TAGUNGSBERICHT 31/1996

REELLE ANALYSIS
18.-24.08.1996

Die Tagung fand unter der Leitung von D. Müller (Kiel), E.M. Stein (Princeton) und H. Triebel (Jena) statt. Insgesamt wurden 33 Vorträge über neuere Ergebnisse aus dem Bereich der Reellen Analysis gehalten. Diese Zahl ist Ergebnis einer während der Tagung vorgenommenen Straffung des Vortragsprogramms, um einen Kompromiß zwischen der traditionellen Programmgestaltung und der neuen Leitlinie des Forschungsinstituts zu finden. Neben den Vorträgen wurde die Tagung in jeweils kleineren Gruppen zu Diskussionen über neue Ideen und Methoden und zur lebhaften Zusammenarbeit genutzt.

Ein großer Teil der Vorträge behandelte Probleme der Fourieranalysis auf euklidischen Räumen (Konvergenzprobleme, Maximaloperatoren, singuläre Integrale, oszillierende Integrale), mehrere Beiträge beschäftigten sich mit der Analysis auf Lieschen Gruppen und symmetrischen Räumen. Ein wichtiger Aspekt der Tagung war, daß zunehmend moderne Methoden der Reellen Analysis (Restriktionssätze, Muckenhoupt–Gewichte, etc.) auf dem Gebiet der Partiellen Differentialgleichungen verwendet werden, insbesondere auch in der Theorie nichtlinearer Gleichungen. Dies wurde in mehreren Vorträgen aufgezeigt. Eine Reihe von Vortragenden stellte Ergebnisse aus der Theorie der Funktionenräume vor. Schließlich gab es auch einige Beiträge zur komplexen Analysis und deren Behandlung mit reellen Methoden.

Die folgenden Vortragsauszüge sind in alphabtischer Reihenfolge der Autoren zusammengestellt.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

J.-P. ANKER:

Heat kernel, Green function and the Martin boundary of symmetric spaces

Let G/K be a Riemannian symmetric space of noncompact type. In joint work with Lizhen Ji, we conjecture the following behavior for the heat kernel on G/K:

$$h_t(x) \asymp t^{-\frac{\ell}{2}} \Big\{ \prod_{\alpha \in \Sigma^{++}} \tfrac{1+\langle\alpha,H\rangle}{t} \left(1 + \tfrac{1+\langle\alpha,H\rangle}{t}\right)^{\frac{m_\alpha + m_{2\alpha}}{2} - 1} \Big\}\, e^{-|\varrho|^2 t - \langle\varrho,H\rangle - \frac{|H|^2}{4t}}$$

$$\forall\, t > 0 \quad \text{and} \quad \forall\, x = k_1(\exp H)\, k_2 \in G \quad \text{with} \quad k_1 \in K,\, H \in \overline{\mathfrak{a}^+},\, k_2 \in K\,,$$

and establish this when $|H| \leq C_1 + C_2 t$ (C_1 and C_2 are arbitrary positive constants) or when H stays away from the Weyl chamber walls. (Our notation is standard, except possibly for ℓ and Σ^{++}, which denote respectively the rank of G/K and the set of positive indivisible roots.) As a consequence we obtain the exact kernel behavior at infinity of the Bessel–Green–Riesz potentials $(-\Delta - |\varrho|^2 + \zeta^2)^{-\frac{\sigma}{2}}$:

$$g_{\sigma,\zeta}(\exp H) \asymp$$

$$\asymp \begin{cases} |H|^{\frac{\sigma-\ell-1}{2} - |\Sigma^{++}|} \Big\{ \prod_{\alpha \in \Sigma^{++}} (1 + \langle\alpha, H\rangle) \Big\}\, e^{-\langle\varrho,H\rangle - \zeta|H|} & \text{if } \sigma > 0 \text{ and } \zeta > 0 \\ |H|^{\sigma-\ell-2|\Sigma^{++}|} \Big\{ \prod_{\alpha \in \Sigma^{++}} (1 + \langle\alpha, H\rangle) \Big\}\, e^{-\langle\varrho,H\rangle} & \text{if } 0 < \sigma < \ell + 2\,|\Sigma^{++}| \text{ and } \zeta = 0 \end{cases}$$

for $H \in \overline{\mathfrak{a}^+}$, $|H| \to +\infty$.

This result for $\sigma = 2$ is the key analytic ingredient for the determination of the Martin compactification $\overline{G/K}^M$ of G/K, which was recently achieved by Y. Guivarc'h, L. Ji and J. Taylor:

$\overline{G/K}^M$ coincides when $\zeta = 0$ with the maximal Furstenberg–Satake compactification $\overline{G/K}^{FS}$, and when $\zeta > 0$ with the closure of G/K diagonally embedded into the product $\overline{G/K}^{FS} \times \overline{G/K}^{\text{geod}}$, where $\overline{G/K}^{\text{geod}}$ denotes the geodesic compactification of G/K.

W. BECKNER:

Logarithmic Sobolev inequalities: from uncertainty to geometric invariants

Analysis of the constants in sharp inequalities can determine explicit geometric information about a manifold. An important example is the Moser-Trudinger inequality where limiting Sobolev behaviour for critical exponents provides significant understanding of geometric analysis for conformal deformation on a Riemannian manifold. A second example is the relation between the uncertainty principle and the Gaussian logarithmic Sobolev inequality where the form of these estimates is essentially determined by the interplay between the dilation structure and the product structure. The framework for characterizing this phenomenon includes "local invariants" (Hardy-Littlewood-Sobolev inequalities, Stein-Weiss integrals, multilinear forms) and "global invariants" (zeta functions, trace inequalities, partition functions).

Two themes are developed in this talk: 1) a derivation of the logarithmic Sobolev inequality from a new "logarithmic uncertainty principle" and 2) an extension of the Hardy-Littlewood-Sobolev inequaliy for the full range of L^p classes on $\mathbb{R}^n$, $0 < p$. These topics are related in the sense that they both follow from the "local invariants" described above, though they depend on different forms of symmetry structure (product structure for the first and conformal structure for the second). In addition, the logarithmic uncertainty principle implies a non-sharp form of the limiting $p = 2$ case of the Hardy-Littlewood-Sobolev inequality. Perhaps the most incisive aspect of the notion of logarithmic Sobolev inequality given by L. Gross in 1974 is the idea of drawing out information from the limiting behaviour of the parameter dependence of global inequalities

Using Pitt's inequality and analysis of a Stein-Weiss integral, a new form of the uncertainly principle can be derived which implies the classical Heisenberg-Weyl inequality. For functions in the Schwartz class $S(\mathbb{R}^n)$ and for the Fourier transform defined by $(\mathcal{F}f)(x) = \int_{\mathbb{R}^n} e^{2\pi i x y} f(y) dy$

$$(*) \qquad \int_{\mathbb{R}^n} \ln|x| \, |f(x)|^2 \, dx + \int_{\mathbb{R}^n} \ln|\xi| \, |(\mathcal{F}f)(\xi)|^2 \, d\xi \geq D_n \int_{\mathbb{R}^n} |f(x)|^2 \, dx$$

where $D_n = \psi(n/4) - \ln \pi$ and $\psi = (\ln \Gamma)'$. Using canonical variables P, Q for the position-momentum representation with $\hat{P}, \hat{Q}$ denoting average values, this corresponds to

$$\langle \ln|Q - \hat{Q}| \rangle + \langle \ln|P - \hat{P}| \rangle \ \geq \ \text{constant}$$

Theorem 1. *The logarithmic uncertainty principle $(*)$ implies the Sobolev inequality for* $||f||_{L^2(\mathbb{R}^n)} = 1$

$$\frac{n}{4} \ln \int_{\mathbb{R}^n} |\nabla f|^2 dx \geq \int_{\mathbb{R}^n} |f|^2 \ln|f| dx + A_n$$

from which the logarithmic Sobolev inequality for Gaussian measure can be derived

$$\int_{\mathbb{R}^n} |g|^2 \ln|g| \, d\mu \leq \int_{\mathbb{R}^n} |\nabla g|^2 \, d\mu$$

where $d\mu = (2\pi)^{-n/2} \exp(-x^2/2) \, dx$ *and* $||g||_{L^2(d\mu)} = 1$.

Theorem 2. *The Hardy-Littlewood-Sobolev inequality extends to the $L^p(\mathbb{R}^n)$ classes for $0 < p < 1$ when restricted to non-negative functions and the usual inequality reverses. For the case where the map is between a space and its dual, conformal invariance can be used to calculate the sharp constant. For functions $f, g \geq 0$, $0 < p < 1$ and $\lambda = 2n/p'$ $(1/p + 1/p' = 1)$*

$$\int_{\mathbb{R}^n \times \mathbb{R}^n} f(x)|x-y|^\lambda g(y) \, dx dy \geq A_p ||f||_{L^p(\mathbb{R}^n)} ||g||_{L^p(\mathbb{R}^n)}$$

and the extremal functions are given by $A(1+|x|^2)^{-n/p}$ up to conformal automorphism.

Observe that the $p=1$ limit of this inequality gives the free energy-entropy inequality dual to the Moser-Trudinger inequality.

G. BOURDAUD:

Homomorphims which acts on Besov spaces

(joint work with Winfried Sickel (Jena)) We give a (almost) full description of the homomorphisms φ of $\mathbb{R}$ onto $\mathbb{R}$ such that $f \in \dot{B}_p^{s,q}(\mathbb{R})$ (resp. $B_p^{s,q}(\mathbb{R})$) $\Rightarrow f \circ \varphi \in \dot{B}$ (resp. B). This work continues – and, for one of the theorems, corrects – the results of S. Vodop'yanov (Novosibirsk):

1 Non critical ($s \neq 1/p$, $0 < s < 1$) Besov spaces.

1–a For $s < 1/p$ the NSC is $\frac{1}{\varphi'} \in L^\infty$ (for both homogeneous and inhomogeneous spaces)

1–b For $s > 1/p$ the NSC is

- $\varphi' \in L^\infty$ for homogeneous spaces.
- $\varphi' \in L^\infty$ and $\sup\limits_{x\in\mathbb{R}} \left(\varphi^{-1}(x+1) - \varphi^{-1}(x)\right) < +\infty$ for inhomogeneous spaces.

2 Critical ($s = 1/p$, $q = p$) Besov spaces.

2–a Homogeneous case. The Vodop'yanov result is the following: φ acts on $\dot{B}_r^{1/p,p}(\mathbb{R})$ iff φ is a quasi conformal transform of the line (which means exactly that φ' is a doubling measure)

2–b Inhomogeneous case. We obtain a sufficient (but not necessary) condition for φ to act. It is the "local quasi-conformality":

$$\exists M \geq 1 \quad \forall x \in \mathbb{R} \quad \forall h \in]0,1] \quad \frac{1}{M} \leq \frac{\varphi(x+h) - \varphi(x)}{\varphi(x) - \varphi(x-h)} \leq M.$$

A. CARBERY:

Sublevelsets of functions with large derivatives

(joint work with M. Christ, J. Wright.) Let $u : Q^n \to \mathbb{R}$ be of class C^∞ on the unit cube Q^n in $\mathbb{R}^n$, and suppose $D^\beta u \geq 1$ on Q^n. We consider whether there exist $\epsilon = \epsilon(\beta) > 0$ and an absolute $C_{\epsilon,\beta}$ such that

$$|\{x \in Q^n : |u(x)| \leq \alpha\}| \leq C_{\epsilon,\beta}\alpha^\epsilon \tag{1}$$

Theorem 1. *For all β there is $\epsilon(\beta)$ and $C_{\epsilon,\beta}$ such that (1) holds. When $n = 2$ and $\beta = (1,k)$ the right hand side of (1) is $\alpha^{\frac{1}{k+1}} \log^{\frac{k}{k+1}}(\frac{1}{\alpha})$; when $\beta = (j,k)$ with $j \geq 2$, the right hand side of (1) is $\alpha^{\frac{1}{j(k+1)}}$.*

Theorem 2. *If in addition u satisfies a "convexity" hypothesis (e.g. for $n = 2$, $\beta = (j, k)$, $\frac{\partial^N u}{\partial x^N}$ is single signed for some $N > j$) then (1) holds with $\epsilon = \frac{1}{|\beta|}$.*

We also consider oscillatory integrals of the first kind. Under the same hypotheses on u, let

$$I(\lambda) = \int_{Q^n} e^{i\lambda u(x)}\, dx$$

and we ask whether, for some $\epsilon = \epsilon(\beta)$ and an absolute $C_{\epsilon,\beta}$

(2) $$|I(\lambda)| \leq C_{\epsilon,\beta}\lambda^{-\epsilon}$$

Theorem 3. *If β contains at least one entry of order ≥ 2, there is an $\epsilon = \epsilon(\beta)$ and $C = C(\epsilon, \beta)$ such that (2) holds. If β contains only zeroes and ones there is no such estimate.*

Theorem 4. *If in addition u satisfies a "convexity" hypothesis, then (2) holds with $\epsilon = \frac{1}{|\beta|}$. (even if β's entries are $\in \{0, 1\}$.)*

Corollary. *If u is a polynomial of degree N, $D^\beta u \geq 1$, then (2) holds with $C_{\epsilon,\beta} = C_{N,\beta}$ and $\epsilon = 1/|\beta|$.*

The proofs of Theorems 1 and 3 are operator theoretic and in fact yield:

Theorem 1'. *If $\frac{\partial^{j+k} u}{\partial x^j \partial y^k}$ on $Q^2 \subset \mathbb{R}^2$ and*

$$S_\alpha = \int \chi_{\{(x,y):|u(x,y)|\leq\alpha\}} f(y)\, dy\ ,$$

then

$$\|S_\alpha f\|_{L^{k+1}} \leq C_k \alpha^{\frac{1}{k+1}} \log^{\frac{k}{k+1}}(\frac{1}{\alpha})\|f\|_{k+1}$$

if $j = 1$, and

$$\|S_\alpha f\|_{L^{k+1}} \leq C_{j,k}\alpha^{\frac{1}{j(k+1)}}\|f\|_{\frac{j(k+1)}{j(k+1)-k}}$$

if $j \geq 2$.

Moreover, modulo the logarithm in the case $j = 1$ this is sharp.

Theorem 3'. *Let $T_\lambda f = \int_{Q^{n_1}} e^{i\lambda u(x,y)} f(y)\, dy$ for $x \in Q^{n_2}$. If $D^\beta u \geq 1$, Then*

$$\|T_\lambda f\|_2 \leq \frac{C}{\lambda^\epsilon}\|f\|_2$$

for some $\epsilon > 0$ and absolute C. If $n = 2$ and $\beta = (1, k)$, we may take $\epsilon = \frac{1}{2k}$, which is sharp. (of course we assume β contains at least one entry ≥ 2).

S. CHANILLO:

Symmetry of solutions of non-linear PDE

We describe some work done jointly with Michael Kiessling to establish the rotational symmetry of solutions to some non-linear systems. Our methods are based on the Pohozaev identity coupled with the iso-perimetric inequality. Our principal applications are to Ginzburg-Landau equations and Liouville systems.

§1. For Ginzburg-Landau equations we have the following theorem.

Theorem. *Let u be a C^2 vector field, $u : \mathbb{R}^2 \to \mathbb{R}^2$, and satisfying the Ginzburg-Landau equation,*

$$-\Delta u = u(1 - |u|^2), \textit{ in all } \mathbb{R}^2$$

with three additional hypotheses, (H1) $\int_{\mathbb{R}^2}(1 - |u|^2)^2 < \infty$, (H2) *$u$ has a single zero and* (H3) *there is a rotation $\mathcal{R}_\beta \in SO(2)$, by the angle $\beta \in [0, 2\pi)$ such that $\nabla \times (\mathcal{R}_\beta u) = 0$, then there exists a point x_0 and a unique, non-negative, radial function f such that,*

$$(\mathcal{R}_\beta u)(x + x_0) = \pm \frac{x}{|x|} f(|x|).$$

§2. The following theorem describes the sort of symmetry results we can prove for Liouville systems.

Theorem. *Let $\{\gamma_{ij}\}$ denote a symmetric $N \times N$ matrix of numbers. Furthermore assume in addition that the matrix entries satisfy three additional properties.* (A) $\sum_{i=1}^N \gamma_{ij} = 1$, (B) $\gamma_{ii} \neq 0$, *for all $i = 1, \cdots, N$,* (C) $\gamma_{ij} \geq 0$. *Now consider the system of equations,*

$$-\Delta u_i(x) = e^{\sum_{j=1}^N \gamma_{ij} u_j(x)}, \ x \in \mathbb{R}^2.$$

Then if $\int_{\mathbb{R}^2} e^{u_i} dx < \infty$ for all $i = 1, \cdots, N$.
1. We can conclude that **all** *the functions $u_i(x)$ are radial and decreasing about some point, x_0^i (i.e. not necessarily the same point for all $i = 1, \cdots, N$).*
2. If, we strengthen hypothesis (C) above and have instead that $\gamma_{ij} > 0$, then $u_i(x)$ are radial and decreasing about the **same** *point x_0.*

J. COLLIANDER:

Wellposedness for the Zakharov system

This talk describes the main results of my thesis written under the direction of Jean Bourgain. We consider the initial value problem (IVP) for the Zakharov system (1):

$$i\partial_t u + \Delta u = nu$$
$$\partial_t^2 n - \Delta n = \Delta(|u|^2)$$
$$(u, n, \dot{n})(0) = (u_0, n_0, n_1)$$
$$u : \mathbb{R}^d \times \mathbb{R} \to \mathbb{C}$$
$$n : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}$$

(in the Hamiltonian setting) and some polynomial generalizations. By formally solving the second equation for n in terms of u and n_0, n_1, and using Duhanel's formula, we see (1) is equivalent to

(2) $$u(t) = s(t)u_0 - i\int_0^t s(t-\tau)\left((\Box^{-1}\Delta(|u|^2)(\tau) + W(\tau)(n_0, n_1))u(\tau)\right)\, d\tau.$$

This equation is shown to have a unique local (in time) solution in the space X_{sb} of functions of space-time satisfying

(3) $$(1+|k|)^s(1+|\lambda+|k|^2|)^b|\hat{u}(k,\lambda)| \in L^2(dkd\lambda)$$

for appropriate s, b. The consequences of this analysis are:

Wellposedness Theorem. *In $d = 1, 2, 3$, the IVP (1) is locally well posed for data $(u_0, n_0, n_1) \in H^{s_d} \times L^2 \times \hat{H}^{-1}$ with $\frac{1}{2} < s_d < 1$. For data giving*

$$\|u(t), n(t), \dot{n}(t)\|_{H^{s_d} \times L^2 \times \hat{H}^{-1}} \leq C$$

for all time t the problem (1) is globally wellposed.

Smoothness Preservation. *If $(u_0, n_0, n_1) \in \bigcap_{\sigma<s} H^\sigma \times H^{\sigma-1} \times \hat{H}^{\sigma-2}$ leads to a global solution then $(u(t), n(t), n(t)) \in \bigcap_{\sigma<s} H^\sigma \times H^{\sigma-1} \times \hat{H}^{\sigma-2}$ for all time*

Corresponding results for the polynomial generalizations are also valid for $d = 1, 2$. The $d = 3$ result exploits Strichartz inequality for the wave-equation and a refined estimate for the convolution of spherical measures due to Bourgain.

M. COWLING:

The Kunze-Stein phenomenon

This is an account of joint work with S. Meda and A. Setti, both from Milan.

In 1960, Kunze and Stein showed that the principal series representations of the group $G = SL(2, \mathbb{R})$ admit an analytic continuation as uniformly bounded representations in a strip. As a consequence, they showed that $L^p(G) * L^2(G) \subseteq L^2(G)$ where $1 \leq p < 2$. Subsequently, this was generalized to other semisimple Lie groups G. All proofs involved analytic continuation and the Plucheral formula; it was considered unsatisfactory to need to use complex analysis and Fourier analysis to prove the purely measure theoretic convolution theorem $L^p(G) * L^2(G) \subseteq L^2(G)$.

We are now able to give a measure theoretic proof. In this talk, we consider the case where G is of real rank one, and prove that $L^{p,1}(G) * L^p(G) \subseteq L^p(G)$ where $1 \leq p < 2$. We start by showing that $L^{p,1}(K \setminus G/K) * L^p(G) \subseteq L^p(G)$, using Herz's principe de majoration, and then use multilinear real interpolation to deduce that $L^p(K \setminus G/K) * L^{p,1}(G) \subseteq L^p(G)$. A simple trick shows that $L^p(G/K) * L^p(G) \subseteq L^p(G)$, and using multilinear interpolation again shows that $L^{p,1}(G/K) * L^p(G) \subseteq L^p(G)$. An argument of Lohoué and Rychener then shows that $L^{p,1}(K \setminus G) * L^p(G) \subseteq L^p(G)$, and then further applications of real interpolation and the trick show that $L^{p,1}(G) * L^p(G) \subseteq L^p(G)$.

E. DAMEK:

Pointwise estimates for fundamental solutions for differential operators on homogeneous manifolds of negative curvature

Let S be a connected, simply connected homogeneous manifold of negative curvature. Such manifolds are solvable Lie groups with a left-invariant Riemannian structure. Moreover, by a result due to Heintze, S is a semi-direct product of its maximal nilpotent normal subgroup N and $A = \mathbb{R}^+$ with the following property: there is an H in the Lie algebra $\mathcal{A}$ of A such that the real parts of eigenvalues of $ad_H \in End(\mathcal{N})$ are strictly positive.

We apply Ancona's potential theory of negatively curved manifolds to obtain sharp pointwise estimates for the fundamental solution G of a left-invariant second order subelliptic operator L as well as for the Poisson kernel P on N corresponding to L. The estimates for G imply weak type $(1,1)$ of the first and second order Riesz transforms for the Laplace-Beltrami operator on S. Moreover, we obtain pointwise estimates for the derivatives of P.

R. FARWIG:

Analysis of the Navier-Stokes equations in weighted function spaces

In the study of stationary and instationary solutions of partial differential equations Calderón-Zygmund estimates and multiplier theorems in L^p-spaces with or without weights are essential tools. In this talk we present several results (jointly with H. Sohr (Univ. Paderborn)) concerning weighted estimates of solutions of the Stokes and Navier-Stokes equations in unbounded domains.

- Solution of the stationary Oseen and Navier-Stokes equations for viscous flow past a body in anisotropically weighted L^2-spaces
- Analysis of the Stokes resolvent problem in weighted L^p-spaces with Muckenhoupt

weights for the whole space or an exterior domain implying that the Stokes operator generates a bounded analytic semigroup.

- Weighted energy estimates with power weights $|x|^\alpha$ of global weak solutions of the Navier-Stokes equations in an exterior domain of $\mathbb{R}^3$ yielding also new partial regularity results
- Weighted $L^s(L^q)$-estimates with power weights of global weak solutions in an exterior domain of $\mathbb{R}^3$ leading to decay estimates for $|x| \to \infty$ and $t \to \infty$

J. GARCIA-CUERVA:

Maximal operators and Banach lattices

(joint work with R. Marcias and J.L. Torrea) We investigate the behaviour of the Hardy-Littlewood maximal operator for functions having values in a Banach lattice. In our paper "The Hardy-Littlewood property of Banach Lattices" (1) we introduce a class of Banach lattices characterized by the boundedness of the lattice version of the maximal function and its smooth variants in several function spaces. The main tool in that paper was the vector-valued Calderon-Zygmund theory. In a recent preprint we analyze thc endpoint results ($L^\infty \to BMO$ or even $BMO \to BMO$) for the classical (non-smooth) version of the maximal operator. These estimates also characterize the Hardy-Littlewood lattices, but lie beyond the Calderon-Zygmund theory. They depend very strongly on the convexity of the lattice.

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G. GAUDRY:

Vector Multipliers

We consider continous linear operators T on the space $L^p(\mathbb{R}^n; H)$, $1 \leq p < \infty$, that commute with translations. Here H is a separable Hilbert space.

Versions of the Hörmander-Calderón-Zygmund theory have been proved by various authors. These treat an operator -valued kernel K that satisfies, for example, the Hörmander condition

$$\left\{ \int_{|x| \geq 2\delta} |K(x-y) - K(x)|^q dx \right\}^{1/q} \leq C, \qquad |y| \leq \delta.$$

There are versions of the Marcinkiewicz multiplier theorem. These results use the operator norm on the kernel or on the corresponding multiplier. This is perhaps unnatural form the point of view of the general theory, and masks some essential features.

Theorem 1. (J. L. Torrea, Tewari, Dutta, Vaidya) *Let T be a continuous linear operator on $L^1(\mathbb{R}, H)$. Then T commutes with translations if and only if it is defined by convolution with a regular Borel measure $\mu : B(\mathbb{R}^n) \to \mathcal{L}H)$ that is σ-additive with respect to the strong operator topology and of pointwise finite variation.*

Pointwise finite variation means that for every $h \in H$, $\sup \sum \|\mu(E_j)h\| < +\infty$, the supremum being taken over all finite Borel partitions of $\mathbb{R}^n$.

Theorem 2. (i) *Suppose $a > 2$. There exists a regular operator-valued measure $\mu : B(\mathbb{R}) \to \mathfrak{L}(L^2(\mathbb{R}))$ such that $T_{\hat{\mu}}$ is a Fourier p-multiplier if $1 \leq p < a$, and is not a Fourier p-multiplier if $p > a$.*

(ii) *There exists a regular operator-valued measure $\mu : B(\mathbb{R}) \to \mathfrak{L}(H)$ such that $T_{\hat{\mu}}$ is a p-multiplier iff $1 \leq p \leq 2$.*

This is joint work with B. Jefferies and W. Ricker.

D. GELLER:

Analytic Weyl calculus and spaces of entire functions

For $j \in \mathbb{R}$, we define a space, AS^j, consisting of certain symbols $r(p,q)$ ($(p,q) \in \mathbb{R}^n \times \mathbb{R}^n$) which have extensions to entire functions of exponential developments at ∞, in terms of homogeneous functions r_ℓ, in a sectorial neighborhood of $\mathbb{R}^{2n}$ in $\mathbb{C}^{2n}$, with precise estimates on the growth of the terms and on the error. We write $r \sim \Sigma r_\ell$.

If $r \in AS^j$, we let $Op(r)$ be the operator in the Weyl calculus with symbol r. If $r \sim \Sigma r_\ell$, we say $Op(r)$ is Hermite-like if $r_0(p,q) \neq 0$ for $(p,q) \neq 0$. For any m, we let $Z_2^2(\mathbb{R}^m)$ be the Gelfand-Šilov space on $\mathbb{R}^m$. We have $r \sim 0 \iff r \in Z_2^2(\mathbb{R}^{2n})$.

Theorem. *Say $r \in AS^j$, $R = Op(r)$. Then:* (a) $R : Z_2^2(\mathbb{R}^n) \to Z_2^2(\mathbb{R}^n)$.

(b) *If $r \sim 0$ then $R : \mathfrak{S}' \to Z_2^2(\mathbb{R}^n)$.*

c) *Say R is Hermite-like. If $f \in \mathfrak{S}'$ and $Rf \in Z_2^2(\mathbb{R}^n)$; there exists $a \in AS^0$ with $a \sim 0$ such that $P = Op(a)$ is the projection in L^2 onto $ker(R)$; and there exists $S \in Op(AS^{-j})$ with $SR = I - P$.*

This calculus underlies some representation-theoretic aspects of analytic pseudodifferential operators on the Heisenberg group, and certain generalizations of the Greiner-Kohn-Stein phenomenon.

D. GORGES:

Convergence a.e. of multiple Fourier series and Fourier integrals

The Carleson-Hunt theorem proves convergence a.e. of the partial sums of one dimensional Fourier series and of partial Fourier integrals for L^p functions, $1 <$

$p < \infty$. In several dimensions there is still no proof of an analogous theorem for partial sums or integrals being taken with respect to Euclidean balls (p=2). In my talk I give some sufficient conditions on the considered functions which imply convergence a.e., when partial sums or integrals are taken with respect to a family $B(R) : R > 0$ of sufficiently growing, 0-star-shaped and bounded subsets of $\mathbb{R}^d$. In the case of spherical summation these results are contained in a paper by Carbery and Soria.

A. GREENLEAF:

Characteristic space-time estimates for the wave equation

(joint work with Gunther Uhlmann) Let $\Box = \partial_t^2 - \Delta_x$ be the d'Alembertian on $\mathbb{R}^{n+1}$ and, for each $w \in S^{n-1}$, $\delta(t - x \cdot w)$ the associated singular plane wave. We consider the operator

$$(Uf)(x, t, w) = \Box^{-1}(f(x) \cdot \delta(t - x \cdot w)),$$

which arises naturally in inverse scattering. (U has previously been considered by A. Melin.) We prove:

Theorem. *Let $n \geq 2$. a) $U : L^2_{s,comp}(\mathbb{R}^n) \to L^2_{s+1,loc}(\mathbb{R}^{n+1} \times S^{n-1})$, $s < 0$ and maps $L^2_{comp} \to B^1_{2,\infty}$ (standard Besov space).*

b) $U : L^2_{comp}(\mathbb{R}^n) \to L^{p_n - \varepsilon}_{s_n + \varepsilon', loc}(\mathbb{R}^{n+1} \times S^{n-1})$ where $p_n = \frac{2(n-1)}{n-2}$, $s_n = \frac{n-2}{n-1}$ and ε' tends to ∞ as ε tends to 0.

c) Globally, $U : L^2(\mathbb{R}^n) \to \overset{\circ}{L}{}^p_s(\mathbb{R}^{n+1} \times S^{n-1})$ for $2 \leq p < p_n$ and $s = \frac{n+1}{p} - \frac{n-2}{2}$,

where

$$\|v\|_{\overset{\circ}{L}{}^p_s(\mathbb{R}^{n+1} \times S^{n-1})} = \|(-\Delta_{x,z})^{s/2} v\|_{L^p(\mathbb{R}^{n+1} \times S^{n-1})}.$$

The limited regularity in a) is a reflection of the fact that U is a Fourier integral operator associated with two cleanly intersecting Lagrangians $\Lambda_1, \Lambda_2 \subset T^*(\mathbb{R}^{n+1} \times S^{n-1} \times \mathbb{R}^n) \setminus 0$, both of which have points sitting over the zero section of $T^*\mathbb{R}^n$. We develop techniques to deal with this difficulty.

D. HAROSKE:

Entropy numbers of some compact embeddings of weighted function spaces; applications

In recent times we have studied compact embeddings of weighted function spaces on $\mathbb{R}^n$, like

(1) $$id : H^s_q(w(x), \mathbb{R}^n) \longrightarrow L_p(\mathbb{R}^n), \quad s > 0,\ 1 < q \leq p < \infty,\ s - \frac{n}{q} + \frac{n}{p} > 0$$

where $w(x)$ might be specified as $w(x) = \langle x\rangle^\alpha$, $\alpha > 0$, or $w(x) = \log^\beta\langle x\rangle$, $\beta > 0$, say, with $\langle x\rangle = \left(2 + |x|^2\right)^{\frac{1}{2}}$. Moreover, we have determined the (asymptotic) behaviour of their entropy numbers $e_k(id)$.

We are now interested in the limiting case $\frac{1}{q} = \frac{1}{p} + \frac{s}{n}$ of (1). Let $w(x) = \log^\beta\langle x\rangle$, $\beta > 0$. Former considerations imply that id cannot be compact in that case independent of $\beta > 0$. Thus we have to replace the target space $L_p(\mathbb{R}^n)$ by a 'slightly' larger one, but keeping the integrability p, $1 < p < \infty$, unchanged. In view of similar investigations related to compact limiting embeddings of (unweighted) function spaces on bounded domains it appears reasonable to replace $L_p(\mathbb{R}^n)$ by some logarithmic space $L_p(\log L)_{-a}(\mathbb{R}^n)$. Due to the lack of a suitable definition we first introduce counterparts of $L_p(\log L)_{-a}(\Omega)$, $1 < p < \infty$, $a > 0$, on $\mathbb{R}^n$, called $L_{p,\infty}(\log L)_{-a}(\mathbb{R}^n)$. Afterwards we study the entropy numbers of the compact embedding

$$id_{a,\beta} : H^s_q\left(\log^\beta\langle x\rangle, \mathbb{R}^n\right) \longrightarrow L_{p,\infty}(\log L)_{-a}(\mathbb{R}^n) \tag{2}$$

where $a > 0$, $\beta > 0$, $s > 0$, $1 < q \le p < \infty$ and $\frac{1}{q} = \frac{1}{p} + \frac{s}{n}$. In case of $a > 2\frac{s}{n}$ and $\beta > \frac{s}{n} + 1$ we obtain for $id_{a,\beta}$ from (2)

$$e_k(id_{a,\beta}) \sim k^{-\frac{s}{n}}, \quad k \in \mathbb{N}\,.$$

Finally we apply our result to estimate eigenvalues of the compact operator

$$B = b_2 \circ b(\cdot, D) \circ b_1$$

acting in some L_p space, where $b(\cdot, D)$ belongs to some Hörmander class $\Psi^{-\varkappa}_{1,\gamma}$, $\varkappa > 0$, $0 \le \gamma < 1$, and b_1, b_2 are in some (weighted) logarithmic Lebesgue spaces on $\mathbb{R}^n$ as introduced above. The crucial link between eigenvalues and entropy numbers is given by Carl's inequality, $|\mu_k| \le \sqrt{2}\; e_k(B)$, where $\{\mu_k\}$ is the eigenvalue sequence of the linear and compact operator B, and $e_k(B)$ are its respective entropy numbers. This additionally explains our interest in studying entropy numbers of such embeddings.

W. HEBISCH:

Spectral multipliers on exponential growth solvable Lie groups

We present a new class of groups such that compactly supported smooth functions operate on the (sub)laplacian giving operators bounded on L^1. Let N be a nilpotent Lie group with dilations e^{sA}. We assume that N is stratified. Let $G = \mathbb{R} \times N$, with the multiplication given by the formula

$$(u_1, n_1)(u_2, n_2) = (u_1 + u_2, e^{-u_2 A} n_1 n_2).$$

Let Q be the homogeneous dimension of N. Assume $X_1, ..., X_n$ generate N and are of order 1 (that is $AX_j = X_j$, $j = 1, ..., n$). We denote by $\tilde{X}_j$ the corresponding right invariant vector fields on G. We put $\tilde{X}_0 = \partial_u$ and we write

$$L = \sum_{j=0}^{n} \tilde{X}_j^2.$$

The heat kernel p_t is defined by the formula $e^{tL} f = p_t * f$.

Theorem 1.1. *There exists C such that for every $s \in \mathbb{R}$ we have*

$$\|p_{1+is}\|_{L^1(G)} \leq C(1 + |s|^{\frac{Q+4}{2}}).$$

Theorem 1.2. *For every compactly supported $F \in C^{[\frac{Q+7}{2}]}$ (or F in the Sobolev space $H^{(\frac{Q+5}{2}+\epsilon)}$) the operator $F(-L)$ is bounded on $L^1(G)$*

A. HULANICKI:

Harmonic and pluriharmonic functions on homogeneous Siegel domains

Let D be a homogeneous Siegel domain. D admits a singly transitive solvable group S of biholomorphic mappings of D onto itself. We transfer the complex structure from D to S, the left translates act as biholomorphic mappings, while the elements from the enveloping algebra of the complexified Lie algebra $\mathcal{S}^{\mathbb{C}}$ of S act on the right.

As every Siegel domain is biholomorphic with a bounded domain, it admits a Kählerian structure induced e.g. by the Bergman metric. This in turn gives rise to a Riemannian connection ∇.

$$\nabla_X : \mathcal{S} \to \mathcal{S} \text{ for } X \in \mathcal{S}$$

We extend ∇ to $\mathcal{S}^{\mathbb{C}}$ by complex linearity. We select an orthonormal basis $Z_1, ..., Z_n$ in $\mathcal{S}^{(1,0)}$.

We define operators on functions on D by

$$\Delta_{Z_k, Z_l} = Z_k \bar{Z}_l - \nabla_{Z_k} \bar{Z}_l \text{ and } \Delta_Z = Z\bar{Z} - \nabla_Z \bar{Z} \text{ for } Z \in \mathcal{S}^{(1,0)}.$$

We see that all these operators annihilate holomorphic functions on D.

As usual, we define the curvature operator by

$$R(X,Y) = \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]}$$

where X and Y are complex vector fields. For given $X, Y \in \mathcal{S}^{\mathbb{C}}$ we interpret $R(X,Y)$ as an element of $\mathrm{End}(\mathcal{S}^{\mathbb{C}}, \mathcal{S}^{\mathbb{C}})$.

We define an elliptic system of operators, we call it the Hua system, by putting

$$\mathcal{H}_{s,t}f = -\sum_{k,l}(Z_s, R(\bar{Z}_k, Z_l)\bar{Z}_t)\Delta_{Z_k,Z_l}f.$$

It is easily seen that this is independent of the orthonormal frame, $Z_1, ..., Z_n$.

Often, the study of the behavior of holomorphic function on a Siegel domain near the Shilov boundary involves defining a suitable class of real valued "harmonic" functions. We restrict our attention to bounded functions. Ideally, such a class should:

(1) Contain all real and imaginary parts of bounded holomorphic functions.
(2) Be describable as "Poisson integral" over the Shilov boundary against a real kernel (the "Poisson" kernel).
(3) Be invariant under all or at least a transitive group of biholomorphisms of the domain.
(4) Be describable as the nullspace of a degenerate-elliptic system $\mathcal{L}$ of second order differential operators, or better of one such operator.

Let M be the space of symmetric real $n \times n$ matrices and let Ω be the open cone in M consisting of positive definite matrices. Let $\mathbf{D}_n = M + i\Omega \subset \mathbf{C}^d$, $d = \frac{(n+1)n}{2}$.

Theorem [E.Damek, A.Hulanicki, R.Penney]. *There exists a single elliptic, second order S-invariant operator L on $\mathbf{D}_n$ for which the space of bounded L-harmonic functions is precisely the space of Poisson-Szegö integral of L^∞-functions on B.*

The operator L is a sum of some "diagonal elements" of the Hua system.

Theorem [E.Damek, A.Hulanicki, R.Penney]. *For every homogeneous Siegel domain, symmetric or not, there is a linear combination with positive coefficients L of operators Δ_{Z_j} such that bounded L-harmonic functions satisfy (1)-(4).*

Theorem [A.Hulanicki, D.Müller, M.Peloso]. *Let D be one of the three Siegel domains: upper half-plane, unit ball in $\mathbb{C}^2$, $\mathbf{D}_2$. Let F be a real valued function on D such that*

(*) $$\sup_{t\in\Omega}\int_{N(\mathcal{F})}|F(x\cdot t)|^2 < \infty$$

and

(**) $$\Delta_{Z_j}F = 0 \text{ for all } j = 1, ..., n.$$

Then there exists a real function G which satisfies (*) and $F+iG$ is holomorphic on D.

N. JACOB:

Balayage theory for a class of pseudo–differential operators

(joint work with W. Hoh) Let $p : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be a continuous function such that $p(x,.) : \mathbb{R}^n \to \mathbb{R}$ is negative definite in the sense of Beurling and Deny. In addition suppose that $-p(x,D)$ generates a symmetric Feller semigroup $(T_t)_{t\geq 0}$ which is even a strong Feller semigroup and admits sufficiently many excessive functions. Then the general theory of Bliedtner and Hansen allows a balayage theory with respect to the potential kernel V_0, $V_0 u = \int_0^\infty T_t u\, dt$. In a first theorem conditions on $p(x,\xi)$ are given in order that this (abstract) balayage theory is applicable, in particular a type of Dirichlet problem is solved. Next the bilinear form B associated with $-p(x,D)$ is used to solve a weak Dirichlet problem. Since the form B is a Dirichlet form one can associate a Hunt process with it (Fukushimas theory). This Hunt process is equivalent to the Feller process associated with the semigroup $(T_t)_{t\geq 0}$. Using this equivalence and certain probabilistic representation formulas it is possible to identify the solutions of both formulations of the Dirichlet problem and prove some regularity results for them.

H.-G. LEOPOLD:

Spectral invariance for pseudodifferential operators on function spaces

In joint works with E. Schrohe and H. Triebel the relation between invertibility in the algebra of pseudodifferential operators and invertibility in the operator theoretical sense with respect to Besov-Triebel-Lizorkin spaces without and with admissible weights were discussed. We proved the following results:

$$\psi^{0\;-1}_{1,\delta} = \mathfrak{L}(B^S_{p,q})^{-1} \cap \psi^0_{1,\delta} = \mathfrak{L}(F^S_{p,q})^{-1} \cap \psi^0_{1,\delta}$$

if $1 < p < \infty, 0 < q < \infty$, s real and $0 \leq \delta < 1$;

$$\psi^{0\;\;\;-1}_{\gamma,sym} = \mathfrak{L}(B^S_{p,q}(w(x))^{-1} \cap \psi^0_{\gamma,sym} = \mathfrak{L}(F^S_{p,q}(w(x)))^{-1} \cap \psi^0_{\gamma,sym}$$

if $0 < p \leq \infty$ ($p < \infty$ in the F-case), $0 < q \leq \infty$, s real, $w(x) \in W$, $0 \leq \gamma < 1$ and $\psi^0_{\gamma,sym} = \{A \in \psi^0_{1,\gamma} :\ A = A' = A^*\}$ – the elements of $\psi_{\gamma,sym}$ are self-dual in the $s - s'$ pairing and self-adjoint in L_2. Here $\psi^{0\;-1}_{1,\delta}$, $\mathfrak{L}(B_{p,q})^{-1}, \ldots$ denote the groups of invertible elements in $\psi^0_{1,\delta}$, $\mathfrak{L}(B^S_{p,q}) \ldots$, respectively.

As a consequence it was shown, that for each pseudodifferential operator, belonging to one of these classes, the spectra of the induced operators acting on the different function spaces are independent of the choice of space parameters and coincide with its L_2-spectrum.

G. MOCKENHAUPT:

On restriction of Fourier transforms

First, we present an analogue of the Stein-Tomas restriction theorem for the Fourier transform on the real line. It has been shown by R. Salem in the 50's that there are compactly supported measures μ on $\mathbb{R}$ such that for $\epsilon > 0$ we have $|\widehat{d\mu}(x)| \leq C_\epsilon |x|^{-\alpha/2+\epsilon}$, where α is the Hausdorff dimension of the support of $d\mu$. For the particular measures above we have

Theorem 1. *If $\alpha > \frac{n-1}{n}$, $n > 1$, then*

$$\int |\hat{f}|^2 \, d\mu \leq C \, \|f\|_p^2$$

for $1 \leq p < \frac{2(n+1)}{n+3}$.

Second, we introduce corresponding to a subset E in the dual of a finite abelian group G the constants

$$B^p(G,E) = \sup_{|a_k|\leq 1} \frac{\|\sum_{\chi\in E} a_k\chi_k\|_p}{\|\sum_{\chi\in E} \chi_k\|_p} \qquad \text{and} \qquad B^p(G) = \sup_{E\subset G} B^p(G,E).$$

The classical Hardy-Littlewood conjecture on majorants of Fouries series does imply that $B^p(\mathbb{Z}/N\mathbb{Z})$ is bounded in N for p not an even integer –if p is an even integer we have $B^p(G) = 1$. Now, the conjecture of Hardy and Littlewood is known to be false. However, the growth of the constants $B^p(\mathbb{Z}/N\mathbb{Z})$ is not known. We show

Theorem 2. *For $2 < p < 4$ there is a constant $c_p > 0$ s.t.*

$$N^{\frac{c_p}{\log\log N}} \leq B^p(\mathbb{Z}/N\mathbb{Z}).$$

By convexity we have also the (rough) upper bound $B^p(\mathbb{Z}/N\mathbb{Z}) \leq N^{\beta(p)}$ where $\beta(p) = (1-p/4)(1-2/p)$, $2 < p < 4$. Since a conjecture by H. Montgomery in number theory and a week form of the *restriction problem for the Fourier transform* state essentially that for particular sets E and exponents p, $B^p(\mathbb{Z}/N\mathbb{Z}, E)$ grows at most as the lower bound given in the theorem above we point out the problem whether this lower bound can be improved. We note also that in case of $G = (\mathbb{Z}/2\mathbb{Z})^n$ there is $\gamma(p) \in (0, \beta(p))$ s.t. $B^p(G) \geq |G|^{\gamma(p)}$, $2 < p < 4$.

A. OLEVSKII:

High frequency oscillation with nonlinear phase

We prove that if $\varphi : \mathbb{R}^d \to [0, 2\pi[$ is any measurable function such that

$$\|e^{i\lambda\varphi}\|_{M_p(\mathbb{R}^d)} = \mathcal{O}(1) \qquad (\lambda \in \mathbb{Z})$$

(M_p – Fourier multipliers algebra) for some $p \neq 2$ then φ is affine in a neighbourhood of almost each point $x \in \mathbb{R}^d$ and the gradient $\nabla\varphi$ takes only a finite number of values. This result yields a specific spectral decomposition of translation invariant operators in L^p with bounded powers.

One of the main points in the proof is the following new general property of multipliers: if $m \in M_p(\mathbb{R}^d)$ for some $p \neq 2$ then m is continous almost everywhere.

The paper represents joint results with V. Lebedev.

D. H. PHONG:

Degenerate oscillatory integrals and oscillatory integral operators

It is an old hypothesis due to V. I. Arnold that the numerical invariants of an analytic function $S(x)$ should be expressible in terms of its Newton polyhedron. For the decay rate of scalar oscillatory integrals with phase $S(x)$, and generic S, this had been confirmed by A. Varchenko in the mid 1970's. In this talk, we describe some recent joint work with E. M. Stein, which establishes that the sharp decay rate

$$||T|| \leq C|\lambda|^{-\frac{1}{2}\delta}$$

for the norm of the oscillatory integral operator on $L^2(\mathbb{R})$

$$Tf(x) = \int_{\infty}^{\infty} e^{i\lambda S(x,y)}\chi(x,y)f(y)\,dy$$

is also given in terms of the Newton polyhedron of the phase $S(x,y)$. Interestingly, this statement holds without any genericity restriction, and without any extraneous logarithmic factor $(\log|\lambda|)^\beta$, as can occur in the scalar case.

H. M. REIMANN:

Conformally invariant differential operators

(joint work with A. Korányi, CUNY) Conformally invariant differential operators on the unit ball $B \subset \mathbb{C}^n$ and its boundary $\Sigma = \partial B$ are considered. Σ has a natural contact structure given by $\theta = Jd\rho$ with ρ a defining function for B. It is well known that a vector field v on Σ generates a flow of contact transforms if it is of the form $v = D^\Sigma p := pT - J\,\mathrm{grad}\,p$ for some real valued function p on Σ (T is the Reeb vectorfield determined by θ and $\mathrm{grad}\,p$ is the horizontal gradient).

1) The vectorfield $D^\Sigma p$ generates a flow of $e^{C|t|}$- quasiconformal mappings if $\|S^\Sigma v\|_\infty \leq C$. S^Σ is the Heisenberg version of the Ahlfors operator. Both D^Σ and S^Σ are $SU(n,1)$- invariant differential operators.

2) Vectorfields $v = D^\Sigma p$ satisfying $\|S^\Sigma v\|_\infty \leq C$ can be extended equivariantly with respect to the $SU(n,1)$- action to vectorfields w on B which are Hamiltonian

with respect to the Kähler -Bergmann form. They satisfy $\|S^B w\|_\infty \leq C'$ and therefore generate flows of quasiconformal mappings with respect to the Bergmann metric.

T RUNST:

A general approach to solvability conditions for semilinear elliptic boundary value problems

The talk deals with some recent results of a joint work with S. B. Robinson, Winston-Salem. The purpose is to show that many well-known and several new existence results for semilinear elliptic boundary value problems at resonance with bounded and unbounded nonlinear perturbations can be understood as simple corollaries of abstract existence theorems.

T. SCHOTT:

Function spaces of Triebel-Lizorkin type with exponential weights

The scale of Lizorkin-Triebel spaces F^s_{pq} with $s \in \mathbb{R}$, $0 < p < \infty$ and $0 < q \leq \infty$ covers the inhomogeneous Hardy spaces h_p and the fractional Sobolev spaces H^s_p with the classical Sobolev spaces W^k_p as a subclass. We study their weighted counterparts $F^s_{pq}(u)$ on the Euclidean space $\mathbb{R}^n$, where $s \in \mathbb{R}$, $0 < p < \infty$ and $0 < q < \infty$. The weight function u satisfies

$$0 < u(x) \leq cu(x-y)\exp(d|y|), \qquad x, y \in \mathbb{R}^n$$

for some $c > 0$ and $d \geq 0$. This involves the weight

$$u(x) = \exp(\pm|y|) \qquad x, y \in \mathbb{R}^n$$

which was excluded from previous theory.

First of all, we study a suitable space of distributions. Then we define the spaces $F^s_{pq}(u)$ and list some basic properties, such as completeness and density of the smooth functions. Furthermore, $f \to uf$ yields a topological isomorphism from $F^s_{pq}(u)$ onto F^s_{pq}. Hence, if k is a non-negative integer, then $F^k_{pq}(u)$ can be identified with the weighted Sobolev spaces $W^k_p(u)$. We give atomic and molecular decompositions of $F^s_{pq}(u)$ in the sense of M. Frazier and B. Jawerth.

Finally, we deal with the boundedness of certain pseudodifferential operators in the above function spaces. The corresponding symbols belong to the Hörmander class S^μ, where $\mu \in \mathbb{R}$. It follows that the spaces $F^s_{pq}(u)$ have the lift property.

A. SEEGER:

Some endpoint bounds for rough operators

The following is joint work with Terence Tao.

(1) Let

$$T_\Omega f(x) = p.v. \int \Omega(y/|y|)|y|^{-d} f(x-y)dy$$

where $\int_{S^{d-1}} \Omega(y')d\sigma(y') = 0$ and $\Omega \in L^r(S^{d-1})$, $r > 1$. Then T_Ω maps the Hardy space $H^1(\mathbb{R}^d)$ to the Lorentz space $L^{1,2}$ (but not necessarily to $L^{1,q}$ for $q < 2$).

(2) Let $\mathcal{T}$ be the Hilbert transform along the curve $\gamma_m(t) = (t, t^m)$, i.e.

$$\mathcal{T} f(x) = p.v. \int f(x - \gamma_m(t)) \frac{dt}{t}$$

Then $\mathcal{T}$ maps the multiparameter Hardy-space $H^1_{\mathrm{prod}}(\mathbb{R} \times \mathbb{R})$ to the Lorentz-space $L^{1,2}$. This result can be used to prove a sharp bound for the averaging operator

$$Af(x) = \int \chi(t) f(x - \gamma_m(t))\, dt$$

where $\chi \in C_0^\infty$. Namely, $D^{1/m}A$ maps $L^{m,2}$ to L^m.

W. SICKEL:

Mapping properties of composition operators

Let $G : \mathbb{R} \to \mathbb{R}$ be continuous. Then we derive the following necessary condition on the corresponding composition operator $T_G : f \to G(f)$ to act between a Bessel potential space and a Besov space or Sobolev space, respectively.

Let $1 < p < \infty$ and $1 < s < \frac{n}{p}$.

a) Assume $T_G(H_p^s) \subset W_p^m$ for some $m \le s$, then G has to satisfy

$$\sup_{w \ge 1} w^{-\gamma} \left(\int_{-w}^{w} |G^{(m)}(y)|^p dy \right)^{1/p} < \infty$$

for all $\gamma > \frac{\frac{n}{p} + \frac{1}{p}(\frac{n}{p} - s) - m(\frac{n}{p} - s + 1)}{\frac{n}{p} - s}$.

b) Let $0 < r \le s$ and assume $T_G(H_p^s) \subset B_{pp}^r$. Then G has to satisfy

$$\sup_{w \ge 1} w^{-\gamma} \left(\int_{-w}^{w} |h|^{-rp-1} \left(\int_{-w}^{w} |\Delta_h^M G(y)|^p \, dy \right) dh \right)^{1/p} < \infty$$

for all $\gamma > \frac{\frac{n}{p}+\frac{1}{p}(\frac{n}{p}-s)-r(\frac{n}{p}-s+1)}{\frac{n}{p}-s}$. It is shown in two special situations that these conditions are unimprovable. The above assertions cover earlier results of Dahlberg, Bourdaud and Runst.

P. SJÖGREN:

Rough singular integral kernels and rough maximal operators applied to radial functions

A homogeneous Calderón-Zygmund kernel is defined by its restriction to the unit sphere. This restriction is always assumed integrable, with vanishing integral. Rather weak additional assumptions imply L^p boundedness and weak type (1,1) for the corresponding convolution operator. When the kernel is odd, it is an open question whether weak type (1,1) follows without extra assumptions. In joint work with F. Soria, we prove the weak type when the operator is applied only to radial functions.

The proof relies on a corresponding result for an analogous maximal operator, defined as a supremum of weighted means in balls. The weight is homogeneous of degree 0 and integrable on the unit sphere. Again, it is not known whether this implies weak type (1,1) without further assumptions. We prove the weak type (1,1) estimate for the restriction to radial functions. This result extends to the larger operator obtained by taking the supremum before integrating in the argument variable, when the integral over each ball is written in polar coordinates. The proof goes via results like $yG(\cdot/y) * f(x) \in L^{1,\infty}(dxdy/y)$, for G and f in L^1 and G satisfying some regularity conditions.

P. SJÖLIN:

Estimates of averages of Fourier transforms of measures with finite energy

Estimates of Fourier transforms of measures with finite energy are considered. In earlier papers spherical means of the Fourier transforms have been studied. Here more general means are considered. Also in particular the case when the measure is given by a radial function is studied.

H. SMITH:

Strichartz estimates for low regularity wave equations

We consider L^p estimates for solutions to wave equations of the form

$$(\partial_t^2 - \sum_{i,j=1}^{n} a_{i,j}(t,x)\partial x_i \partial x_j)u(t,x) = F(t,x)$$

$$u(0,x) = f(x)$$

$$\partial_t u(0,x) = g(x)$$

under minimal regularity hypotheses on the coefficients $a_{i,j}(t,x)$.

Theorem 1. *If $\nabla_x^2 a_{i,j}(t,x) \in L^\infty$, and $\partial_t a_{i,j}(t,x) \in L^\infty$, then the Strichartz and Pecher estimates hold for space dimension $n = 2,3$.*

Counterexamples of Smith-Sogge show that this is sharp, in the sense that $\nabla_x^2 a_{i,j}(t,x) \in L^q$ is not sufficient if $q < \infty$. On the other hand, the following improvement holds:

Theorem 1. *If the Riemann curvature tensor associated to $a_{i,j}(x)$ belongs to L^∞, and $a_{i,j}(x) \in H^{1,p}$, some $p > n$, then the Strichartz and Pecher estimates hold for $n = 2,3$.*

The proof modifies the paraproduct theory of Coifman, Bony, and Meyer to construct the wave group for $\partial_t^2 - P(t,x,\partial x)$, modulo an invertible error, within a class of oscillatory integral operators with $S_{\frac{1}{2},\frac{1}{2}}$ symbols and $S_{\frac{1}{2},\frac{1}{2}}$ phases. This new class of operators, which uses a different phase on different dyadic energy shells, is a model class for Fourier integral operators with nonsmooth (Lipschitz) Lagrangean manifolds.

C. THIELE:

Multilinear singular integrals

We present some joint work with M. Lacey. Let $f_1, f_2, f_3 \in \mathcal{S}(\mathbb{R})$ and consider the trilinear form

$$\Lambda(f_1,f_2,f_3) := \int \left[p.v. \int f_1(x-t)f_2(x+t)\frac{dt}{t} \right] f_3(x)\,dx$$

We show the following theorem:

Theorem. *For all $2 < p_1, p_2, p_3 < \infty$ satisfying $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = 1$ there is a constant C such that for all $f_1, f_2, f_3 \in \mathcal{S}(\mathbb{R})$:*

$$|\Lambda(f_1,f_2,f_3)| \leq C\|f_1\|_{p_1}\|f_2\|_{p_2}\|f_3\|_{p_3}$$

The proof is based on a method that is derived from the classical proofs of Carleson and Fefferman of Carleson's theorem on almost everywhere convergence of Fourier series. In the talk we describe the trilinear form in terms of the phase plane and demonstrate how geometric arguments in the phase plane enter into the proof of the theorem.

A. VARGAS:

An extension theorem for the Fourier transform

This is the result of joint work with Adela Moyua and Luis Vega.

The classical Stein-Tomas restriction theorem says that for $f \in L^p(\mathbb{R}^3)$, $p \leq 4/3$ we have $\|\hat{f}|_{S^2}\|_{L^2(S^2)} \leq C\|f\|_{L^p(\mathbb{R}^3)}$. Its dual form is the extension theorem, which in the extremal case is $\|\widehat{fd\sigma}\|_{L^4(\mathbb{R}^3)} \leq \|f\|_{L^2(S^2)}$. Here we present an improvement of this theorem. We will consider only characteristic functions, $f = \chi_\Omega$, $\Omega \subset S^2$. Since $L^2(S^2)$ is the sharp L^p space corresponding to $L^4(\mathbb{R}^3)$, we need a different norm in S^2. Following Bourgain, for $\Omega \subset S^2$, we define

$$|||\chi_\Omega|||_p = \left[\sum_{dyadic\,\delta} \delta^4 \sum_{\tau \in C_\delta} \left(\frac{|\Omega \cap \tau|}{|\tau|} \right)^{4/p} \right]^{1/4}$$

where, for each $\delta = 2^{-j}$, C_δ is a family of caps of radius δ covering S^2.

It can be shown that $|||\chi_\Omega|||_p \leq \|f\|_{L^2}$. Moreover $|||\chi_\Omega|||_p$ is comparable to $\|f\|_{L^2}$ only when Ω is essentially a cap.

We prove the following

Theorem. $\|\widehat{fd\sigma}\|_{L^4(\mathbb{R}^3)} \leq C|||\chi_\Omega|||_p$ *for all* $p > \frac{4}{1+\sqrt{2}}$. *The result is false for* $p < \frac{4}{1+\sqrt{2}}$.

This theorem can be applied to the problem of pointwise convergence to the initial datum of the solution of the Schrödinger equation. We can prove that the convergence holds for datum in a Sobolev space $H^s(\mathbb{R}^2)$ for some $s < 1/2$, which is smaller than the exponents on the previous theorems of this type.

A. YOUSSFI:

Regularity properties of bilinear operators

We consider the regularity properties of the bilinear operators

$$B_j(f,g) = fR_j(g) + gR_j(f)$$

where $R_j, j = 1, \ldots, n$, are the Riesz transforms in $\mathbb{R}^n$.

We prove existence or nonexistence of the space

$$\xi_{s_1,s_2} = \overline{vect\ (B_j(\dot{H}^{s_1} \times \dot{H}^{s_2}))}$$

where $\dot{H}^{s_i}$, $(i = 1, 2)$ is the homogeneous Sobolev space. In the positive case we give a characterization of ξ_{s_1,s_2}.

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