MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 45/1996

Nonlinear Equations in Many-Particle Systems

1. - 7.12.1996

The Mathematics Research Institute at Oberwolfach (Germany) hosted the third meeting of the series entitled "Nonlinear Equations in Many-Particle Systems", which was organized by J. Batt (Munich) and C. Cercignani (Milano). The meeting was attended by forty speakers from Germany, Canada, Italy, Sweden, France, Spain, Russia, India, Japan, and the United States. There were four main themes:

(1) particle dynamics at a microscopic level (e.g. molecular motion governed by classical analytic mechanics) and the connection with macroscopic motion of fluids governed by laws of continuum mechanics. This theme was accentuated in the talks of M. Pulvirenti (Rome), W. Wagner (Berlin), R. Esposito (L'Aquila, Italy), F. Golse (Paris), H. Spohn (Munich).

(2) classical problems arising from the Boltzmann equation: Talks in this area aimed at both application of the Boltzmann equation in fluid mechanics or other fields and mathematical theory of the Boltzmann equation. Speakers were S. Ukai (Tokyo), M. Slemrod (Madison, WI), S. Bobylev (Moscow), L. Arkeryd (Goteborg, Sweden), L. Desvillettes (Orleans, France), S. Deshpande (Bangalore, India), A. Nouri (Nice, France), B. Wennberg (Goteborg, Sweden), H. Babovsky (Ilmenau), A. Klar (Kaiserslautern), J. Struckmeier (Kaiserslautern), H. Cabannes (Paris), R. Marra (Rome), Y. Brenier (Paris), F. Poupaud (Nice).

(3) plasma equations (Vlasov-Poisson-, Vlasov-Maxwell-, Vlasov-Einstein system and related systems): The main results here concerned (global) existence and qualitative behaviour of solutions. Speakers here included G. Rein (Munich), A. Rendall (Potsdam), P. Braasch (Munich), R. Illner (Victoria, B. C.), H. Andréasson (Goteborg, Sweden), E. Caglioti (Rome), J. Soler (Granada), Y. Guo (Providence, RI), R. Glassey (Bloomington), N. Mauser (Berlin), E. Grenier (Paris), J. Dolbeault (Paris), G. Toscani (Pavia), J. Batt (Munich).

(4) semiconductor modeling: Models of semiconductor dynamics (because of the small space-time scales involved) are often done at the kinetic level. Several speakers addressed this topic and the mathematical issue of obtaining macroscopic fluid-like models from the mesoscopic kinetic models. The tools used are

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the Wigner transform and the Murat-Tartar theory of compensated compactness. Speakers included P. Degond (Toulouse), I. Gasser (Berlin), P. Markowich (Berlin).

Finally, there were two lectures not included in these four main themes: P. Morrison (Austin, Texas) gave a lecture "Necessary and Sufficient Conditions for Stability of Plane Parallel Shear Flow". He obtained conditions for instability which improve on classical results of Rayleigh. C. Bardos (Paris) spoke on "Propagation of Defect Measures and Applications". Bardos showed how to use the defect measure to quantify the fact that solutions to the wave equation can converge to zero weakly but not strongly.

Despite of the extensive lecture program, there were many discussions between participants outside the lecture room on scientific topics of mutual interest. The participants expressed their wish for the possibility of a similar meeting in three years.

VORTRAGSAUSZÜGE

M. PULVIRENTI:

Kinetic equations for granular media

[Joint work with D. Benedetto and E. Caglioti. To appear in *MMAN* (1997).]

A granular medium is a large ensemble of solid particles interacting inelastically. A widely investigated simple model is that of a one-dimensional point particle system with the collision law

$$\begin{aligned} v_1' &= v_1 - \varepsilon(v_1 - v_2), \\ v_2' &= v_2 + \varepsilon(v_1 - v_2), \end{aligned}$$

where (v_1, v_2) and (v_1', v_2') are the incoming and outgoing velocities, before and after the collision, and $\varepsilon \in (0, 1/2)$ is the inelasticity parameter. If N is the number of particles, we establish a kinetic equation which describes the system in the limit $N \to \infty, \varepsilon \to 0$, $N\varepsilon \to \lambda$, where $\lambda > 0$ is a parameter. A preliminary study of this equation is presented.

S. UKAI:

On the half space stationary problem for the discrete velocity model of the Boltzmann equation

We study the 1D stationary boundary value problem for the discrete velocity model of the Boltzmann equation on the semi-infinite interval $[0, \infty)$ with Dirichlet boundary conditions at $x = 0$ for the components of the solution corresponding to positive velocities. The point of our aim is to find a solution which tends to a prescribed Maxwellian at $x = \infty$. This is a nonlinear problem closely related to the boundary layer problem, the Milne and Kramers problems, the evaporation-condensation problem and so on. We will show that a solution exists uniquely for the Dirichlet data at $x = 0$ sufficiently close to the Maxwellian prescribed at $x = \infty$ if, in particular, the number of the Dirichlet conditions is equal to the number of positive eigenvalues of a matrix associated with the linearized operator of the nonlinear collision operator arround this Maxwellian.

M. SLEMROD:

The Chapman-Enskog expansion revisited

This lecture introduces a procedure for rewriting the well-known Chapman-Enskog expansion used in the kinetic theory of gases. The usual Chapman-Enskog procedure will exhibit instability of the rest state for some finite truncations of the expansion. The rewritten expansion eliminates instabilities and suggests a method for summation of the expansion as a nonlocal (generally nonlinear) operator in space.

A. V. BOBYLEV:

Moment inequalities for the Boltzmann equation and applications to spatially homogeneous problems

Some inequalities for the Boltzmann collision integral are proved. These inequalities can be considered as a generalization of the well-known Povzner inequality. The inequalities are used to obtain estimates for the moments of a solution to the spatially homogeneous Boltzmann equation for a wide class of intermolecular potentials. We obtain simple necessary and sufficient conditions (on the potential) for uniform boundedness of all moments. For potentials with compact support the following statement is proved: All moments of a solution are bounded by the corresponding moments of a certain Maxwellian $A e^{-B|v|^2}$ for all $t > 0$, if this condition is fulfilled at $t = 0$; moreover $B(t)$ is constant for hard spheres. Estimates for a collision frequency are also obtained.

L. ARKERYD:

A functional inequality related to kinetic theory

Let S be a (locally) weakly precompact set of positive L^1-functions in $\mathbb{R}^n$. Let $(v_1, v_2) \in \mathbb{R}^{2n}$ be precollisional and (v_1', v_2') corresponding postcollisional velocities under binary collisions. We prove results of the type "If $f \in S$ and $f(v_1') f(v_2') - f(v_1) f(v_2)$ small on large sets, then f is close to a Maxwellian (on large sets or) in L^1".

G. REIN:

The Vlasov equation as a matter model in cosmology

The Vlasov-Poisson system in the stellar dynamics case,

$$\partial_t f + v \cdot \partial_x f - \partial_x U \cdot \partial_v f = 0,$$

$$\Delta U = 4\pi\rho, \;\; \rho(t,x) = \int f(t,x,v)\, dv,$$

where $t \geq 0$ and $x, v \in \mathbb{R}^3$, admits the following spatially homogeneous solutions:

$$f_0(t,x,v) = H(|a(t)\, v - \dot{a}(t)\, x|^2),$$

$$\rho_0(t) = a^{-3}(t), \;\; U_0(t,x) = \frac{2\pi}{3}\, a^{-3}(t)\, x^2.$$

Here $H \in C_c^1(\mathbb{R})$, $H \geq 0$ with $\int H(v^2)\, dv = 1$, and a solves

$$\ddot{a} + \frac{4\pi}{3}\, a^{-2} = 0.$$

We prove that these solutions are nonlinearly stable against spatially periodic perturbations, which are posed at some time t_0 with $a(t_0) > 0$ and $\dot{a}(t_0) > 0$, provided $\frac{1}{2}\, \dot{a}^2(t_0) - \frac{4\pi}{3}\, a^{-1}(t_0) > 0$. The question whether such homogeneous solutions are stable or not is related to the formation of large scale structure in the universe. This result is an example of problems from cosmology for which the Vlasov equation as a matter model can be useful.

A. RENDALL:

Global solutions of the Einstein-Vlasov system with $U(1) \times U(1)$ symmetry

In the past four years various global existence results for solutions of the Einstein-Vlasov equations with symmetry have been proved. The symmetry assumptions were always so strong as to suppress the hyperbolic nature of the equations. The strongest symmetry where this does not happen is $U(1) \times U(1)$ symmetry, which leads to a situation analogous to the $1\frac{1}{2}$-dimensional Vlasov-Maxwell system. Proving a global existence theorem for the case of $U(1) \times U(1)$ symmetry involves a combination of analytic and geometric techniques. The main analytic steps in the argument are modifications of arguments used by Gu in the study of wave maps and by Glassey, Schaeffer and Strauss in the study of the Vlasov-Maxwell system.

P. BRAASCH:

Semilinear elliptic equations and the Vlasov-Maxwell system

An existence theory for [classical] G-symmetric solutions of semi-linear elliptic boundary-value problems is developed both by variational and sub-supersolution methods. Here, a G-symmetric solution is one that is invariant with respect to a group G of congruence mappings. First, this theory is applied to the construction of stationary solutions of the Vlasov-Poisson and Vlasov-Maxwell system that are periodic with respect to all or some space coordinates. These stationary solutions induce time-periodic travelling wave solutions. It turns out that — due to the boundary conditions of an ideal conductor — one obtains nontrivial solutions if and only if there is a nontrivial external force field or a background density. Secondly, the existence of stationary and quasistationary solutions of the Vlasov-Maxwell system, where the distribution functions are functions of the particle energy and the canonical momentum, is established by using the sub-supersolution method. For the quasistationary case we need again the presence of an external force field.

P. J. MORRISON:

Necessary and sufficient conditions for stability of plane parallel shear flow

We derive necessary and sufficient conditions for the stability of inviscid, two-dimensional shear flow with profiles $U(y)$ that are monotonic, analytic functions of the cross stream coordinate y. The analysis, which is based upon the Nyquist method, includes a means for delineating the possible kink of bifurcations that involve the presence of the continuous spectrum, including those that occur at nonzero wavenumbers. Several examples are given.

L. DESVILLETTES:

Validity of the linear non cut-off Boltzmann equation

[Joint work with M. Pulvirenti.]

In a paper of 1972, Gallavotti proved that a Hamiltonian equation of the type

$$\begin{cases} \dot{x} &= v, \\ \dot{v} &= -\sum_{i=1}^{N} \nabla V^{(\varepsilon)}(x-c_i), \end{cases}$$

where $V^{(\varepsilon)}(x) = V(x/\varepsilon)$ and the c_i are scatterers distributed according to Poisson's law with parameter μ/ε, generated a flow $f_0(T^{+}_{\varepsilon,c}(x,v))$ which expectation converges, when $\varepsilon \to 0$ and f_0 is Lipschitz bounded, towards the solution of a linear Boltzmann equation with initial datum f_0. The result is true for hard spheres or potentials V with compact support. We extend this result in the case when V has an infinite range (e. g. when $V(r) = r^{-s}$). The corresponding Boltzmann equation is singular.

R. ILLNER:

Stellardynamic equations with Manev type force terms

Consider a stellardynamic equation

$$\partial_t f + v \cdot \nabla_x f + (E_1[\rho] + E_2[\rho]) \cdot \nabla_v f = 0$$

with

$$E_1[\rho](x,t) = -\gamma \int \frac{x-y}{|x-y|^3}\, \rho(y,t)\, dy$$

and

$$E_2[\rho](x,t) = -2\,\varepsilon \int \frac{x-y}{|x-y|^4}\, \rho(y,t)\, dy.$$

The force term $E_1 + E_2$ is formally generated by a point potential $U(r) = -\gamma/r - \varepsilon/r^2$, known in the literature as a Manev potential. We focus mostly on the case $\gamma = 0$, $\varepsilon > 0$ ("pure" Manev potential) and deal with issues like nonexistence of global solutions, transformation properties of the equation, linear stability analysis and a "justification" of the term E_2. A local existence proof requires delicate estimates, some of which we show. The significance of additional Boltzmann collision terms is also discussed.

H. ANDRÉASSON:

Controlling the propagation of the support for the relativistic Vlasov equation with a selfconsistent Lorentz invariant field

Successful techniques have been developed for controlling the propagation of the support (preventing blow up) for the classical Vlasov-Poisson equation, leading to global existence of smooth solutions. It is well known that these techniques all fail for the *relativistic* Vlasov-Poisson equation. This equation is a hybrid of a relativistic transport equation and a classical, Galilean invariant, field equation. In this paper we introduce a new equation for the field making it Lorentz invariant. We show that the propagation of the support, for solutions satisfying this equation and the relativistic Vlasov equation, may be controlled.

E. CAGLIOTI:

Landau damping for the one-dimensional Vlasov-Poisson equation

[Joint work with C. Maffei.]

It is proved that Landau damping occurs for a class of solutions of the Vlasov-Poisson equation on a 1D torus. The proof is achieved by a scattering theory approach. A consequence of this result is that a large class of stationary solutions of the Vlasov-Poisson equation (e. g. the Maxwellian) is unstable in a weak norm.

J. SOLER:

Asymptotics for Schrödinger-Poisson and Wigner-Poisson systems

[Joint work with J. L. López.]

Using an appropriate scaling group to the 3D Schrödinger equation and the equivalence between the Schrödinger formalism and the Wigner representation of quantum mechanics it is proved that, when time goes to infinity, the limit of the self-consistent potential can be identified as the Coulomb potential. As a consequence, Schrödinger-Poisson and Wigner-Poisson systems are simplified asymptotically and their long time behaviour is explained through the solutions to the corresponding linear limit problems.

Y. GUO:

Smooth irrotational flow in the large to the Euler-Poisson system in $\mathbb{R}^{3+1}$

A simple two-fluid model to describe the dynamics of a plasma is the Euler-Poisson system, where the compressible electron fluid interacts with its own electric field against a constant charged ion background. The plasma frequency produced by the electric field plays the role of a 'mass' term to the linearized system. Based on this 'Klein-Gordon' effect, we construct global smooth irrotational flows with small amplitude of the electron fluid.

H. CABANNES:

Analytical solution for a semi-continuous model of the Boltzmann equation

A two-dimensional model of the Boltzmann equation is (cp. [1]):

$$\frac{\partial N}{\partial t} + \cos(\theta)\,\frac{\partial N}{\partial x} = \frac{1}{2\,\pi}\int_0^{2\,\pi}\{N(\varphi)\,N(\varphi+\pi) - N(\theta)\,N(\theta+\pi)\}\,d\varphi, \qquad (1)$$

where we wrote $N(\varphi)$ for $N(t,x;\varphi)$, $N(\varphi+\pi)$ for $N(t,x;\varphi+\pi)$, ... The functions $N(t,x;\theta)$ are periodic in θ with period $2\,\pi$.

For the homogeneous solutions, i. e., $(\partial/\partial x)\,N(t,x;\theta) \equiv 0$, if the initial density is symmetric, i. e. $N(0;\theta+\pi) \equiv N(0;\theta)$, so is the solution, and equation (1) becomes

$$\frac{\partial N(t;\theta)}{\partial t} + N^2(t;\theta) = \frac{1}{2\,\pi}\int_0^{2\,\pi} N^2(t;\varphi)\,d\varphi. \qquad (2)$$

The equation (2) is a nonlinear integro-differential equation, the integration of which can be reduced to the integration of a linear ordinary differential equation (see [3]). Then we have obtained a family of solutions which depend on an arbitrary function $f(\theta)$ and which exist globally in time, even, in some cases, with partially negative initial data.

[1] H. Cabannes. Global solution of the discrete Boltzmann equation. In: H. Neunzert and D. C. Pack (ed.), *Mathematical problems in the kinectic theory of gases*, p. 25–44. Peter Lang-Verlag, 1980.

[2] R. Gatignol. *Théorie cinétique d'un gaz à répartition discrète de vitesses*. Lecture notes in physics 36. Springer-Verlag, 1975.

[3] H. Cabannes and N. Sibgatullin. Homogeneous solutions for a semicontinuous model of the Boltzmann equation. *Math. Mod. Meth. Appl. Sci.* 5:1129–1138, 1995.

S. M. DESHPANDE:

New developments in kinetic schemes

A Moment Method strategy has been followed for several years at the CFD Centre for developing kinetic schemes for obtaining numerical simulations of PDE of fluid dynamics. Here we consider the Least Squares Kinetic Upwind Method (LSKUM) in which space derivatives in the Boltzmann equation are discretised using Least Squares. A corresponding numerical scheme for the Euler equations has been obtained by taking moments. This scheme has been proved to be extremely robust and has been tested on a large number of problems with speed varying from subsonic to hypersonic. Also, the usual treatment of outer boundary conditions has been replaced by a kinetic treatment of boundary conditions. The kinetic treatment helps in implementing flow tangency, inflow and outflow boundary conditions and surface transpiration conditions. The latter boundary condition is very useful in computing aerodynamic force coefficients for a configuration with control surface deflection. A large number of results will be presented to demonstrate the success of the above ideas.

W. WAGNER:

Convergence results for stochastic particle systems related to the Boltzmann equation

[Joint work with S. Caprino and M. Pulvirenti.]

We consider a stochastic n-particle system in a bounded domain with diffusive boundary conditions at a possibly not constant temperature. We study the distance between the stationary measure for the particle system and the measure given by the n-fold product of the solution to the regularized stationary Boltzmann equation with the same boundary conditions. We show that, if the mean free path inverse is sufficiently small, the L_1-difference between the k-particle distribution functions of such two measures vanishes in the limit $n \longrightarrow \infty$, for any fixed k, with the order $1/n$.

R. ESPOSITO:

Derivation of the incompressible Navier-Stokes equation from particle systems on the lattice

We consider a stochastic particle system on the $\mathbb{Z}^d$ lattice, based on the simple exclusion process. In the diffusive scaling and the low Mach number limit we show that the non-equilibrium distribution stays close to a suitable modification of the local equilibrium with parameters ruled by the incompressible Navier-Stokes equation. The transport coefficients are obtained via a variational formula which is the rigorous counterpart of the Green-Kubo formula.

A. NOURI:

On the stationary Boltzmann equation in two space variables

We consider the stationary Boltzmann equation in a bounded convex set of $\mathbb{R}^n$, with given indata on the boundary. The collision kernel in the Boltzmann operator is truncated for small velocities. An existence theorem is proved far from equilibrium. A-priori bounds on the energy and the entropy are obtained by controlling the distribution function in the domain by its value on the outgoing boundary.

F. GOLSE:

On the distribution of free path lengths for the periodic Lorentz gas

[Joint work with H. S. Dumas and L. Dumas and then with J. Bourgain and B. Wennberg.]

For all $0 < \varepsilon < 1$, $0 < r < 1/2$ and $\gamma \geq 1$, let

$$Z_\varepsilon := \{x \in \mathbb{R}^n \,|\, \mathrm{dist}(x, \varepsilon\,\mathbb{Z}^n) > r\varepsilon^\gamma\}.$$

Consider the evolution of a gas of point particles moving at speed 1 in Z_ε. Collisions between particles are neglected; only the reflections of the particles at the boundary are taken into account. Hence the free path length is, for all $x \in \overline{Z}_\varepsilon$ and $\omega \in S^{n-1}$:

$$\tau_\varepsilon(x,\omega) = \inf\{t > 0 \,|\, x - t\,\omega \in \partial Z_\varepsilon\}.$$

The phase space $Z_\varepsilon \times S^{n-1}/\varepsilon\,\mathbb{Z}^n$ being endowed with the natural normalized Lebesgue measure, the distribution of τ_ε as $\varepsilon \longrightarrow 0$ is investigated. It is found that the limiting behaviour of this distribution depends on γ, precisely upon whether γ is bigger or smaller then the critical value $\gamma_c = \frac{n}{n-1}$ (this value γ_c corresponds exactly to the Boltzmann-Grad scaling).

B. WENNBERG:

Uniqueness and existence for the spatially homogeneous Boltzmann equation

[Joint work with S. Mischler.]

We prove existence and uniqueness of solutions to the spatially homogeneous Boltzmann equation under the hypothesis that the initial data have bounded mass and energy. Previous results in this direction have required additional hypothesisses on the initial data, such as entropy bounds and bounds on higher moments. The entropy condition is replaced by a regularity estimate for the gain term, and the moment conditions are avoided by the use of a very precise Povzner-like inequality. The same Povzner inequality can be used to prove if there is a solution for which certain lower moments are bounded for some $t > 0$, then the energy must be bounded at $t = 0$.

G. TOSCANI:

Entropy production for the Fokker-Planck equation and consequences

We reckon the rate of exponential convergence to equilibrium, both in relative entropy and in relative Fisher information, for the solution to the spatially homogeneous Fokker-Planck equation

$$\partial_t f = \nabla_v \cdot (\nabla_v f + v f).$$

The result follows from lower bounds on the entropy production which are explicitly computable. In particular, it is shown that Gross's logarithmic Sobolev inequality is a direct consequence of the lower bound for the entropy production relative to the Fisher information. Due to the Csizlar-Kullback inequality, we obtain exponential convergence in the strong L^1-norm with a "sharp" rate.

R. MARRA:

The hydrodynamic limit of the Boltzmann equation in the Boussinesq approximation

We consider a flow in a slab subject to the gravitational force. The walls are at constant temperature. The system is described by the Boltzmann equation. We consider the behaviour of the system when the Knudsen number ε is small, the velocity field is of order ε, as well as the temperature difference, and the force is of order ε^2. We prove that there exists a solution on a finite time interval, converging to a global Maxwellian, while its moments are close up to order ε^2 to the density, velocity and temperature solutions of the Boussinesq equations.

C. BARDOS:

Propagation of defect measures and applications

Let (u_ε) be a sequence of solutions of a partial differential equation $P u_\varepsilon = 0$, converging weakly to zero. The non-strong convergence is described by an H measure. To take into account the notion of propagation, a microlocal version of this object, closely related to the Wigner measure, is introduced. Applications of this notion to classical boundary value problems for wave and Schrödinger equations are given.

R. GLASSEY:

Large–data initial–value problems for the Vlasov–Maxwell system in higher dimensions

The motion of a collisionless plasma is modeled by the Vlasov–Maxwell system. For the relativistic Vlasov–Maxwell system in two space and momentum variables, new integral representations of the electromagnetic fields are derived. Using these representations it is shown that smooth solutions can break down only if particles of the plasma approach the speed of light. Then, for smooth Cauchy data of unrestricted size, a priori bounds on particle speeds are derived, from which it follows that solutions remain smooth for all time. A generalization to the case in which the velocity distribution function depends on two space variables and three velocity variables will also be treated.

P. DEGOND:

Kinetic layers in semiconductor macroscopic models

[Joint work with C. Schmeiser.]

A heterojunction is a junction between two semiconductor materials of different crystalline structure. This results in an abrupt change of the band characteristics (namely the mass of the electrons, their potential energy at rest) and in the introduction of scattering mechanisms at the interface. Our goal is to derive interface conditions at the heterojunction for the various macroscopic models currently used by the electronical engineers. To that purpose, we start with the interface problem at the level of the kinetic model. The interface conditions are expressed by means of the reflection and transmission probabilities that are derived from the scattering operator associated with the microscopic particle dynamics. These interface conditions preserve the particle flux and are identically satisfied by functions of the energy only. This allows to perform a diffusion approximation of the kinetic model which provides natural interface conditions to the first considered macroscopic model, namely the SHE equation (SHE for spherical harmonic expansion). These interface conditions depend on a parameter, the extrapolation length, which contains all the information about the microscopic scattering dynamics. From these interface conditions at the level of the SHE equation, we can derive, by a moment method, interface conditions for the energy-transport and drift-diffusion models that are the two other macroscopic models considered here. Again, an extrapolation length or matrix appears in these conditions, which provide improvements to the standard interface conditions used by the engineers.

I. GASSER:

The energy transport and drift-diffusion equations as relaxation limits of the hydrodynamic model for semiconductors

Two relaxation limits of the hydrodynamic model for semiconductors are investigated. Using compensated compactness tools we show the convergence of scaled entropy solutions of the hydrodynamic model to the solutions of the energy-transport model and the drift-diffusion equations, according to different time scales.

H. SPOHN:

Long time asymptotics for quantum particles in a periodic potential

After a sufficiently long time the position of a classical particle in a periodic potential will either stay bounded or grow diffusively ($\cong \sqrt{t}$), resp. ballistically ($\cong t$). In fact, since in general the single cell phase space is mixed any intermediate behaviour may also occur. In contrast, a quantum particle in a periodic potential moves always ballistically. To support our claim we consider a quantum particle subject to a periodic potential $U(x)$ and a slowly varying potential $V(\varepsilon x)$. We prove that on the time scale $\varepsilon^{-1} t$ the distribution in phase space is well approximated by the semiclassical equations of motion with a kinetic energy determined through the band structure. For a single band, techniques from semi-classical approximations are used. The transition between bands are proven to be suppressed by a fast time averaging. As applications we study the hydrodynamic limit for independent bosons and fermions in a crystal potential and the small k, ω behaviour of the equilibrium structure function.

N. J. MAUSER:

The Dirac equation: (Semi) classical / nonrelativistic limits

The description of electrons in a relativistic and quantum-mechanical framework is based on the Dirac equation governing the time evolution of the 4-component spinor. In a self-consistent model the Dirac equation is coupled to the Maxwell equations. In this short presentation we deal with the linear Dirac equation only, i. e. the time dependant electromagnetic potential is given. The limit of vanishing Planck's constant (with fixed velocity of light) yields a relativistic

Vlasov equation with a Lorentz force and is obtained as a special case of the general theory of homogenization limits using Wigner transforms. In the limit of infinite velocity of light (with fixed Planck's constant) we recover Schrödinger equations.

Y. BRENIER:

Two particle methods related to the theory of scalar conservation laws

In the first part of the talk it is shown that sticky particles moving on the real line can be entirely described by scalar conservation laws, when the initial density tends to some measure that is absolutely continuous with respect to the Lebesgue measure. In the second part, a different model of particles that exchange their velocities according to their order along the real line is shown to have the same macroscopic behaviour as sticky particles, although there is no dissipation of energy at the microscopic level. Instead of getting stuck together, particles move around each other with the same time averaged speed. In the discrete case of a finite number of particles, the velocity exchange is performed according to an asymptotic permutation that can be precisely characterized, as time goes to infinity.

F. POUPAUD:

Transport equations with discontinuous coefficients

[Joint work with M. Ruscle. To appear in *CPDE* (1997).]

We study the multidimensional linear transport equation in the case where the coefficients are not smooth. Then Dirac distributions appear in the solution in finite time. Therefore the solution cannot be defined in the distributional sense. We use generalized characteristics introduced by Fillipov (1962). The measure solutions are defined as the image of the initial measure data under the flow. When the coefficients satisfy a one-sided Lipschitz bound we prove existence, uniqueness and stability of the solutions. This result is a generalization of a result obtained in 1D by Bouchut and James using functionalanalytic arguments. We point out that the method of Di Perna and Lions (1988) to obtain renormalized solutions is of no use in this context. Indeed, the characteristics obtained by the previous authors are defined a. e., so they cannot describe the transport of δ-distributions.

E. GRENIER:

The quasineutral limit of the Euler-Poisson system

We consider the quasineutral limit of the Euler-Poisson system (on the ions), with massless electrons as the (rescaled) electric permittivity goes to zero. Using pseudodifferential techniques to split the electric field and extract its most "dangerous" part in order to put it in the hyperbolic part of the Euler equations, we prove strong convergence of solutions for smooth initial data, as long as the corresponding solution of the limit system remains smooth.

H. BABOVSKY:

Kinetic boundary layers and their numerical coupling to fluid flows

Two problems are addressed: The design of a rigorous numerical scheme for the steady Boltzmann equation and its coupling to fluid flows. The justification of such a coupling is discussed.

A recently developed scheme for 1D slab problems for the Boltzmann equation is introduced. After discretization of velocity space, the Boltzmann equation turns into a discrete-velocity system. The slab problem is then an ODE system with two-point boundary conditions and is solved via a shooting method combined with a fixed-point iteration. The resulting code is capable of treating small Knudsen numbers generating a kinetic boundary layer with fluid dynamic asymptotic state.

Two generalizations of the scheme are discussed. First, the introduction of velocities with vanishing normal components turns the ODE system into a differential-algebraic one. Secondly, an appropriate semi-discretization of (e. g. periodic) 2D and 3D problems leads to a higher dimensional modified ODE (resp. DAE) system. The numerical scheme can be readily extended to these situations.

A. KLAR:

A numerical method for instationary transport equations in diffusive regimes

An asymptotic-induced scheme for instationary transport equations with the diffusion scaling is developed.

The main problem for numerical work on transport equations in these regimes is the stiffness of the equations for small mean free paths. For standard numerical schemes one has to use a very fine and expensive discretization with a discretization size depending on the mean free path. The presented scheme works uniformly for all ranges of mean free paths with a discretization which is independent of the mean free path. It is based on the asymptotic analysis of the diffusion limit. The results of a boundary layer analysis are also included in the scheme.

A theoretical investigation of the behaviour of the scheme in the diffusion limit is given and an approximation property is proven. Moreover, numerical results for different physical situations are shown and the uniform convergence of the scheme is established numerically.

An indication is given how to extend the scheme to a more complicated situation like the Boltzmann equation in the incompressible Navier-Stokes limit. Here a more careful use has to be made of the perturbation procedure leading from the Boltzmann equation to the incompressible Navier-Stokes equation.

J. DOLBEAULT:

Confinement for the Vlasov-Poisson-Fokker-Planck system

The problem of finding the stationary solutions of the Vlasov-Poisson-Fokker-Planck system

$$\partial_t f + v \cdot \nabla_x f - \nabla_x \phi \cdot \nabla_v f = \nabla_v \cdot (\beta\, v\, f + \sigma\, \nabla_v f),$$

$$-\Delta\phi(t,x) = \int f(t,x,v)\, dv - n(x)$$

has been addressed first by K. Dressler. Here $n(x)$ represents a background density of particles with opposite sign charges that tend to thermalize the system at the temperature $\theta = \sigma/\beta$. When U_0 is an external potential such that $-\Delta U_0 = n$, he gave a sufficient condition on the growth of U_0 (linear growth) such that a stationary solution exists (and is a Maxwellian function with zero mean velocity and temperature θ). It turns out that a necessary and sufficient condition is $\exp(-U_0/\theta) \in \mathrm{L}^1$. Such an external potential will be called "confining". The purpose of this lecture is to investigate also the properties of the solutions of the evolution problem when the potential U_0 is confining by using the free energy and the configurational free energy.

J. STRUCKMEIER:

Some estimates for the collision operator of the Boltzmann equation

The paper presents some new estimates on the gain term of the Boltzmann collision operator. For Maxwellian molecules, it is shown that the L^∞-norm of the gain term can be bounded in terms of the L^1- and L^∞-norm of the density function f. In the case of more general collision kernels, like the hard-sphere interaction potential, the gain term is estimated pointwise by the L^∞-norm of the density function and the loss term of the Boltzmann collision operator.

J. BATT:

The one-dimensional Vlasov-Poisson system – rescaling and asymptotic behaviour

[Joint work with M. Kunze and G. Rein. To appear in *Adv. Diff. Eqns.* (1997).]

We consider a one-dimensional, monocharged plasma as described by the Vlasov-Poisson system and investigate the behaviour of the solutions for large times.

Using a rescaling method we are able to determine an explicit solution of the system which corresponds to a globally attractive steady state for the rescaled system. We investigate in which sense and at which rate the solutions of the rescaled system converge to this global attractor and interpret the results for the original system.

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