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Topologie

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The conference was organized by R. Kirby (Berkeley), W. Lück (Münster) and E.G. Rees (Edinburgh). About 45 participants from Europe and the United States attended the meeting. The topics of 17 talks dealt with new developments in algebraic and geometric topology, with a focus on low dimensional topology and on homotopy theory.

A special feature of the conference were two talks in honour of Albrecht Dold and Dieter Puppe, who retired in 1996.

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Vortragsauszüge

Mladen Bestvina: *"Morse theory and finiteness properties of groups"*

A group has type F_n if it acts freely and cocompactly on an $(n-1)$-connected cell complex. It has type FH_n if it acts freely and cocompactly on a homologically $(n-1)$-connected complex.

Stallings and Bieri have given examples of groups that are of type F_n but not of type FH_{n+1}. In the talk the speaker outlined a Morse theoretic proof of this fact. The space in question is the cube complex T^n, the n-fold product of the universal covering of the figur eight. Then he states a generalization, in a joint work with Noel Brady, that in particular gives examples of groups that are of type FH_2, but not F_2 (i.e. not finitely presented).

Dvi file can be downloaded from http://www.math.utah.edu/ bestvina

Carl Friedrich Bödigheimer: *"Rede auf Albrecht Dold"*

Anlaßlich der Emeritierung wurde ein Abriß von Leben und Werk Albrecht Dolds gegeben. Zunächst wurde ein kurzer Lebenslauf durch die wichtigsten Daten und Stationen skizziert. Dann wurde die Situation der Topologie in Deutschland am Ende der 40er Jahre beleuchtet, wobei die Rolle Oberwolfachs besonders berücksichtigt wurde. Im mathematischen Werk Albrechts Dolds wurde der Begriff der Faserung als eine Art Leitfaden herausgearbeitet, und das Werk insgesamt in die Abschnitte "Mannigfaltigkeiten", "Allgemeines" und "Fixpunkttheorie" gegliedert. Einzelne Arbeiten wie die Dissertation (1954), die Arbeiten zur Thomschen Bordismustheorie (1956) und zum symmetrischen Produkt wurden erläutert und ihre Bedeutung für spätere Entwicklungen unterstrichen. Die Gegenstände der zweiten Periode haben die Topologie bis heute mitgeprägt, und fanden Niederschlag in dem Buch "Lectures on Algebraic Topology". In der dritten Periode wurden die Begriffe Index und Transfer geklärt, Höhepunkt ist eine gemeinsame Arbeit mit D. Puppe (1978). Nicht vergessen wurde die Tätigkeit als Herausgeber der Lecture Notes in Mathematics. Der Redner schloß mit seinem Dank für die Betreuung, der er als Schüler von Albrecht Dold erfahren hat.

Thomas Fiedler: *"New invariants in knot theory"*

An oriented smooth link in S^3 is called *invertible* if it is isotopic to itself with reversed orientation. In recent years, many new knot invariants appeared, notably the Vassiliev invariants. Netherless, there were no numerical (i.e. integer valued) invariants which could detect non-invertibility.

The speaker considers 2-component oriented links $K \cup T$, where T is a trivial knot (e.g. a knot and its meridian). He replaces the space of all knots —studied in Vassiliev's theory— by the (non-connnected) space of all diagrams of a given knot K with T being a line of projection. *Diagram* means that the projection of the knot in $\mathbb{R}^2$ is an immersion.

Using his small state sums (which can be considered as a refinement of the Gauss-diagram sums of Viro and Polyak) he obtains a new class of invariants, which contains Vassiliev invariants, but with two crucial properties:

(1) These invariants can detect non-invertibility of links $K \cup T$ (and, probably, of also of knots, using a cabling operation)

(2) They depend not only on the isotopy class of the link, but in an essential way on the regular isotopy class of the link $K \cup T$ (always with T as line of projection)

The speaker expressed his hopes that these new invariants are strong enough ot recover the peripheral system of the knot K and, hence, to classify knots.

Ian Hambleton: *"The spherical space form problem"*

The spherical space forms (in dimension 3) are quotients S^3/Γ of the standard 3-sphere by a fixed point free, finite group of isometries $\Gamma \subset SO(4)$. In 1925 Hopf listed the possible finite groups Γ admitting such representations and asked wether every finite $\pi_1(M^3)$, M closed oriented 3-manifold, occurs in this list. The speaker describes his method (joint with Ronnie Lee) which answers Hopf's question with "yes". There are 3 parts. In the first part, due to several people (Milnor, Lee, ...) over the period 1957–1975 the question is reduced to deciding wether any of the groups $Q(8p,q)$ can be the fundamental group of a 3-manifold. These groups are certain extensions of the cyclic groups C_{pq} of order pq ($p > q$ distinct odd primes) by the quaternion group of order 8. In the second part, under the assumption that M^3 exists with $\pi_1 M = Q(8p,q)$, a smooth 4-manifold $(Z, \partial Z)$ with an action of $Q(8p,q)$ is constructed whose boundary components are copies of Σ^3 (homotopy spheres), (homology) spheres invariant under free actions of subgroups of $Q(8p,q)$, and a free orbit $Q(8p,q) \times N$ of a 3-manifold N. In addition, $b_1(Z) = 0 = b_2^+(Z)$. In the third part, cylindrical ends (resp. D^4 with linear actions) are attached to the various boundary components of Z, and then the speaker studies ASD L^2-finite connections over the non-compact manifold Z_+, with $c_2 = 1$ (rel. to framings over the ends). Then techniques developed by Donaldson, Taubes, Morgan, Mrowka and Ruberman are used to derive a contradiction.

V. Kharlamov: *"On the moduli space of real Enriques surfaces, Donaldson's trick and Pontryagin-Viro form"*

A *real Enriques surface* is a complex Enriques surface E equipped with an anti-holomorphic involution c; the fixed point set $E_{\mathbf{R}} = \mathrm{Fix}(c)$ is called the *real part*. The study of a real Enriques surface is equivalent to the study of a real $K3$-surface X (the double covering of E) equipped with a fixed point free holomorphic involution τ (called the *Enriques involution*) commuting with the real structure.

Two real Enriques surfaces are said to have the same *deformation type* if they can be included into a continuous one-parameter family of real Enriques surfaces. Clearly, the topological type of the real part of a surface is preserved under deformation. Another immediate deformation invariant, which is called the *sign decomposition*, comes from the covering $K3$-surface X: since c lifts to two real structures c_1, c_2 on X whose real parts $X_{\mathbf{R}1}$, $X_{\mathbf{R}2}$ are disjoint, $E_{\mathbf{R}}$ naturally splits into disjoint union of two halves $E_{\mathbf{R}i} = X_{\mathbf{R}i}/\tau$, each half being a union of whole components of $E_{\mathbf{R}}$.

The classification of the real parts $E_{\mathbf{R}}$ and decompositions $(E_{\mathbf{R}}; E_{\mathbf{R}1}, E_{\mathbf{R}2})$ up to homeomorphism was started by V. Nikulin and completed jointly by the speaker and A. Degtyarev.

The real Enriques surfaces are divided in three groups: *hyperbolic*, *parabolic*, or *elliptic* if the minimal Euler characteristic of the components of $E_{\mathbf{R}}$ is negative, zero, or positive, respectively. Here, the surfaces of hyperbolic and parabolic type are classified up to deformation equivalence. In addition to the sign decomposition and its homological properties a new invariant, so called *Pontrjagin-Viro form* is necessary (and sufficient) to distinguish the M-surfaces, i.e. those with maximal total $\mathbb{Z}/2$-Betti number $\beta_*(E_{\mathbf{R}}) = 16$.

The approach is based on an equivariant version of a trick by Donaldson, which transforms a real Enriques surface to a real rational surface with a certain curve on it.

Proposition. *The Donaldson construction establishes a one-to-one correspondence between the deformation classes of real Enriques surfaces with distinguished nonempty half, and deformation classes of pairs (Y, B), where Y is a real rational surface and $B \subset Y$ is a nonsingular real curve such that the following holds: $B \in |-2K_Y|$ (K being the canonical class); the real point set of B is empty; and B is not linked with the real point set of Y.*

Theorem. *With few exceptions the deformation type of a hyperbolic or parabolic surface is determined by its sign decomposition. The exceptions are surfaces with $E_{\mathbf{R}} = V_{10}$; M-surfaces of parabolic type with $E_{\mathbf{R}} = 2V_2 \amalg 4S$ or $E_{\mathbf{R}} = V_2 \amalg 2V_1 \amalg 3S$; and surfaces of parabolic type with $E_{\mathbf{R}2} = 4S$. A list of possible deformation types is available in these cases, too.*

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Gerd Laures: *"Topological q-expansion theory"*

Imitating the classical q-expansion principle the speaker uses the elliptic character map to develop the relation between elements in elliptic cohomology and their q-series in K-theory. He shows that, under certain exactness conditions, the integrality of elliptic objects is completely controlled by their characters.

As an application, he obtains an interpretation of the cooperations in elliptic cohomology as was conjectured by F. Clarke and K. Johnson. This enables to give a description of the elliptic based Adams-Novikov spectral sequence in terms of cyclic cohomology of modular forms in several variables, and to set up a higher e-invariant with values in N. Katz's ring of divided congruences..

Martin Lustig: *"The structure and dynamics of a free group automorphism"*

Every automorphism α of a free group F_n of finite rank $n > 2$ acts naturally on two important spaces:

(a) Culler-Vogtmann space (= "Outer space") CV_n , an analogue of Teichmüller space.

(b) A metric and in general non-simplicial tree $\mathcal{T} = \mathcal{T}(\alpha)$ with an F_n-action by isometries.

The speaker gave a self-contained exposition of these actions, and showed how a dynamic study of these actions leads to a structural analysis of α. In particular concentrated on the two basic types of automorphisms and explain the following results:

1. *Dehn twist automorphisms* have parabolic dynamics on CV_n. The analysis of its fixed point leads to a normal form and a solution of the conjugacy problem of those automorphisms, see [CL]. An extension to roots of Dehn twists and thus to all linear growth automorphisms is described in [KLV].

2. *Partial pseudo-Anosov automorphisms* have simple north-southpole dynamics on CV_n, see [L2]. In [GJLL] an analysis of $\mathcal{T}(\alpha)$ is given, which improves the Bestvina-Handel estimate for rank(Fix(α)). Our methods are based on Bestvina-Handel's train track representatives, simplified recently in [LOe]. They lead to a complete solution of the conjugacy problem in Out(F_n), see [L1].

[CL] (& M. M. Cohen) The conjugacy problem for Dehn twist automorphisms of free groups, preprint 1995

[GJLL] (& D. Gaboriau, A. J (b) A metric and in general non-simplicial tree $\mathcal{T} = \mathcal{T}(\alpha)$ with an F_n-action by isometries. ger, G. Levitt) An index for counting fixed points of automorphisms of free groups, preprint 1996

[KLV] (& S. Krstic und K. Vogtmann) An equivariant Whitehead algorithm and conjugacy for roots of Dehn twists, preprint 1996

[LOe] (& U. Oertel) Invariant Laminations for Diffeomorphisms of Handlebodies, preprint 1996

[L1] Prime decomposition and conjugacy problem for $\mathrm{Out}(F_n)$, survey preprint 1994

[L2] Perron-Frobenius actions of free groups on e-trees, manuscript 1995

Marc Mahowald: *"Algebraic curves of genus $(p-1)(p-2)/2$ and the Hopkins-Miller spectrum EO_{p-1}"*

The content of the talk is joint work with M. Hopkins

The spectrum, EO_{p-1} is constructed by considering the action of the Morava stabilizer group, S_n $(n = p-1)$ on E_n, the spectrum whose homotopy is the Lubin Tate moduli space of lifts on the height n formal group over F_p. Then $E_{n*} = W_{F_{p^n}}[[x_1, \cdots, x_{n-1}]][u, u^{-1}]$. Since $n = p-1$, S_n has a finite subroup, G, which is a semi-direct product of Z/p and a group prime to p. Let $EO_n = E_n^{hG}$.

Understanding the connection between EO_2 and elliptic curves proved very useful.. This work was moditivated by an effort to find an analogous situation for all $n = p-1$.

Theorem. *Consider the curve*

$$y^n = x^{n+1} + \cdots + a_n x + a_{n+1}$$

with coefficients in Z_p. Then the Jacobian variety has a 1-dim subgroup and the associated formal group has G as an automorphism group. The equation is corepresented by the ring $Z_5[a_1, \cdots, a_{n+1}] = R$ and the operations by $R[r]$. The homology of the resulting Hopf algebroid is the E_2 term of an Adams-Novikov spectral sequence which computes the homotopy of eo_n, a connected version of EO_n.

Bob Oliver: *"Fixed point sets and tangent bundles of finite group actions on disks"*

The main result discussed was the determination, for any finite group G not of prime power order, exactly which smooth manifolds can occur as fixed point sets of smooth G-actions on disks or on euclidean spaces. More precisely, let $n_G \geq 0$ be the integer such that

$$n_G \cdot \mathbf{Z} = \{\chi(X) - 1 \,|\, X \text{ a finite contractible } G\text{-CW complex}\};$$

and let $\mathrm{Fix}(G)$ be the class of all smooth manifolds F for which there is a real G-vector bundle $\eta \downarrow F$ satisfying

(1) $\eta^G \cong \tau(F)$,

(2) η is non-equivariantly a product bundle,

(3) for each prime p dividing $|G|$ and each p-subgroup $P \subseteq G$, $[\eta|P] \in \widetilde{KO}_P(F)$ is infinitely p-divisible.

Then F can occur as the fixed point set of a smooth G-action on a disk if and only if F is compact, $\chi(F) \equiv 1 \pmod{n_G}$, and $F \in \mathrm{Fix}(G)$. And F can occur as the fixed point set of a smooth G-action on a euclidean space if and only if $\partial F = \emptyset$ and $F \in \mathrm{Fix}(G)$.

The numbers n_G and the classes $\mathrm{Fix}(G)$ have been determined explicitly. For example, if G has a subquotient which is dihedral of order $2n$, where n is not a prime power, then all smooth manifolds lie in $\mathrm{Fix}(G)$.

Justin Roberts: *"Perturbative 3-manifold invariants"*

The many-faceted theory of finite-type (Vassiliev) invariants of knots is now quite well developed, and recent efforts have concentrated on extending the theory, by analogy, to 3-manifolds.

Ohtsuki defined a class of finite-type invariants (for integer homology spheres) by using ± 1-surgery on a knot as an analogue of the crossing change operation used to define Vassiliev invariants of knots. He also introduced a diagrammatic classification in terms of trivalent graphs, which play the role that chord diagrams do for knots. Kontsevich's integral, which is a universal Vassiliev invariant for knots and links, has been used by Le, J. Murakami and Ohtsuki to produce a universal invariant of 3-manifolds with values in these trivalent graphs (modulo relations). Kricker and Spence have shown that a kind of algebraic asymptotic expansion of the Reshetikhin-Turaev invariant of a 3-manifold (due to H. Murakami and Ohtsuki) has terms which are of finite type, just as Taylor expansions of the Jones polynomial yield Vassiliev invariants.

For a topological interpretation it seems much better to use the Chern-Simons perturbation theory in its intersection theoretic incarnation (invented by D. Thurston for knots, and Fukaya for rational homology spheres). Invariants are obtained by counting certain configurations of Morse flow arcs in a 3-manifold. This approach separates the topology from the algebra completely, demonstrates integrality of the invariants, and ought to be quite accessible via cutting and gluing techniques.

Brian Sanderson: *"Compressing embeddings"*

Theorem. *Given a compact manifold M^n embedded in $Q^q \times \mathbb{R}$ with normal field α $(n < q)$, then there is an isotopy of (M, α) so that α is straight up.*

This compression theorem solves a 20 year old problem posed by Bruce Williams at the Stanford conference, 1976, problem 6. Applications include short new proofs for immersion theory and for the loops-suspension theorem of James et al and a new approach to classifying embeddings of manifolds in codimension one or more, which leads to theoretical solutions. The proof introduces a novel technique in differential topology: proof by dynamical systems. We define flows which straighten vector fields and which then allow a given embedding or immersion to be 'compressed' to an immersion in a lower dimension. The technique gives explicit descriptions of the resulting immersions and can be seen as a way of desingularising certain maps. An example is the transition from the non-immersion of the projective plane in 3-space as a sphere with cross-cap to Boy's surface.

The result also gives a geometric meaning to the homotopy groups of the rack space and in particular we can construct a classifying space for codimension two embeddings with fundamental group mapping to a given group. It has also been applied by Wiest to prove structure theorems for rack spaces.

The paper is available from: http://www.maths.warwick.ac.uk/ bjs/

Stefan Stolz: *"Manifolds of positive scalar curvature"*

This is the abstract of a mini-series of three talks. It presents a survey about what is known concerning the question which compact manifolds admit metrics of positive scalar curvature. The central conjecture in the subject is the Gromov-Lawson-Rosenberg conjecture which claims that a spin manifold M of dimension $n \geq 5$ admits a positive scalar curvature metric if and only if an index obstruction $\alpha(M) \in KO_n(C^*\pi)$ vanishes. Here π is the fundamental group of M, and $KO_n(C^*\pi)$ is the K-theory of the C^*-algebra of π (a completion of the real group ring). Stable homotopy theory is the essential ingredient for the proof of this conjecture for those groups π which have periodic cohomology. The speaker outlines the proof of a very recent result saying that if the "Baum-Connes map" is injective for a group π, then a stable version of the Gromov-Lawson-Rosenberg Conjecture holds for spin manifolds with fundamental group π. The Baum-Connes map is known to be injective e.g. for discrete subgroups of Lie groups. More generally, the Baum-Connes Conjecture claims that this map is an isomorphism for *all* discrete groups π (the injectivity part of that statement is one form of the Novikov-Conjecture).

Tammo tom Dieck: *„Betrachtungen zum mathematischen Werk von Dieter Puppe"*

Dieter Puppe wurde zum Sommersemester 1996 emeritiert. Aus diesem Anlaß wurde im Vortrag an ausgewählte Arbeiten von Dieter Puppe erinnert. Dabei wurde auf die historische und gegenwärtige Bedeutung eingegangen, und einige Entwicklungslinien und Gedankenkreise wurden herausgearbeitet. Die Arbeit "Homotopiemengen und ihre induzierten Abbildungen" erschien im Jahr 1958 in der mathematischen Zeitschrift in zwei Teilen. Die Ergebnisse dieser vorbildlichen Publikation gehören seitdem zur Standardausbildung in der Topologie. Außerdem haben die darin niedergelegten Methoden in vielfältiger Weise in der Topologie weitergewirkt. Systematische Schwerpunkte des Werkes von Dieter Puppe sind: Homotopietheorie, stabile Homotopietheorie, Kategorientheorie, simpliziale Topologie, Fixpunkttheorie. Zu diesen Bereichen wurden im Vortrag einzelne Arbeiten herausgehoben. Allgemeine Betrachtungen wurden daran angeknüpft, zum Beispiel über mögliche oder wünschenswerte Weiterentwicklungen. Eine Fülle von schönen und bedeutenden Einzelresultaten blieben im Vortrag leider unerwähnt. Da der Vortragende zu den ersten Diplomanden und Doktoranden von Dieter Puppe gehört, schloß er mit einem herzlichen persönlichen Dank für die vorbildliche und nachhaltige Förderung, die ihm in jungen Jahren durch Herrn Puppe zuteil wurde.

Tammo tom Dieck: *„Braids in the cylinder — knots, tensor categories, quantum groups"*

The talk presented joint work with R. Häring-Oldenburg (Göttingen). The braid group ZB_n associated to the Coxeter graph B_n describes braids with n strings in the cylinder and has generators $t, g_1, \dots, g_{n-1}$ and relations $tg_1tg_1 = g_1tg_1t$, $g_ig_jg_i = g_jg_ig_j$ for $|i-j| = 1$, $tg_j = g_jt$ for $j > 1$, $g_ig_j = g_jg_i$ for $|i-j| > 1$. A *four braid pair* (X, F) consists of an automorphism $F : V \to V$ and a Yang-Baxter operator $X : V \otimes V \to V \otimes V$ on the K-module V such that the *four braid relation* $X(F \otimes 1)X(F \otimes 1) = (F \otimes 1)X(F \otimes 1)X$ holds. Such a pair yields a tensor representation of ZB_n on $V^{\otimes n}$ via $t \mapsto F \otimes 1 \cdots \otimes 1$ and $g_i \mapsto 1 \otimes \cdots X \cdots \otimes 1$. A *tensor category with cylinder braiding* consists of the following data: A braided tensor category $\mathcal{A}$ with braidings $z_{M,N} : M \otimes N \to N \otimes M$, a category $\mathcal{B}$ with the same objects as $\mathcal{A}$ which is an $\mathcal{A}$-module tensor category and an isomorphism $t_M : M \to M$ in $\mathcal{B}$, called *cylinder twist*, for each object M such that

$$t_{M \otimes N} = (t_M \otimes 1)z_{N,M}(t_N \otimes 1)z_{M,N} = z_{N,M}(t_N \otimes 1)z_{M,N}(t_M \otimes 1).$$

Let (X, F) be a four braid pair. The FRT-construction yields a cobraided bialgebra $(A(X), r_X)$ with braid form r_X.

Theorem. The algebra $A(X)$ carries a cylinder form f induced by F. The category of $A(X)$-comodules carries a cylinder braiding. In the dual situation of $U = U_q(sl_2)$-modules the category of integrable modules carries a cylinder braiding. There exists a universal cylinder twist of the form $t = L \circ z$ with Lusztig's operator L and $z = \sum_{k \geq 0} \gamma_k E^{(k)}$. The γ_k are certain polynomials which are defined by a three-term recursion.

Elmar Vogt: *"Foliations with few non-compact leaves"*

The speaker tried to give some substance to the following

Belief. *If a foliation of a compact manifold contains a non-compact leaf then it will contain many non-compact leaves.*

This is definitely true for foliations of codimension 1, where the union of non-compact leaves is an open set. Also for suspension foliations one can show that there must be uncountably many non-compact leaves if there is one at all.

The question whether the union of non-compact leaves of a C^1-foliation must have positive measure if it is not empty has to be denied in view of an example of G. Reeb (1948) who describes in his thesis a real analytic foliation on $S^1 \times S^1 \times S^n$ of codimension 2 with $\{pt\} \times S^1 \times S^n$ as the closure of the union of non-compact leaves.

Let M be a compact manifold, F a foliation of M and let N be the union of non-compact leaves. For a union A of leaves denote by I_A the union of leaves in closure $\mathrm{Cl}(A)$ which are open as subsets of $\mathrm{Cl}(A)$. Define by transfinite induction

$I_1 = I_{\mathrm{Cl}(N)}, I_{\alpha+1} = I_\alpha \cup (I_{\mathrm{Cl}(N) \setminus I_\alpha} \cap N)$, and $I_\alpha = \bigcup_{\beta<\alpha} I_\beta$, if α is a limit ordinal.

Theorem. *Let M be a compact 3-manifold and F a 1-dimensional foliation of M which contains non-compact leaves. Then one of the following is true:*

(1) F contains uncountably many leaves.

(2) The closure of the union of non-compact leaves has infinitely many components.

(3) N is properly contained in $\bigcup I_\alpha$.

The proof uses the fact that for any connected 3-manifold with finite second Betti number which supports a circle foliation the first Betti number is also finite and the Euler characteristic is non-negative. One considers a component W of $M \setminus \mathrm{Cl}(N)$ whose closure intersects the union of the I_α. W

is foliated by circles and has finite second Betti number if (2) does not hold. By the above statement and Alexander duality the Euler characteristic of the complement A of W calculated via Alexander cohomology is non-negative. Using results about connected compact subsets which are unions of compact leaves of a 1-dimensional foliation of a 3-manifold one can show that this Euler characteristic is negative if in addition (1) and (3) do not hold.

Since for any foliation with finitely many non-compact leaves (1) and (2) do not hold, and I_1 is non-empty if (1) does not hold, we get the following

Corollary. *On a compact 3-manifold M no foliation exists with exactly one non-compact leaf, or with finitely many non-compact leaves whose union is a submanifold of M.*

R. Vogt: *"Formal properties of topological Hochschild homology and applications"*

Let R be a commutative S-algebra spectrum, A an R-algebra spectrum and M an A-bimodule spectrum in the notation of Elmendorf, Kriz, Mandell, and May. Topological Hochschild homology $THH^R(A,M)$ is the spectrum obtained by realizing the simplicial spectrum

$$[n] \mapsto A \wedge_R A \wedge_R \cdots \wedge_R A \wedge_R M \qquad n \text{ copies of } A$$

with the classical boundary and degeneracy maps.

We define a cofibration structure on the category ${}_AMod$ of left A-module spectra relative to the forgetful map ${}_AMod \to {}_RMod$ which allows to do relative homological algebra in the non-additive setting of A-module spectra, and show that

$$THH^R(A,M) \simeq Tor^{A^e/R}(M,A)$$

with $A^e = A \wedge_R A^{op}$, i.e. $THH^R(A,M)$ can be interpreted as a relative Tor-functor.

Using the original definition and this equivalence we are able to verify a list of formal properties of Hochschild homology also for THH. An immediate application is

Theorem. *Let k be a commutative classical ring, K a k-algebra, Hk and HK their corresponding Eilenberg-MacLane spectra. Suppose that K is flat over k and that there is a cell S-algebra spectrum E and a weak equivalence $E \wedge_S Hk \to HK$ of Hk-algebra spectra. Then $THH_*(K) \cong HH^k(K) \overset{L}{\otimes}_k THH(k)$ where $THH(K) := THH^S(HK,HK)$ and HH^k is Hochschild homology over the ground ring k.*

This result together with Böckstedt's calculation of $THH(\mathbb{Z})$ and $THH(F_p)$ gives $THH(K)$ for $K = \mathbb{Z}[x]/(x^n)$, $\mathbb{Z}[G]$, G any group, subrings of $\mathbb{Q}$, $F_p[x]/(f)$, f any monic polynomial, $R(\xi)$ with $R \subset \mathbb{Q}$, ξ an n-th primitive root of unity, the prime factors of n invertible in R.

(Some of these results have previously been obtained by tedious spectral sequence arguments.)

Michael Weiss: *"Applications of index theory without operators"*

The index theory in the title relates certain "indices" (global invariants of manifolds, such as Euler characteristic and signature) to local invariants such as characteristic classes. Conceptually it is meant to resemble Atiyah–Singer index theory, but the technical ingredients come from topology, not from analysis.

I. Signature (joint work with B. Williams)

Theorem. *(Atiyah, Frank 69) Suppose M^n closed smooth oriented with q everywhere lin. indep. vector fields. (♣) If $4|n$ then $2a_q|sig(M)$. If also $4|q$ then $4a_q|sig(M)$.*

Here $a_q = \dim(E^0)$ where E irred. $\mathbb{Z}/2$–graded mod. over the Clifford algebra of $\mathbb{R}^q$. Explicitly: $a_q = 1, 2, 4, 4, 8, 8, 8, 8$ for $q = 1, \ldots, 8$ and $a_{q+8} = 16a_q$.

Var.1. *Assume M^n cl. smooth or. and the spherical fibration $S(\tau(M))$ is stably equivalent to fibration with spheres of dim $n-q-1$ as fibers. (Case $n = q$ allowed.) Then (♣) holds.*

Example. Suppose M^4 closed topological spin. By obstruction theory, stabilization of $\tau(M)$ pulls back from 4–sphere under a degree one map, so the degree one map is a degree one normal map. Then $S(\tau(M))$ is stably (fiber homotopy) trivial. If M is smooth, can apply Var.1 with $q = 4$. Find that $16|\text{sig}(M)$ (Rochlin's theorem). If M not smooth, then $\text{sig}(M)$ need not be div. by 16 (Freedman), showing that smoothness is necessary in Var.1.

Var.2. *Assume M^n cl. or. top. n–mfld with stable reduction of structure gp. of $\tau(M)$ from $TOP(n)$ to $TOP(n-q)$. Then (♣) holds.*

Var.3. *Assume M^n closed or. top. $M = U \cup V$ where U, V open, U smooth. Suppose restriction of $S(\tau(M))$ to U is equipped with stable equivalence to fibration with spheres of dim $n-q-1$ as fibers. Suppose restriction of $\tau(M)$ to V is equipped with stable reduction of structure gp. from $TOP(n)$ to $TOP(n-q)$. Suppose that these reductions are compatible over $U \cap V$, and that the second is a (stable) vector bundle reduction, from $O(n)$ to $O(n-q)$, over $U \cap V$. Then (♣) holds.*

Corollary. *N smooth comp. oriented parallelized n-mfld and ∂N a homotopy sphere Σ. Suppose Σ has Gromoll filtration $\geq q-1$ (defined below). Then ($\clubsuit$) holds for $M := N \cup D^n$.*

Gromoll filtration $\geq q-1$ means: the connected sum $\Sigma \# \mathbb{R}^{n-1}$ has a smooth submersion to $\mathbb{R}^{q-1}$ which, away from a compact subset of $\Sigma \# \mathbb{R}^{n-1}$, agrees with a linear surjection $\mathbb{R}^{n-1} \to \mathbb{R}^{q-1}$. *Note:* Big G. f. $\Rightarrow$ small exoticity.

Example. Milnor's exotic sphere (in dim 7, 11, 15, ...) is ∂N where $\operatorname{sig}(N) = 8$. So it has Gromoll filtration ≤ 2, therefore $= 2$ because nobody has G. f. less than 2. Very exotic.

II. Euler characteristic (Joint work with W.Dwyer and B.Williams)

Suppose $p : E \to B$ smooth fiber bundle: smooth cpct. fiber F, structure gp. DIFF(F). Let R be a ring, V bundle of f.g. left proj. R-modules on E. Determines $[V] : E \to K(R)$ (alg. K-theory *space* of R). Here $K(R)$ = group completion of class. space for such module bundles. Assume $H_i(p^{-1}(b); V)$ is proj. over R, $\forall b$ in B, $\forall i \geq 0$. Let $V_i :=$ bundle on B with fiber $H_i(p^{-1}(b); V)$ over b. Determines $[V_i] : B \to K(R)$.

Theorem. *$tr^*[V] = \sum(-1)^i[V_i]$ in $[B, K(R)]$, where $tr : \Sigma^\infty B_+ \to \Sigma^\infty E_+$ is the Becker–Gottlieb–Dold transfer, a stable map determined by p.*

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