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Cohomology of Moduli Spaces

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Die Tagung fand unter der Leitung von C.-F. Bödigheimer (Bonn), Ralph Cohen (Stanford) und Frances C. Kirwan (Oxford) statt. F. Kirwan konnte nicht selbst teilnehmen.

Das besondere Anliegen dieser Konferenz bestand darin, den Begriff „Modulraum" möglichst offen zu lassen und so Mathematiker aus den verschiedensten Bereichen wie Algebraischer Topologie, Algebraischer Geometrie, Analysis und Differentialgeometrie zusammenzubringen. Entsprechend facettenreich waren dann auch die insgesamt 19 Vorträge.

Abstracts

Structure of the mapping class group of surfaces – interplays of Topology, Algebraic Geometry and Number Theory –

Shigeyuki Morita

The mapping class group, denoteg by Γ_g, of a closed oriented surface Σ_g of genus $g \geq 2$ acts on the Teichmüller space of Σ_g and the quotient space $\mathcal{M}_g$ is the moduli space of curves of genus g. The structures of Γ_g as well as of $\mathcal{M}_g$ have been investigated from various points of view, e.g. algebraic geometry, complex analysis, topology, differential geometry, mathematical physics and so on. In this talk, we discussed recent results concerning the cohomology of Γ_g and/or $\mathcal{M}_g$ which are obtained by combining those of Hain, Looijenga and Pikaart in the context of algebraic geometry on the one hand and those of Kawazumi and myself in the context of topology on the other. The result can be summarized as

Theorem. The continuos part of the stable cohomology of the moduli space $\mathcal{M}_g$ is exactly equal to the polynomial algebra generated by the Mumford-Morita-Miller tautological classes.

Here the "continuous part" means the image of the continuous cohomology of Γ_g, with respect to certain canonical filtrations on it, in the ordinary cohomology under the forgetful homomorphism. In the latter half of the talk, we discussed more recent developments including a joint work with Nakamura. There were five directions which are closely related to each other:

1. a topological approach to the Faber conjecture using an explicit description of cocycles for the MMM classes due to Kawazumi and myself.

2. a proposal of certain "higher geometry" of the moduli space which should be related to the secondary characteristic classes of Γ_g introduced by myself recently.

3. relations between the Torelli Lie algebra and the universal 3-manifold invariants of L-J.Murakami-Ohtsuki along the line of recent work of Garoufalias and Levine.

4. a close connection between the structure of the derivation Lie algebra, which is the Lie algebra version of Γ_g in a naive sense, and unstable homology of $OutFree(n)$ (the outer automorphism group of the free group of rank ≥ 2), in particular, $H_{2n-3}(OutFree(n))$ for $n \geq 4$.

5. (joint work witch Nakamura) a topological approach to understand

the outer representation of $Gal(\overline{\mathbb{Q}}/\mathbb{Q})$, in particular a (possible) topological construction of the "Galois obstructions" over $\mathbf{Z}$.

Homology of moduli spaces via operads

Alexander A. Voronov

We propose a duality between the cohomology of compactified and noncompactified moduli spaces of algebraic curves with punctures. This duality generalizes one between commutative and Lie algebras suggested by Lazard, Quillen, Kontsevich, and Ginzburg and Kapranov. More explicitly, we show that the cohomology of noncompactified moduli space is the graph complex decorated with the cohomology of the compactified ones. This in its turn generalizes a result of Getzler for the genus zero moduli space.

Construction of Gromov-Witten Invariants for general symplectic manifolds

Kenji Fukaya

This is joint work witch Kaoru Ono of Ochanomizu University. We study (M, J, n) where (M, n) is a symplectic manifold and J is a compatible almost complex structure. For $\beta \in H_2(M, \mathbf{Z})$ we consider pairs (Σ, h), where Σ is a Riemannian surface of genus g with m marked points, and $h : \Sigma \to M$ is a (pseudo) holomorphic map s.t. $h_*\Sigma = \beta$. Let $\mathcal{M}_{g,m,\beta}(M)$ be the moduli space of such pairs up to ismorphism.

There is a map

$$\mathcal{M}_{g,m,\beta}(M) \xrightarrow{\pi} \mathcal{M}_{g,m}(M) \times M^m.$$

The problem we discuss here is to find a class which is

$$\pi_*[\mathcal{M}_{g,m,\beta}] \in H_*(C\mathcal{M}_{g,m} \times M^m)$$

when $\mathcal{M}_{g,m,\beta}(M)$ is a compact and smooth manifold. In general, $\mathcal{M}_{g,m,\beta}(M)$ is neither smooth nor compact. So the problem is to find an appropriate compactification and some perturbation of $\bar{\partial}h = 0$ (the PDE) to obtain an appropriate perturbed moduli space.

In symplectic setting the problem is a version of the "negative multiple cover problem", which was a major trouble to apply Gromov's pseudo holomorphic curve technique to general symplectic manifolds. We solved the problem by using the following idea:

1. Compactify $\mathcal{M}_{g,m,\beta}(M)$ by using the notion of stable map due to Kontsevich.

2. Work out Kuranishi theory for $C\mathcal{M}_{g,m,\beta}(M)$. Namely present it locally as $\frac{f^{-1}(0)}{\Gamma}$ where $f : \mathbb{R}^n \to \mathbb{R}^{n'}$, Γ acts on $\mathbb{R}^n$ and $\mathbb{R}^{n'}$, and $\Gamma < \infty$.
3. Glue the "Kuranishi chart" in 2) to obtain a topological analogue of a scheme (or more precisely stack).
4. Use multivalued perturbation to obatin a fundamental class of the "toplogical stack" (which we call Kuranishi structure) as a cycle over $\mathbb{Q}$.

The same idea is used to solve a version of Arnold's conjecture on periodic Hamiltonian systems.

Bi-invariant Grassmanians and Instanton moduli spaces

Giorgio Valli

Let $\mathcal{M}_{k,n}$ be the moduli space of holomorphic vector bundles over $\mathbb{P}^2$, of rank n, $c_2 = k$, with a trivialization at the $\mathbb{P}^1$ at infinity; and let $\overline{\mathcal{M}}_{k,n}$ be it's "completion" of framed torsion-free sheaves. We compute the homology of $\overline{\mathcal{M}}_{k,n}$: it is torsion-free, zero in odd dimension and it approximates the homology of $BU(k)$ up to a range growing with k and n. This is proved by studying its deformation retract: the "bi-invariant Grassmanian" of k-dimensional vector subspaces (of a space of vector polynomials) stable under two commuting nilpotent operators. We then define a $\mathbb{C}^*$-action and we use the Bialynicky-Birula theorem. We use these results to get an Atiyah-Jones type statement:

$$H_j(\mathcal{M}_{k,n}) \cong H_j(\Omega^3 SU(n))$$

for $j < k$ $(n = 2)$ or $j < 2k + 1$ $(n \geq 3)$ with surjections in the limiting cases. The proof is by a Segal-Arnold argument.

Intersection theory in moduli spaces of holomorphic bundles on a Riemann surface

L. Jeffrey

The moduli space $M(n, d)$ of semistable holomorphic vector bundles of rank n and degree d (assumed coprime) on a Riemann surface of genus $g \geq 2$ is a smooth Kähler manifold. We outline joint work with F. Kirwan (alg-geom/9608029) in which we prove formulas for intersection numbers of cohomology classes in $M(n, d)$. These formulas were found originally by Witten using physical methods (*J. Geom. Phys.* 1992) and encode (at least in principle) the complete structure of the cohomology ring of $M(n, d)$.

Quantization and Symplectic Reduction

Youliang Tian

In this joint work with Weiping Zhang, we present a direct analytic approach to the Guillemin-Sternberg geometric quantization conjecture. Besides deriving an alternative approach to this conjecture in full nonabelian group action case, our method also leads immediately to generalizations in various contexts. Among other things, we obtain a "dual version" of the conjecture and the invariance of Todd genus under symplectic reduction. When the underlying symplectic manifold is Kähler, we also obtain a set of holomorphic Morse type inequalities.

Topological Stability Theorems for Spaces of Rational Curves

Charles Boyer

In joint work with Jacques Hurtubise and Jim Milgram we describe topological stability theorems for holomorphic maps (based) from the Riemann sphere $\mathbb{P}^1$ to certain rational varieties X. These varieties are such that one has a generalized poles and principal parts description of these maps. For this one needs X to have a dense open set N on which solvable algebraic group acts transitively and freely. So X is an equivariant compactification of N. In this case $X\backslash N$ is a divisor and the points in $X\backslash N$ which are in the image of a given holomorphic map describe a "pole". The type of varieties X which satisfies these conditions includes generalized flag varieties G/P, tonic varieties, spherical varieties and equivariant "blow-ups" of these.

The topological stability theorem is:

Let X be a variety as described above (a principal almost solvable one) and let $Hol_k(\mathbb{P}^1, X)^*$ denote the space of based holomorphic maps of multi-degree k. Let $\Omega^2_k X$ denote the space of based continuous maps, the two-fold loop space. Then there are constants $c_0 > 0$, c_1 and an increasing function $q(|k|)$ such that the natural inclusion $\iota_k : Hol_k(\mathbb{P}^1, X)^* \to \Omega^2_k X$ induces isomorphisms

$$H_t(Hol_k(\mathbb{P}^1, X)^*) \cong H_t(\Omega^2_k X) \text{ for all } t \leq q(|k|).$$

One can obtain similar statements in homotopy.

A splitting for the stable mapping class group

Ulrike Tillmann

We show that the classifying space of the stable mapping class group after plus construction, $B\Gamma_\infty^+$, is an infinite loop space by considering the classifying space of the symmetric monoidal 2-category of one dimensional manifolds, cobordisms and diffeomorphisms of cobordisms. Furthermore, we show that the classical representation of the mapping class group $\Gamma_{g,1} \to \mathrm{SP}_{2g}\mathbf{Z} \to \mathrm{GL}_{2g}\mathbf{Z}$ induces a map of infinite loop spaces from $B\Gamma_\infty^+$ to the algebraic K-theory of the integers. Indeed, there is a natural lift of of this map to Waldhausen's K-theory of a point, $A(*) = \mathbf{Z} \times BGL_\infty QS^0+$ which is also a map of infinite loop spaces. Here $QS^0 := \lim_n \Omega^n S^n$ is the ring up to homotopy of stable selfmaps of the sphere. This result is used to show that, localized away from two, a connected component of QS^0 splits off $B\Gamma_\infty^+$. As an immediate consequence, the homology with coefficients in $\mathbf{Z}[1/2]$ of the infinite symmetric group is a direct summand of the homology of the stable mapping class group. The classes corresponding to the factor $CokJ$ in $QS^0 = ImJ \times CokJ$ are torsion classes which have not been detected so far.

Intersection theory on $\overline{\mathcal{M}}_{1,n}$ and Gromov-Witten invariants in genus 1

E. Getzler

Let V be a smooth projective variety of dimension d (or more generally, a compact symplectic manifold). The Gromov-Witten invariants of V are S_n-invariant linear maps

$$I^V_{g,n,\beta} : H^*(V, \mathbb{Q})^{\otimes n} \to H^*(\overline{\mathcal{M}}_{g,n}, \mathbb{Q}),$$

where $\beta \in H_2(V, \mathbf{Z})$, reducing degree by $2d(1-g) + \langle c_1(V), \beta\rangle$. These invariants "measure" in a generalized sense the number of projective curves of genus g with n marked points
passing through prescribed cycles on V.

The interest in Gromov-Witten invariants motivates the study of the cohomology of $\overline{\mathcal{M}}_{g,n}$. This has been done in detail for $g = 0$, and in our talk, we did the same for $g = 1$. It turns out that the even-degree homology is spanned by cycles associated to the strata defined by Deligne, Mumford and Knudsen, and that the odd-degree homology has a similar description involving spaces of cusp forms for $\mathrm{PSL}(2, \mathbf{Z})$. The relations among these homology classes may be calculated by means of the Serre spectral sequence for the fibration $\mathcal{M}_{1,n} \to \mathcal{M}_{1,1}$, using Saito's theory of mixed Hodge modules. The main conclusion is that in even degree, there is a unique relation, in $H_4(\overline{\mathcal{M}}_{1,4}, \mathbb{Q})$, which gives rise by means of maps $\overline{\mathcal{M}}_{1,n} \to \overline{\mathcal{M}}_{1,4}$ to $\binom{n}{4}$ relations.

Any relation among cycles in $\overline{\mathcal{M}}_{1,n}$ must give rise to a relation among the Gromov-Witten invariants $I^V_{0,m,\beta}$ and $I^V_{1,m,\beta}$. Using this observation, applied to the Gromov-Witten invariants of an elliptic curve, together with some intersection theory, we calculate the new relation. (It is not very simple, having 7 terms.) It is also possible to calculate it by showing that it is the only relation among the Gromov-Witten invariants associated to $\mathbb{CP}^2$. We conjecture that all relations among Gromov-Witten invariants of $\mathbb{CP}^2$, for higher genus, reflect relations among cycles in $\overline{\mathcal{M}}_{g,n}$.

The new relation allows us to calculate the $g = 1$ Gromov-Witten invariants of $\mathbb{CP}^n$. For $\mathbb{CP}^2$, there are $N_k^{(1)}$ irreducible plane curves of genus 1 and degree k which meet $3k$ generic points, where

k	$N_k^{(1)}$
1	0
2	0
3	1
4	225
5	87192
6	57435240
7	60478511040

These numbers agree with those obtained, in degrees up to 6, by Vainsencher, and presumably also agree with the recent formula of Caporaso and Harris, good in all degrees, although unlike them, we have nothing to say about the number of plane curves of higher genus.

On the L^2-geometry of the moduli space of locally conformally flat structures

Lutz Habermann

Proceeding in a similar way as Leutwiler for Euclidean domains and Arakelov for Riemann surfaces, now using the Green function for the Yamabe operator, we construct a canonical Riemannian metric for each closed, locally conformally flat n-manifold, $n \geq 3$, that supports a Riemannian metric of positive scalar curvature. The construction depends on the positive mass theorem of Schoen and Yan. The resulting metric differs from the canonical metrics of Apanasov-Kulkarni-Pinkall and Nayatani. In particular, our metric is always of class C^∞ and (locally) distance nondecreasing under conformal maps.

We study the Riemannian L^2-metric h on the moduli space ${B_0}^+$ of scalar positive locally conformally flat structures which is obtained by taking the L^2-product of tangent vectors to the moduli space w.r.t. the canonical metric. By a surgery construction similar as for Riemannian surfaces, we find curves of finite length on ${B_0}^+$ along which the conformal structure degenerates. Therefore, (B_0^+, h) is not complete.

In the case where the underlying manifold is $S^1 \times S^2$, a more detailed description of the L^2-metric h can be obtained.

Integrable Systems and Surfaces

Jacques Hurtubise

Many of the classical integrable systems can be considered as special cases of the integrable systems defined by Hitchin and generalized by Markman and Botachin to the moduli space $\mathcal{M}(D)$ of stable pairs on a Riemann surface Σ (with meromorphic Higgs fields whose divisor of poles is dominated by D, a fixed divisor on Σ). One particular feature of the Hitchin systems is that they are locally symplectomorphic to a symmetric product of a blow-up of the total space of the line bundle $K_\Sigma(D)$, or rather its symplectic desingularisation which is a Hilbert scheme.

This fact indicates a way to classify some integrable systems. Let $\mathbb{J}^{2g} \to U^g$ be a (local) algebraically integrable system of Jacobians, that is, a symplectic manifold $\mathbb{J}$ with a Langrangian fibering over U, a ball in $\mathbb{C}^g$, with fibers which are Jacobians of curves. If one considers the Abel map $A : \mathbb{S} \to \mathbb{J}$ from the corresponding family $\mathbb{S}$ of curves, one can ask that $A^*\Omega \wedge A^*\Omega = 0$. One then has a symplectic surface (Q, ω) and a projection $\rho : \mathbb{S} \to Q$ with $\rho^*\omega = A^*\Omega$. The curves of $\mathbb{S}$ embed into Q. The system $\mathbb{J}$ is then birationally the Hilbert scheme of g points of Q. Under favorable conditions one can show that Q completes to an algebraic surface $\overline{Q}$, with $kod(\overline{Q}) = -\infty$ or 0. Examples of such systems include not only the Hitchin systems, but also the systems considered by Skylanin (rational $\overline{Q}$) and the symmetric products of a $K-3$ surface or an Abelian variety.

In joint work with E.Markman I have also developped an analogous theory for Prym-Tyurin varieties. Examples here include the Hitchin systems for arbitrary semi-simple groups.

Degenerating metrics and instantons on S^4.

Paul Norbury

This is a report on joint work with Stuart Jarvis.

We give a direct proof of Atiyah's theorem relating instantons over the four-sphere with holomorphic maps from the two-sphere to the loop group. Our approach uses the non-linear heat flow equation for Hermitian metrics as used in the study of Kähler manifolds. The proof generalises immediately to a larger class of four-manifolds.

Theorem 1 (Atiyah). *For any classical group G and positive integer k, the following two spaces are diffeomorphic:*

(1) the parameter space of Yang-Mills k-instantons over S^4 with group G, modulo based gauge transformations,

(2) the parameter space of all based holomorphic maps $S^2 \to \Omega G$ of degree k.

The map $f : S^2 \to \Omega U(n)$ is holomorphic when $f^{-1}\partial_{\bar{w}} f$ extends to a holomorphic map of the disk to $\mathbf{gl}(n, \mathbf{C})$ for each $w \in S^2$. Put η equal to this extension. Over $S^2 \times D = \{(w, z) = (u + iv, x + iy)\}$ define the connection

$$A_f = \eta d\bar{w} - \bar{\eta}^T dw \tag{0.1}$$

so A_f is flat on each $\{w\} \times D$. Furthermore,

$$\left.\begin{array}{l} [\partial_u^A, \partial_x^A] = [\partial_v^A, \partial_y^A] \\ [\partial_u^A, \partial_y^A] = -[\partial_v^A, \partial_x^A] \\ [\partial_x^A, \partial_y^A] = 0 \end{array}\right\} \tag{0.2}$$

which resembles the anti-self-dual equations with respect to the product Kähler metric on $S^2 \times D$. We will use the round metric and the hyperbolic metric on S^2 and D respectively. It so happens that $S^2 \times D \cong S^4 - S^1$ and the product metric is conformally equivalent to the round metric on S^4. That means that (0.2) also resembles the anti-self-dual equations over S^4.

Atiyah remarked that his proof merely gives existence without a direct means of associating an instanton to a holomorphic map. Donaldson suggested that there ought to be some type of adiabatic limit proof that avoids Atiyah's roundabout route. The following theorem addresses these two comments and gives an alternative proof of Atiyah's theorem.

Theorem 2. *For each based holomorphic map $f : S^2 \to \Omega U(n)$, there exists a unique gauge equivalence class of anti-self-dual connections on a framed $U(n)$-bundle over S^4 and a canonical representative A that is in some sense close to A_f. This correspondence defines a diffeomorphism between the respective moduli spaces.*

Remarks. (i) The connections are close in the sense that we use A_f as the initial data in a non-linear heat flow that converges to an instanton. We get a bound on the distance traveled during the flow.

(ii) We can think of the equations (0.2) as describing the anti-self-dual equations with respect to a metric that is infinite in the disk factor. Theorem 2 essentially describes the limit of the moduli space of instantons as we stretch the metric on S^4 so that the area of the disk goes to infinity.

(iii) The $U(1)$-invariant conections are hyperbolic monopoles. The approximate instantons we give can be interpreted as the renormalised zero mass limit of the monopoles. The results here confirm the suggestion of

Atiyah that in the zero mass limit the correspondence between monopoles and rational maps should become transparent. These techniques also give us information about the charge distribution of hyperbolic monopoles enabling us to confirm the conjecture of Atiyah that the infinite mass limit of hyperbolic monopoles yields the Euclidean monopoles.

The novelty of the decomposition $S^4 = S^1 \times B^3 \cup S^2 \times D^2$ rather than the more usual picture of S^4 as $\mathbf{CP}^2$ with a divisor

collapsed, allows us to generalise the result. We can replace the loop group and S^4 in Theorem 2 respectively by $LGL(n, \mathbf{C})/L^+_\Sigma GL(n, \mathbf{C})$ and $X_\Sigma = S^1 \times B^3 \cup S^2 \times \Sigma$ for a Riemann surface Σ with $\partial\Sigma = S^1$.

Theorem 3. *The moduli space of instantons on a framed $U(n)$-bundle over X_Σ is diffeomorphic to the space of based holomorphic maps from S^2 to $LGL(n, \mathbf{C})/L^+_\Sigma GL(n, \mathbf{C})$.*

The origin of the shift of the dual Coxeter number

Martin Schottenloher

The shift in question is

$$k \mapsto k + h, \quad k \in \mathbb{N},$$

or

$$c \mapsto c + h, \quad c \quad \text{a central charge},$$

where $h = h(G)$ is the dual Coxeter number of a compact simple Lie group. This shift occurs in various formulas in quantum theory and in representation theory, in particular in the Verlinde formula. In the case of $G = SU(2)$ the Verlinde formula is

$$d_k^{SU(2)}(g) = \left(\frac{k+2}{2}\right)^{g-1} \sum_{j=1}^{k+2-1} \left(\sin^2 \frac{j\pi}{k+2}\right)^{1-g},$$

and the dual Coxeter number of $SU(r)$ is r. Hence, we have the obvious shift $k \mapsto k + 2$. This shift is often attributed to the appearance of an anomaly of the quantization procedure or of a central charge. In this talk we want to point out that the shift may be explained as to arise from an incomplete quantization. In fact, if one replaces the incorrected geometric quantization by a metaplectic quantization the shift disappears.

The number $d_k^{SU(r)}(g)$ is the dimension of the space $Z = H^0(\mathcal{M}^{SU(r)}, \mathcal{L}^k)$ of conformal blocks, where $\mathcal{M}^{SU(r)}$ is the moduli space of flat connections on a surface S of genus g. Z can be considered to be the state space of the quantized Chern-Simons theory: Introducing a complex structure $J \in \mathcal{M}_g$ on S the space

$\mathcal{M}^{SU(r)}$ acquires the interpretation of the moduli space $\mathcal{M}_J^{SU(r)}$ of semistable holomorphic rank r vector bundle on the Riemann surface S_J with trivial Chern classes and trivial determinant. In this way $\mathcal{M}_J^{SU(r)}$ obtains a symplectic (Kähler) structure on the regular locus of $\mathcal{M}_J^{SU(r)}$ together with a holomorphic determinant line bundle $\mathcal{L}$. Now, the uncorrected geometric quantization of the symplectic space $\mathcal{M}_J^{SU(r)}$ with $\mathcal{L}^k$ as prequantum bundle and with the holomorphic polarization is $H^0(\mathcal{M}_J^{SU(r)}, \mathcal{L}^k)$, the space of generalized theta functions of level k, and this space can be identified with the above space Z of conformal blocks. In particular,

$$d_k^{SU(r)}(g) = \dim H^0(\mathcal{M}_J^{SU(r)}, \mathcal{L}^k).$$

Now, the metaplectic correction yields the state space

$$Z^m := H^0(\mathcal{M}_J^{SU(r)}, \mathcal{L}^k \otimes \mathcal{K}^{\frac{1}{2}}),$$

where $\mathcal{K}^{\frac{1}{2}}$ is a square root of the canonical bundle $\mathcal{K}$ of $\mathcal{M}_J^{SU(r)}$ which represents a metaplectic structure on $\mathcal{M}_J^{SU(r)}$. Since $\mathcal{K}$ is isomorphic to the dual of $\mathcal{L}^{2r}$, a natural choice for $\mathcal{K}^{\frac{1}{2}}$ is $\mathcal{L}^{-r}$. Hence,

$$Z^m := H^0(\mathcal{M}_J^{SU(r)}, \mathcal{L}^{k-r}),$$

and the corrected dimension is

$$d_k^{m,SU(r)}(g) = \dim H^0(\mathcal{M}_J^{SU(r)}, \mathcal{L}^{k-r}) = d_{k-r}^{SU(r)}(g)$$

explaining the shift.

This is true not only for $SU(r)$ but also for general compact simple groups G since $\mathcal{K}$ is isomorphic to the dual of $\mathcal{L}^h$ in this more general situation. Moreover, it turns out that if one wants to generalize the deformation independence of the spaces of sections $H^0(\mathcal{M}_J^{SU(r)}, \mathcal{L}^k)$ from Riemann surfaces to compact Kähler manifolds one is forced to take the metaplectic quantization from the beginning.

Moduli spaces of framed manifolds, hyperbolicity and asymptotics of Kähler-Einstein metrics on open manifolds

Georg Schumacher

We give a report on an approach to provide the moduli space of homogeneously polarized (non-uniruled) projective varieties with a generalized Petersson-Weil metric and related results. A pair (X, C), where C is a smooth divisor on a compact complex manifold, is called a framed manifold.

We give a notion for non-uniruledness, show the statement of the Matsusaka-Mumford theorem in the category of polarized framed Kähler manifolds and obtain the existence of a moduli space.

By Fujita's theorem, we can assign to any homogeneously polarized projective variety a framed manifold (X, C) with the condition

$$(N): \quad K_X + C > 0.$$

On the other hand, any (X, C) with this condition possesses the natural polarization $K_X + C$. Using the existence of a complete Kähler-Einstein metric $\omega_{X'}$ of negative curvature on $X' = X \backslash C$ together with L^2-Hodge theory, we get a generalized Petersson-Weil metric on the moduli space $\mathcal{M}_{(N)}$. It provides a symplectic structure. The curvature is explicitly computed. As an application, we see that the moduli space of smooth divisors $C \subset X_0$ in a fixed manifold X_0 modulo $\mathrm{Aut}(X_0)$ is hyperbolic. Condition (N) also implies the existence of a Kähler-Einstein metric ω_C on C. We show that $\omega_{X'}$ converges to ω_C, when restricted to directions parallel to C. To do so, we have to solve the Monge-Ampere equation with functions of logarithmic decay. As an application, we get that any biholomorphic map $f : X\backslash C \to Y \backslash D$ of manifolds with (N) and $(C), (D) > 0$ extends to $f : X \to Y$.

Algebraic cohomology of moduli of vector bundles on curves

Alastair King

For a smooth projective variety X over $\mathbb{C}$, the algebraic cohomology ring $H^*_A(X) \subset H^*(X; \mathbb{Q})$ is the subspace spanned by the classes of algebraic subvarieties of X. In joint work with V.Balaji and P.E. Newstead we investigate the relationship between H^*_A for the Jacobian J of a smooth projective curve C and for the moduli space N of stable holomorphic vector bundles over C of rank 2 and fixed determinant of odd degree.

First, we have just a numerical relationship which generalizes the known formula for the Betti numbers/Poincare polynomial of N.

Theorem 1. If

$$P_A(X;t) = \sum_{i=0}^{\infty} t^i \dim H^i_A(X)$$

is the algebraic Poincare polynomial of X, then

$$P_A(N;t) = \frac{P_A(J;t^3) - t^{2g} P_A(J;t)}{(1-t^2)(1-t^4)}.$$

This is proved by first relating $P_A(N)$ to $P_A(S^k C)$ for the symmetric products $S^k C$ of C using Thadden's chain of flips, and then relating $P_A(S^k C)$ to $P_A(J)$ by an algebraic version of Mac Donald's formula.

Second, we have a more direct relationship. The generators of $H^*(N)$ described by Newstead '72 may be interpreted as giving a surjective ring homomorphism

$$\nu : H^*(J) \otimes \mathbb{Q}[\alpha, \beta] \to H^*(N)$$

where ν multiplies the degree of classes in $H^*(J)$ by 3.

Theorem 2.

$$\nu(H_A^*(J) \otimes \mathbb{Q}[\alpha, \beta]) = H_A^*(N).$$

Given Theorem 1, the main point is to show that $\nu(H_A^*(J)) \subset H_A^*(N)$, which is done by showing that ν is given by an algebraic correspondence in $J \times N$.

One corollary is that numerical and homological equivalence coincide for N, because they do for J [Liebermann'68]. It is reasonable to hope for a version of Theorem 2 with H_A^* replaced by the Chow ring A^*, but this method fails for lack of a version of Theorem 1.

Purity of Mixed Hodge Structures on the stable cohomology of the moduli space

Martin Pikaart

Harer's stability theorem states that for $k \leq \frac{2}{3}g$, we have an isomorphism

$$H^k(\Gamma_g, \mathbb{Q}) \cong H^k(\Gamma_{g+1}, \mathbb{Q}).$$

Here Γ_g is the mapping class group of a closed oriented surface of genus g. Using the fact that Γ_g and the moduli space of curves of genus g, $\mathcal{M}_g$ have the same rational cohomology and an algebro-geometric description of the "stability map", we obtain a canonical mixed Hodge structure (MHS) on $H^k(\Gamma_\infty, \mathbb{Q})$.

Looijenga observed that the cohomology of M_g^n (= the moduli space of curves of genus g with n marked points) in low degree with respect to g appears as a free polynomial algebra over the cohomology of $\mathcal{M}_g$ with generators $c_1(\mathcal{L}_i)$, $i = 1, \ldots, n$. Here $\mathcal{L}_i|_{[C,x_1,\ldots,x_n]} \cong T_{x_i}C$ for $[C, x_1, \ldots, x_n] \in \mathcal{M}_g^n$. Using the above as main input, we prove that the canonical MHS on $H^k(\Gamma_\infty)$ is actually pure of weight n.

A corollary is that for any symplectic representation S_λ of Γ_g – where $\lambda = (\lambda_1, \ldots, \lambda_g)$ is a partition of $\sum_{i=1}^g \lambda_i$ – the cohomology group $H^k(\Gamma_g, S_\lambda)$, which stabilizes according to Ivanov and carries a canonical MHS according to Saito, actually carries a pure HS of weight $k + \sum_{i=1}^g \lambda_i$ in the stable range.

Holomorphic Curves and Three-manifolds

Helmut Hofer

Let M be a closed oriented three-manifold equipped with a contact form λ. The contact form determines the so-called contact structure ξ defined by $\xi = \text{kern}(\lambda)$ and the Reeb vectorfield X defined by $i_X\lambda = 1$ and $i_X d\lambda = 0$. We assume that M is equipped with the orientation induced by $\lambda \wedge d\lambda$. Observe that replacing λ by $-\lambda$ does not change the induced orientation. Let us assume that $\xi \to M$ is a trivial vector bundle.

For a periodic solution x of $\dot{x} = X(x)$ denote by $T_c \in (0, \infty]$ the minimum of all numbers t such that $x(0) = x(t)$ and $x_{T_c} : \mathbb{R}/(t\mathbf{Z}) \to M$ is contractible. We call x a contractible periodic orbit if $T_c < \infty$. To a contractible periodic orbit (x, T_c) we can associate the so-called self-linking number $sl(x, T_c)$ as follows. We take a map $u : D \to M$ satisfying $u(e^{2\pi i t}) = x(tT_c)$ and take a nowhere vanishing section Z of $u^*\xi \to D$. We push the loop x_{T_c} into the direction of Z and obtain a new loop y. The oriented intersection number $\text{int}(u, y)$ is called the self-linking number of (x, T_c). It does not depend on the choices involved. We also note that Z gives a framing of the symplectic vector bundle $(x^*_{T_c}\xi, d\lambda) \to \mathbb{R}/(T_c\mathbf{Z})$. We call a contact form strictly convex if for every contractible periodic orbit (x, T_c) the following holds. For every infinitesimally close orbit the winding with respect to Z over the period T_c is strictly larger than 2π.

The main result is now the following:

Theorem. Assume M is equipped with a positive contact structure ξ. Then the following statements are equivalent:

- (M, ξ) is diffeomorphic to S^3 with the standard contact structure of complex lines in TS^3.
- There exists a strictly convex contact form λ inducing ξ and there exists an unknotted periodic orbit P with self-linking number -1.

The proof relies on holomorphic curve techniques for a suitable $\mathbb{R}$-invariant almost complex structure $\tilde{J}$ on $\mathbb{R} \times M$ associated to λ and an admissable complex multiplication for $\xi \to M$. Using suitable families of holomorphic curves in $\mathbb{R} \times M$ and their projections into M one constructs a suitable open book decomposition of M with disk-like pages.

The topology of real and quaternionic algebraic cycles

Paul Lima-Filho

We apply techniques from equivariant homotopy theory to study spaces of algebraic cycles on projective varieties, and to study properties of holomorphic mappings into such spaces. As an outcome we associate to each

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algebraic variety X on which a finite group G acts, an equivariant cohomology theory $\mathfrak{Z}^X$ (connective spectrum), which closely reflects the geometry of X. Even in the particular case where X is a point and $G = \mathbf{Z}_2$ we already have highly non-trivial infinite loop space structures which carry deep relations with the theory of characteristic classes. In this context we present a unified approach to delooping various "total characteristic classes maps" which encompasses Stiefel-Whitney, Chern, Pontrjagin and symplectic characteristic classes, along with various related cohomology operations.

Cohomology of moduli spaces and Schur Q-functions

Jack Morava

There is a classical map

$$B\Gamma_g \to BSp(2g, \mathbf{Z}) \to BSp(2g, \mathbb{R}) \simeq BU(g)$$

which stabilizes to a map, hopefully of H-spaces

$$B\Gamma \to BU \to B(U/O) \xrightarrow{\sim} Sp/U,$$

the latter map being induced by a Bott isomorphism. This seems to induce a splitting

$$H^*(B\Gamma, F) \cong H^*(Sp/U, F) \times ?$$

as $H^*(Sp/U, F)$-modules (cf. Milnor-Moore Thm 4.9); in any case $H(Sp/U)$ defines a large interesting subalgebra of $H^*(B\Gamma)$. The cohomology $H^*(Sp/U)$ can be identified with the algebra of J.Schur's Q-functions, cf. Humphreys & Hoffmann. Remarkably, the free energy function of Witten, considered as a map from $\overline{M_g}$ to $\underline{MU}$, also can be defined to take values in an algebra of Schur functions [D.Francesco & others in CMP151(1993) and Josefich in Lett MP33(1995)]. These constructions can be expressed by

$$q^{\nu}(z^{1/2}) = exp(-\sum_{i\geq 0} \frac{t^i}{(2i-1)!!} z^{i+1/2})$$

or in terms of the Kontsevich-Witten genus of the form

$$\log_{KW}(T) \text{ “ = “ } -(\frac{\pi}{2})^{1/2} \text{ trace } E_{1/2}(-(2T)^{-1/2}\Lambda),$$

with $k_i^{\nu} = \text{trace}\Lambda^{2i-1}$, $\Lambda \in$ {positive-definite hermitian symmetric matrices} and

$$E_\alpha(x) = \sum_{n\geq 0} \frac{x^n}{\Gamma(\alpha n + 1)},$$

a classical entire function studied by Mittag-Leffler (the " = " has to be taken in the sense of asymptotic expansions). It seems reasonable to expect a commutative diagram of the form

$$
\begin{tikzcd}
B\Gamma_g \arrow[r] \arrow[d] & \overline{M_g} \arrow[d, "F_g = \text{Witten's free energy}"] \\
BU \arrow[d] & \underline{MU_{\mathbb{Q}}} \\
Sp/U \arrow[ur] &
\end{tikzcd}
$$

Note, the Schur Q-functions are Hall-Littlewood symmetric functions at $t = -1$. It seems reasonable to hope for a version this result related to Witten's W_p-algebra conjectures at other roots of unity.

Reported by Stefan J. Wisbauer

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