental cycle $\subset$ sandwiched singularities (de Jong, van Straten).

Riemenschneider defined a singularity to be quasideterminantal if equations can be obtained by taking quasiminors of

$$\begin{pmatrix} f_1 & & f_2 & & \dots & & f_m \\ g_1 & & g_2 & & \dots & & g_m \\ & h_1 & & h_2 & \dots & h_{m-1} & \end{pmatrix}$$

Rational quasideterminantal surface singularities were characterized (in terms of the resolution) by Wahl and Röhr. We indicate how to get equations.

JONATHAN WAHL

## Graded $T^2$ of cones over curves

Let $A$ be the cone over the embedding of a projective smooth complex curve $C \hookrightarrow \mathbb{P}^n$ via a projectively normal line bundle $L = \mathcal{O}_X(1)$. By our 1985 theorem,

$$(T^1_{-1})^* \cong \mathrm{Coker}\,(\Phi_{K,L} : \mathrm{Ker}\,(\Gamma(K) \otimes \Gamma(L) \to \Gamma(K \otimes L)) \longrightarrow \Gamma(K \otimes K \otimes L))$$
$$T^1_{-2} = 0 \ \text{ if } L \text{ satisfies } (N_2).$$

Here, $\Phi$ is a "Gaussian-W" map.

**Theorem:** *If $L$ satisfies $(N_2)$ and $T^2_{-i} = 0$ for $i \geq 2$, then $C$ is the hyperplane section of a non-conical $X \subset \mathbb{P}^{n+1}$ if and only if $T^1_{-1} \neq 0$.*

**Proposition:** *Let $C \hookrightarrow \mathbb{P}^{g-1}$ be the canonical embedding of a non-hyperelliptic curve, $\mathcal{J}_C$ the ideal sheaf of $C$. Then $(T^2_{-i})^* \cong H^1(\mathcal{J}_C^2(i+1))$ for $i \geq 1$.*

**Theorem:** *The cone over a general canonical curve of genus $g \geq 3$ satisfies $T^2_{-i} = 0$ for $i \geq 2$.*

**Conjecture:** If the Clifford index of $C$ is $\geq 3$, then $T^2_{-i} = 0$ for $i \geq 2$.

**Proposition:** *Suppose $L$ satisfies $(N_2)$, giving $C \hookrightarrow \mathbb{P}^n$. Then*

$$(T^2_{-i})^* \cong Coker\left(\Gamma(\mathcal{J}_C(2)) \otimes \Gamma(K_C(i-2)) \longrightarrow \Gamma(\mathcal{J}_C/\mathcal{J}_C^2 \otimes K_C(i))\right),$$

*hence* $(T^2_{-1})^* \cong Ker(\Phi_{K,L})$.

One should be able to prove a vanishing theorem for non–special $C$ if $\deg L \geq 2g+4$, say.

YURI A. DROZD

## Module–theoretic properties of "good" curve singularities

Let $x$ be a point of an algebraic curve, $R = R_x$ the completion of its local ring, $Q = Q_x$ the corresponding quadratic form. We consider the relations between *"deformation"* properties of $x$ (in particular, the *modality*), the type of $Q$ $(= (\mu_+, \mu_0, \mu_-)$, $\mu_\pm$ being positive/negative indices, $\mu_0$ the defect) and the Cohen–Macaulay type of $R$. The results for plane singularities are summarised in the following table:

| type of $x$ | type of $Q$ | CM–type of $R$ |
|---|---|---|
| simple | negative definite | finite |
| parabolic unimodal ($T_{44}$ or $T_{36}$) | $(0,2,n)$ | tame bounded |
| hyperbolic unimodal (other $T_{pq}$) | $(1,1,n)$ | tame unbounded |
| exceptional unimodal or bimodal | $(2,0,n)$ | wild with $p_1 = 1$ |
| others | $\mu_+ + \mu_0 > 2$ | wild with $p_1 > 1$ |

In general, we have the following "planarization principle":

**Theorem:** *$R$ is (at worst) of CM–$\langle$type$\rangle$ if and only if it dominates a plane singularity of CM–$\langle$type$\rangle$ (where $\langle$type$\rangle$ = $\langle$ finite | tame bounded | tame unbounded | $p_1 = 1\rangle$ ).*

Here $R$ is said to be *CM–finite* if it has only finitely many indecomposable non–isomorphic CM–modules; *CM–tame* if it possesses at most 1–parameter families of non–isomorphic indecomposable CM–modules (*bounded* if the number of components in such families is bounded), *CM–wild* if it possesses $p$–parameter families of non–isomorphic indecomposable CM–modules for any $p$. $p_1$ denotes the maximal number of parameters in families of non–isomorphic ideals.

DAVID MOND

## Symplectic Milnor fibres associated to stable mappings

Let $f : (\mathbb{C}^n, s) \to (\mathbb{C}^{n+1}, 0)$ be a stable map–germ of corank $\leq 1$ (i.e. $\dim \operatorname{Ker} df_{|s} \leq 1$). Let $D$ be the image of $f$. Then

**Theorem 1:** *If $A$ is an adjoint divisor for $D$ then $D \cup A$ is a free divisor.*

(By definition, $A$ is an adjoint divisor if $|A \cap D| = |D_{\mathrm{Sing}}|$ (i.e. as sets); more precisely, if $f^{-1}(A) = \text{closure}\,\{x \in \mathbb{C}^n \mid f^{-1}(f(x)) \neq \{x\}\}$ with reduced structure.)

**Examples:** i) $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$, $(x,y) \mapsto (x, y^2, xy)$ parametrises the Whitney umbrella. With coordinates $X, Y, Z$ on target, $A = \{X = 0\}$.
ii) $D = \{X_1 \ldots X_{n+1} = 0\}$ (normal crossing divisor), $A = \{\sigma_n(X_1, \ldots, X_{n+1}) = 0\}$.

**Theorem 2:** (Holland, Mond) *If $n$ is even, then there exists $\omega \in \Omega^2(\log D)$ such that $\omega|_{D_t}$ is non–degenerate for each $t \neq 0$.* (Here $D_t$ is the Milnor fibre of $D$.)

**Conjecture:** $d(\omega|_{D_t}) = 0$, i.e. $\omega|_{D_t}$ is symplectic.

Using Macaulay we have checked the conjecture for $n = 4$ and $6$. For $n = 2$ it is trivial!

Proofs of both theorems come from relating Der $(\log D)$ to the ramification ring $\mathcal{O}_{\mathbb{C}^n}/R_f$, where $R_f$ denotes the ideal of $n \times n$–minors of $df$. This ring is Gorenstein (because $f$ has corank 1) and has grade 3 as $\mathcal{O}_{\mathbb{C}^{n+1}}$–algebra. By a variant of the Buchsbaum–Eisenbud structure theorem, $\mathcal{O}_{\mathbb{C}^n}/R_f$ has a free resolution over $\mathcal{O}_{\mathbb{C}^{n+1}}$ with skew–symmetric central matrix. This skew matrix gives rise to a perfect pairing on Der $(\log h)$ (the module of vector fields tangent to all $D_t$) which by general theory must be induced by a logarithmic 2–form with the properties required in Theorem 2.

ALEXANDER VARCHENKO

## Verlinde algebras and monodromy groups

Consider the Verlinde algebra of the $sl(N)_k$ model of conformal field theory. Following Zuber, we define its Dynkin diagram and reflection group.
We consider a real vector space with a basis $\alpha_a$ where $a$ runs through the set of vertices. We define an intersection form by $\langle\alpha_a, \alpha_b\rangle = 2\delta_{ab} + (G)_{ab}$, where

$$(G)_{ab} = \begin{cases} -1 & \text{if the vertices } a, b \text{ are connected by an edge} \\ 0 & \text{otherwise}. \end{cases}$$

We define the reflection group of $sl(3)_k$ as the group generated by the reflections at $\{\alpha_a\}$.

**Theorem:** (Zuber, Warner, Gusein–Zade, Varchenko) *The Dynkin diagram and the reflection group is a Dynkin diagram and the monodromy group of a quasihomogeneous polynomial $V(x, y)$ of degree $k+3$ with weights of $x, y$ equal to $1, 2$ respectively.*

MAXIM E. KAZARIAN

## Fibre singularities and invariants of circle bundles

Let $\pi : W \to M$ be a smooth locally trivial bundle and $f : W \to \mathbb{R}$ a generic smooth function. We study the relation between the fibre singularities of $f$ and the topology of the bundle $\pi$. The simplest result is the following

**Theorem:** *Let $\pi : S^3 \to S^2$ be the Hopf bundle. Consider the fibres such that the restriction of $f$ to these fibres takes a global maximum at two different points, a global minimum at two different points, and the points of global maximum and minimum alternate. Then the number of such fibres counted with proper signs is the Chern number of $\pi$.*

This theorem and its variations appeared as some versions of Arnold's symplectic generalizations of the four–vertices theorem. In higher dimensions the calculations lead to complexes similar to those appearing in the cyclic homology theory. The main conclusion is that all powers of the first Chern class of a circle bundle can be expressed as classes Poincaré dual to some linear combinations of strata on the base $M$ given by certain degenerations of the restriction of $f$ to the fibres.

ANDRAS NÉMETHI

## The signature of some hypersurface singularities

The first goal of the talk is to present proofs of some conjectures of A. Durfee for singularities of the type $g = f(x, y) + z^N : (\mathbb{C}^3, 0) \to (\mathbb{C}, 0)$. Durfee conjectured that in the case of a hypersurface singularity $g : (\mathbb{C}^3, 0) \to (\mathbb{C}, 0)$, the invariants $p_g$, $\mu$ and $\sigma$ satisfy $p_g \leq \mu/6$ (strong version), in particular $\sigma \leq 0$ (weak version).
We present a simple geometrical proof for $\sigma \leq 0$ in the case $f$ irreducible (or $f$ reducible, but with another weak restriction of the pair $(f, N)$); we present a proof for the strong version as well. Doing this, we make connections with eta–invariants, spectral pairs and

generalized Dedekind sums, and we analyse the resolution graph of plane curve singularities. This is helpful if we want to answer the following question (our second goal): what is special in hypersurface singularities? Why do they satisfy these conjectures?
For singularities $f : (X, x) \to (\mathbb{C}, 0)$ (where $(X, x)$ is a normal surface singularity), we formulate some properties which distinguish the plane curve singularities (and for plane curve singularities they are (almost) equivalent to the property codefied in Durfee's conjectures for $f + z^N$).

JOSEPH H. M. STEENBRINK

## The spectrum of isolated complete intersection singularities

Let $(X, x)$ be an isolated complete intersection singularity. Its spectrum $\mathrm{Sp}(X, x) \in \mathbb{Z}[\mathbb{Q}]$ is defined as follows. Consider two parameter deformations $F = (f, g) : (\mathfrak{X}, x) \to (\mathbb{C}^2, 0)$ of $X$ with $\mathfrak{X}$ and $X' := g^{-1}(0)$ icis. One obtains the mixed Hodge structure $\varphi_f \psi_g \mathbb{Q}^H_{\mathfrak{X}}$ with its automorphism $T_{f,s}$, whose spectrum is denoted by $\mathrm{Sp}(F)$. For all such $F$, $\mathrm{Sp}(F)$ is symmetric under the involution $(\alpha) \mapsto (n + 1 - \alpha)$ of $\mathbb{Z}[\mathbb{Q}]$. By a suspension construction $X \mapsto \widetilde{X} = \widetilde{F}^{-1}(0)$ with $\widetilde{F}(z, w) = (f(z) + w^m, g(z))$ one can prove that $\mathrm{Sp}(F)$ is semicontinuous under deformations of $F$. By considering a generic $F$, one obtains $\mathrm{Sp}(X)$. It depends semicontinuously on $X$, and is constant under deformations with $\mu(X)$ and $\mu(X')$ constant. With W. Ebeling, I found a formula for $\mathrm{Sp}(\widetilde{X})$. Also, I computed all spectra for the $K$-unimodal isolated complete intersections. Methods which are used: resolution, suspension, Newton diagrams and series methods.

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