MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 39/1996

Combinatorial Optimization

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The meeting followed a sequence of related Oberwolfach conferences on similar topics in the years 1991 and 1993. It was attended by 41 participants from eight countries (Austria, Belgium, France, Germany, Italy, the Netherlands, Switzerland, USA). Several people that were expected could not come and canceled on short notice.

31 talks of 30–45 minutes length were given, partially in the traditional style on the blackboard, and partially using projected slides, as appropriate for the type and material of presentation. (One talk relied exclusively on a presentation system by computer.)

The topics covered a wide spectrum, from structural insights of predominantly theoretical interest, over results of an algorithmic nature whose practical use remains to be seen, to applicable results whose successful application to real-world problems was reported.

In the area of graph algorithms, Schrijver presented a new edge-coloring algorithm for bipartite graphs. Quite a few papers treated scheduling problems, most of them presenting approximation algorithms (Bräsel, Möhring, Schulz, Woeginger). There were talks about location problems (Hamacher, Welzl), packing, covering and partitioning problems (Borndörfer, Hochstättler, Tinhofer), and network design problems (Du, Grötschel and Wessäly),

Many talks dealt with various techniques for combinatorial optimization problems. A few presentations were devoted to solution techniques for difficult and large combinatorial optimization problems, including polyhedral combinatorics (Fischetti, Nemhauser, Rinaldi, Wolsey). Continuous approaches to combinatorial optimization problems were discussed by Pardalos, Rendl (quadratic and semidefinite programming) and Starke (dynamical systems). Hammer treated Boolean methods, and Hochstättler investigated test sets and Gröbner bases techniques.

There were talks exploring the connections between graphs and games (Boros, Faigle) and search problems (Fekete). Two talks dealt with optimization problems in specially structured graphs related to circulant and Toeplitz matrices, as well as characterizations of such graphs (Burkard, Euler). Several talks treated optimization problems in a geometric setting (Fekete, Liebling, Rote, Welzl).

On the applied side, there were talks about network design problems in telecommunication (Grötschel and Wessäly), transportation planning (Borndörfer, Moll, Zimmermann), and problems in computational biology (Jünger). In addition to the talk whose abstract is documented in this report, George Nemhauser also gave an overview of various planning problems in the airline industry.

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On Wednesday night, there was an open problem session chaired by Gerhard Woeginger. Most of the problems that were posed are listed at the end of this report.

Between the scheduled talks, ample time was left in the lunch breaks and evenings to facilitate the exchange of ideas in informal scientific discussions and the familiarization between the participants. In the traditional Oberwolfach style, the relaxed atmosphere and the nice surrounding provided an excellent setting for this.

The usual hiking excursion went to the village of St. Roman. Thanks to the wisdom of the organizers, the excursion was planned already on Tuesday afternoon, since it rained a lot on Wednesday and Thursday.

In summary, the meeting provided the participants with a cross-selection of the recent results that were obtained in the various subfields, it showed the activity of theoretical research, and it documented the importance of combinatorial optimization for real-world problems. In this sense, the meeting was very successful.

Abstracts of talks

Solving set partitioning problems to route vehicles in handicapped people's transport

Ralf Borndörfer, Berlin

Telebus is Berlin's dial-a-ride system for handicapped people. Every day, about 1,500 transportation requests have to be scheduled into 100 minibuses such that punctual service is provided and total costs are minimal. This optimization problem can be modeled as a *set partitioning problem.*

Our approach to solve these set partitioning problems is by branch-and-cut. One important part of such an algorithm is the use of *preprocessing.* Inside of a branch-and-cut algorithm, one would like to remove iteratively as much as possible from the formulation with these techniques. To maintain dual feasibility of the basis, however, one can eliminate only constraints with zero dual price right away.

We show in this talk that basic variables that are fixed to their bounds inhibit removal of redundant rows to a significant extent by causing nonzero prices. We suggest to eliminate these variables form the basis by simple *pivoting techniques* that can be implemented using standard LP software. We give computational results for both the ACS problems by Hoffman and Padberg (1993) and Telebus set partitioning problems.

Graphs and games

Endre Boros, New Brunswick; joint work with Vladimir Gurvich

In this talk we recall some game theoretic results, show how to assign a game to a graph, and using this connection and deep results of game theory, we prove that perfect graphs are kernel solvable, as it was conjectured by Berge and Duchet (1983).

The converse statement, i.e. that kernel solvable graphs are perfect, was also conjectured in the same paper, and is still open. In this direction we can show that it is always possible to substitute some of the vertices of a non-perfect graph by cliques so that the resulting graph is not kernel solvable. The above results allow us to reformulate the open question, and prove that the following statements are equivalent:

1. Kernel solvable graphs are perfect.
2. If a graph is kernel solvable, then its complement is also kernel solvable.
3. A graph, obtained from a kernel solvable graph by the substitution of a vertex by an edge is kernel solvable, too.

Finally, to further the similarity between perfectness and kernel solvability, we show that odd holes are the only connected edge minimally non-kernel solvable graphs. This theorem is analogous to a result of Olaru (1972) claiming that odd holes are the only connected edge minimally non-perfect graphs.

On the hardness of the classical job-shop problem

Heidemarie Bräsel, Magdeburg

We consider the classical job-shop problem: n jobs have to be processed on m machines. The processing time for the operation (ij), i.e. for job i on machine j, is given for all i

and j. Each machine can process at most one job at a time and each job can be processed on at most one machine at a time.

The order of machines for the job i is called *machine order* of job i, the order of jobs on machine j is called *job order* on machine j. In the case of a job-shop problem all machine orders are given and all job orders have to be determined in such a way, that the makespan is minimal.

Computational experiments show that there are differences in the hardness of the job-shop problem depending on the given machine orders. We will explain this by means of the concept of reducibility of sequences. Therefore we consider a partial order ($\prec$) on the set of all sequences with the following property: Independent of the given processing times there exists an optimal sequence in the set of minimal elements (irreducible sequences). If the sequences A and B satisfy the relation $A \preceq B$ we call B reducible to A. For two given sequences A and B we present a polynomial algorithm which decides if B is reducible to A. Furthermore, an enumeration algorithm of all irreducible sequences will be given. This algorithm can also be applied to show the irreducibility of a sequence. Then we specify these considerations for the job-shop problem and combine the results with investigations of isomorphic sequences and isomorphic machine orders.

We demonstrate the results and the open questions in detail for the job-shop problem with 3 jobs and 3 machines. From $3!^3 = 216$ different machine orders we obtain only 7 orders which are structurally different. The distribution of the irreducible sequences on these classes yields an explanation of the above mentioned computational experiments.

The circulant traveling salesman problem

Rainer Burkard, Graz; joint work with W. Sandholzer, Q. Yang, E. Çela and G. Woeginger

Let the $(n \times n)$ matrix C with 0-1 entries be a circulant matrix. It is completely determined by the entries in its first row. C can be seen as adjacency matrix of a graph with n vertices denoted by $G(n; a_1, ..., a_k)$. G has an arc from node i to node j if $j - i \equiv a_t \pmod n$ for some $t \in \{1, 2, ..., k\}$. If $G(n; a_1, ..., a_k)$ is hamiltonian, then $\gcd(n, a_1, ..., a_k) = 1$. This condition is also sufficient for hamiltonicity if C is symmetric. For nonsymmetric circulant digraphs no necessary and sufficient condition for the hamiltonicity of G is known in general.

If G is an arbitrary circulant matrix, the shortest hamiltonian path problem can be solved by the nearest neighbour rule. It is not known, however, if the TSP in circulant graphs is an NP-complete problem or not. In symmetric circulant graphs the bottleneck TSP can be solved efficiently (Burkard and Sandholzer, 1991).

For circulant matrices with only two nonzero stripes, i.e. $G(n; a_1, a_2)$ we get the following results. If $\gcd(n, a_1, a_2) = 1$, then $G(n; a_1, a_2)$ is hamiltonian if there exists an integer $h, 0 \le h \le t := gcd(n, a_2 - a_1)$, such that $\gcd(n', a_1 + a'b) = 1$ for $n' = n/t$ and $a' = \frac{a_2 - a_1}{t}$. This condition can be checked in $O(log^4 n)$ time, a Hamiltonian cycle can be constructed in $O(n)$ time. Hamiltonian cycles in 2-stripe circulant digraphs show a regular periodic pattern which can be used to solve the TSP in these graphs efficiently (joint work with Q. Yang, E. Çela and G. Woeginger).

Approximation for subset interconnection designs

Ding-Zhu Du, Minneapolis; joint work with Xiufeng Du, Dean F. Kelley, and Guoliang Xue

Given a complete weighted graph on the set X of n vertices and subsets $X_1, \cdots, X_m$ of X, we consider the problem of finding a minimum total weight subgraph G such that for every $i = 1, \cdots, m$, G contains a spanning tree for X_i. We will refer this problem as the SID (subset interconnection designs). The SID has applications in computer science and statistics and has been studied extensively in the literature. It has been proved that the SID is NP-complete. Prisner introduced a polynomial-time heuristic with performance ratio $\ln m + O(1)$ in 1992. However, his proof is incorrect. In this note, we will give a correct proof. In addition, we show that the SID has no polynomial-time approximation with performance ratio $\rho \log m$ for $0 < \rho < 1/4$ unless NP $\subset$ DTIME($n^{\text{polylog} n}$). This means that Prisner's heuristic has almost best possible performance ratio. We also present another heuristc with performance ratio depending on only the maximum size of X_i's.

Characterizing bipartite Toeplitz graphs

Reinhard Euler, Brest

A non-oriented, simple graph is a *Toeplitz graph*, if its adjacency matrix A is a Toeplitz matrix, i.e. $A_{ij} = A_{i-1,j-1}$ for $2 \leq i, j \leq n$. Toeplitz-graphs are thus uniquely defined by the first line of A, a 0-1 vector. In this talk we address the question of characterizing bipartite such graphs in terms of the base-circuit language from matroid theory. A complete answer can be given for the infinite case by introducing the notion of an *odd T-cycle*, a 0-1 vector possessing exactly two 1-entries in certain positions, and which is minimal w.r.t. inducing an odd cycle in the corresponding Toeplitz-graph. A maximal 0-1 sequence defining a bipartite Toeplitz-graph can be described by $B(\alpha) = (0 \cdots 010 \cdots 010 \cdots 0 \cdots)$, a sequence containing α 0's first followed by blocks of 2α elements whose first is a 1, α being a power of 2. It turns out that an infinite 0-1 sequence I defines a bipartite Toeplitz-graph iff I does not contain an odd T-cycle iff I is contained in one of the $B(\alpha)$.

We also present our results on the finite case.

Bin packing games

Ulrich Faigle, Enschede; joint work with Walter Kern

A *bin packing game* is a cooperative N-person game, where the set of players consists of k bins of size 1 and n items of sizes $a_1, \ldots, a_n$. The value of a coalition of bins and items is the maximum total size of items in the coalition that can be packed into the bins of the coalition. Our main result asserts that for every $\epsilon > 0$, there exist ϵ-approximate core allocations provided k is large enough.

Conjecture: There exists a universal constant C such that for every bin packing game an allocation vector x can be found that $x(S) \geq v(S)$ holds for every coalition S and the gap $x(N) - v(N)$ is bounded by C (v denote the value).

The conjecture has been verified (with $C = 1/4$) for the case where each item i has size $a_i > 1/3$.

Rendezvous search

Sándor Fekete, Köln

In the problem of *Rendezvous Search*, we are to find good strategies for two players who wish to get together as quickly as possible. Depending on the scenario, these players may have visibility bounded by obstacles, by some finite radius, or visibility may be zero. In particular, we describe an optimal strategy for two players who know that the other player is at a distance of d; the direction is unknown, and the players will only see each other when they meet. This strategy also works when the players have different speed, or when player i can only travel a distance s_i and we want to maximize the probability of a meeting.

Lifted cycle inequalities for the asymmetric traveling salesman problem

Matteo Fischetti, Udine; joint work with Egon Balas

Lifted cycle inequalities for the asymmetric traveling salesman problem have been introduced by Grötschel and Padberg in 1977. We analyze the combinatorial structure of these inequalities. In particular, we give a complete characterization of the cycle chords that may receive a lifting coefficient of 2 in a lifted cycle inequality, and a partial characterization of the chords with coefficient equal to 0. We also investigate the Chvátal rank of lifted cycle inequalities, and exhibit cases with arbitrarily large rank. New large subclasses of lifted cycle inequalities are then described explicitly. Finally, we discuss a possible way of extending facet-defining inequalities for the asymmetric TSP to facet-defining inequalities for the symmetric TSP. The extension is based on the property of the asymmetric traveling salesman polytope on n nodes of being a face of the symmetric traveling salesman polytope on $2n$ nodes.

Design of survivable telecommunication networks I, II

Martin Grötschel and Roland Wessäly, Berlin

Designing low-cost networks that survive certain failure situations is one of the prime tasks in the telecommunication industry. In this series of two talks we survey the development of models for network survivability used in practice in the last ten years. We show how algorithms integrating polyhedral combinatorics, linear programming, and various heuristic ideas can help solve real-world network dimensioning instances to optimality or within reasonable quality guarantees in acceptable running times.

The most general problem type we address is the following. Let a communication demand between each pair of nodes of a telecommunication network be given. We consider the problem of choosing, among a discrete set of possible capacities, which capacity to install on each of the possible edges of the network in order to

(i) minimize the building cost of the network,

(ii) satisfy all demands.

In addition to determining the network topology and the edge capacities we have to provide, for each demand, a routing such that

(iii) no path in the routing exceeds a given length,

(iv) no path can carry more than a given percentage of the demand.

We also have to make sure that

(v) for every single node or edge failure, a certain percentage of the demand is reroutable.

Moreover, for all failure situations feasible routings must be computed.

The model described above has been developed in cooperation with e-plus, one of the German mobile phone providers. We describe our MIP-formulation of this model and report on computational results with data from practice.

Polynomial algorithms for finding all lexicographical and Pareto locations in a network with sum objectives

Horst Hamacher, Kaiserslautern; joint work with Martine Labbé and Stefan Nickel

Network location problems with several objectives are discussed, where every single objective is of the median (or sum) type.

The problem of finding all lexicographical optimal locations reduces to the following problem: Given a finite set of vectors, find for all possible permutations of the components the lexicographically minimal vectors. Moreover it is shown that the problem of finding all Pareto locations can be reduced to a combinatorial optimization problem, where only so-called bottleneck points on the edges need to be investigated. Polynomial algorithms for both problems are presented.

Connectedness in Boolean functions

Peter Hammer, New Brunswick; joint work with Oya Ekin and Alexander Kogan

A Boolean function is called (co-) connected if the subgraph of the Boolean hypercube induced by its (false) true points is connected; it is called strongly connected if it is both connected and co-connected. The concept of (co-)geodetic Boolean functions is defined in a similar way by requiring that at least one of the shortest paths connecting two (false) true points should consist only of (false) true points. This concept is further strengthened to that of convexity where every shortest path connecting two points of the same kind should consist of points of the same kind. This paper studies the relationships between these properties and the DNF representations of the associated Boolean functions.

Gröbner bases of vertex cover problems

Winfried Hochstättler, Köln

We consider the family $\mathcal{I} = (I(b))_{b \in \mathbb{Z}^n}$ of integer programs

$$\begin{array}{rl} \text{minimize} & 1^\top x \\ \text{subject to} & (A^\top, -I)\, a = b \\ & a_i \in \mathbb{Z}^{\geq 0} \end{array} \qquad (I(b))$$

where A is the incidence matrix of

1. the complete bipartite graph $K_{n,n}$,
2. the complete graph K_n.

Using a result of Conti and Traverso we give — for specific lexicographic orders on the variables — simple combinatorial characterizations of the Gröbner bases of the ideal

$$\langle x^{\alpha^+} - x^{\alpha^-} \mid (A, I)\, \alpha = 0, \alpha = \alpha^+ - \alpha^-, \alpha^+, \alpha^- \in \mathbb{Z}^{\geq 0} \rangle.$$

In the case where b is a 0-1-vector the program $I(b)$ is the minimum vertex cover problem for the graph $G = (V, E))$ where $e \in E \Leftrightarrow b_e = 1$.

Physical mapping of chromosomes using end-probes: Exact solution of a betweenness formulation by branch and cut

Michael Jünger, Köln; joint work with Thomas Christof, John Kececioglu, Petra Mutzel, and Gerhard Reinelt

A fundamental problem in computational biology is the construction of physical maps of chromosomes from hybridization experiments between unique probes and clones of chromosome fragments in the presence of error. Alizadeh, Karp, Weisser and Zweig (1994) first considered a maximum-likelihood model of the problem that is equivalent to finding an ordering of the probes that minimizes a weighted sum of errors, and developed several effective heuristics. We show that by exploiting information about the end-probes of clones, this model can be formulated as a weighted Betweenness Problem. This affords the significant advantage of allowing the well-developed tools of integer linear-programming and branch-and-cut algorithms to be brought to bear on physical mapping, enabling us for the first time to solve small mapping instances to optimality even in the presence of high error.

Voronoi diagrams in piecewise linear surfaces

Thomas Liebling, Lausanne; jointly with C. Indermitte, H. Clemençon and M. Troyanov

We consider a triangulated piecewise linear surface and present a generalization of the well-known flip algorithm to construct the Voronoi partition and its dual Delaunay graph generated by the vertices of the given triangulation. This construction is this valid for arbitrary, not necessarily orientable, possibly self intersecting surfaces and it also covers the particular case of the flat torus. The construction relies on the notions of universal branched covering and development. and on an efficient description of geodesics using dcel data structures. This work was motivated by mycelial wall growth modeling.

On the scheduling of one-dimensional transport systems

Christoph Moll, München; joint work with Udo Heinrichs

We consider the problem of finding a feasible routing for one-dimensional transport systems. This problem arises as a subproblem in production planning, when a mono-rail crane with several crabs is part of the production lane. Practical restrictions like finite speed of crabs and a minimal distance between crabs lead to a special routing problem.

Each transport task is defined by a start position, an end position, a starting time and a duration. Thus a task corresponds to a segment in the Euclidean plane, where the x-axis corresponds to positions and the y-axis represents time. "Life lines" of crabs correspond to monotonic paths in this diagram. In practice crabs can only move with finite speed. Thus we consider monotonic, disjoint, angle-restricted curves as a representation of possible life lines.

This leads to an angle-restriction on possible curves. We call these curves super-monotonic. Using these assumptions and notation we can formulate the problem of finding a feasible schedule for a crane with k crabs as the following geometric problem:

Given a set of super-monotonic disjoint line segments in the plane. Is there a set S of k super monotonic, disjoint curves covering all segments?

We prove that each set S corresponds to a partition of a partial order into anti-chains, where the partial order is induced by the geometric situation. This characterization leads to an efficient algorithm to solve the decision problem.

If we take into account an uniform width of crabs, we have to study sets S of curves that have at least x-distance 1. We prove that this question can be formulated as a shortest path problem on a graph induced by the geometric situation.

Finally we discuss how the presented algorithms can be used as a module for a greedy-algorithm solving no-wait scheduling problems.

Approximation algorithms for scheduling problems with communication delays

Rolf Möhring, Berlin; joint work with Markus Schäffter and Andreas Schulz

In the last few years, multi-processor scheduling with interprocessor communication delays has received increasing attention. This is due to the more realistic constraints in modeling parallel processor systems. Most research in this vein is concerned with the makespan criterion.

We contribute to this work by presenting a new and simple $(2-\frac{1}{m})$-approximation algorithm for scheduling to minimize the makespan on identical parallel processors subject to series-parallel precedence constraints and both unit processing times and communication delays. This meets the best known performance guarantee for the same problem but without communication delays. For the same problem but with (non-trivial) release dates, arbitrary precedence constraints, arbitrary processing times and "locally small" communication delays we obtain a simple $\frac{7}{3}$-approximation algorithm compared with the involved $(\frac{7}{3}-\frac{4}{3m})$-approximation algorithm by Hanen and Munier for the case with identical release dates.

Another quite important goal in real-world scheduling is to optimize average performance. We study for the first time scheduling with communication delays to minimize the average weighted completion time. Specifically, based on an LP relaxation we give the first constant-factor polynomial-time approximation algorithm for scheduling identical parallel processors subject to release dates and locally small communication delays. Moreover, the optimal LP value provides a lower bound on the optimum with the same worst-case performance guarantee.

The common underlying idea of our algorithms is to compute first a schedule that regards all constraints except for the processor restrictions. This schedule is then used to construct a provably good feasible schedule for a given number of processors and as a tool in the analysis of our algorithms. Complementing our approximation results, we also show that minimizing the makespan on an unrestricted number of identical parallel processors subject to series-parallel precedence constraints, unit-time jobs, and zero-one communication delays is NP-hard.

Lifted flow cover inequalities for 0-1 mixed integer programs

George Nemhauser, Atlanta; joint work with Z. Gu and M. W. P. Savelsbergh

We investigate strong inequalities for mixed 0-1 integer programs derived from flow cover inequalities. Flow cover inequalities are usually not facet defining and need to be lifted to obtain stronger inequalities. However, because of the sequential nature of the standard lifting techniques and the complexity of the optimization problems that have to be solved to obtain lifting coefficients, lifting of flow cover inequalities is computationally very demanding. We present a computationally efficient way to lift flow cover inequalities based on sequence independent lifting techniques and computational results that justify the effectiveness of our lifting procedures.

Global optimization approaches for general maximum clique problems

Panos Pardalos, Gainesville; joint work with L. Gibbons, D. Hearn, and M. Ramana

Given a graph G whose adjacency matrix is A, the Motzkin-Strauss formulation of the Maximum-Clique Problem is the quadratic program $\max\{x^TAx \mid x^Te = 1, x \geq 0\}$. Based on this formulation an efficient heuristic has been developed (Gibbons, Hearn and Pardalos 1996). Furthermore, we study the quadratic program and provide polynomial time recognition algorithms for verifying if a given point is a first order point, a second order point, or a local maximum (for general quadratic programs these are NP-hard problems). Finally, a parametrization of the Motzkin-Strauss is introduced and an extension for the Motzkin-Strauss formulation is provided for the weighted clique number of a graph (Gibbons, Hearn, Pardalos, and Ramana 1996). This work is part of our efforts to study continuous approaches for several discrete optimization problems.

Some equivalent bounds for Quadratic Boolean Programs

Franz Rendl, Graz; joint work with Svata Poljak and Henry Wolkowicz

Various ways have been proposed to derive tractable relaxations of

$$\text{maximize } x^tQx - 2c^tx \text{ subjet to } x_i^2 = 1.$$

These include convex quadratic programming,

$$B_{cp} := \min_{Q+U\preceq 0} \max_{|x_i|\leq 1} q_u(x), \text{ where } q_u(x) := x^t(Q+U)x - 2c^tx - u^te$$

and U =diag(u). The trust region model leads to

$$B_{tr} := \min_u \max_{x^tx=n} q_u(x).$$

Homogenization yields the eigenvalue bound

$$B_{eo} := \min_v (n+1)\lambda_{\max}(Q^c - V) + v^te, \text{ where } Q^c := \begin{pmatrix} 0 & -c^t \\ -c & Q \end{pmatrix}.$$

The semidefinite relaxation is

$$B_{sd} := \max_{Y\succeq 0, \operatorname{diag}(Y)=e} \operatorname{trace} Q^cY.$$

It is shown that these bounds are all equal to the Lagrangian relaxation

$$B_{ld} := \min_u \max_x q_u(x).$$

Sparse maximum cuts

Giovanni Rinaldi, Rome; joint work with Michael Jünger and Gerhard Reinelt

In the past few years many new results have been obtained on the maximum cut problem in complete graphs that make it now possible to find an optimal solution for instances of size up to one hundred nodes in a moderate amount of time. On the other hand, very little work has been done on this problem for arbitrary (sparse) graphs. One of the motivations for studying the maximum cut problem for sparse graph comes from a classical application to Statistical Physics: the exact determination of a minimal energy configuration of a spin glass under no exterior field and under a continuously varying exterior magnetic field. In the first case, typically one has to solve very large but sparse instances of max-cut to optimality; in the second one has to provide an optimal solution for all members of a family of objective functions.

To solve these kinds of problems, we introduce new separation and lifting procedures for the cut polytope on arbitrary (sparse) graphs. These procedures exploit any algorithmic and structural results known for the cut polytope on complete graphs to generate valid (and sometimes facet defining) inequalities for the cut polytope on arbitrary graphs, which are violated by the current fractional solution of a cutting-plane or of a branch and cut algorithm. Based on these procedures, we developed a branch and cut algorithm that we used to run experiments on 2-dimensional toroidal instances of spin glasses of sizes up to 22 500 spins. The results of these experiments are sometimes in agreement with those that can be found in the Physics literature. Surprising enough, in some cases they disagree with the theoretical predictions.

Two-dimensional voltages, geometric tensions in graphs, and matching of point sets with reference points

Günter Rote, Graz; joint work with Helmut Alt and Oswin Aichholzer

1. Geometric tensions. Given a graph G with an *offset* vector $d_{ij} \in \mathbb{R}^n$ and a tolerance $\varepsilon_{ij} \geq 0$ for each arc ij, we want to find a point $x_i \in \mathbb{R}^n$ for each vertex i such that, for all ij, $\|(x_j - x_i) - d_{ij}\| \leq \varepsilon_{ij}$. (We assume that for every arc ij, the reverse arc ji also belongs to the graph, with $d_{ji} = -d_{ij}$ and $\varepsilon_{ji} = \varepsilon_{ij}$.) In $n = 1$ dimension, such a solution exists if and only if, for every directed cycle C in the graph, $\left|\sum_{ij\in C} d_{ij}\right| \leq \sum_{ij\in C} \varepsilon_{ij}$. (With $\varepsilon_{ij} \equiv 0$, such values x_i are called *voltages*, and the d_{ij} are called *tensions* or potential differences.) In higher dimensions the condition must be strengthened in order to be sufficient: An n-dimensional voltage x_i exists if, for every cycle C,

$$\left\|\sum_{ij\in C} d_{ij}\right\|_2 \leq \left(\sum_{ij\in C} \varepsilon_{ij}\right) / \chi_n, \qquad (*)$$

with $\chi_n = \frac{2\Gamma(n/2+1)}{\sqrt{\pi}\Gamma(n/2+1/2)} \approx \sqrt{n/(2\pi)}$.

2. Shape matching with reference points.
Given two "shapes" $A, B \subset \mathbb{R}^n$, we want to translate them in such a way that their Hausdorff distance is as small as possible:

$$\min_{t\in\mathcal{T}} \delta^{\mathrm{H}}(A, B + t), \qquad (**)$$

where $\mathcal{T}$ is the set of translations or a more general set of transformations. This problem arises is pattern recognition, image processing, and computer vision. An important class of heuristics for this optimization problem uses *reference points*: A reference point assigns to every set $A \subset \mathbb{R}^n$ in a certain class of sets (e.g., compact convex sets) a characteristic point $r(A)$ such that

$$\|r(A) - r(B)\|_2 \leq K \cdot \delta^{\mathrm{H}}(A, B),$$

for some constant K. The reference point heuristic for shape matching restricts the possible transformations t in $(**)$ to those transformations that map $r(A)$ to $r(B)$. This reduces $\mathcal{T}$ by two degrees of freedom and makes the optimization problem $(**)$ easier to solve. On the other hand, the solution gets worse by a factor of at most $1 + K$, for translations and rigid motions, and at most $3+K$, for similarity transformations. The best constant K for convex sets A is achieved by the *Steiner point*

$$s(A) = n \cdot \int_{u \in S^{n-1}} h_A(u) u \, d\mu(u),$$

where $h_A(u) = \max\{\langle u, x\rangle \mid x \in A\}$ is the support function of A and integration is with respect to the normalized surface area measure on the $(n-1)$-dimensional sphere S^{n-1}. The constant K for the Steiner point is χ_n. Optimality of the Steiner point was proved by Przesławski and Yost (1965), extending a special case of Rutovitz (1965). These proofs are nonconstructive since they use tools that are related to the axiom of choice.

3. The relation between reference points and voltages.
For a set of (convex or non-convex) shapes $A_1, A_2, \ldots$ we can define a vertex i for each set A_i and set $d_{ij} = \arg\min_{t \in \mathcal{T}} \delta^{\mathrm{H}}(A_i, A_j + t)$ and $\varepsilon_{ij} := K \cdot \min_{t\in\mathcal{T}} \delta^{\mathrm{H}}(A_i, A_j + t)$. ($\mathcal{T}$ is the set of translations.) Then $(*)$ is fulfilled. So every geometric voltage would give a set of reference points for the given set of figures. A constructive proof of the lower bound on K would proceed as follows.

(1) Find a graph with given tensions and tolerances satisfying $(*)$ for which the constant K cannot be decreased beyond a bound.

(2) Find such a graph that comes from geometric shapes $A_1, A_2, \ldots$ as described above.

(3) Find such a graph that comes from *convex* shapes.

We know from the theorem of Przesławski and Yost that such a graph showing a lower bound of $4/\pi$ must exist. Currently we have only a solution of (2) with a lower bound of $\sqrt{4/3}$. The corresponding graph has two nodes and three parallel edges whose vectors d_{ij} are three unit vectors with angles of 120°, and $\varepsilon_{ij} = 1$.

Bipartite edge coloring

Alexander Schrijver, Amsterdam

We show that a minimum edge-colouring of a bipartite graph can be found in $O(\Delta m)$ time, where Δ and m denote the maximum degree and the number of edges of G, respectively. It is equivalent to finding a perfect matching in a k-regular bipartite graph in $O(km)$ time.

By sharpening the methods, a minimum edge-colouring of a bipartite graph can be found in $O((p_{\max}(\Delta) + \log\Delta)m)$ time, where $p_{\max}(\Delta)$ is the largest prime factor of Δ. Moreover, a perfect matching in a k-regular bipartite graph can be found in $O(p_{\max}(k)m)$ time.

Advanced approximation techniques in min-sum criteria scheduling: off-line and on-line algorithms

Andreas Schulz, Berlin

The last year has seen an amazing improvement in approximation results for NP-hard scheduling problems in which the objective is to minimize the weighted sum of the job completion times. These results are based on the development of several new techniques. For a variety of scheduling models, these techniques yield the first algorithms that are guaranteed to find schedules that have objective function value within a constant factor of the optimum.

The progress on these problems follows essentially from two basic approaches. In the first approach a linear programming relaxation of the scheduling problem is solved, and then a schedule is constructed simply by list scheduling in a natural order dictated by the linear program solution. The second approach is a general on-line framework, in which one attempts to pack the most profitable jobs into successive intervals of geometrically increasing size. These two approaches have also been combined, by using the LP relaxation to assign jobs to intervals.

The aim of this talk is to present and highlight in a unifying way the most important ideas from these approaches.

Combinatorial optimization based on coupled selection equations

Jens Starke, Stuttgart

A new approach for combinatorial optimization problems based on coupled selection equations will be introduced. This method works with a specifically constructed nonlinear dynamical system with suitable stable points and suitable basins of attraction. The idea will be explained using the example of assignment problems.

The choice between several decisions of the combinatorial optimization problem is mapped to the competition between stable points, i. e. to the stay in one of the basins of attraction of the dynamical system. There exists a bijective mapping from the set of feasible solutions to the set of stable points. Additional constraints can easily be considered by extending the dynamical system. To obtain the necessary adaptation, specific coupling terms are used to result in a suitable selection of decisions and feasible solutions as stable points.

In comparison to many other methods this approach has the advantage that even complicated additional constraints of the optimization problem can easily be considered. Furthermore, parallel hardware realizations of this approach are possible because of the similarity to models of complex physical and chemical systems.

Set packing under tolerance constraints

Gottfried Tinhofer, München

A k-packing problem under tolerance constraints is a set packing problem on a family $\mathcal{T}$ of subsets of $N = \{1, 2, \ldots, n\}$ where, given some vector $z \in \mathbb{R}^n$ with $z_1 \leq \ldots \leq z_n$, the family $\mathcal{T}$ is defined by

$$\mathcal{T} = \left\{ T \subset N \;\middle|\; |T| = k \wedge \sum_{j \in T} z_j \in [-1, 1] \right\}.$$

Such a $\mathcal{T}$ is a poset (sort the elements of $T \in \mathcal{T}$ according to their z-value and compare T and T' componentwise) and has the following property:

$$T, T' \in \mathcal{T} \wedge T \leq T'' \leq T' \Longrightarrow T'' \in \mathcal{T}$$

which is called *order compatibility*. Order compatible problems may be presented by their sets $\mathcal{T}_{\min}$ and $\mathcal{T}_{\max}$ of minimal and maximal elements.

In this talk we consider order compatible packing problems and discuss a solution method which is applied on $(\mathcal{T}_{\min}, \mathcal{T}_{\max})$ and does not use an explicit representation of the intersection graph of $\mathcal{T}$.

Rectilinear p-centers in the plane

Emo Welzl, Zürich; joint work with Micha Sharir

We consider the *p-piercing problem*, in which we are given a collection of regions, and wish to determine whether there exists a set of p points that intersects each of the given regions. We give linear or near-linear algorithms for small values of p in cases where the given regions are either axis-parallel rectangles or convex c-oriented polygons in the plane (i.e., convex polygons with sides from a fixed finite set of directions).

We also investigate the planar *rectilinear (and polygonal) p-center problem*, in which we are given a set S of n points in the plane, and wish to find p axis-parallel congruent squares (isothetic copies of some given convex polygon, respectively) of smallest possible size whose union covers S. We also study several generalizations of these problems.

New results are a linear-time solution for the rectilinear 3-center problem (by showing that this problem can be formulated as an LP-type problem and by exhibiting a relation to Helly numbers). We give $O(n \log n)$-time solutions for 4-piercing of translates of a square, as well as for the rectilinear 4-center problem; this is worst-case optimal. We give $O(n \log^{O(1)} n)$-time solutions for 4- and 5-piercing of axis-parallel rectangles, for more general rectilinear 4-center problems, and for rectilinear 5-center problems. 2-pierceability of a set of n convex c-oriented polygons can be decided in time $O(c^2 n \log n)$, and the 2-center problem for a convex c-gon can be solved in $O(c^5 n \log n)$ time. The first solution is worst-case optimal when c is fixed.

Approximation algorithms for scheduling

Gerhard Woeginger, Graz; joint work with Noga Alon, Yossi Azar, and Tal Yadid

We consider the classic scheduling/load balancing problem where there are m identical machines and n jobs, and each job should be assigned to some machine. Traditionally, the assignment of jobs to machines is measured by the makespan (maximum load) i.e., the L_∞ norm of the assignment. An ϵ-approximation scheme for minimizing the L_∞ norm was given by Hochbaum and Shmoys (1987).

In several applications, such as in storage allocation, a more appropriate measure is the sum of the squares of the loads (which is equivalent to the L_2 norm). This problem was considered e.g. by Chandra and Wong (1975), by Cody and Coffman (1976), and by Leung and Wei (1995) who showed how to approximate the optimum value by a factor of about 1.04. In fact, a more general measure, which is the L_p norm (for any $p \geq 1$) can also be approximated to some constant (see Chandra and Wong) which may be as large as 3/2. We improve these results by providing an ϵ-approximation scheme for the general L_p norm (and in particular for the L_2 norm).

Heuristics for mixed-integer programming

Laurence Wolsey, Louvain-la-Neuve

Using a general purpose mixed integer programming system, we consider different ways to try to find good quality feasible solutions when the default strategies fail. Ideas discussed include the use of surrogate constraints and variables, as well as heuristics based on the relaxation of integrality, fixing of variables and tightening of bounds, each involving the solution of two or more mixed integer programs simpler than the original problem. The performance of such ideas on a set of large practical problems arising in network design, multiplexer assignment, electricity generation and production planning models are presented.

Assignment of local transport vehicles to to routes and sidings minimizing shunting cost in storage yards

Uwe Zimmermann, Braunschweig; joint work with Thomas Winter

Assigning vehicles to positions in a depot as well as to the subsequent departure from the depot is part of the daily routine in public transportation systems. For example, trams are parked on tracks within storage buildings on sidings which can be modeled by stacks or queues. Since the daily timetable requests different types of trams at different times, it is crucial to plan the assignments well ahead. In order to avoid unnecessary movements (shuntings) on the storage yard which are very costly, the dispatcher tries to find a shunting-free assignment. We present two models for minimizing shunting. The first generalizes a container shipment model, the second is a specially structured quadratic assignment model with additional constraints. Both 0-1 models are not only NP-hard but seem to be quite difficult to attack by known methods even for the practical data we have at the time being. The second model easily decomposes into an arrival and a departure model of symmetric structure. Although even the departure problem is NP-hard, it can polynomially be solved if the number of stacks is fixed. The corresponding DFS algorithms constructs a shunting free assignment provided there exists one. For the minimization of shunting, reactive tabu search is a fast alternative, at least for the given practical data from several German storage yards. A simultaneous solution of arrivals and departures is under investigation and has to include a study of the online effects of daily dispatching.

Open Problems

1. Large neighborhoods for the traveling salesman problem.

   The well-known *pyramidal tours* form a large set of cyclic permutations over which we can optimize in polynomial time. The set of all cyclic permutations is even larger, but in this case we do not know how to optimize in polynomial time. The question is to find something in between: Very large (= $\exp(n \log n)$ permutations) and easy to handle (polynomial-time optimization routine). In other words, you should complete line 3 of the following table.

Neighborhood	Size	log(Size)	Time for Optim.
PYRAMIDAL	$\Theta(2^n)$	$\Theta(n)$	$O(n^2)$
PQ-TREE	$2^{\Theta(n \log\log n)}$	$\Theta(n \log\log n)$	$O(n^3)$
?????	$2^{\Theta(n \log n)}$	$\Theta(n \log n)$	$O(\text{poly}(n))$
ALL TOURS	$(n-1)!$	$\Theta(n \log n)$	$O(n^2 2^n)$

   [BDW] R. E. Burkard, V. G. Deĭneko, and G. J. Woeginger, The Traveling Salesman and the PQ-Tree, *Proceedings of IPCO V*, 1996, Springer LNCS **1084**, 490–504.

   G. Woeginger

2. INSTANCE: Integers d, n, m; n points and m half-spaces in $\mathbb{R}^d$.

   QUESTION: Is the convex hull of the points equal to the intersection of the half-spaces?

   Is there a polynomial-time algorithm for this problem? Is the problem NP-hard?

   G. Rote

3. The master tour problem for the asymmetric traveling salesman problem.

   An optimal traveling salesman tour T in a directed graph is called a *master tour* if the optimal tour for every smaller problem that is obtained by deleting some cities (i.e., deleting corresponding rows and columns of the cost matrix) is obtained by deleting these cities from T and visiting the remaining cities in the same order as on T.

   Characterize those cost matrices for which a master tour exists, and find a polynomial algorithm for checking this condition.

   R. Burkard

4. Let $G = (V, E)$ be a bipartite graph with a partial order $P = (E, \leq)$ on the edge set E. A *P-matching* in G is a subset $M \subseteq E$ such that any two members of M are either disjoint or comparable relative to P. Give an efficient algorithm for finding a P-matching of maximal cardinality.

   U. Faigle

5. Voronoi diagrams in piecewise linear surfaces.

   Find an efficient algorithm that computes the Voronoi diagram of a set of given points (sites) on a piecewise linear surface, without assuming that all vertices of the surface belong to the set of sites.

   T. Liebling

6. For a 0/1-polytope P in n dimensions given by a subroutine for optimization, the *adjacent augmentation problem* consists in asserting, for a given 0/1-point $x \in P$ and an objective function vector c, whether x is optimal with respect to c and, if not, in determining another 0/1-point $y \in P$ with $c \cdot y > c \cdot x$ and which is adjacent to x on P. Can one solve the adjacent augmentation problem in polynomial time, given a subroutine that solves the *optimization problem* for P (for any linear objective function) in polynomial time?
A. Schulz

7. For a planar graph $G = (V, E)$ embedded in the plane and a positive integer K, we are looking for a subset E' of E whose removal transforms G into an even graph, i.e., a graph with only vertices of even degree, such that property (a) resp. properties (a) and (b) are satisfied.

   (a) E' contains no path of length K where two subsequent edges on the path belong to the same face.

   (b) E' contains no star of degree K such that two subsequent edges w.r.t. the planar embedding belong to the same face, except the first and the last edge.

   1. What is the complexity status of this problem? 2. Is there a K for which such a set E' always exists?
D. Wagner

8. Let $A \in \mathbb{N}_0^{m \times n}$ be a matrix with non-negative integers as entries. A partial order on A's columns can be defined by setting

$$A_{\cdot j} \leq A_{\cdot k} :\Longleftrightarrow a_{ij} \leq a_{ik} \quad \forall i = 1, \ldots, m.$$

   Is it possible to find the minimal elements of this partial order in less than $O(n^2)$ time?

   A fast algorithm for this problem would be useful in

   1. preprocessing for set partitioning problems

$$\begin{aligned} \min c^T x & \\ Ax &= 1 \\ x &\in \{0,1\}^n, \end{aligned}$$

   where one would be interested in finding all minimal columns of A^T (i.e., the non-dominated rows of A), and in the

   2. computation of test sets, where one iteratively constructs a larger and larger set of test vectors and in turn eliminates all dominated ones until the test set is found (here, also negative entries would appear, but the problem is in principle the same).
R. Borndörfer

9. Geometric wire routing with few bends.

   Suppose we are given $2n$ point, grouped in n point pairs, such that point pair i consists of the two points $p_{i,1} = (0, i)$ and $p_{i,2} = (1, \pi(i))$, where π is any permutation. A *feasible wire routing* consists of n pairwise disjoint polygonal paths ("wires") P_i, such that path P_i has end points $p_{i,1}$ and $p_{i,2}$. For a given feasible wire routing, let

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$b(P_i)$ be the number of edges of path P_i. Furthermore, let $B(n)$ be the largest number of edges in a wire that may be necessary, i. e.,

$$B(i) = \max_{\pi \in S(n)} \min\{ \max_{1 \leq i \leq n} b(P_i) : (P_i) \text{ feasible} \}.$$

Currently, the best known upper bound on $B(n)$ is the obvious $O(n)$, the best lower bound [BF] is $\Omega(\log n)$. Can you improve these bounds? A conjecture is $B(n) = \sqrt{n}$.

Context: It has been shown [BF] that it is NP-hard to decide whether a given arrangement of point pairs has a feasible wire routing with at most two edges per wire; if the edges have to run axis-parallel, feasibility of a layout with at most two edges per path is a polynomial problem, while checking the existence of a feasible axis-parallel routing with at most three edges per wire is NP-complete. It would be interesting to get good performance bounds for an approximation method; the general case can be reduced to the specific ("bipartite") layout of the point pairs described above.

[BF] Oliver Bastert, Sándor Fekete. *Geometrische Verdrahtungsprobleme*. Manuscript, 1996.

S. Fekete

10. An *Abstract Optimization Problem* (AOP) is a triple $(n, <, \Phi)$, where $<$ is a total order on the subsets of $H := \{1, \ldots, n\}$ and Φ is an oracle that, given two subsets $F \subseteq G$ of H, either reports that $F = \min_< \{ F' \subseteq G \}$ or returns a set $F' \subseteq G$ with $F' < F$. There is a randomized algorithm for finding the minimum subset of H in an expected number of $\exp(2\sqrt{n} + o(\sqrt{n}))$ oracle calls, whereas any deterministic algorithm has to make $2^n - 1$ oracle calls in the worst case [G].

    Problem: Find nontrivial lower bounds or improved upper bounds in the randomized setting. A better upper bound would imply a new combinatorial bound for linear programming and related problems.

    [G] Bernd Gärtner, A subexponential algorithm for abstract optimization problems, *SIAM J. Comput* **24** (1995), 1018–1035.

    E. Welzl

11. Find a polynomial-time algorithm for the following problem. Given a plane graph and vertices A_1, B_1, A_2, B_2, A_3, A_4, B_4, B_3 which lie on the boundary of the outer face in this order, compute four vertex-disjoint paths between A_i and B_i of smallest total length (number of edges).

    A. Schrijver

14. The Traveling Scientist's Problem.

    The scientist's home university is located at p_0 in the unit square. In addition there is a set N of n points in the unit square (the universities he wants to visit). Given a subset S of N (a "coalition"), let $T(S)$ denote the length of a shortest tour starting and ending in p_0 and visiting all nodes in S. Let $R(N)$ be the maximum total amount of travel cost refunding the scientist can get for visiting all nodes in N, i.e.,

$$R(N) = \text{maximize } x_1 + \cdots x_n \\ \text{subject to } x(S) \leq T(S) \text{ for each coalition } S,$$

assuming that no coalition is willing to contribute more than $T(S)$. We can show that

$$\text{HK}(N) \leq R(N) \leq T(N),$$

where HK denotes the Held-Karp lower bound. This seems to suggest that there is a limit theorem for $R(N)$, i.e., that

$$R(N) \to \text{const} \times \sqrt{n} \quad \text{(a.s.)}$$

if n tends to infinity and the n points are uniformly distributed in the unit square, but we miss a rigorous proof of this.

A second (more interesting) problem is to investigate constant. Personally I would not be surprised if this constant equals the corresponding constant for TSP tour lengths, i.e., if it turns out that with high probability the scientist can get an arbitrarily large percentage of his travel costs refunded.

M. Steele: Probabilistic and worst-case analyses of classical problems of combinatorial optimization in Euclidean space, *Math. of Oper. Res.* **15** (4), 1990, 749–770.

M. Goemans, D. Bertsimas: Probabilistic analysis of the Held and Karp lower bound for the euclidean TSP, *Math. of Oper. Res.* **16** (1), 1991, 72–89.

U. Faigle et al.: On approximately fair cost allocation in euclidean TSP games, *OR Spectrum* (to appear soon). *W. Kern*

15. Consider the integer lattice $L(M)$ generated by the incidence vectors of the cocycles of a binary matroid M on E. The dual lattice L^* of L is defined as

$$L^* = \{x \in \mathbb{Q}^E \mid \forall y \in L : x^\top y \in \mathbb{Z}\}.$$

In [LS] it is proven that the following two statements are equivalent:

(i) A vector $w \in \mathbb{R}^E$ is in $L^*(M)$ if and only if $w = w' + (1/2)C$, where w' is integral and C is a cycle ("the obvious sufficient conditions for $w \in L^*(M)$ are necessary");

(ii) M does not contain the binary sum of any set of planes of F^2 (the Fano plane) as a restriction minor.

Clearly this property is not preserved under taking minors, but preserved under deleting elements. Since cocircuits behave much nicer w.r.t. to contraction an affirmative answer to the following problem would help to get more insight into this class of binary matroids.

Problem: If a binary matroid M satisfies (i) resp. (ii), does there always exist some $x \in M$ such that contracting x preserves the property?

[LS] L. Lovász and Á. Seress: The cocycle lattice of binary matroids, *Europ. J. Comb.* **14** (1993), 241–250. *W. Hochstättler*

16. Determine the number of paths with length r, $1 \le r \le nm$, in the Hamming graph $K_n \times K_m$. *H. Bräsel*

17. Given are n points p_i in $\mathbb{R}^d$, where d is small, say $d \le 10$, and n can be quite large, perhaps $n \approx 1000$. Moreover, $\|p_i\| \le 1$. Furthermore, there are m query points $q_j \in \mathbb{R}^d$. Typically, m is much larger than n, perhaps $m = O(n^2)$.

Now I would like to find for each q_j all points p_i, satisfying $\|q_j - p_i\|_\infty \le \varepsilon$, where $\varepsilon > 0$ is a given, small constant. *F. Rendl*

18. Let $G = (V, E)$ be a graph. For $S \subset V$ let $\sigma(S)$ be the size of the largest component of $G - S$, and
$$s(G) = \min\{\, |S| : S \subset V \wedge \sigma(S) < \frac{|V|}{2} \,\}.$$
I would like to have a sequence $G_t = (V_t, E_t)$ of graphs with the properties
$$|V_t| = O(s(G_t)), \quad |E_t| = O(|V_t|),$$
preferably, the G_t's should be cubic graphs. The series should be explicitly defined, an existence proof only does not help. *G. Tinhofer*

19. Can you optimize/separate in polynomial time over the first Chvátal closure of the 0-1 Knapsack polytope? *M. Fischetti*

20. The complexity status of the following scheduling problem is unknown: n unit time jobs have to be scheduled on 2 machines subject to precedence constraints and unit-time communication delays. The makespan (completion time of the last job) has to be minimized. The communication delay means that, if job i has to precede job j according to the given precedence relation, job j must wait one time unit after the completion of job i before it can be started, unless job j is scheduled on the same machine as job i. *J. K. Lenstra*

21. The previous problem is open even when the precedence constraints are restricted to series-parallel partial orders. *R. Möhring*

22. A Toeplitz graph is a graph whose adjacency matrix is a Toeplitz matrix. What is the complexity of calculating the stability number and the chromatic number in an arbitrary Toeplitz graph? *R. Euler*

23. Simultaneous motion planning on a line

    In the talk we used segments as a model for transport tasks. This corresponds to the case that the routing of transports is fixed. In practice crabs transporting goods may make way for other crabs, too. The corresponding geometric problem is the following:

    A curve is called super-monotonic if, for any two points (x_1, y_1) and (x_2, y_2) on the curve, $|x_1 - x_2| \leq |y_1 - y_2|$. Given a set of super-monotonic line segments in the plane. Is there a set of k super monotonic, disjoint curves with the following properties:

    1. The endpoints of all segments are covered.
    2. If a curve covers an endpoint of a segment it covers the other endpoint, too.
    3. The subcurve induced by two endpoints of a segment, does not cover an endpoint of an other segment.
    4. All curves have at least x-distance 1.

An efficient algorithm for this problem would be helpful in a lot of practical cran scheduling problems. The results presented in the talk suggest to study the problem without property (4) first. Even in this case no polynomial time algorithm is known. *C. Moll*

24. Prove that the chromatic number of a graph is bounded from above by the rank of its adjacency matrix, or provide a counter-example. *P. Pardalos*

25. The Klee-Minty Game.

What is the expected number of steps taken by the following *Klee-Minty game* KM_n? Is it quadratic, $\Theta(n^2)$? That is, is there a constant $c > 0$ such that the expected number of steps of the Klee-Minty game KM_d, started at the zero string, and selecting a random 0 for each step, is at least cd^2?

Start with a string of d 0s (corresponding to the vertex at the origin). Then for each step, one selects one 0 in the string, and flips this 0 *together with all the bits to its right.* (Here a *flip* changes a 0 into a 1, and a 1 into a 0.) For example, for $d = 8$ one might get

$$00000000 \longrightarrow 00000011 \longrightarrow 00111100 \longrightarrow 00111101 \longrightarrow 01000010 \longrightarrow \cdots$$

The game stops when one reaches the string $1111\ldots11$ that does not have a 0.

One can show that the expected number of steps is less than $.27d^2$ for large d from any starting vertex (an upper bound of $\binom{d+1}{2}$ is very easy to see), while it is more than d when starting from the zero string/vertex. That leaves a gap: what is the expected number of steps the RANDOM EDGE rule on the Klee-Minty game KM_d, if one starts with the zero string? We don't know! However, Gärtner and Ziegler [GZ,GHZ] established that the expected number of steps is at least $\frac{d^2}{4(\ln(d)-1)}$ *when starting at a random starting vertex.*

[GHZ] B. Gärtner, M. Henk, and G. M. Ziegler: Randomized simplex algorithms on Klee-Minty cubes, Preprint, TU Berlin, May 1996, 21 pages.

[GZ] B. Gärtner and G. M. Ziegler: Randomized simplex algorithms on Klee-Minty cubes, in: Proc. 35th Annual "Symposium on Foundations of Computer Science" (FOCS), IEEE Computer Society Press, Los Alamitos CA, 1994, pp. 502-510.

G. Ziegler

Reporter: *Günter Rote*

This report is available on the world-wide web at
`http://www.opt.math.tu-graz.ac.at/oberwolfach1996/abstracts+problems.ps[.gz]`

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