MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 29/1996

Complex Geometry: Topological and Transcendental Aspects

04.08. bis 10.08.96

Die Tagung fand unter der Leitung von A. Beauville (ENS), F. Catanese (Pisa) and Ch. Okonek (Zürich) statt. Die Vorträge bezogen sich auf wichtige klassische komplex-geometrische Themen (Riemannsche Flächen, Brill-Noether loci, Hilbert Schemata, Klassifikation von Flächen, Linearsysteme, Untermannigfaltigkeiten von Abelschen Mannigfaltigkeiten, Deformationstheorie, Modulräume von Vektorbündeln, Shafarevich Vermutung, Fundamentalgruppen von Kählerschen Mannigfaltigkeiten) oder auf die neuen Theorien (Gromov-Witten Theorie, Donaldson und Seiberg-Witten Theorie, Mirror-Symmetrie), die Probleme aus anderen Gebieten (Differential- & Symplektische Geometrie, Topologie und Physik) mit komplex-geometrischen Methoden behandeln.

L. EIN

Singularities of theta divisors and birational geometry of irregular varieties

In joint work with Rob Lazarsfeld, we prove a conjecture of Kollàr, which says that if X is a subvariety of an abelian variety and X is of general type, then $\chi(\omega_{\tilde{X}}) > 0$, when $\tilde{X}$ is a desingularization of X. As an application, we show that if (A, Θ) is a principally polarized abelian variety and Θ is reducible, then Θ has at most rational singularities. Using the generic vanishing theorem of Green and Lazarsfeld, we give a new simple proof of the fact that Albanese map of a variety of Kodaira dimension zero is always surjective with connected fibers.

A. TELEMAN

The coupling principle and Seiberg-Witten theory

(joint work with Ch. Okonek) Many important problems (e.g. the computation of Donaldson invariants of a projective surface, Verlinde formulas, the computation of Gromov-Witten invariants of a projective manifold) reduce to the computation of "correlation functions" on a GIT-quotient. Let $G \longrightarrow GL(A)$ be a a linear representation of a complex reductive group in a finite dimensional vector space, and $Q_A := \mathbb{P}(A)^{ss}//G$. Coupling to a new GIT-problem $G \longrightarrow GL(B)$ means to study the Master space $Q := \mathbb{P}(A \oplus B)^{ss}//G$ as a $\mathbb{C}^*$-space. The *coupling principle* asserts that (under suitable assumptions) the computation of the correlation functions on the initial GIT-quotient Q_A can be reduced to a computation on the new GIT-quotient Q_B and a computation on the space of reductions $R := (Q \setminus (Q_A \cup Q_B))^{\mathbb{C}^*}$. We construct the Master space associated to the coupling of Gieseker stability for torsion free sheaves to morphisms into a fixed reference sheaf (joint result with Ch. Okonek and A. Schmitt). We indicate possible applications and gauge theoretical analoga (in the differentiable category, e.g. in Seiberg-Witten theory) of these constructions and ideas. As a consequence of a differential geometric version of the coupling principle we show that the computation of the Donaldson invariants reduces to a computation on the Seiberg-Witten moduli spaces.

R. HAIN

Locally symmetric families of Jacobians

Let $\mathcal{A}_g$ be the moduli space of principally polarized abelian varieties of dimension g. The period map realizes the moduli space of smooth curves of genus g, $\mathcal{M}_g$, as a (not closed) subvariety of $\mathcal{A}_g$. Denote its Zariski closure in $\mathcal{A}_g$ by $\tilde{\mathcal{M}}_g$. Motivated by a question by Franz Oort, which was in turn motivated by by a conjecture of Robert Coleman, we consider the following problem:

Problem: Are there locally symmetric (i.e. totally geodesic) subvarieties X of $\mathcal{A}_g$ that lie in $\tilde{\mathcal{M}}_g$ and which intersect $\mathcal{M}_g$ non trivially ?

Oort believes that there are no Shimura varieties contained in $\tilde{\mathcal{M}}_g$(except points, of course).

In this talk we presented the following results which bear on Oort's question:

Theorem 1: *Let Γ_g be the mapping class group. If Γ is a discrete group and $\Gamma \longrightarrow \Gamma_g$ is a homomorphism such that $H^1(\Gamma, V(\lambda_3)) = 0$, where $V(\lambda_3)$ is the 3rd fundamental representation of $Sp_g(\mathbb{R})$), then the morphism $H^2(Sp_g(\mathbb{Z}), \mathbb{R}) \to H^2(\Gamma, \mathbb{R})$ is trivial. Here $V(\lambda_3)$ is regarded as a Γ-module via $\Gamma \longrightarrow \Gamma_g \longrightarrow Sp_g(\mathbb{Z})$.*

Corollary *Suppose that G is an almost simple $\mathbb{Q}$-group such that (1) $G \subset Sp_g(\mathbb{Q})$ (2) $\mathrm{rk}_{\mathbb{Q}} G \geq 2$ (3) $G(\mathbb{R})/K$ is Hermitian symmetric. If Γ is an arithmetic subgroup of g and $\Gamma \longrightarrow \Gamma_g$ is a homomorphism, then the image of $\Gamma \longrightarrow Sp_g(\mathbb{Z})$ is finte.*

Theorem 2: *If $X = \Gamma \backslash G(\mathbb{R})/K$ is a locally symmetric subvariety of $\mathcal{A}_g$, $X \subset \tilde{\mathcal{M}}_g$ and $X \cap \mathcal{M}_g \neq \emptyset$, then either $X \cap \{$hyperelliptic curves$\}$ has a component of codimension 1 in X or $\mathrm{rk}_{\mathbb{Q}} G \leq 2$.*

L. KATZARKOV

Non-abelian Hodge theory and the Shafarevich conjecture

We prove the following theorem:

Theorem: *Let X be a smooth projective variety/$\mathbb{C}$, $Y \subset X$ a divisor in X with many components $Y = \bigcup Y_i$ and ρ a representation of $\pi_1(X)$ in $GL(n, \mathbb{C})$. Then, if $\rho|_{\pi_1(Y_i)}$ is finite, $\rho|_{\pi_1(Y)}$ is finite as well.*

As a consequence we get that

Theorem: *Let X be a smooth projective surface with $\pi_1(X)$ linear. Then $\tilde{X}$*

is holomorphically convex.

The above statement is a partial case of the well known Shafarevich conjecture.

L. Li

Algebraic and symplectic geometry of Gromov-Witten invariants

(joint work with G. Tian.) Let X be a smooth projective variety, $\alpha \in H_2(X,\mathbb{Z})$ and n, g integers. Let $\mathcal{M}_{\alpha,g,n}$ be the moduli space of stable morphisms $F : D \subset C \longrightarrow X$, where $D \subset C$ denotes an n-marked genus g curve. There is a tautological topological class $h : H^*(X)^{\times n} \times H^*(\mathcal{M}_{g,n}) \longrightarrow H^*(\mathcal{M}_{\alpha,g,n})$. Let $v :=$vir.dim.$\mathcal{M}_{\alpha,g,n}$. In case $v = \dim \mathcal{M}_{\alpha,g,n}$ the Gromov-Witten invariant is the function $\Phi_{\alpha,g,n} : H^*(X)^{\times n} \times H^*(\mathcal{M}_{g,n}) \longrightarrow H^*(\mathcal{M}_{\alpha,g,n})$ defined by $\Phi_{\alpha,g,n}(\Sigma) = \deg(h(\Sigma)[\mathcal{M}_{\alpha,g,n}])_0$, where $[\mathcal{M}_{\alpha,g,n}]$ is the fundamental class of $\mathcal{M}_{\alpha,g,n}$. In case $v \neq \dim \mathcal{M}_{\alpha,g,n}$ we have found a purely dimensional virtual cycle $[\mathcal{M}_{\alpha,g,n}]^{vir} \in A_v(\mathcal{M}_{\alpha,g,n})$ so that the Gromov-Witten invariants are given by $\Phi_{\alpha,g,n}(\Sigma) = \deg(h(\Sigma)[\mathcal{M}_{\alpha,g,n}]^{vir})_0$. The construction of this cycle cycle has the following important property: if $\mathcal{M}_{\alpha,g,n} \stackrel{\eta}{\longrightarrow} Z$ is a morphism and $y \subset Z$ is a regular embedding, then there is a canonically defined virtual cycle $[\mathcal{M}_{\alpha,g,n} \times_Z Y]^{vir}$ such that $\eta^![\mathcal{M}_{\alpha,g,n}]^{vir} = [\mathcal{M}_{\alpha,g,n} \times_Z Y]^{vir}$. An application of this is a new proof of the associativity law of Gromov-Witten invariants, which implies the existence of quantum cohomology of X.

There is an analytic way to define the virtual moduli cycle $[\mathcal{M}_{\alpha,g,n}]^{vir}_{hom} \in H_*(\mathcal{M}_{\alpha,g,n})$ as follows: Let $\mathcal{W}_{\alpha,g,n}$ be the space of all smooth maps $f : D \subset C \longrightarrow X$, where $d \subset C$ is as before. Let V be an infinite-rank vector bundle whose fiber over f is $\Gamma(\Omega^{01}_C(f^*(TX)))$. There is an obvious section $\xi : \mathcal{W}_{\alpha,g,n} \longrightarrow V$ that sends f to $\bar\partial f$. At each $f \in \mathcal{W}_{\alpha,g,n}$ the derivative $d\xi(f) : T_f\mathcal{W}_{\alpha,g,n} \longrightarrow V_f$ is Fredholm, whose index is the expected real dimension of $\mathcal{M}_{\alpha,g,n}$. Using the Fredholm property one can perturb ξ, obtaining say $\bar\xi$, so that locally the perturbation is always along finite direction, and $\bar\xi^{-1}(0)$ is smooth of the expected dimension. Thus $[\mathcal{M}_{\alpha,g,n}]^{vir}_{hom} = [\bar\xi^{-1}(0)] \in H_*(\mathcal{W}_{\alpha,g,n})$. Once we have such perturbation, we put $C^\nu = \lim_{t\to\infty} t\Gamma_\xi$, which is a cone current in $V|_{\mathcal{M}_{\alpha,g,n}}$, and

$$[\mathcal{M}_{\alpha,g,n}]^{vir}_{hom} = [\bar\xi^{-1}(0)] = [\bar\xi^{-1}(\Gamma_\xi)] = [\bar\xi^{-1}(C^V)] \in H_*(\mathcal{W}_{\alpha,g,n}) .$$

The last expression is the image under "refined Gysin map" of $[C^V] \in H_*(V)$ in $H_*(\mathcal{W}_{\alpha,g,n})$. This formula should lead to the proof of the

Conjecture: *The algebraic and symplectic definitions of the Gromov-Witten invariants do coincide.*

J. LE POTIER
Strange duality, on $\mathbb{P}^2$

Let $K(\mathbb{P}^2)$ be the Grothendieck algebra of coherent sheaves on the projective plane; this algebra is isomorphic to $\mathbb{Z}^3$; an isomorphism is given by (r, c_1, χ), the rank, the Chern class and the Euler-Poincaré characteristic. We have a non-degenerate quadratic form on $K(\mathbb{P}^2)$ given by $c \mapsto \chi(c^2)$. Let $c \in K(\mathbb{P}^2)$ be an element of rank $r > 0$, and M_c the moduli space of semi-stable sheaves of Grothendieck class c. By a result of Drezet, the canonical morphism $\lambda_c : c^\perp \longrightarrow \mathrm{Pic}(M_c)$ given by the determinant of the cohomology is surjective. The subgroup of $c^\perp$ of elements of rank 0 is a cyclic group; the line bundle $\mathcal{D} = \lambda_c(-\mathrm{u})$ associated to the generator u with fundamental class $[\mathrm{u}] > 0$ is called the Donaldson determinant line bundle. The problem is to describe the $SL(3)$–representation $H^0(M_c, \mathcal{D}^{\otimes d})$.
Theorem: *Let $u \in c^\perp$ be a Grothendieck class of dimension 1, and $\mathcal{D}_u = \lambda_c(-u)$. For $q > 0$ and $\deg[u] > -3r$, we have $H^q(M_c, \mathcal{D}_c) = 0$.*
This implies that $h^0(M_c, \mathcal{D}^{\otimes d})$ is a polynomial, and in the case of $r = 2$ the coefficient of the term of highest degree is related to the Donaldson numbers.

Let $u \in K(\mathbb{P}^2)$ be a Grothendieck class of dimension 1, and fondamental class $[u] > 0$. Consider the moduli space M_u of semi-stable sheaves of dimension 1, and Grothendieck class u. We have also a (surjective) morphism $\lambda_c : u^\perp \longrightarrow \mathrm{Pic}(M_u)$, and for $c \in u^\perp$ we can define the line bundle $\mathcal{D}_c$ on M_u by $\mathcal{D}_c = \lambda_u(-c)$.
Theorem: *Suppose that the rank r of c is > 0, and that the Chern class c_2 of c is big enough; then, for $q > 0$ we have $H^q(M_u, \mathcal{D}_c) = 0$.*
Let c and u as above and such that $\langle c, u \rangle = 0$. On $M_c \times M_u$ we have a canonical section of $\mathcal{D}_u \boxtimes \mathcal{D}_e$ and then a linear map $D_{c,u} : H^0(M_u, \mathcal{D}_c)^* \to H^0(M_c, \mathcal{D}_u)$
Conjecture: Let d be the degree of $[u]$. If M_c is not empty, the linear map $D_{c,u}$ is an isomorphism for $d \leq 2$, and surjective for $d \geq 3$.

It is not difficult to verify the conjecture in the case where the rank r is 1, i.e. for the sections of the powers of Donaldson determinant on the Hilbert scheme $\mathrm{Hilb}^n(\mathbb{P}^2)$. With G. Danila we have proved this conjecture for the sections of the Donaldson determinant on the moduli M_n of semi-stables sheaves of rank 2 and Chern classes $c_1 = 0$, and $c_2 = n$, for $n \leq 11$; that gives a partial answer to a question of A. Beauville. In that case, M_u is the projective dual plane, and $D_{c,u}$ is induced by the map $\gamma : M_n \to \mathbb{P}(H^0((\mathbb{P}^2)^*, \mathcal{O}(n)))$ which associates to a semi-stable sheaf the curve of jumping lines; it is known that $\gamma^*(\mathcal{O}(1)) = \mathcal{D}$.

For the sections $\mathcal{D}^{\otimes d}$, the conjecture is also true for $n \leq 3$ and $d \leq 3$. For $n = 3$, the result comes from the computation of Danila of the Poincaré serie

$$\sum_{d\geq 0} h^0(M_3, \mathcal{D}^{\otimes d})t^d = \frac{1-t^6}{(1-t)^{10}(1-t^2)}.$$

H. KNÖRRER

Riemann surfaces of infinite genus

(joint work with J. Feldman, E. Trubowitz) Let $\Gamma = (0, 2\pi)\mathbb{Z} \oplus (\omega_1, \omega_2)\mathbb{Z}$ be a lattice in $\mathbb{R}^2$ and let $q \in L^2(\mathbb{R}^2/\Gamma)$. The heat curve of q is defined as

$$\begin{aligned}\mathcal{H}_q := \{(\xi_1, \xi_2) \in \mathbb{C}^* \times \mathbb{C}^* | & \text{ there exists a nontrivial sol. of} \\ & \left(\tfrac{\partial}{\partial x} - \tfrac{\partial^2}{\partial y^2} + q(x,y)\right)\Psi(x,y) = 0 \quad \text{satisfying} \\ \Psi(x+\omega_1, y+\omega_2) = \xi_1\Psi(x,y)\ , & \quad \Psi(x, y+2\pi) = \xi_2\Psi(x,y)\}\end{aligned}$$

For general q this is a Riemann surface of infinite genus embedded in $\mathbb{C}^* \times \mathbb{C}^*$. It has zero ideal boundary in the sense of Nevanlinna-Ahlfors. Many results of the classical theory of compact Riemann surfaces can be generalized to heat curves and Riemann surfaces "similar" to heat curves. Among these results are:

. convergence of the theta functions on a suitable Banach space.

. Riemann's Vanishing Theorem.

. Torelli's Theorem.

Heat curves are spectral curves for the Kadomcev-Petriashvilii (KP) equation will spatially periodic initial data. The analysis of heat curves is used to give a solution to the initial value problem of (KP) in terms of theta functions and thus to prove that the solution is almost periodic in time.

R. MIRANDA

4 to 1 Covers in Algebraic Geometry

Given a variety Y, a d-to-1 cover of Y is a flat finite map $\pi : X \longrightarrow Y$ of degree d. How to systematically construct such covers is the topic at hand.

Well-known is the case of double covers ($d = 2$), where π is detrmined by a line bundle L on Y and a section of $L^{\otimes 2}$ (determining the branch locus). For $d = 3$, it was worked out about ten years ago: π is given by a rank 2 sheaf E on Y and a section of $S^3E^* \otimes \Lambda^2 E$. Following recent work of Casnati, Ekedahl, and Hahn, I describe the situation for quadruple covers ($d = 4$).

E. LOOIJENGA

A Lie algebra attached to a projective variety

(report on a joint work with V. Lunts) One of the basic facts of Hodge theory is one that the choice of a Kähler class κ on a compact complex manifold X gives its complex cohomology the structure of a $sl(2)$ representation with $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \mapsto e_\kappa$, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mapsto h$, $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \mapsto f_\kappa$, where e_κ is the cupping with κ, h is the multiplication by $l - \dim_{\mathbb{C}} X$ in degree l and f_k is uniquely determined by the condition that is of degree -2 and $[e_\kappa, f_\kappa] = h$. This representation leaves infinitesimally invariant the modified Poincaré pairing on $H^*(X)$ defined by $\langle \alpha, \beta \rangle = (-1)^{[\frac{l - \dim X}{2}]} \int_X \alpha \wedge \beta$, $\alpha \in H^l(X, \mathbb{C})$. This form is $(-1)^{\dim X}$-symmetric and nondegenerate.

Another choice of a Kähler class κ' defines a different $sl(2)$-module structure. We always have $[e_\kappa, e'_\kappa] = 0$, but in general $[f_\kappa, f'_\kappa] \neq 0$. If $\mathfrak{a} \subset H^2(X, \mathbb{C})$ is a subspace containing a Kähler class, and $e : \mathfrak{a} \longrightarrow \mathfrak{gl}(H^\cdot(X))_2$ is the obvious map, then there is a rational map $f : \mathfrak{a} - \to \mathfrak{gl}(H^\cdot(X))_{-2}$ such that $[e_\kappa, f_\kappa] = h$ on the domain of f. Let $\mathfrak{g}(\mathcal{K}, X) \subset \mathfrak{gl}(H^\cdot(X)$ denote the graded Lie subalgebra generated by the images of e and f.

Proposition: *If $\mathfrak{a} \subset H^2(X, \mathbb{C})$ is a Hodge substructure, then $\mathfrak{g}(\mathfrak{a}, X)$ is semisimple.*

Three cases of particular interest to which this proposition applies are $\mathfrak{a} = H^2(X, \mathbb{C})$, $\mathfrak{a} = H^{1,1}(X, \mathbb{C})$, $\mathfrak{a} = NS(X) \otimes \mathbb{C}$ (assuming X projective). In these cases we write $\mathfrak{g}_{tot}(X)$, $\mathfrak{g}_\kappa(X)$, $\mathfrak{g}_{NS}(X)$ for $\mathfrak{g}(\mathfrak{a}, X)$. These Lie algebras can be hard to compute in practice. Here is a sample result:

Proposition: *If $X = G/B$ is the flag space of a simple complex algebraic group, then $\mathfrak{g}_{NS}(X)$ is maximal, namely equal to the Lie algebra of infinitesimal automorphisms of $(H^\cdot(X), \langle , \rangle)$.*

Of special interest are the cases when the f_κ's commute. Then it follows that $\mathfrak{g}(\mathfrak{a}, X)$ has just 3 graded pieces (in degrees -2,0,2) and we are then dealing with the theory of Jordan algebras. In particular, such graded Lie algebras have been classified. The classical cases all appear (as $\mathfrak{g}_{tot}(X)$, $\mathfrak{g}_{\kappa}(X)$, $\mathfrak{g}_{NS}(X)$ of a complex torus or a hyperkähler manifold). It is not known whether the exceptional case (of type E_7) occurs. To make the challenge specific: Let X be a Kähler 3-fold and restrict to H^{ev} only. Assume that $\mathfrak{g}^{ev}_{tot}(X)$ is of simple Jordan type. Then either H^{ev} is as an algebra isomorphic to H^{ev} of a torus or H^{ev} is an irreducible representation (of dimension 56) of $\mathfrak{g}^{ev}_{tot}(X)$ with $\mathfrak{g}^{ev}_{tot}(X)$ is of type E_7. Question: Does this last case occur? (It can be realized in the C^∞-category.)

L. GÖTTSCHE

Modular forms and the structure of Donaldson invariants for $b_+ = 1$

(joint work with Don Zagier) The Donaldson invariants $\Phi^{X,g}_{c,d}$ of a 4-manifold with $b_+ = 1$ depend by definition on the choice of a Riemann metric on X. It turns out (Kotschick-Morgan) that they depend only on the chamber of the period point $\omega(g)$ in the positive cone $H_X := \{H \in H^2(X,\mathbb{R}) |\ H^2 > 0\}/\mathbb{R}^*$. We extend the definition of the invariants to the "boundary" $S_X := \{F \in H^2(X,\mathbb{Q}) |\ F^2 = 0\}/\mathbb{Q}^*$. We show that for period points F on the boundary structure theorems similar to those of Kronheimer and Mrowka in the $b_+ > 1$ case hold.

Theorem: *Let $x \in H_2(X,\mathbb{Z})$, $p \in H_0(X,\mathbb{Z})$ the class of a point and $c \in H^2(X,\mathbb{Z})$. Let $F, G \in S_X$ and put $m := \max\{W^2 | W \in H^2(X,\mathbb{Z})$ characteristic, $(W\cdot F)(W \cdot G) \leq 0\}$. Then*

(1) $\Phi^{X,F}_{c}\left(e^{xz}(p^2-4)^{(m-\sigma(x))/8}\right) - \Phi^{X,G}_{c}\left(e^{xz}(p^2-4)^{(m-\sigma(x))/8}\right) = 0$, i.e. the difference of the invariants are of higher order simpler type.

(2) There is a precise formula for $\Phi^{X,F}_{c} - \Phi^{X,G}_{c}$ in terms of modular forms depending on a set of "basic classes" which are related to Seiberg-Witten invariants.

(3) If X is a rational algebraic surface, then the results apply to $\Phi^{X,F}_{L}$ itself instead of the difference.

In a previous paper of mine I determined a formula for the wall-crossing of the Donaldson invariants in terms of modular forms. Using this we show that $\Phi^{X,F}_{c} - \Phi^{X,G}_{c}$ can be described in terms of theta functions $\theta^{F,G}_{L}$ for indefinite lattices which we define and study. Based on this we can express the difference $\Phi^{X,F}_{c} - \Phi^{X,G}_{c}$ in terms of the poles of certain modular forms for a group $\Gamma_u \subset SL_2(\mathbb{Z})$ at the cusps of the corresponding modular curve $\mathbb{H}/\Gamma_u$.

C. VOISIN

Mirror symmetry conjecture for complete intersections (after Givental) (C. Voisin)

In this talk, we describe the main ideas in the proof by Givental of the mirror symmetry conjecture for Calabi-Yau complete intersections of dimension 3. This conjecture says that the "quantum periods" of X, which are the components of $1 \in H^0(X)$ in a ∇-flat basis of the trivial bundle with fibre $H^{2*}(X)$ on $H^2(X)$, are exactly the normalized periods of the mirror family X^*_λ near $\lambda = \infty$. Here ∇ is the quantum connection constructed using the quantum product given by the cubic derivatives of the Gromov-Witten potential.

The normalization of the periods means the existence of a canonical trivialization of the bundle $\mathcal{H}^{3,0}$, and of a canonical coordinate q centered at infinity. In this statement, q is identified with e^t, where t is the coordinate on $H^2(X)$ given by the generator $P = c_1(\mathcal{O}_X(1))$ of $H^2(X)$.

J. P. LI

Higher rank stable vector bundles over rational surfaces

Let X be $\mathbb{F}_e$ or $\mathbb{P}^2$, and H an ample divisor on X. We are interested in the birational geometry of $\mathcal{M}_H(r, c_1, c_2)$-the moduli space of H-stable rank-r vector bundles V with $c_1(V) = c_1$ and $c_2(V) = c_2$.

Let $H = a\sigma + bf$, where σ is a section of $\mathbb{F}_e \xrightarrow{\pi} \mathbb{P}^1$ with $\sigma^2 = -e$ and f is a fiber of π. Then we can show that if $b/a \geq 0$, $c_1 \cdot f \neq 0$ mod. r, then the moduli space is empty; if $b/a \gg 0$, $c_1 \cdot f = 0$ mod r, the moduli space is not empty ($c_2 \geq r + c_1 \cdot f$), is irreducible and unirational. Next we consider variations of moduli spaces according to different ample divisors H. For rank-3 case, we are able to show that that the moduli spaces are birational if they are not empty.

Using the method of elementary transformations, we are able to show that $\mathcal{M}(\mathbb{P}^2, 3, 1, c)$ is rational for $c \geq 2$. Using the fact that a generic stable bundle in $\mathcal{M}(\mathbb{P}^2, 3, 0, c)$ can be written as extension of a line bundle by a rank-2 stable vector bundle, we can show that $\mathcal{M}(\mathbb{P}^2, 3, 0, c)$ is rational for

$c = 1/2(3n^2 + 4n + 4)$, where n is an odd positive integer.

H. CLEMENS

Normal differential operators and deformation theory

Higher derivatives of normal functions and Abel-Jacobi mappings have a potentially important role to play in many geometric problems related to Hodge theory, such as the problem of rigidity of rational curves on Calabi-Yau threefolds.

With Paul Burchard we develop a sheaf $\mathcal{D}$ of "normal differential operators" to a submanifold Y_0 of Z_0 in a moving family of ambient manifolds $Z_{x'}$, $x' \in$ a parameter space X'. $\mathcal{D}$ is supported on Y. The notion of an "almost multiplicative " map $\varphi : \mathcal{D}_{X',0} \longrightarrow \mathcal{D}_y$, $y \in Y$, is defined. It is a map which is an enveloping algebra module homomorphism with respect to some (non-unique) $\mathfrak{A}_{X'}$-module structure on $\mathcal{D}_y$, where $\mathfrak{A}_{X'}$ is the envelopping algebra of vector fields on X'.

Theorem:$\mathfrak{A}_{X'}$*-module structures on* $\mathcal{D}_y$ *are in one-to-one correspondence with local foliations of* $Z = \bigcup_{x' \in X'} Z_{x'}$ *transverse to* Y_0. *Formal deformations of* Y *parametrized by* X' *are in a natural one-to-one correspondence with almost multiplicative maps*

$$\varphi : \mathcal{D}_{X',0} \longrightarrow H^0(\mathcal{D}) .$$

C. CILIBERTO

Linear systems of plane curves

(joint work in progress with R. Miranda) Let $x, p_1, \ldots, p_n$ be general points in the plane and let $L(d, m_0, m_1, \ldots, m_n)$ be the linear system of plane curves of degree d having multiplicity m_0 at x and m_i at p_i, $i = 1, \ldots, n$. In this talk I address the question of computing the dimension $D(d, m_1, \ldots, m_n)$ of the above linear system. For simplicity I will consider the case $m_1 = \ldots = m_n = m$, writing $L(d, m_0, m, n)$ etc. instead of $L(d, m_0, m_1, \ldots, m_n)$. One can consider the virtual dimension

$$N(d, m_0, m_1, \ldots, m_n) := \frac{d(d+3)}{2} - \binom{m_0 + 1}{2} - n\binom{m+1}{2}$$

of the system and the expected dimension

$$E(d, m_0, m_1, \ldots, m_n) := \max\{-1, N(d, m_0, m_1, \ldots, m_n), \} .$$

Of course $D(d, m_0, m_1, \ldots, m_n) \geq E(d, m_0, m_1, \ldots, m_n)$ and the system is called regular if the equality holds. I will set up a recursion technique based on a degeneration argument in order to prove regularity of the above linear systems, under certain numerical assumptions.

B. FANTECHI

Intrinsic normal cone and virtual fundamental classes

This is a report on a joint work with Kai Behrend.

Let X be a moduli space of expected dimension d. We want to construct a virtual fundamental class $[X] \in A_d(X)$ satisfying suitable assumptions; $[X]$ can then be used to define numerical invariants.

Let X be any Deligne Mumford (DM) stack of finite type over a field k. If $X \to W$ is a closed embedding in a smooth DM stack, define the intrinsic normal cone $\underline{C}_X$ to be the stack quotient of $C_{X/W}$ by the natural action of $T_W|_X$; the intrinsic normal sheaf $\underline{N}_X$ is the same with $N_{X/W}$ instead of $C_{X/W}$.

We prove that $\underline{C}_X$ and $\underline{N}_X$ do not depend on the chosen embedding; in fact, they can be defined even for a DM stack X which does not admit a global embedding in a smooth stack. There is a natural map $\underline{C}_X \to \underline{N}_X$ which is a closed embedding of Artin stacks; $\underline{C}_X$ has pure dimension zero.

Let $E^\bullet$ be an object in $D^-(\mathcal{O}_X)$; we will always assume that $h^i(E^\bullet)$ is zero if $i > 0$ and coherent if $i = 0, -1$. Then locally $\tau_{\geq -1} E^\bullet$ is isomorphic to $E^{-1} \to E^0$, where E^i is coherent and E^0 is locally free. We prove that we can associate to $E^\bullet$ an Artin stack $\underline{E}$ over X, which locally is the stack quotient of *Spec Sym* E^{-1} by the natural action of *Spec Sym* E^0. This construction is functorial, that is a morphism $E^\bullet \to F^\bullet$ in the derived category induces a morphism $\underline{F} \to \underline{E}$. In particular $\underline{N}_X$ is the stack associated to the cotangent complex $L^\bullet_X$ of X.

We define an obstruction theory for X to be a morphism $E^\bullet \to L^\bullet_X$ in $D^-(\mathcal{O}_X)$ such that the induced morphism $\underline{N}_X \to \underline{E}$ is a closed embedding. This is equivalent to requiring that $E^\bullet$ is "as good as" $L^\bullet_X$ when studying infinitesimal deformations of morphisms with X as a target. In particular most moduli spaces carry a natural obstruction theory.

We define the obstruction theory $E^\bullet$ to be perfect if it is locally isomorphic to a complex $E^{-1} \to E^0$ of locally free, coherent sheaves. If the isomorphism is global we call it a global resolution.

If $E^\bullet$ is a perfect obstruction theory admitting a global resolution, we define the virtual fundamental class $[X, E^\bullet] \in A_{rkE^0 - rkE^{-1}}(X)$ to be $0^!([C])$, where $0 : X \to E_1 = Spec\,Sym\,E^{-1}$ is the zero section and C is the fibre product $\underline{C}_X \times_{\underline{E}} E_1$. We prove that $[X, E^\bullet]$ does not depend on the resolution chosen.

This construction can be repeated in a relative context, and it enjoys several nice properties with respect to, e.g., products and base change. As an application, Behrend has completed the program of Kontsevich, developed by Kontsevich–Manin and Behrend–Manin, to construct Gromov Witten invariants for arbitrary smooth projective varieties over a field.

K. ZUO

Kodaira dimension of the Shafarevich maps

Let X be a smooth projective variety over $\mathbb{C}$ and let $Sh : X \longrightarrow Sh(X)$ be the Shafarevich map on X, which is a surjective morphism with connected fibres and has the following property: if $V \subset X$ is a subvariety, then $|im(\pi_1(V) \longrightarrow \pi_1(X))| < \infty$ if and only if V is contained in some fibres of Sh. Kollàr conjectures that that if $\pi_1(X)$ is large (i.e. Sh is birational) then the Kodaira dimension of X is non negative. In this talk we prove the following:

Theorem:*Let $G \subset GL_n(\mathbb{C})$ be an almost simple algebraic group. If $\rho : \pi_1(X) \longrightarrow G$ is a Zariski dense, large representation, then X is of general type.*

Corollary:*Suppose $\kappa(X) = 0$. If $\rho : \pi_1(X) \longrightarrow GL_n(\mathbb{C})$ is reductive, then $\rho = \oplus_i$ 1-dim. representation, after passing to a finite étale cover of X.*

D. NAIE

Numerical Campedelli surfaces cannot have σ_3 as the algebraic fundamental group (work in progress)

Let X be a smooth, minimal, projective surface of general type with $K^2 = 2$ and $p_g = 0$ (called a numerical Campedelli surface). It is known that its algebraic fundamental group is of order ≤ 9. M. Reid has pointed out that the dihedral group of order 8 cannot occur, and constructed an example for the quaternionic group. Therefore the result would show that the quaternionic group is the only non-abelian algebraic fundamental group in this range.

Idea of proof: to study the existence or the non-existence of a surface with given invariants and finite algebraic fundamental group, one considers the Galois cover associated to the fundamental group and then studies the canonical image of this cover. In our case we notice that the image of the canonical map is a surface, that $|K_X|$ has no base points and finally we eliminate, case by case, all the possibilities for the degree of the canonical map.

Remark: I hven't eleiminated yet completely the case $\deg \varphi_{|K_X|} = 1$.

I. BAUER

Irrational pencils on non-compact algebraic varieties

We consider the following situation: let X be a quasiprojective manifold and let $\bar{X}$ be a smooth compactfication of X such that $Y := \bar{X} \setminus X$ is a divisor with normal crossings.

We prove the following result:

Theorem: *Every maximal real isotropic subspace $V \subset H^1(X,\mathbb{C})$ of dim. ≥ 2 determines a logarithmic irrational pencil, i.e. a surjective holomorphic map $f : X \longrightarrow C$ with connected fibres from X to a quasiprojective smooth curve C with log. genus ≥ 2.*

The genus g of a smooth compactification $\bar{C}$ of C equals to $\frac{1}{2}(\dim V \cap H^1(\bar{X},\mathbb{C}))$. C is complete iff $V \subset H^1(\bar{X},C)$; in this case $\dim V = g$. If g is non-complete, then $V = f^* H^1(C,\mathbb{C})$, so $\dim V = g + g^*$.

In this way we have established (in the case where $V \not\subset H^1(\bar{X},C)$) a 1-1 correspondence between the set of maximal real isotropic subspaces of $H^1(X,\mathbb{C})$ of dim. β and the set of log. irrational pencils with first Betti number β.

Consequence: *If X admits or not a fibration over a Riemann surface of log. genus ≥ 2 is a cohomological property.*

Results in this direction were proved by Siu, beauville, Gromov, Catanese, Green-Lazarsfeld.

From the above theorem we deduce:

Theorem: *Assume that $\pi_1(X)$ admits a surjective homomorphism $\pi_1(X) \longrightarrow G := \langle a_1, \ldots, a_n | R_1, \ldots, R_m \rangle$, $n - m \geq 3$. Then there is an integer $\beta \geq$*

$(n-m)$, a smooth Rieman surface C with first Betti number β and a fibration $f : X \longrightarrow C$.

F. CATANESE

An 8-dimensional family of 1-connected Godeaux surfaces (informal talk)

We describe the construction of a family of minimal surfaces S with $\pi_1(S)=0$, $p_g=q=0$, $K_S^2=1$.

This family has an 8-dimensional image in the moduli space, thus it has the expected dimension and should be a component of the moduli space.

A 4-dimensional family had been constructed by R. Barlow in 1982-1983, thus giving a counterexample to a question raised by Severi. These surfaces are interesting yet for

A) Bloch's conjecture: $A_0(S)=\mathbb{Z}$ if $p_g=0$.

B) The classification of C^∞ (symplectic) 4-manifolds with $\pi_1=0$, $b^+=1$.

C) as a cornerstone of surface classification.

We also discussed an approach to show that, if $|K_S|$ does not have a double base point, there exists only our family.

Geometrically, S is the normalization of $Y \subset \mathbb{P}^3 \times \mathbb{P}^1$ (under $\varphi_3 \times \varphi_2$, φ_i being the i-th canonical map) which is a c.i. of type (2,1) (3,3) with 3 singular curves, for $\lambda \in \mathbb{P}^1$, $\lambda = 0,\ 1,\ \infty$, being 3 irreducible twisted cubics. The above correspond to the 3 hyperelliptic curves in $|2K_S|$.

We have shown that the case "$|2K_S|$ contains no hyperelliptic curve" is impossible.

G. ELLINGSRUD

Action of Heisenberg algebras on the cohomology of the Hibert scheme of surfaces

Let S be a smooth projective surface and let $S^{[n]}$ denote the Hilbert scheme parametrising finite subschemes of S of length n. Nakajama and Grojnowski have defined an action of a certain Heisenberg algebra on $\oplus_n H^*(S^{[n]})$ depending on the sequence of integers (c_n) given by $c_n = \int_{S^{[n]}} [M_n]\cdot[M_n(p)]$, where $M_n \subset S^{[n]}$ consists of the subschemes supported in one point, and $M_n(p) \subset S^{[n]}$ consists of those supported at the point $p \in S$. We prove by induction that $\int_{S^{[n]}} [M_n] \cdot [M_n(p)] = (-1)^{n-1} n$

C. FABER

Intersection theory on the moduli spaces of curves

Let M_g be the moduli space of smooth curves of genus $g \geq 2$ and let $C_g = M_{g,1}$ be the universal curve, with natural morphism $\pi : C_g \longrightarrow M_g$. Denote by K the first Chern class of the relative dualizing sheaf ω_π. Define the tautological classes κ_i as $\pi_*(K^{i+1})$; $\kappa_i \in CH^i(M_g)$, the codimension i Chow group with $\mathbb{Q}$-coefficients.

Theorem: $\kappa_{g-2} \neq 0$ *in* $CH^{g-2}(M_g)$.

Denote by $\mathbb{E}$ the Hodge bundle $\pi_*(\omega_\pi)$ of rank g on M_g and let λ_i denote its i-th Chern class. Mumford defined the Chow ring of $\bar{M}_g$ and showed that κ_i and λ_i can be defined naturally as classes in this ring. The proof of the theorem starts with the observation that $\lambda_g \lambda_{g-1}$ vanishes on the boundary $\bar{M}_g \setminus M_g$ of the Deligne-Mumford compactification. This follows easily from $ch_{2k}(\mathbb{E}) = 0$, for all $k \geq 1$. These identities in turn were derived by Mumford by applying G-R-R to π and ω_π. Also $ch_{2g-1}(\mathbb{E})$ is a non-zero multiple of $\lambda_g \lambda_{g-1}$, and by applying Mumford's result one obtains a formula for the number $\kappa_{g-2} \lambda_g \lambda_{g-1}$ in terms of intersection numbers of basic line bundles on moduli spaces $\bar{M}_{g',n'}$. All such numbers can be computed recursively from the so-called Witten conjecture, which was proven by Kontsevich. In this way I was able to conclude the proof of the theorem. Together with Looijenga's recent result, this says that the tautological ring (the subring generated by the κ_i) of M_g is one-dimensional in degree $g-2$ and vanishes in higher degrees. This provides considerable evidence towards the author's conjecture giving a very precise description of the tautological ring, saying among other things that it should be Gorenstein with socle in degree $g-2$. This conjecture is now established for all $g \leq 15$.

G. DASKALOPOULOS

On the Brill-Noether problem for vector bundles

On an arbitrary compact Riemann surface, necessary and sufficient conditions are found for the existence of semistable (stable) vector bundles with slope between 0 and 1 and a prescribed number of linearly independent holomorphic sections. Existence is achieved by minimizing the Yang-Mills-Higgs functional.

B. SIEBERT

Gromov-Witten invariants for general symplectic manifolds

We present a new approach to GW-invariants which takes singular domains into account from the very formulation. This removes the positivity condition that so far had to be imposed on the symplectic manifolds studied. The method describes the relevant moduli spaces of J-pseudo holomorphic curves as zero sets of a Fredholm section of a Banach bundle over a Banach orbifold and uses a theory of localized Euler classes for these. Similar results have been obtained independently by K. Fukaya/ K. Ono, J. Li/ G. Tian and H. Hofer/D. Salamon with different methods.

O. KÜCHLE

Bounds for Seshadri-constants

(joint work with A. Steffens) We present a new approach to the boundedness of Seshadri constants of ample line bundles at very general points of an arbitrary projective variety X over $\mathbb{C}$. The Seshadri constant of the line bundle L at $x \in X$ is a measure for the local positivity of L at x and can be defined by

$$\epsilon(L, x) = \inf_{C \ni x} \{L \cdot C / mult_x C\} \ ,$$

where the infimum is taken over all integral curves C containing x. Our approach is based on the study of deformations of linear systems whose members are highly singular; the method of differentiation in parameter direction is used. The main result of this technique is the following

Theorem: *Let X be a smooth projective variety over $\mathbb{C}$ of dimension n, L an ample line bundle on X with $L^n > \alpha^n > 0$, and $0 = b_1 < b_2 < \ldots < b_n < \alpha$ rational numbers. Let $x \in X$ be a very general point and suppose that for all $k \gg 0$ there is no divisor $D \in |kL|$ with an isolated singularity at x of order $\geq k(\alpha - b_n)$. Then there is a subvariety $V \subset X$ of codimension $c \neq n$ containing x with $\deg_L V = L^{n-c} \cdot V < \frac{\alpha^n}{(b_{c+1} - b_c)^c}$.*

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