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STOCHASTISCHE ANALYSIS

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Die Tagung fand statt unter der Leitung der Herren G. Ben Arous (Paris), J.-D. Deuschel (Berlin) und A.-S. Sznitman (Zürich).

Es nahmen 41 Wissenschaftler teil, und es wurden 26 Haupt- und sieben Kurzvorträge gehalten. Außerdem gab es eine Vielzahl von kleineren Diskussionsrunden und Arbeitsgruppen, in denen Probleme ausgetauscht und Lösungsvorschläge diskutiert wurden, kurz: aktuelle Forschung betrieben wurde.

Die Teilnehmer kamen aus vielen Teilen der Welt, und ihre Forschungsinteressen lagen in vielen verschiedenen Teilgebieten der Theorie der stochastischen Prozesse. Die Hauptrichtungen, die auf dieser Tagung vertreten waren, sind:

- Systeme von interagierenden zufälligen Pfaden,
- isoperimetrische und koerzive Ungleichungen,
- fraktale Brownsche Bewegung,
- maßwertige Prozesse, Superdiffusionen,
- stochastische partielle Differentialgleichungen,
- Geometrie Brownscher Bewegungen auf Mannigfaltigkeiten,
- Gibbssche Beschreibungen zufälliger Felder,
- Oberflächen zufälliger Felder,
- zufällige Spinsysteme auf Gittern,
- zufällige dynamische Systeme,
- Lokalzeitenprozesse.

HAUPTVORTRÄGE

Normal forms for stochastic differential equations

L. Arnold

(joint work with P. Imkeller)

We address the following problem from the intersection of dynamical systems and stochastic analysis: Two SDE $dx_t = \sum_{j=0}^m f_j(x_t) \circ dW_t^j$ and $dx_t = \sum_{j=0}^m g_j(x_t) \circ dW_t^j$ in $\mathbb{R}^d$ are said to be smoothly equivalent if there is a smooth random diffeomorphism $h(\omega)$ with $h(\omega, 0) = 0$ and $Dh(\omega, 0) = \text{id}$ ('near identity coordinate transformation') which conjugates the corresponding flows,

$$\varphi(t, \omega) \circ h(\omega) = h(\theta_t \omega) \circ \psi(t, \omega),$$

where $\theta_t \omega(s) = \omega(t+s) - \omega(t)$ is the shift on the canonical Wiener space. The normal form problem for SDE consists in finding the 'simplest-possible' member in the equivalence class of a given SDE, in particular in giving conditions under which it can be linearized (in which case $g_j(x) = Df_j(0)x$).

We develop a normal form theory which is based on the multiplicative ergodic theorem and uses anticipative calculus. We justify earlier physics and engineering literature on that problem. As a by-product, we prove the existence of a unique invariant measure for a hyperbolic affine SDE with anticipative additive term.

Brownian motion on the pre-Sierpinski carpet

M.T. Barlow

(joint work with R.F. Bass)

The pre-Sieprinski carpet is a Lipshitz domain in $\mathbb{R}^d$ which may be defined as follows. Let C_n be the n-th stage in the construction of the standard Cantor ternary set, and let $B_n = 3^n C_n \subset [0, 3^n]$. Set

$$H_n = \{(x_1, \ldots, x_d) \in [0, 3^n]^d : x_i \in B_n \text{ for at least one } i.\},$$

and let $H = \cup_{n=0}^{\infty} H_n$. Thus $H \subset \mathbb{R}^d$, and $H \cap [0, 3^n]^d = H_n$ consists of $[0, 3^n]^d$ with a number of cubical holes removed, of sizes between 1 and 3^{n-1}.

Let $\{W_t, t \geq 0\}$ be Brownian motion in H, with normal reflection on the boundaries. We show there exists a constant ρ_d (which can be estimated using 'short-cut' methods from electrical resistance) such that if $d_w = \log(\rho_d(3^d - 1))/\log 3$ then we have the following estimates on the density $q(t, x, y)$ of W.

Theorem. *There exist* $c_1, \ldots, c_8 \in (0,\infty)$ *such that if* $x, y \in \tilde{F}_0$ *and*

(a) $t \geq 1$, $|x-y| \leq t$, *then,*

$$c_1 t^{-d_s/2} \exp\left(- c_2 \left(\frac{|x-y|^{d_w}}{t}\right)^{1/(d_w-1)}\right) \leq q(t,x,y) \leq c_3 t^{-d_s/2} \exp\left(- c_4 \left(\frac{|x-y|^{d_w}}{t}\right)^{1/(d_w-1)}\right);$$

(b) *if* $t \geq 1$, $|x-y| \geq t$, *or if* $t \leq 1$ *then*

$$c_5 t^{-d/2} \exp(-c_6 |x-y|^2/t) \leq q(t,x,y) \leq c_7 t^{-d/2} \exp(-c_8 |x-y|^2/t).$$

Since $\tilde{F}_0$ is locally similar to $\mathbb{R}^d$, but has a 'fractal' global structure, we would expect $q(t,x,y)$ to have different behaviour for small and large t. We would also expect, in view of standard large-deviation theory for Brownian motion, that, if $|x-y|$ is large in comparison with t then $q(t,x,y)$ will exhibit Gaussian behaviour. (Very roughly, if $|x-y| >> t$ then for the process W to move from x to y in time t it will with high probability stay close to the shortest path connecting x and y, so that it will have no time to feel the fractal structure of $\tilde{F}_0$.)

Wetting phenomena for the lattice free field

E. Bolthausen

(joint work with D. Ioffe)

We consider d-dimensional random surfaces interacting with a wall. The random surface is described by a random field $X(i), i \in V_N \equiv N[-1,1]^d \cap \mathbb{Z}^d$, whose law P_N on $\mathbb{R}^{V_N}$ is given by

$$P_N(dx) = \frac{1}{Z_N} \exp\left(-\frac{1}{2} \sum_{\substack{i,j \in V_N \cup \partial V_N \\ |i-j|=1}} \phi(x_i - x_j) \right) \prod_{k \in V_N} dx_k.$$

Here ∂V_N is the outer boundary of V_N and $x \equiv 0$ on ∂V_N. The lattice free field is the special case where $\phi(x) = x^2$.

The 'wall' is the configuration identical to 0. We consider three types of interactions of the surface with the wall:

- a hard wall condition, which is the condition for the surface to stay positive on V_N,
- an attractive surface-wall interaction,
- macroscopic volume constraints for the region between the surface and the wall.

For a model where all three effects enter, it is proved that macroscopically, i.e., after appropriate rescaling, an optimal droplet forms which solves a variational problem. This is a joint work with D. Ioffe. We also discuss various aspects connected with this model, for instance the open problem of the existence of a wetting transition.

Hot Bodies

K. Burdzy

(joint work with R. Bañuelos)

Suppose that $D \subset \mathbb{R}^n$ is an open and connected set. The 'hot spot' conjecture of Rauch asserts that the second eigenfunction for the Laplacian in D with Neumann boundary conditions attains its maximum on the boundary of D.

(i) The conjecture is true when D is a triangle with an obtuse angle.

(ii) Suppose that D is a convex planar domain with a line of symmetry S which intersects ∂D at x and y. Assume that $|x-y| = \text{diam} D$. The conjecture is true when D is 'sufficiently long.'

Heat kernel measures on loop groups

B. K. Driver

Let $\{\Sigma_t\}_{t\in[0,T]}$ be a Brownian motion on a compact Riemannian manifold M with a fixed starting point $o \in M$, $\mu \equiv \text{Law}(\Sigma_\cdot)$ be Wiener measure on the based path space $W(M)$ of M, and $p_t(o,dx) = \text{Law}(\Sigma_t)$ be a heat kernel measure on M. Using a generalization, to the Riemannian manifold setting, of Cameron's 1951 integration by parts formula, one can give a proof of the well known fact that the gradient operator $\vec{\nabla}$ is closable when thought of as an unbounded operator on $L^2(M, p_t(o,dx))$. A more involved application of this same infinite dimensional integration by parts formula yields the following more refined result:

$$E[(\text{div}Y)(\Sigma_T)] = T^{-1}E[\langle //_T^{-1}Y(\Sigma_T), b_T - \frac{1}{2}\int_0^T tRic_{//_t} \overset{\leftarrow}{\rightarrow} db(t)\rangle].$$

In this last formula Y is a smooth vector field on M, $//_t$ is stochastic parallel translation along Σ, $b_t = \int_0^t //_\tau^{-1} \circ d\Sigma_\tau$ is the 'undeveloped' Brownian motion associated to Σ, $Ric_{//_t} \equiv //_t^{-1}Ric//_t$ is the equi-invariant form of the Ricci tensor, and $\overset{\leftarrow}{\rightarrow} db(t)$ denotes the backwards Itô differential. These same results may also be proved in the case that M is replaced by the infinite dimensional loop group $\mathcal{L}(\mathcal{G})$ of based loops in a compact Lie group G. In particular, this shows that the quadratic form appearing in the Logarithmic Sobolev inequality considered by Driver and Lohrenz is closable.

Geometry of stochastic differential equations

D. Elworthy

(joint work with Yves Le Jan, Xue-Mei Li)

The representation of the generator $\mathcal{A}$ of a diffusion in Hörmander form $\mathcal{A} = \frac{1}{2}\sum_{j=1}^{m} \mathcal{L}_{X^j}\mathcal{L}_{X^j} + \mathcal{L}_A$ associates additional structure to $\mathcal{A}$. (Here $X^1, \ldots, X^m$ and A are C^∞ vector fields with bounded derivatives.) For example the Lie derivatives $\mathcal{L}_{X^j}$, $\mathcal{L}_A$ also act on q-forms and our expression also defines an operator on q-forms, $\mathcal{A}^q$, say. Corresponding to this representation is the stochastic differential equation $dx_t = X(x_t) \circ dB_t + A(x_t)\,dt$ where $X(x)$ maps $\mathbb{R}^m$ to the tangent space at x with $X^j(x) = X(x)e_j$ for $(e_j)_{j=1,\ldots,m}$ the standard basis for $\mathbb{R}^m$. The solution will be a Markov process with generator $\mathcal{A}$. Additional structure appears with the solution flow $\{\xi_t : t \geq 0\}$. Here we will examine some of this extern structure under the assumption that $X(x)$ has constant rank and $A(x) \in \mathrm{Image}X(x)$ for each x. We show that there is a connection on the image of X, the *Le Jan-Watanabe connection* of the SDE, and apply the construction to filtering and redundant noise (in case $X(x)$ is not injective for every x) for the derivative of the flow and of the Itô map determined by the SDE. This extends earlier results by Elworthy and Yor. This is used to give an integration by parts theorem for such, possibly degenerate, diffusion measures on path spaces. Expressions are also given for $\mathcal{A}^q$, including a Weitzenböck formula.

On the longtime behavior of measure-valued processes

A. Greven

(joint work with D. Dawson)

Genetics models incorporating resampling and migration are now fairly well understood. Problems arise in the analysis, if both selection and mutation is incorporated. This talk addresses aspects of this problem namely analyses the longtime behaviour before the equilibrium is reached (quasi equilibrium, which is the time range of interest in most applications). As first model we use a countable system of Fleming-Viot processes with selection and interaction between components via migration. Each component of the system takes values in $\mathcal{P}(\mathbb{N})$, the probability measures on $\mathbb{N}$ since types are labelled by natural numbers, and represents the frequency of genetic types which occur in this colony. Furthermore we discuss in a second model the effect of adding mutation and recombination to the system already incorporating selection.

Of particular interest in such evolutionary models is the nonequilibrium behaviour. The latter can be studied rigorously by exhibiting quasi equilibria after taking a parameter in the evolution to infinity and looking at the system in various different time scales. In these limits the analysis simplifies and the behaviour of the space-time rescaled system is described by a sequence of time dependent Markov chains with values in $\mathcal{P}(\mathbb{N})$ and initial points which vary with the used time scale. The properties of these collections of chains correspond to properties in the longtime behaviour of the original system.

The nonequilibrium behaviour displays a very rich structure, we focus on the following phenomena: influence of the migration on the quasi equilibria, the increased variety of species under selection, the influence of migration on the speed of selection, and the role of mutation.

Finally we set up with this talk the framework needed to discuss in future work a number of unresolved issues in evolutionary models on a rigorous level, at the same time the analysis provides an interesting example for a rigorous renormalisation analysis.

Decay to equilibrium in random spin systems on a lattice

A. Guionnet

(joint work with B. Zegarlinski)

We study the decay to equilibrium of continuous and discrete Glauber dynamics for spin systems on a lattice with random interactions of finite range in a high temperature phase.

Even in this regime, it has been known since [1] that the generator of Glauber dynamics for the Ising type discrete spin systems on $\mathbb{Z}^d$ cannot have a spectral gap with probability one, if the couplings are allowed to take sufficiently large values. We extend this result to continuous models as the 2D rotator model.

However the relaxation to equilibrium is not exponential, one should expect it to be fast in the high temperature phase. In two dimensions, we indeed show that, if the probability of the interaction to be weak is large enough, we have an almost sure stretched exponential upper bound for the decay to equilibrium. This result generalizes the 1D result proved in [1]. In higher dimensions, we can consider ferromagnetic continuous systems and show that the decay has a subexponential upper bound.

References

[1] Zegarlinski B., *Strong decay to equilibrium in one-dimensional random spin systems*, J. Stat. Phys. Vol 77, 717–732 (1994)

Spinor fields over the free loop space

R. Leandre

Killingback and Witten have introduced for smooth loop the notion of string structure over the free loop space. J.D.S. Jones and the speaker have constructed an approximation of the Dirac operator over the free loop space, whose the index is very important for the so-called elliptid cohomology. Unfortunately, the fiber was not the good one. We construct the good fiber, which is related to the considerations of Killingback and Witten. We construct Hilbert space of sections of spinors over the brownian bridge. We begin by elementary statements, by considering line bundle over the Brownian bridge.

Lévy-Gromov's isoperimetric inequality for an infinite dimensional diffusion generator

M. Ledoux

(joint work with D. Bakry)

We establish, by simple semigroup arguments, a Lévy-Gromov isoperimetric inequality for the invariant measure of an infinite dimensional diffusion generator of positive curvature with isoperimetric model the Gaussian measure. This produces in particular a new proof of the Gaussian isoperimetric inequality. This isoperimetric inequality strengthens the classical logarithmic Sobolev inequality in this context. A local version for the heat kernel measures is also proved, which may then be extended into an isoperimetric inequality for the Wiener measure on the paths of a Riemannian manifold with bounded Ricci curvature.

Branching processes in Lévy processes

J.-F. Le Gall

(joint work with Y. Le Jan)

The genealogy of a discrete-time Galton-Watson branching process is described by a discrete tree, the genealogical tree of the population. The main goal of this talk is to give a similar description for the genealogy of continuous-state branching processes, which are the possible scaling limits of discrete-time Galton-Watson branching processes. To this end, we determine the so-called exploration process (or contour process) associated with the genealogical structure of a continuous-state branching process. Informally, the exploration process of a tree gives the motion of a particle that explores the tree by moving up and down along its branches. In the special case of Feller's diffusion, the simplest continuous-state branching process, it has been known for a long time that the associated exploration process is reflecting Brownian motion on the positive real line. In general, we determine the exploration process as a simple functional of a spectrally positive Lévy process. This leads to a new deep connection between branching processes and Lévy processes, which can be used to derive new results in both theories. In particular, we get an extension for general spectrally positive Lévy processes of the classical Ray-Knight theorem on Brownian local times.

Continuous-state branching processes give the evolution of the total mass of superpocesses. From our description of their genealogical structure, we can easily derive a path-valued process construction of superprocesses with a general branching mechanism, which extends the construction of super-Brownian motion from the Brownian snake. This new construction has many potential applications to path properties of superprocesses as well as their connections with partial differential equations.

Stochastic cascades and 3-dimensional Navier-Stokes equations

Y. Le Jan

(joint work with A.-S. Sznitman)

In this work, we introduce a probabilistic interpretation of the Navier-Stokes equation in the full three dimensional space and use it to derive same global existence results in classes where uniqueness holds. To this end we investigate a Fourier representation of the Navier-Stokes equation, namely the integral equation

$$\chi_t(\xi) = \exp(-\nu|\xi|^2 t)\chi_0(\xi) + \int_0^t \nu|\xi|^2 e^{-\nu|\xi|^2(t-s)} \left(\frac{1}{2}\chi_s \circ \chi_s(\xi) + \frac{1}{2}\varphi(s,\xi)\right) ds$$

with

$$f_1 \circ f_2(\xi) = -i \int \left(f_1(\xi_1) \cdot \frac{\xi}{|\xi|}\right) p_{\xi^\perp} f_2(\xi - \xi_1)\, K_\xi(d\xi_1)$$

and

$$K_\xi(d\xi_1) = \frac{1}{\pi^3} \int \frac{|\xi|\, d\xi_1}{|\xi_1|^2\, |\xi - \xi_1|^2}.$$

(χ_t is proportional to the Fourier transform of the Laplacian of the velocity field.)

The global existence and uniqueness holds for example in classes of functions dominated by 1 or by $\frac{\pi^2}{2}|\xi|^3 e^{-|\xi|}$, provided χ_0 and φ are in that class.

Flows on spaces of rough paths

T. Lyons

Let S be the space of piecewise smooth paths in a Banach space V; and ΩG_p its closure in the topology where a sequence is Cauchy if it converges in p-variation and in addition its iterated integrals of degree less than or equal to $[p]$. Consider a vector field on smooth paths defined by $X.(\xi)$:

$$dX_t = f(X_t)\, d\xi_t, \qquad X_0 = 0.$$

Then the flow

$$\frac{d\xi^\tau}{d\tau} = X(\xi^\tau)$$

is defined for a non-trivial time interval depending on the roughness of ξ and the Lipschitz norm of f.

Applications include deterministic constructions of Driver-type flows on pinned paths on a manifold.

What could be the tangent space to a probability space?

P. Malliavin

An infinitesimal deformation is said to be *admissible* if it preserves the underlying structures and the notion of vector field corresponds to the notion of admissible deformation. For a probability space the underlying structure is the class of the probability measure: an infinitesimal deformation is therefore admissible if it preserves *almost sure properties*. For the case of abstract Wiener space, it is sufficient that the vector field sits in the Cameron-Martin space; working with this type of vector fields it is possible to construct differential analysis on any abstract Wiener space (see [3]). See [1] for the case of Poisson spaces.

Let us put these notions in the concrete framework of the probability space X of an $\mathbb{R}^d$ valued Brownian motion defined for the time $\tau \in [0,1]$. Then the Cameron-Martin space is the space of $\mathbb{R}^d$-valued absolutely continuous function h such that $h(0) = 0$ and $\int |\dot{h}|^2 \, d\tau = \|h\|_H^2 < \infty$. A specificity of X compared to an abstract Wiener space is the existence of the Itô filtration $\mathcal{N}_\tau$. The most general deformation adapted to this two structure is the data of $h(x,\tau)$ adapted to the filtration and such that $\int_X \|h(x,\cdot)\|_H^2 < \infty$.

The concept of *tangent process* is obtained extending this quasi invariance taking advantage of the unitary invariance of the Brownian motion: a tangent process is an $\mathbb{R}^d$-valued semi martingale ζ which has an Itô differential of the form $d\zeta^\alpha = a^\alpha_\beta \, dx^\beta + c^\alpha \, d\tau$, where $a^\alpha_\beta + a^\beta_\alpha = 0$ plus growth conditions. The Wiener measure is infinitesimally preserved by a tangent process and the infinitesimal modulus being given by the Itô integral $\sum_\alpha \int_0^1 c^\alpha \, dx^\alpha$.

Given a Riemannian manifold M its Brownian motion defines on the path space $P_{m_0}(M)$ a structure of filtered probability space. The stochastic parallel transport along the path $p(\cdot)$ let $t^p_{0\leftarrow\tau}$, defines an unitary isomorphism $T_{p(\tau)}(M) \mapsto T_{m_0}(M)$; therefore any infinitesimal deformation given by a vector field along the path $Z(p,\tau) \in T_{p(\tau)}(M)$ can be transferred to an $T_{m_0}(M)$-valued function $\zeta^*(p,\tau) = t^p_{0\leftarrow\tau}(Z(p,\tau))$.

The Itô map $\mathcal{I} : X \mapsto P_{m_0}(M)$ is defined by the Stratonovich SDE, $dp_x = t^{p_x}_{\tau\leftarrow 0}(\circ dx)$.

The following theorem is a basic result for the geometry of path spaces.

Theorem ([2]). *Given ζ a tangent process on X, then $\mathcal{I}'(x)\zeta$ is defined by*

$$\mathcal{I}'(x)\zeta := t^{p_x}_{0\leftarrow *}\left[\frac{d}{d\epsilon}\mathcal{I}(x+\epsilon\zeta)\Big|_{\epsilon=0}\right] = \zeta^*.$$

Then $\mathcal{I}$ realizes an isomorphism of the space of tangent processes; this isomorphism is given through the system of SDE

$$d\zeta = d\zeta^* - \rho \circ dx, \qquad d\rho = -\Omega(\circ dx, \zeta^*)$$

where Ω is the curvature tensor of the manifold M.

References

[1] Albeverio, S., Kondratiev, Y.G., Röckner, M., Differential geometry of Poisson spaces: C.R. Acad. Sci. Paris 1996.

[2] Cruzeiro, A.B. and Malliavin, P., *Renormalized differential geometry on path spaces: Structural equation and curvature,* J. Funct. Analysis 139 (1996), 119-181.

[3] Malliavin, P., *Stochastic Analysis:* Grundlehren der Mathematik, Springer, 375 pages.

Random evolution equations and anticipating stochastic integrals

D. Nualart

(joint work with J. Leòn)

The purpose of this talk is to present some results on stochastic evolution equations on a Hilbert space H of the form:

$$X_t = x_0 + \int_0^t (A_s X_s + F(X_s))\,ds + \int_0^t B(X_s)\,dW_s, \quad t \in [0,T], \tag{1}$$

where x_0 is an element in H. We assume that $W = \{W_t, t \in [0,T]\}$ is a cylindrical Wiener process on a Hilbert space U. The coefficients of the equation, F and B, are supposed to be Lipschitz functions on H and on the space of Hilbert-Schmidt operators $L_2(U,H)$, respectively. On the other hand, we assume that $\{A_s, s \in [0,T]\}$ is a random and adapted family of unbounded operators on H, such that $\mathrm{Dom}A_s^*$ contains a fixed dense subspace H_0. A weak solution to such equation is, by definition, a continuous and adapted H-valued stochastic process which satisfies the equation in the distribution sense.

Assuming that A_s generates a random evolution system $\{S(t,s), 0 \le s \le t \le T\}$, we prove that there is a unique mild solution to the equation (1) that is also a weak solution. In order to show this result we require the random evolution system $S(t,s)$ to be twice differentiable in the sense of the Malliavin calculus, and we assume that $S(t,s)$ and its derivatives satisfy some integrability and continuity conditions. The proof is based on the following maximal inequality for the forward stochastic integral:

$$E\left(\sup_{0\le t\le T}\left|\int_0^t S(t,s)\Phi_s\,dW_s^-\right|^p\right) \le CE\left(\int_0^T \|\Phi_s\|_{HS}^p\,ds\right),$$

where $\Phi = \{\Phi_s, s \in [0,T]\}$ is an adapted stochastic process with values in $L_2(U,H)$, and the constant C depends on p and on the evolution system $S(t,s)$. This maximal inequality is deduced from the Itô's formula for the Skorohod stochastic integral.

As an application we discuss the case of a second order differential operator with random and adapted coefficients.

Einstein diffusion of an asymmetric tagged particle in the one-dimensional nearest-neighbor symmetric simple exclusion

S. Olla

(joint work with C. Landim and S.B. Volchan)

We investigate the behavior of a tagged particle under the action of an external constant driving force in an infinite one dimensional system of point particles evolving according to symmetric random walks with hard core interaction. We prove the position $\epsilon X(\epsilon^{-2}t)$ of the test particle diffusively rescaled converges in probability, as $\epsilon \to 0$, to a deterministic function $v(t)$, for a large class of initial distributions of the random environment. The function $v(\cdot)$ depends only on the initial distribution of the random environment through a non linear parabolic equation. This law of large numbers for the position of the tracer particle is deduced from the hydrodynamical limit of an inhomogeneous one dimensional symmetric zero range process with an asymmetry at the origin.

A family of 'Brownian motions' on d-dimensional Sierpinski carpets

H. Osada

Let $d_{\mathrm{H}} = \log 3(3^d - 1)$ be the Hausdorff dimension of the Sierpinski carpet S. For each $\beta > d_{\mathrm{H}}$ we construct self-similar diffusions $\{P_x\}$ satisfying

- 'cell'-translation invariance,
- symmetry w.r.t. μ_β where μ_β is a self-similar measure on S,
- invariance under transformations of S.

We construct such diffusion by using the Dirichlet form

$$\mathcal{E}^{\mu_\beta}(f,g) = \frac{1}{2}\int \sum_{i=1}^{d} \partial_i f \cdot \partial_i g \, d\mu_\beta$$

on $L^2(\mu_\beta)$ with a suitable domain $\mathcal{D}_0$ which is not $C_0^\infty(\mathbb{R}^d)$.

In addition, when $d_{\mathrm{H}} < \beta < d_{\mathrm{H}} + 2$, by using time change we construct a diffusion such that the Hausdorff measure on S is the invariant measure.

We use isoperimetric inequalities for the self-similar measure μ_β which yields a Gaussian estimate for $\{P_x\}$,

$$c_1 t^{-\frac{\beta}{2}} e^{-c_2 \frac{|x-y|^2}{2}} \leq P_t(x,y) \leq c_3 t^{-\frac{\beta}{2}} e^{-c_4 \frac{|x-y|^2}{2}}.$$

Time-ergodicity of stochastic dynamics and extremality of Gibbs states in lattice and continuum models

M. Röckner

(joint work with S. Albeverio and Y.G. Kondratiev)

The convex set $\mathcal{M}^a$ of quasi–invariant measures on a locally convex space E with given "shift"–Radon-Nikodym derivatives (i.e., cocycles) $A = (a_{tk})_{k\in K_0, t\in\mathbb{R}}$ is analyzed. The extreme points of $\mathcal{M}^a$ are characterized and proved to be non-empty. A specification (of lattice type) is constructed so that $\mathcal{M}^a$ coincides with the set of the corresponding Gibbs states. As a consequence, via a well-known method due to Dynkin–Föllmer a unique representation of an arbitrary element in $\mathcal{M}^a$ in terms of extreme ones is derived. Furthermore, the corresponding classical Dirichlet forms $(\mathcal{E}_\nu, D(\mathcal{E}_\nu))$ and their associated semigroups $(T_t^\nu)_{t>0}$ on $L^2(E;\nu)$ are discussed. Under a mild positivity condition it is shown that $\nu \in \mathcal{M}^a$ is extreme if and only if $(\mathcal{E}^\nu, D(\mathcal{E}^\nu))$ is irreducible or equivalently, $(T_t^\nu)_{t>0}$ is ergodic (resp. mixing). This implies time–ergodicity of associated diffusions. Applications to Gibbs states of classical and quantum lattice models as well as those occurring in Euclidean quantum field theory are presented. In particular, it is proved that the stochastic quantization of a Guerra–Rosen–Simon Gibbs state on $\mathcal{D}'(\mathbb{R}^2)$ in *infinite volume* with polynomial interaction is ergodic, if the Gibbs state is extreme (i.e., is a pure phase).

Sample Lyapunov exponent for a class of linear Markovian systems over $\mathbb{Z}$

T. Shiga

Let $\{Y_i(t)\}_{i\in\mathbb{Z}}$ be independent copies of a one-dimensional Lévy process $Y(t)$, and we consider the following linear SPDE over $\mathbb{Z}$;

$$d\xi_i(t) = \kappa \sum_{|j-i|=1} (\xi_j(t) - \xi_i(t))dt + \xi_i(t-)dY_i(t) \quad (i \in \mathbb{Z}), \tag{1}$$

where $\kappa > 0$ is a constant. Under the assumption that (1) admits a nonnegative solution one can define its sample Lyapunov exponent $\lambda(\kappa, d)$ as follows;

$$\lim_{t\to\infty} \frac{1}{t} \log \xi_i(t) = \lambda(\kappa, d) \quad \text{in} \quad L^1(P^Y)\text{-sense.}$$

When $\{Y_i(t)\}$ are independent copies of standard Brownian motions, (1) is called *parabolic Anderson model*, and when $Y_i(t) = -N_i(t) + t$ and $\{N_i(t)\}_{i\in\mathbb{Z}}$ are independent copies of a Poisson process with parameter one, (1) is a dual object of survival probability of a continuous time random walk in a temporally varying random environment. In the lecture we discussed aymptotical analysis of $\lambda(\kappa, d)$ for small $\kappa > 0$ in two extremal cases with some interpretation of the result by the survival probability problem.

The methods we use for analysis are Feynman-Kac representation, comparison theorems and some large deviation arguments together with discretization of the continuous time random walk.

Gibbs description of the 'non-Gibbs' states

S. Shlosman

This is the paper on which late Professor Dobrushin started working in the last few month of his life. In this paper the new approach to the Gibbseanity is proposed, which is ment to enable one to give Gibbs description to the fields which are not Gibbs in the usual sence. The idea is that one does not need to define the energy of every finite box configuration with respect to every boundary condition; it is enough to do it with respect to almost every boundary condition according to the measure in question. This general idea is applied to the example of "non-Gibbs" field, constructed by Roberto Schonmann. The result is that the projection of the +-phase of the 2D low temperature Ising model on the one dimensional sulattice $\mathbb{Z}^1$ is a Gibbs field according to the new definition.

Motion of a tagged particle

S.R.S. Varadhan

We consider the motion of a tagged particle in a system of interacting particles, particularly a system of random walks interacting by simple exclusion. The interacting process is assumed to be in equilibrium at some density. We investigate the motion of the tagged particle and show that under certain conditions, with proper diffusive scaling of space and time, the motion of the tagged particle converges to a Brownian motion. This was established earlier by Kipnis-Varadhan for the symmetric case, by Varadhan for the asymmetric mean zero case, and now by Sethuraman, Varadhan and Yau for the general asymmetric case in dimension three or more. The methods involve writing the motion as the sum of a martingale and an additive functional and then proving a central limit theorem for the additive functional by rewriting it as the sum of another martingale and a negligible additive functional.

Critical exponents for planar Brownian motion

W. Werner

The subject of this talk are the critical exponents for non-intersection or non-disconnecting probabilities of planar random walks or Brownian motions. It is believed that these well-defined exponents take rational values, some of which are predicted by the representation theory of the Virasoro Algebra.

We first define these exponents and recall some estimates and conjectures. We then briefly explain the link with fine properties of planar Brownian motion (including some recent results by G. Lawler) and self-avoiding walks. Finally, we derive some asymptotic results for these exponents, when the number of considered walks becomes large.

Relaxation to equilibrium for conservative dynamics

H.T. Yau

(joint work with S.L. Lu)

The Kawasaki dynamics, also known as symmetric simple exclusion with speed change or lattice gases dynamics, describe systems of interacting random walks with Gibbs states as equilibria. For such systems the relaxation rates in infinite volume are expected to be power laws. Together with S.L. Lu, I have proved Poincaré inequalities, or spectral gap, in finite volume assuming mixing conditions for the Gibbs states. The corresponding logarithmic Sobolev inequalities are also established in a paper by H.T. Yau under similiar assumptions. Based on these estimates and under similiar assumptions as in the previous work, we prove that the relaxation rate to equilibrium in L^2-norm in infinite volume is bounded above by $t^{-d/2}\log t^4$ for $d \leq 2$ and by $t^{-1}\log t^4$ for $d \geq 2$.

Perturbed Bessel processes and Ray-Knight theorems

M. Yor

(joint work with R. Doney and J. Warner)

The family of linear combinations of one-dimensional Brownian motion $\{B_t, t \geq 0\}$ and its supremum process $\{S_t = \sup_{s\leq t} B_s, t \geq 0\}$ enjoys many important properties; two particular examples are: $\{S_t - B_t, t \geq 0\}$ which is a reflecting Brownian motion, and $\{2S_t - B_t, t \geq 0\}$ a 3-dimensional Bessel process; these classical results are due respectively to P. Lévy and J. Pitman. It was also proven by J.-F. Le Gall and the author that the process of local times of $\{\alpha S_t - B_t, t \geq 0\}$ for $\alpha > 1$, is a square Bessel process with dimension $\delta = \frac{2}{\alpha-1}$.

In this lecture, the following theorem was presented: both the processes of local times of $\{M_t^{\beta/\overline{\beta}} R_t, t \geq 0\}$ and $\{J_t^{\beta/\overline{\beta}} R_t, t \geq 0\}$, where $\{R_t, t \geq 0\}$ is a 3-dimensional Bessel process, $M_t = \sup_{s\leq t} R_s$ and $J_t = \inf_{s\geq t} R_s$, are distributed as square Bessel processes with dimension $\delta = 2(1-\beta) = 2\overline{\beta}$. An explanation of their 'coincidence' is provided by the following relationship:

$$\frac{1}{R_{3,\beta}(t)} = \Sigma_\delta\left(\int_t^\infty \frac{ds}{(R_{3,\beta}(s))^4}\right),$$

where $(R_{3,\beta}(t), t \geq 0)$ is a 3-dimensional perturbed Bessel process, and $\Sigma_\delta(t) = |B_t| + \frac{2\ell_t}{\delta}$. ($R_{3,\beta}$ is the solution of $X_t = B_t + \int_0^t \frac{ds}{X_s} + \beta M_t^X, t \geq 0$.)

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Ergodicity via coercive inequality

B. Zegarlinski

At the beginning the general relations between Spectral Gap, Logarithmic Sobolev, General Nash and Logarithmic Nash inequalities and ergodic properties of infinite dimensional systems has been discussed. In the main part of the talk a general strategy for proving the General Nash and Logarithmic Nash inequalities for Gibbs measures together a nontrivial example of application to Kawasaki dynamics have been presented.

The corresponding preprints one can find at
http://www.ma.ic.ac.uk/ bertini/preprints.html

On roots of random polynomials

O. Zeitouni

(joint work with I. Ibragimov)

We study the distribution of the complex roots of random polynomials of degree n with i.i.d. coefficients. Using integral geometric techniques, we derive an exact formula for the mean density of this distribution, which yields appropriate limit average densities. Further, using a different technique, we prove limit distributions results for coefficients in the domain of attraction of the stable law. Finally, some variance computations allow us to show that, at least in the Gaussian case, the root closest to the unit circle is at distance of order n^{-2}.

KURZVORTRÄGE

Large Deviations for heavy tails and random walk in random environment

N. Gantert

1) Consider the following random walk in random environment (called RWRE): $\omega = (\omega_x)_{x\in\mathbf{Z}}$ is a collection of i.i.d. random variables taking values in the unit interval, which serves as an environment, and (X_k) is the random walk which, when at x, moves one step to the right with probability ω_x and one step to the left with probability $1-\omega_x$. Solomon (1975) determined the asymptotic speed of the RWRE, i.e., the almost sure limit of X_n/n. We will assume that $\lim X_n/n = c > 0$. Let $T_k := \inf\{i : X_i = k\}$ be the first hitting time in k and $\tau_i := T_i - T_{i-1}$. The sequence $(\tau_1, \tau_2, \ldots)$ is stationary and has good mixing properties. For certain environment distributions, the random variables τ_i are heavy-tailed, i.e., $E[\exp(\lambda\tau_i)] = \infty$ for all $\lambda > 0$. In these cases, the probability of a

deviation of the RWRE below its asymptotic speed decays slower than exponential in n, see Dembo, Peres and Zeitouni (1996). We prove large deviation theorems for a heavy-tailed, mixing sequence; they can be applied to to the sequence $(\tau_1, \tau_2, \ldots)$. A deviation of the arithmetic mean of the τ_i from above corresponds to a deviation of the RWRE from below.

2) We give a functional limit law for random walks whose increments have a stretched exponential distribution. Let $Y_1, Y_2, \ldots$ be i.i.d. and $Z_n(t) := \sum_{i=1}^{[nt]} Y_i + (t - \frac{[nt]}{n}) Y_{[nt]+1}$ $(0 \leq t \leq 1)$. It is well-known that, if $E[\exp(\lambda Y_1)] < \infty$ for all $\lambda \in \mathbb{R}$, then the distributions of Z_n satisfy a large deviation principle (in $C[0,1]$), where the rate function is concentrated on the set of absolutely continuous functions. We show that, if Y_1 is bounded below and $P[Y_1 \geq t]$ decays like $\exp(-ct^r)$ (with $0 < r < 1$), then the distributions of Z_n satisfy a large deviation principle (in $M([0,1])$ where the rate function is concentrated on the set of pure jump functions. We indicate what is expexted to be the corresponding functional limit law for RWRE.

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The smoothness of the laws of Oseledets spacees of linear stochastic differential equations

P. Imkeller

For a random dynamical system generated by a linear SDE the Oseledet spaces carry the random invariant measures crucial for their asymptotic properties. They are obtained by intersecting the nested random linear spaces of a forward and a backward flag. We study existence and smoothness of densities and conditional densities of their laws considered as random elements taking their values on appropriate Grassmann manifolds with respect to Riemannian volume. Their laws are seen to be closely related to the laws of certain flows of diffeomorphisms which decompose the linear flow coming from the SDE. The latter are investigated using the tools of Malliavin's calculus and Hörmander's hypoelliptic conditions for the vector fields generating the flows. As an application we find that the semimartingale property is well preserved if the Wiener filtration is enlarged by the information present in the flag or the Oseledets spaces. Hence the tools of stochastic analysis are applicable in a number of problems arising in the study of random dynamical systems despite the fact that the invariant measures are not adapted to the history of the Wiener process.

Central limit theorems for one-dimensional polymer models

W. König

(joint work with R. van der Hofstad and F. den Hollander)

A polymer model is a transformed path measure that suppresses self-intersections. Fix a parameter $\beta > 0$, the strength of self-repellence. For simple random walk $(S_n)_{n\in\mathbb{N}_0}$ on $\mathbb{Z}$ let $dQ_n^\beta = \exp(-\beta\sum_{i,j=0}^n 1_{\{S_i=S_j\}})\, dP/Z_n^\beta$ be the *Domb-Joyce model*, and for Brownian motion $(B_T)_{T\geq 0}$ on $\mathbb{R}$ let $d\widehat{Q}_T^\beta = \exp(-\beta\int_{\mathbb{R}} L(T,y)^2\,dy)\, d\widehat{P}/\widehat{Z}_T^\beta$ be the *Edwards model* where $(L(T,y))_{T\geq 0, y\in\mathbb{R}}$ is the Brownian local time process. Our main question is: How does S_n typically behave under $Q_n^{\beta_n}$ as $n\to\infty$ if $\beta_n\to 0$ and $\beta_n n^{\frac{3}{2}}\to\infty$? We state three central limit theorems: for Q_n^β, $Q_n^{\beta_n}$, and $\widehat{Q}_T^\beta$ and relate them to each other.

It turns out that, for some $\theta^*(\beta)\in(0,1)$ and some $\sigma^*(\beta)>0$, $\mathcal{L}_{Q_n^\beta}\left(\frac{|S_n|-\theta^*(\beta)n}{\sigma^*(\beta)\sqrt{n}}\right)\Rightarrow^w\mathcal{N}$ and for some b^*, $c^*\in(0,\infty)$, we have $\mathcal{L}_{\widehat{Q}_T^\beta}\left(\frac{|B_T|-b^*T}{c^*\sqrt{T}}\right)\Rightarrow^w\mathcal{N}$. Furthermore, one has $\theta^*(\beta)\beta^{-\frac{1}{3}}\to b^*$ and $\sigma^*(\beta)\to c^*$ as $\beta\to 0$. Coupling the limits $n\to\infty$ and $\beta\to 0$, we get our main result: If $\beta_n\to 0$ and $\beta_n n^{\frac{3}{2}}\to\infty$, then $\mathcal{L}_{Q_n^{\beta_n}}\left(\frac{|S_n|-\theta^*(\beta_n)n}{\sigma^*(\beta_n)\sqrt{n}}\right)\Rightarrow^w\mathcal{N}$.

As an illustration of this result, if $\beta_n=\beta n^{-p}$ with some $p\in(0,\frac{3}{2})$, then, under $Q_n^{\beta_n}$, $|S_n|\sim b^*\beta^{\frac{1}{3}}n^{1-\frac{p}{3}}$ and $|S_n|-E_{Q_n^{\beta_n}}|S_n|\sim c^*\sqrt{n}\mathcal{N}$, which shows a perfect linear interpolation between diffusive and ballistic behavior as p varies from $\frac{3}{2}$ to 0.

On the generation of one-sided random dynamical systems by stochastic differential equations which are driven by (discontinuous) semimartingales with stationary increments

M. Scheutzow

(joint work with G. Kager)

Let Z be an $\mathbb{R}^m$-valued semimartingale with stationary increments which is realized as a helix over a filtered metric dynamical system S. Consider a stochastic differential equation with Lipschitz coefficients which is driven by Z. We show that its solution semiflow ϕ has a version for which $\varphi(t,\omega)=\phi(0,t,\omega)$ is a cocycle and therefore (S,φ) is a random dynamical system. Our results generalize previous results which required Z to be continuous. We also address the case of local Lipschitz coefficients with possible explosions in finite time. Our abstract perfection theorems for semiflows are designed to cover also potential applications to infinite dimensional equations.

On the intrinsic distance of the Fleming-Viot process

A. Schied

The intrinsic metric of a diffusion process appears, for example, as rate function for a Varadhan-type large deviation theorem. It is shown that, under certain assumptions, such a theorem holds for the Fleming-Viot process. The intrinsic distance is identified as the Bhattacharya metric

$$\overline{\rho}(\nu,\mu) = 2\arccos \int \sqrt{\frac{d\nu}{d\eta}\frac{d\mu}{d\eta}}\, d\eta, \quad \text{where } \nu,\mu << \eta.$$

Moreover, a geometric interpretation of the relations between Fleming-Viot and Dawson-Watanabe models is given.

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