Tagungsbericht 30/1996

High Dimensional Probability

11.-17.8.1996

This meeting was organized by E. Eberlein (Freiburg), M. Hahn (Medford) and M. Talagrand (Paris). It emerged from a series of conferences which were held during the last twenty years under the title Probability in Banach Spaces. The broader title reflects the expansion of the topic, which includes many different directions such as isoperimetric inequalities and concentration of measure, empirical processes and likelihood ratios, Gaussian processes and applications, large deviations, random graphs and random trees, new inequalities and asymptotic results as well as applications to number theory.

Approximately 50 mathematicians attended the conference. Many contributed via discussions and comments to the formal lectures. There was a highly interactive atmosphere. A proceedings volume will appear next year in the Birkhäuser series "Progress in Probability". The group plans to meet again in three years in Seattle/USA.

Abstracts/Vortragsauszüge

A. de Acosta

Abstract non convex large deviations

We prove a general large deviation result for a sequence of random vectors $Y_n : \Omega \to E$, a Banach space. The main assumption on $\{Y_n\}$ is an "abstract exponential martingale condition" $E\exp[\langle Y_n, \xi\rangle - \Phi_n(Y_n, \xi)] = 1$ for all $\xi \in E^*$, where $\Phi_n, \Phi : E \times E^* \to \mathbb{R}$, $\sup_{x \in E} |\frac{1}{n}\Phi_n(x, n\xi) - \Phi(x, \xi)| \to 0$. Φ is assumed to satisfy $\Phi(x, 0) = 0$ for all x, $\Phi(\cdot, \xi)$ is continuous for all ξ, $\sup_{x \in E} |\Phi(x, \xi)| < \infty$. Under the assumption of exponential tightness, the large deviation upper bound follows:

$$\overline{\lim}\ 1/n \log P[Y_n \in A] \leq -\inf_{x \in \bar{A}} \Psi(x),$$

where $\Psi(x) = \sup_{x \in E^*}[\langle x, \xi\rangle - \Phi(x, \xi)]$. If, in addition suitable regularity assumptions are imposed on Φ, the corresponding large deviation lower bound holds:

$$\underline{\lim}\ 1/n \log P[Y_n \in A] \geq -\inf_{x \in A^\circ} \Psi(x).$$

This general result may be applied to large deviations of trajectories of Markov processes and in that context yields significant improvements of results of Wentzell (Kluwer, 1990); in particular, it gives a new proof of the classical result of Freidlin and Wentzell on small random perturbations of dynamical systems.

O.E. Barndorff-Nielsen (joint with A.E. Koudou)

Trees with Random Conductivities and the (Reciprocal) Inverse Gaussian Distribution

Equipping the edges of a finite rooted tree with independent resistances that are inverse Gaussian for interior edges and reciprocal inverse Gaussian for endpoint edges makes it possible, for suitable choice of the parameters, to show that the total resistance is reciprocal inverse Gaussian (Barndorff-Nielsen, 1994). This result is here extended to infinite trees. Also, a connection to Brownian diffusion is established and, for the case of finite trees, an exact distributional and independence result is derived for the conditional model given the total resistance.

S. Bobkov

A discrete version of the Gaussian isoperimetric inequality

An isoperimetric inequality on the discrete cube which is the precise analogue of

M. Talagrand's logarithmic inequality (1993) is proved. The presented inequality contains in limit case the isoperimetric inequality in Gauss space (theorem by V.N. Sudakov and B.S. Tsirel'son (1974) and C. Borell (1975)).

A. Dembo

Information inequalities and concentration of measure

We derive inequalities of the form $d(P,Q) \leq H(P|R) + H(Q|R)$ which hold for every choice of probability measures P, Q, R, where $H(P|R)$ denotes the relative entropy of P with respect to R and $d(P,Q)$ stands for a coupling type "distance" between P and Q. Using the chain rule for relative entropies and then specializing to Q of a given support we recover some of Talagrand's concentration of measure inequalities for product spaces.

E. Dettweiler

Series Expansions of Banach Space valued Random Vectors

Suppose that X is a random vector defined on some probability space $(\Omega, \mathcal{F}, P)$ and taking values in a real separable Banach space E such that $\mathbb{E}\|X\|^2 < \infty$ and $\mathbb{E}X = 0$. It is proved that there exists a continuous, square integrable, E-valued martingale $(M_t(X), \overline{\mathcal{F}}_t)_{0\leq t\leq 1}$ on an extension $(\overline{\Omega}, \overline{\mathcal{F}}, \overline{P})$ of Ω with the following properties:

(a) $M_0(X) = 0$ and $M_1(X) = X$,

(b) there exists a $(\overline{\mathcal{F}}_t)$-adapted, cadlag process $(Q_s)_{0\leq s\leq 1}$ taking values in the set of covariance operators from E' into E, such that the quadratic variation $([M(X)]_t)_{0\leq t\leq 1}$ of $M(X)$ is of the form

$$[M(X)]_t = \int_0^t Q_s ds.$$

From this embedding result it is derived that there exist

(i) a sequence (β_k) of independent, $(\overline{\mathcal{F}}_t)$-Brownian motions, and

(ii) a sequence (f_k) of $(\overline{\mathcal{F}}_t)$-adapted cadlag processes $(f_k(s))_{0\leq s\leq 1}$ taking values in E,

such that for all $x' \in E'$ (the dual of E)

$$\langle X, x'\rangle = \sum_k \int_0^1 \langle f_k(s), x'\rangle d\beta_k(s) \qquad \overline{P} - \text{a.s.}.$$

R. Dudley

On Fréchet differentiability for p-variation norms

Given a function $f : [a,b] \to \mathbb{R}$ and $1 \leq p < \infty$, let $\|f\|_{(p)} := \sup\{[\sum_{i=1}^{k} |f(x_i) - f(x_{i-1})|^p]^{1/p} : a = x_0 < x_1 < \ldots < x_k = b, k = 1,2,\ldots\}$, $\|f\|_{[p]} := \|f\|_{(p)} + \|f\|_\infty$. Let F be a distribution function and F_n an empirical d.f. for it. Let $B_{n,F}(t) := B_n(F(t)), B_n$ a Brownian bridge. Then for $\infty > p > 2$, there exist B_n such that $E\|\sqrt{n}(F_n - F) - B_{n,F}\|_{[p]} = 0(n^{\frac{1}{p}-\frac{1}{2}})$ (Yen-Chin Huang). For $1 \leq p < 2$, $\|F_n - F\|_{[p]} = O_p(n^{\frac{1}{p}-1})$ (Jinghua Qian). Consider the six operators (0) $(F,G) \mapsto FG$, (1) $(F,G) \mapsto \int FdG$, (2) $(F,G) \mapsto F * G := \int_{-\infty}^{\infty} F(\cdot - y)dG(y)$, (3) $(F,G) \mapsto F \circ G$ where $(F \circ G)(x) \equiv F(G(x))$, (4) $F \mapsto F^{\leftarrow}$ where $F^{\leftarrow}(y) := \inf\{x : F(x) \geq y\}$, (5) $f \to P_a^b(1+df) := \lim\{\prod_{i=1}^{k}(1 + f(x_i) - f(x_{i-1}))\}$ under refinement of partitions. It is shown that all six operators are Fréchet differentiable for some p-variation norms, with analyticity (holomorphy) for (0), (1), (2) and (5) (work with Rimas Norvaiša on (5)), and optimal remainder bounds $0(\|\cdot\|_{[p]}^{\alpha})$, $1 < \alpha < 2$, for (3), (4).

U. Einmahl

Strong approximations for local empirical processes

A general notion of "local empirical process" is introduced which includes the so-called tail empirical process, kernel density and regression function estimators among others. We will then discuss how one can obtain a meaningful strong approximation of such processes by suitable Gaussian processes. As a corollary of our strong approximation result we obtain a general compact LIL for local empirical processes. (Joint work with David M. Mason, University of Delaware)

X. Fernique

Random Fourier series

Let $A = \{A_k, k \in \mathbb{N}\}$ be a sequence of symmetric real r.v.'s. We present a condition for the series $\sum A_k \exp[2i\pi kt]$ to converge uniformly a.s. This sufficient condition is also necessary if the A_k's have the following form: $A_k = a_k X_k$ where $\{a_k, k \in \mathbb{N}\}$ is nonnegative and nonincreasing and $\{X_k, k \in \mathbb{N}\}$ is i.i.d. The proofs use the spectral criteria for the regularity of stationary Gaussian random functions.

P. Gaenssler

Random Measure Processes

We consider function-indexed Random Measure Processes $S_n = (S_n(f))_{f\in\mathcal{F}}$ defined by $S_n(f) = \sum_{j\leq j(n)} W_{nj}(f)\cdot\xi_{nj}$, where (W_{nj}) is a triangular array of random probability measures on an arbitrary measurable space (X,Ξ), (ξ_{nj}) is a triangular array of real-valued random variables, and where $\mathcal{F}$ is a class of real-valued measurable functions f on X (with $W_{nj}(f) = \int_X f dW_{nj}$). A uniform law of large numbers and a functional central limit theorem for $(S_n)_{n\in\mathbb{N}}$ are presented containing various results for empirical and partial-sum processes as special cases. Special emphasis will be given to smoothed empirical processes.

F. Götze (joint with V. Bentkus)

Functional limit theorems and lattice point problems

Consider a non-void ellipsoid $E\subset R^d$. Let $A(r)$ denote the number of lattice points in rE, i.e. $A(r) = \#\{z\in Z^d\cap(rE)\}$. We show that, for $d\geq 9$,

$$\left|\frac{A(r)-V(r)}{V(r)}\right| = 0\left(\frac{1}{r^2}\right), \quad as \quad r\to\infty,$$

where $V(r)$ denotes the volume of rE. That solves the lattice point problem for general ellipsoids for dimensions $d\geq 9$, and improves a result $0\left(\frac{1}{r^{2d/(d+1)}}\right)$ of Landau (1915). Relations to probability theory are discussed, and corresponding results are presented as well.

V. Goodman

LIL Behavior of Second Order U-Statistics

Giné and Zhang have recently given examples of degenerate second order U-statistics with infinite second moments and yet satisfying the following bounded LIL:

$$\text{(LIL)} \qquad \overline{\lim}_{n\to\infty}\frac{1}{nLLn}\sum_{i<j\leq n} h(X_i,X_j) < \infty \quad a.s.$$

They also obtained sufficient conditions for the LIL which involve a combination of moment assumptions and almost sure behavior random matrix norms. We obtain a complementary set of sufficient conditions for the (LIL). Our sufficient conditions are the following:

$$E\left[\frac{E'[h^2(X,X')]}{LLE'[h^2(X,X')]}\right] < \infty \qquad \text{and} \qquad \sup_{\|g(X)\|_2\leq 1} E[(E'[h(X,X')g(X')])^2] < \infty.$$

M. Hahn

Comments on Distinctions between the Regular and Self-Normalized CLT's

Giné, Götze and Mason have established the equivalence of the central limit theorem (CLT) and self-normalized CLT for i.i.d. random variables. In particular, this means that characterizations of the CLT are also characterizations of the self-normalized CLT for i.i.d. random variables. However, if the independence assumption is relaxed only slightly, as for example in the case of exchangeable random variables, then the equivalence no longer holds. In fact, conditions such as asymptotic negligibility of the maximal term of absolute value to the partial sums or to the self-normalizers turn out to characterize the self-normalized CLT rather than the CLT for exchangeable random variables with symmetric mixands.

B. Heinkel

Laws of large numbers and continuity of processes

T.L. Lai has shown that for a sequence $(X_k)_{k\geq 0}$ of independent, identically distributed, real-valued random variables (r.v.), the strong law of large numbers (i.e. the almost sure (a.s.) convergence to 0 of the sequence $\left(\frac{1}{n}\sum_{k=0}^{n} X_k\right)_{n\geq 1}$) is equivalent to the fact that $(X_k)_{k\geq 0}$ converges a.s. to 0 in the sense of Abel. Several extensions of this result are given; for example:
"For a sequence $(X_k)_{k\geq 0}$ of independent, symmetrically distributed, real valued r.v., such that $\left(\frac{X_k}{k}\right)_{k\geq 1}$ converges a.s. to 0, the strong law of large numbers is equivalent to the following two properties:

i) $\left(\frac{1}{n^2}\sum_{k=0}^{n} X_k^2\right)_{n>1}$ converges a.s. to 0;

ii) The process $(\xi(t), t \in [0,1])$ has a.s. continuous paths, where:

$$\forall t \in [0,1[, \quad \xi(t) = (1-t)^2 \sum_{n=1}^{\infty} t^n \sum_{0\leq i<n/2} X_i X_{n-i}, \quad \xi(1) = 0 \quad \text{a.s.}"$$

P. Hitczenko

Moments of linear combinations of iid symmetric random variables

A rather general method of establishing upper bounds for moments of linear combinations of iid symmetric random variables will be discussed. As a main application we obtain tight upper and lower bounds for moments of linear combinations of symmetrized Weibull random variables with parameter $r, 0 < r \leq 1$. This complements earlier work of Gluskin and Kwapien who have done the same for $r \geq 1$. This is a joint work with S.J. Montgomery-Smith and K. Oleszkiewicz. Recently, Latala obtained a generalization to general sums of independent, symmetric random variables.

J. Hoffmann-Jørgensen

A new concept of convergence in law

Considering continuity properties of set valued functions such as argmax and argzero one is naturally led to the so-called upper Fell topology on the set of all subsets of a given topological space T. The upper Fell topology has the nasty property that only constant functions can be continuous. This means that the classical notion of convergence in law (i.e. $\lim E^*\phi(X_n) = E_*\phi(X) \quad \forall\phi \in C(T)$) trivializes for the upper Fell topology (every sequence of random sets converges in law to every random set). To handle this situation the following new notion of convergence in law is suggested. If T is a topological space, X and X_n are T-valued random elements, and Usc (T) is the set of upper semicontinuous functions $f : T \to \overline{\mathbb{R}}$ with $\sup f < \infty$, then we say that (X_n) converges in Borel law to X and we write $X_n \xrightarrow{\sim} X$ if

$$\limsup_n E^* f(X_n) \leq E_* f(X) \qquad \forall f \in \text{Usc}\ (T)$$

With this new notion it becomes meaningful and interesting to consider weak convergence of random sets.

V. I. Koltchinskii

Asymptotics of the Spectra of Random Matrices Approximating Integral Operators

Let $(S, \mathcal{S}, P)$ be a probability space and let $h : S \times S \to \mathbf{R}^1$ be a symmetric measurable kernel, which defines a compact integral operator H from $L_2(P)$ into $L_2(P)$:

$$Hg(x) := \int_S h(x,y)g(y)P(dy),\ g \in L_2(P).$$

Let $X_1, X_2, \ldots$ be independent random elements in S with common distribution P. Consider an $n \times n$-matrix $\widetilde{H}_n$ with the entries $h(X_i, X_j), 1 \leq i,j \leq n$ (which is a matrix of an empirical version of the operator H with P replaced by the empirical measure P_n). Let H_n denote the modification of $\widetilde{H}_n$, obtained by deleting its diagonal.

Given a compact operator (matrix) A, let $\sigma(A)$ be the spectrum of A (i.e. the sequence of its eigenvalues arranged in decreasing order). We study the convergence

of $\sigma(\widetilde{H}_n)$ and $\sigma(H_n)$ to $\sigma(H)$ as $n \to \infty$. The laws of large numbers, rates of convergence results and central limit theorems (for particular eigenvalues as well as in the spaces of sequences ℓ_2 and c_0) are considered.
This is joint work with E. Giné and J. Zinn.

J. Kuelbs (joint with U. Einmahl)

Dominating Points and Large Deviations for Random Vectors

We establish a representation formula useful for obtaining precise large deviation probabilities for convex open subsets of a Banach space. These estimates are based on the existence of dominating points in this setting. In particular, assume $X, X_1, X_2, \ldots$ are i.i.d with values in a separable Hilbert space H, $E(X) = 0$, $E(e^{t\|X\|}) < \infty$ for all $t > 0$, and that $D = \{x \in H : \|x-a\| < R\}$ where $0 < R < \|a\|$. Let $\lambda(x) = \sup_{g\in H}[\langle x, g\rangle - \log E(e^{\langle X,g\rangle})]$, dom $(\lambda) = \{x \in H : \lambda(x) < \infty\}$, and assume $D \cap \operatorname{dom}(\lambda) \neq \emptyset$. Then there exists a unique point $a_0 \in \partial D$ such that $a_0 \in$ dom (λ) and constants $C_1, C_2 \in (0,\infty)$ such that for all $n \geq 1$

$$C_1 \;\leq\; P(X_1 + \cdots + X_n \in nD)\exp\{n\lambda(a_0)\}\sqrt{n} \;\leq\; C_2.$$

S. Kwapien

Isoperimetric inequalities and concentration of measure

We present a recent approach due to M. Ledoux to some of Talagrand's concentration inequalities. The approach is based on inequalities of logarithmic Sobolev type. The advantage of the approach is the simplicity and generality.

R. Latala

Moments of sums of independent random variables

(second part of the talk joint with P. Hitczenko)

We give an explicit formula for moments of sums of independent symmetric random variables for $p \geq 2$ (and nonnegative for $p \geq 1$):

$$\frac{e-1}{2e^2}|||(X_i)|||_p \;\leq\; \|\sum X_i\|_p \;\leq\; e|||(X_i)|||_p$$

$$\text{where} \quad |||(X_i)|||_p \;=\; \inf\{t > 0 : \prod_i \mathbb{E}|1 + \frac{X_i}{t}|^p \leq e^p\}.$$

As an application we get results mentioned by P. Hitczenko in the first part of the talk.

W. Li

Professor Xiangchen Wang's contributions to Probability on Banach spaces

This is a tribute to Professor Xiangchen Wang who died tragically last summer in Jilins Province, China, at the age of 52. Professor Wang introduced the topic of probability on Banach space to China and had many students who are actively working on the subject. His most important research was in strong limit theorems for B-valued random variables with multidimensional indices.

M.A. Lifshits

Some new results on Strassen functional law

We discuss some recent results on the convergence rate in Strassen law for Brownian motion and partial sum processes. Though the correct rate for Brownian motion was found recently the answer turns out to be very unstable with respect to the variation of normalizing function (iterated logarithm). This phenomenon implies very spectacular behavior of convergence rates in partial sum processes. The "Brownian" rate appears not to be the best possible (U. Einmahl, D. Mason) but there exists an integral test for possible rates and one can describe the rate for given increment distribution.

Some examples of Strassen-type results with nonconvex limit sets will be given.

W. Linde (joint with Th. Dunker)

An example of unexpected small ball behaviour

In 1984 V. M. Zolotarev gave a formula for the asymptotic behaviour of

$$\mathbf{P}\left(\sum_{j=1}^{\infty} f(j)\xi_j^2 < r\right) \quad \text{as} \quad r \downarrow 0\,.$$

Here $\xi_1, \xi_2, \dots$ are i.i.d. $\mathcal{N}(0,1)$ and $f : [1,\infty) \to \mathbf{R}^+$ is a decreasing, integrable and log–convex function. We state a corrected version of Zolotarev's result and show that for $f(x) = e^{1-x}/2$, i.e. for $\mathbf{P}(\sum_{j=0}^{\infty} e^{-j}\xi_j^2 < 2r)$, a periodic function of r has to be added to Zolotarev's formula.

M. Marcus

High order Gaussian chaos processes and self-intersections of Lévy processes

In this joint work with J. Rosen we show that the continuity of a $2n$-th Wick power Gaussian chaos implies the continuity of the n-fold renormalized self-intersection local time of a corresponding Lévy process.

D. M. Mason

When is the Student t-statistic asymptotically standard normal?

Let $X_1, X_2, \ldots,$ be a sequence of independent and identically distributed random variables. For each integer $n \geq 1$, let $S_n = X_1 + \cdots + X_n$ and $V_n^2 = X_1^2 + \cdots + X_n^2$. Consider the self-normalized sum $U_n = S_n / V_n$. This self-normalized sum has a number of unexpected large sample distributional properties. Among other things, these properties are shown to lead to an 'elementary' solution of a 23 year old conjecture concerning when the classical Student's t-statistic is asymptotically standard normal. Also as a by-product, a very general bounded law of the iterated logarithm for U_n is obtained.

S. A. Murphy (joint with A. van der Vaart)

Empirical Likelihood/Likelihood Ratio Tests in Semiparametric Models

Likelihood ratio tests and related confidence intervals for a real parameter in the presence of an infinite dimensional nuisance parameter are considered. In all cases, the estimator of the real parameter has an asymptotic normal distribution. However, the estimator of the nuisance parameter may not be asymptotically Gaussian or may converge to the true parameter value at a slower rate than the square root of the sample size. Nevertheless the likelihood ratio statistic is shown to possess an asymptotic χ-squared distribution. In particular the proportional odds model arising in survival analysis is discussed.

V. de la Pena (joint with N. Eisenbaum)

Inequalities for Randomly Stopped Processes with cadlag and independent increments

In this talk we discuss the following inequality: Let $\{X_t\}$ be a continuous time process with cadlag (right continuous with left limits) paths and independent increments taking values in an arbitrary Banach space $(B, \|\cdot\|)$. Let T be a stopping time adapted to $\sigma\{X_t\}$. Consider an independent copy $\{\widetilde{X}_t\}$ of $\{X_t\}$, with $\{\widetilde{X}_t\}$ independent of T as well, then for all $p > 0$, there exist universal constants $0 < c_p, c < \infty$ such that

$$c_p E \sup_{s\leq T} \|\widetilde{X}_s\|^p \leq E \sup_{s\leq T} \|X_s\|^p \leq c^p E \sup_{s\leq T} \|\widetilde{X}_s\|^p.$$

Applications of these results are given to

1) The extension of the Burkholder-Gundy inequality for randomly stopped Brownian motion to Banach Spaces as well as exponential inequalities
2) Bessel Processes
3) Results of Klass 88,90 for sums of independent variables

G. Peskir

Optimal Stopping and Maximal Inequalities for Geometric Brownian Motion

The explicit formulas are found for the payoff and the optimal stopping strategy of the optimal stopping problem:

$$\sup_{\tau} E\left(\max_{0\leq t\leq \tau} X_t - c\tau\right)$$

where $X = (X_t)_{t\geq 0}$ is geometric Brownian motion with drift $\mu < 0$ and volatility $\sigma > 0$, and the supremum is taken over all stopping times for X. The payoff is shown to be finite, if and only if $\mu < 0$. The optimal stopping time is given by:

$$\tau_* = \inf\left\{t > 0 | X_t = g_*\left(\max_{0\leq s\leq t} X_s\right)\right\}$$

where $s \mapsto g_*(s)$ is the *maximal* solution of the (nonlinear) differential equation:

$$\frac{\partial g}{\partial s} = K\frac{g^{\Delta+1}}{s^{\Delta} - g^{\Delta}} \quad (s > 0)$$

under the condition $0 < g(s) < s$, where $\Delta = 1 - 2\mu/\sigma^2$ and $K = \Delta\sigma^2/2c$. The estimate is shown to be valid:

$$g_*(s) \sim \left(\frac{\Delta - 1}{K\Delta}\right)^{1/\Delta} s^{1-1/\Delta}$$

as $s \to \infty$. Applying these results we prove the following maximal inequality:

$$E\left(\max_{0\leq t\leq\tau} X_t\right) \;\leq\; 1-\frac{\sigma^2}{2\mu}+\frac{\sigma^2}{2\mu}\exp\left(-\frac{(\sigma^2-2\mu)^2}{2\sigma^2}E(\tau)-1\right)$$

where τ may be any stopping time for X. This extends the well-known identity:

$$E\left(\sup_{t>0} X_t\right) \;=\; 1-\frac{\sigma^2}{2\mu}$$

and is shown to be sharp. The method of proof relies upon a smooth pasting guess (for the Stephan problem with moving boundary) and Itô-Tanaka's formula (being applied two-dimensionally). The key point and main novelty in our approach is the maximality principle for the moving boundary (the optimal stopping boundary is the maximal solution of the differential equation obtained by a smooth pasting guess). We think that this principle is by itself of theoretical and practical interest.

I. Pinelis

Inverse problems for moment comparison inequalities

Based on moment comparisons of the form $E\varphi(\xi) \leq E\varphi(\eta) \quad \forall\varphi \in \mathcal{F}$ for a set $\mathcal{F}$ of functions and random variables ξ and η, tail comparisons of the form $P(\xi > x) \leq cP(\eta > x) \quad \forall x$ are deduced. Applications include comparison inequalities for Gaussian measures, an improved Hoeffding-Azuma inequality, and a generalized mass transportation duality.

G. Pritchard

Empirical processes and sorting

The scaled-sample problem asks the following question: given a distribution on a normed linear space E, when do there exist constants $\{\gamma_n\}$ such that $\{X_j/\gamma_n\}_{j=1}^n$ converges as $n \to \infty$ (in the Hausdorff metric given by the norm) to a fixed set K ? (Here $\{X_j\}$ are i.i.d. with the given distribution.) We relate this property to a large deviation principle for dilations of the distribution, and consider an application to distributions represented by random series.

D. Radulović

The Bootstrap for Empirical Processes under dependence

It is shown that the blockwise bootstrap of the empirical processes for stationary sequences converges to an appropriate Gaussian limit. In the case of indexing class $\mathcal{F}$ being indicators of the half lines we use the α-mixing and in the case of VC-subgraph we use the β-mixing conditions. In both cases the conditions imposed are only marginally stronger than the best possible.

J. Rosinski

Certain problems of ergodic theory arising in the study of stable processes

Structural analysis of stable processes is based on an explicit form of groups of isometries on L^p-spaces. These groups are determined by nonsingular deterministic flows and, related to such flows, cocycles. Consequently, many questions concerning stationary stable process can be formulated as ergodic theory questions related to flows and cocycles. Using this relationship, decompositions of stationary stable processes into simpler independent components can be obtained and chaotic properties of such processes can be determined.

G. Schechtman (joint with V. Milman)

An "Isomorphic" Version of Dvoretzky's Theorem

Let $\log n < k < n$ and let $C \subset \mathbf{R}^n$ be a symmetric (about the origin), bounded, convex set with non empty interior. Then there is a k-dimensional subspace L of $\mathbf{R}^n$ such that the ratio between the outer and inner radii of $C_0 = C \cap L$ is bounded by $K\sqrt{\frac{k}{\log(n/k)}}$, for some universal constant K. (i.e., if B denotes the Euclidean ball in L, then $rB \subset C_0 \subset RB$ for some r, R with $R/r \leq K\sqrt{\frac{k}{\log(n/k)}}$).
This extends a celebrated theorem of Dvoretzky. Except for the determination of the absolute constant K, the result is best possible for each value of k and n.

Qi-Man Shao

Self-normalized Limit Theorems

The normalizing constants in classical limit theorems are usually a sequence of real numbers. It is well-known that moment conditions or other related assumptions are necessary and sometimes sufficient for many classical limit theorems. For instance, a necessary and sufficient condition for large deviation is that the moment generating function is finite in a neighborhood of zero. The law of the iterated logarithm holds for i.i.d. random variables if and only if the second moment is finite. However, the situation becomes very different if the normalizing constants are a sequence of random variables. In this talk, by using a suitable sequence of random variables as normalizing constants, we establish a self-normalized large deviation without any moment conditions. As a consequence, we obtain a precise constant for the self-normalized law of the iterated logarithm of Griffin and Kuelbs (1989).

M. Weber

Spectral regularization in ergodic theory and probability

We show that the idea of spectral regularization introduced by M. Talagrand in the study of covering numbers of averages of contractions in a Hilbert space H, can be concentrated in one inequality which turns out to be a suitable tool for the study of other characteristics of the set of averages. This inequality generates an intrinsic Lipschitz embedding of the circle and yields many useful corollaries. In particular, we recover some recent results for the Littlewood-Paley square functions in ergodic theory due to R. Jones, I. Ostrovskii and J. Rosenblatt. We also easily deduce original Talagrand's estimate of covering numbers and provide better estimates for geometric subsequences of the averages. Using majorizing measures technique, we prove a new criterion of the a.s. convergence of random sequences defined on a lacunary index set under suitable incremental conditions. Combining this criterion with our Lipschitz embedding, we obtain as a corollary the classical theorem of Rademacher-Menshov on orthogonal series and the famous spectral criterion for the strong law of large numbers due to V.F. Gaposhkin.

J. A. Wellner (joint with E. Giné)

Empirical Process methods for multiple particle systems

For n particles diffusing throughout R (or R^d), let $\eta_{n,t}(A)$, $A \in \mathcal{B}$, $t \geq 0$, be the random measure that counts the number of particles in A at time t. It is shown that for some basic models (Brownian particles with or without branching and diffusion with a simple interaction) the processes $\{(\eta_{n,t}(\phi) - E\eta_{n,t}(\phi))/\sqrt{n} : t \in [0, M], \phi \in C_L^\alpha(R)\}$, $n \in \mathbb{N}$, converge in law uniformly in (t, ϕ). Previous results consider only convergence in law uniform in t but not in ϕ. The methods used are from empirical process theory: we use a bracketing lemma due to Van der Vaart and Ossiander's uniform CLT.

J.E. Yukich

Laws of Large Numbers for Random Graphs and Random Matchings

Let $X_i, i \geq 1$, be iid with values in $\mathbb{R}^d, d \geq 2$. We show that for a wide class of functions $L = L(X_1, \ldots, X_n)$ arising as solutions to problems in combinatorial optimization, mathematical statistics, operations research and computational geometry that

$$\frac{L(X_1, \ldots, X_n)}{n^{(d-1)/d}} \longrightarrow \beta(L, d) \int f(x)^{(d-1)/d} dx \qquad a.s.$$

where $\beta(L, d)$ is a finite positive constant and f is the density of the absolutely continuous part of $\mathcal{L}(X)$. For example, we may let $L := L(X_1, \ldots, X_n)$ be the length of the random graph representing the shortest tour through $X_1, \ldots, X_n$, the length of the Minimal spanning tree through $X_1, \ldots, X_n$, as well as the length of numerous other graphs including bi-partite graphs, k nearest neighbors, and minimal triangulations.

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