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Nichtlineare Eigenwertaufgaben

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The meeting has been organized by T. Küpper (Köln) and Ch. Stuart (Lausanne). The organizers feel that this was an extraordinarily successful meeting from two points of view. The scientific discussion was rich and well-informed and, to an unusual degree, it involved almost the whole group at various stages. This probably reflects a lucky choice of participants drawn from different research groups, but having important common factors both in terms of mathematical content and individuals concerned. The other aspect which struck us was the wonderfully friendly, open and constructive atmosphere which pervaded the meeting from the very first day. It certainly contributed greatly to the success of the scientific component.

Let us mention some of the themes which provoked particularly stimulating and fruitful exchanges:

- existence and bifurcation of bound states for periodic and almost-periodic elliptic equations on $\mathbb{R}^n$ (Troestler, Ambrosetti, Séré, Coti Zelati, Buffoni)
- bifurcation from points in the essential spectrum (Troestler, Ambrosetti, Ruppen, Esteban, Reichel)
- multiplicity results for elliptic equations based on geometric properties of the domain (Heinz, Maier-Paape, Cao)
- global aspects of bifurcation for elliptic equations (Arcoya, Gámez, Rynne)
- special problems from physics involving the above phenomena (Alama, Séré, Tarantello, Toland, Kielhöfer)

S. ALAMA:
Heteroclinic solutions in $\mathbb{R} \times \mathbb{R}$ for elliptic systems with multiple well potentials
We study entire solutions on $\mathbb{R}^2$ of the elliptic system $-\Delta U + \nabla W(U) = 0$

where $W : \mathbb{R}^2 \to \mathbb{R}^2$ is a multiple-well potential. We seek solutions $U(x_1, x_2)$ which are heteroclinic, in the sense that not only do they connect for each fixed $x_2 \in \mathbb{R}$ a pair of constant global minima of W, but also they connect a pair of distinct one dimensional solutions when x_2 varies from $-\infty$ to ∞. These solutions describe the local structure of smooth phase boundary curves for a vector-valued Allen-Cahn equation, and their existence implies that the transition profiles may vary tangentially along the interface. Our method involves approximation by boundary-value problems, *a priori* estimates, and original global variational arguments which ensure that the limiting solution attains the desired asymptotic conditions. These results are joint work with Lia Bronsard and Changfeng Gui.

M. WILLEM:

Solitary waves with prescribed speed on infinite lattices

We consider the system

$$\ddot{q}_k(t) = V'(q_{k+1}(t) - q_k(t)) - V'(q_k(t) - q_{k-1}(t)), \quad k \in \mathbb{Z}.$$

A solitary wave is a solution of the form $q_k(t) = u(k - ct)$ where $c > 0$ is fixed. Using variational methods we prove the existence of a non-trivial solitary wave such that $\dot{u} \in L^2(\mathbb{R})$ and $\dot{u} \geq 0$, if $V(u) = c_0^2 u^2/2 - W(u)$, $W(0) = W'(0) = W''(0) = 0$, $c > c_0$, $\sup_{u \geq 0} W(u) > 0$, and, for some $\alpha > 2$, $u \geq 0 \Rightarrow 0 \leq \alpha W(u) \leq W'(u)u$.

CH. TROESTLER:

Bifurcation in spectral gaps for a Schrödinger equation

The aim of this talk is to show that the nonlinear Schrödinger eigenvalue problem

$$-\Delta u + V(x)u - \partial_u F(x, u) = \lambda u, \quad u \in H^1(\mathbb{R}^N),$$

where $V \in L^\infty$ and $F \in C^1$ are both $\mathbb{Z}^N$-periodic in x, bifurcates from the trivial solution at the right endpoint of every spectral gap of $-\Delta + V$. Bifurcation here means that there is a sequence of nontrivial solutions u_λ going to zero as λ approaches the right endpoint. Actually, some convergence rate on u_λ is established. No convexity condition on F is assumed.

Our method is a combination of variational arguments, concentration-compactness techniques, and estimates.

The following interesting fact is also proven: if $P : H^1(\mathbb{R}^N) \to H^1(\mathbb{R}^N)$ denotes the projection on the eigenspace associated to a component of the spectrum of $-\Delta + V$, then $P : H^1 \cap L^p \to H^1 \cap L^p$ is L^p-continuous.

D. ARCOYA:

Quasilinear elliptic problems, resonant and asymmetric nonlinearities

Bifurcation theory is applied to study existence of solutions for different elliptic nonlinear boundary value problems. First, resonant problems for the p-Laplace operator ($p > 1$) in a bounded domain $\Omega \subset \mathbb{R}^N$ are considered. Specifically, it is proved existence and multiplicity for the b.v.p.

$$\left.\begin{array}{c} -\Delta_p u \equiv -\mathrm{div}\,(|\nabla u|^{p-2}\nabla u) = \lambda_1 |u|^{p-2}u + f(x,u),\ x \in \Omega \\ u = 0,\ x \in \partial\Omega \end{array}\right\}$$

with a continuous function $f(x,s)$ satisfying $\lim_{s\to+\infty} f(x,s) = 0$, uniformly in $x \in \Omega$. Among others arguments, our results (joint work with A. Ambrosetti) are based on a suitable estimate on the behavior at infinity of the continua emanating from infinity. This motivates the exhaustive study (joint work with J.L. Gámez) of this kind of estimates near the bifurcation point. Rabinowitz' global bifurcation theorem is also used to prove (joint work with S. Villegas) existence of solutions for asymmetric semilinear problems (depending on the parameter λ) like

$$\left.\begin{array}{c} -\Delta u = f(x,u),\ x \in \Omega \\ u = 0,\ x \in \partial\Omega \end{array}\right\}$$

with $\lambda_1 < \lambda \equiv \lim_{s\to-\infty} f(x,s)/s \not\leftarrow \lim_{s\to+\infty} f(x,s)/s = +\infty$.

J.L. GÁMEZ:

Global bifurcation of positive solutions for an elliptic equation with indefinite homogeneous nonlinearity. An example of the second alternative of Rabinowitz's Theorem.

We consider an elliptic problem of the type

$$\begin{array}{rl} -\Delta u = \lambda u + g(x)|u|^{p-2}u & \text{in } \Omega\,, \\ u = 0 & \text{on } \partial\Omega\,, \end{array} \tag{1}$$

where Ω is a bounded and regular domain in $\mathbb{R}^N$, and $g \in L^\infty(\Omega)$. Depending on the properties of the sign of g, we study the local and global behavior of the unbounded branch of positive solutions of (1) bifurcating from $(\lambda_1(\Omega), 0) \in \mathbb{R} \times H_0^1(\Omega)$. We also consider the case of a weighted linear part, i.e.

$$\begin{aligned} -\Delta u = \lambda h(x)u + g(x)|u|^{p-2}u \quad & \text{in } \Omega, \\ u = 0 \quad & \text{on } \partial\Omega. \end{aligned} \tag{2}$$

Under suitable hypotheses on the sign behavior of $g, h \in L^\infty(\Omega)$, we find a case where the second alternative of Rabinowitz's theorem is satisfied, this is, the branch of positive solutions of (2) emanating from $(\lambda_1(\Omega, h), 0)$ meets another (principal) eigenvalue $(\lambda_{-1}(\Omega, h), 0)$. We finally give conditions to extend previous results to the case $\Omega = \mathbb{R}^N$.

B. RYNNE:

Bifurcation from zero or infinity in nonlinear Sturm-Liouville problems which are not linearizable

We consider the nonlinear Sturm-Liouville problem

$$\begin{gathered} Lu := -(pu')' + qu = \lambda au + h(\cdot, u, u', \lambda), \quad \text{in } (0, \pi), \\ a_0\, u(0) + b_0\, u'(0) = 0, \quad a_1\, u(\pi) + b_1\, u'(\pi) = 0, \end{gathered}$$

where a_i, b_i are real numbers with $|a_i| + |b_i| > 0$, $i = 0, 1$. Suppose that the non-linearity h satisfies a condition of the form

$$|h(x, \xi, \eta, \lambda)| \leq M_0|\xi| + M_1|\eta|, \quad (x, \xi, \eta, \lambda) \in [0, \pi] \times \mathbb{R}^3,$$

as either $|(\xi, \eta)| \to 0$ or $|(\xi, \eta)| \to \infty$, for some constants M_0, M_1. Then we show that there exists global continua of non-trivial solutions (λ, u) bifurcating from $u = 0$ or '$u = \infty$' respectively. These global continua have similar properties to those of the continua found in Rabinowitz' well known global bifurcation theorem.

A. AMBROSETTI:

Poincaré-Melnikov theory, a variational approach

We deal with some recent advances obtained in the existence of homoclinics via a variational approach.

As a model problem, let us consider the Duffing equation

$$-u'' + u = u^3 + \varepsilon g(t), \quad t \in \mathbb{R}. \tag{1}$$

It is well known that if the stable and unstable manifold through the origin cross transversely in another point, then there exist homoclinics and (1) has a chaotic dynamic. This transversal intersection property can be verified by checking that the *Poincaré-Melnikov function* has a simple zero.

We will show that this kind of results can be obtained in a greater generality by a variational technique, based on some new perturbation arguments in critical point theory. This abstract setting provides a common framework for many problems, usually considered different in nature and faced by different techniques. Applications include second order Hamiltonian systems, semilinear elliptic equations in $\mathbb{R}^n$, some problems arising in nonlinear optics and the existence of *semiclassical bound states* of a class of Schroedinger equations with a bounded potential Q, such as

$$-\varepsilon^2 \Delta u + \omega u + Q(x)u = u^p , \quad x \in \mathbb{R}^n .$$

We finally shortly survey a remarkable recent result by *Berti* and *Bolle*, showing that the preceding abstract variational set up can be extended to prove that a wide class of perturbed systems have a chaotic behaviour: there exist infinitely many homoclinic solutions, the system has a Bernoulli shift structure, a positive topological entropy and a sensitive dependence on the initial conditions.

D.M. CAO:
Multiplicity of positive solutions for a semilinear equation
We are concerned with the following problems:

$$\begin{aligned} \varepsilon\Delta u + u &= u^p \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega , \end{aligned} \qquad (DP)-\varepsilon$$

and

$$\begin{aligned} \varepsilon\Delta u + u &= u^p \quad \text{in } \Omega, \\ \tfrac{\partial u}{\partial \nu} &= 0 \quad \text{on } \partial\Omega , \end{aligned} \qquad (NP)-\varepsilon$$

where Ω is a bounded smooth domain in $\mathbb{R}^n$ $(n \geq 2)$, $1 \leq p \leq \frac{n+2}{n-2}$ if $n \geq 3$, $1 \leq p \leq +\infty$ if $n = 2$, ε is a positive constant and ν is the unit outward normal to $\partial\Omega$. We introduce some recent results concerning the effect of domain geometry on the number of the single and multi-peaked positive solutions to $(DP)-\varepsilon$ and $(NP)-\varepsilon$ for small ε.

H.J. RUPPEN:

Conflicting nonlinearities

We consider the nonlinear eigenvalue problem

$$\begin{cases} -\Delta u(x) - q(x)|u(x)|^\alpha u(x) + \mu r(x)|u(x)|^\beta u(x) = \lambda u(x), & x \in \mathbb{R}^N \\ u \in H^1(\mathbb{R}^N) \setminus \{0\}, \end{cases} \tag{1}$$

where $N \geq 1$, $\mu > 0$ is kept fixed, $0 < \alpha < \beta$ $(< 4/(N-2))$. The functions q and r are positive and $q \in L^\infty(\mathbb{R}^N) \cap L^{\frac{2+\beta}{\beta-\alpha}}(\mathbb{R}^N)$ whereas $r \in L^\infty(\mathbb{R}^N)$ and $r(x) \geq A > 0$ a.e. We are interested in solutions (λ, u) with $\lambda < 0$. Concerning the nonexistence of results, we can show:

Theorem 1. There exists a decreasing function $c(\mu) = \text{const} \cdot \mu^{-\alpha/(\beta-\alpha)}$ such that the problem (1) has no solutions as soon as $\lambda < -c(\mu)$.

Concerning the existence of solutions, we can show:

Theorem 2. There exists a nonincreasing function $\bar{\mu}_0(\lambda)$ such that the following holds: We put $\lambda_0 = \inf\{\lambda < 0 \,;\, \bar{\mu}_0(\lambda) > \mu\}$. Then the problem (1) has, for this fixed value of μ, (at least) two pairs of solutions pairs $(\lambda, \pm v_{0,\lambda})$ and $(\lambda, \pm w_{0,\lambda})$, $\forall \lambda \in (\lambda_0, 0)$. A similar result holds for multiple solutions:

Theorem 3. There exists a nonincreasing function $\bar{\mu}_k(\lambda)$ (for $k \in \mathbb{N}$) such that the following holds: We put $\lambda_k = \inf\{\lambda < 0 \,;\, \bar{\mu}_k(\lambda) > \mu\}$. Then the problem (1) has, for this fixed value of μ, (at least) $k+1$ pairs of solutions pairs $(\lambda, \pm v_{j,\lambda})$ and $(\lambda, \pm w_{j,\lambda})$ $(j = 0, \ldots, k)$, $\forall \lambda \in (\lambda_0, 0)$.

E. SÉRÉ:

Nonlinear Dirac systems

The Dirac equation is the equation for the wave function of an electron in relativistic quantum mechanics. The nonlinear Dirac models arise when one deals with interactions, either between an electron and a force field, or between several electrons. We are interested in localized stationary solutions of various nonlinear Dirac equations, in particular the Soler model, the Klein-Gordon-Dirac and the Maxwell- Dirac models. For these models, we find stationary solutions with exponential decay in the space variables. These solutions represent "particle-like" states.

Our method of proof is variational. It involves the study of a functional which is strongly indefinite and presents a lack of compactness. This functional present some similarities with the action functional of the homoclinic problem in Hamiltonian systems. (Based on joint work with M.J Esteban and V. Georgiev.)

M.J. ESTEBAN:
Global branch of solutions for some nonlinear problems with lack of compactness

Since there is no theory of bifurcation for non compact problems, we have started a systematic analysis of what happens in a bifurcation diagram when one passes (by going to a limit in some parameter) from a compact to a noncompact situation. We have done this for degenerate elliptic problems as $-|x|^2\Delta u + \lambda u = f(x,u)$ in $B(0,1)$. In dimension $N = 1$ a large amount of connected branches of solutions have been found in a joint work with H. Berestycki. But, for some nonlinearities, with M. Ramaswamy, we have proved nonexistence of positive solutions at all. Then, with J. Giacomoni, we have studied various classes of nonlinearities and analyzed the existence or nonexistence of connected branches of positive solutions and their qualitative behaviour. We have also considered an elliptic problem in the whole space $\mathbb{R}^N$, like $-\Delta u + \lambda u = f(x,u)$. We have considered different types of nonlinearities (superlinear, sublinear, bounded, unbounded, ...) and in all cases we have studied the existence and the behaviour of the branches. A very complete picture follows from this study. (Based on joint work with J. Giacomoni)

A. TERTIKAS:
Existence and bifurcation for a degenerate elliptic problem

We study the existence and nonexistence of principle eigenvalues for linear eigenvalue problems of the form ($N \geq 3$)

$$-\Delta u = \lambda V(x)u \quad \text{in} \quad \mathbb{R}^N, \quad u \in \mathcal{D}^{1,2}(\mathbb{R}^N),$$

when $V \in L^1_{\mathrm{loc}}(\mathbb{R}^N)$ and $V \mathrel{\overset{>}{\not\equiv}} 0$ in $\mathbb{R}^N$, where we allow strong singularities in the potential V. We establish that phenomena appearing under critical exponent nonlinearities (*Brezis-Nirenberg*), are present in linear problems when the potential V has strong singularities.

TH. BARTSCH:

Order relations between solutions of elliptic partial differential equations

We consider the semilinear elliptic boundary value problem

$$-\Delta u = f(u) \quad \text{in} \quad \Omega \subset \mathbb{R}^N, \quad u\big|_{\partial\Omega} = 0, \tag{$*$}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain and $f \in C^1(\mathbb{R})$. Suppose that $f(0) = 0$, f grows subcritically and superlinearly, and that f' is bounded below.

Theorem 1. a) If $f'(0) < \lambda_2$, then there exists a solution u_1 of $(*)$ such that (i) u_1 changes sign, (ii) for any solution $u \neq 0$ of $(*)$ the following implication holds: $u < u_1 \Rightarrow u < 0\,,\ u > u_1 \Rightarrow u > 0$. b) if $f'(0) < \lambda_1$, then there exist solutions u_+, u_- of $(*)$ such that (i) $u_+ > 0$, $u_- < 0$, (ii) for any solution $u \neq 0$ of $(*)$: $u < u_+ \Rightarrow u < 0\,,\ u > u_- \Rightarrow u > 0$. Thus u_+ (u_-) is minimal (maximal) in the set of positive (negative) or sign-changing solutions.

Theorem 2. If f is odd, then $(*)$ has an unbounded sequence of solutions $\pm u_k$, $k \in \mathbb{N}$, such that a) u_k changes sign $\forall k$, b) $u_k - u_l$, $u_k + u_l$ change sign $\forall k \neq l$, c) for any solution $u \neq 0$ of $(*)$: $u < u_k \Rightarrow u < 0\,,\ u > u_k \Rightarrow u > 0$.

H.P. HEINZ:

Eigenfunctions with prescribed geometrical properties of semilinear problems

We consider equations of the form

$$Ay + B(y)y = \lambda y,$$

where A is a positive self-adjoint operator in a Hilbert space H, λ is a real parameter, and B is a continuous map from the domain of $A^{1/2}$ into the space of bounded linear operators in H. The problem is supposed to have variational structure in the sense that $N(y) := B(y)y$ is a gradient operator. Under suitable assumptions we prove the existence of solutions (y, λ) which satisfy a constraint of the form $\| y \|_H = R$ and which have the property that for a prescribed $n \in \mathbb{N}$ the corresponding eigenvalue λ is the n-th eigenvalue μ_n of the problem

$$Ah + B(y)h = \mu h.$$

These solutions also enjoy variational characterizations of Ljusternik-Schnirelman type. We give applications to nonlinear Dirichlet problems on an interval, periodic solutions of second-order systems, and radially symmetric solutions of nonlinear Schrödinger equations. The prescribed relationship between λ and the associated linear problem then entails prescribed geometric properties of the eigenfunctions so obtained (e.g. prescribed number of zeros or nodal lines).

V. COTI ZELATI:
Homoclinic and almost periodic solutions for a class of second order Hamiltonian systems
We consider a class of second order Hamiltonian systems of the form

$$-\ddot{q} + q = \alpha(t)V'(q). \tag{HS}$$

We assume that V is a superquadratic potential having a non degenerate minimum for $x = 0$ and that α depends almost-periodically on time. Under these assumptions we prove that there exist infinitely many homoclinic solutions of (HS).

In case α is a limit periodic function (or a almost periodic perturbation of a limit periodic function), and a suitable non degeneracy condition holds, we show that infinitely many limit (almost) periodic solutions of (HS) exist. (Based on joint work with P. Montecchiari and M. Nolasco.)

B. BUFFONI:
Shooting methods and topological transversality
We show that shooting methods for homoclinic or heteroclinic orbits in dynamical systems may automatically guarantee the topological transversality of the stable and unstable manifolds. The interest of such kind of results is twofold. First these orbits persist under perturbations which destroy the structure allowing the shooting method and, second, topological transversality is often sufficient when some kind of transversality is required to obtain chaotic dynamics. We focus on heteroclinic solutions in the Extended Fisher-Kolmogorov equation.

G. TARANTELLO:
Condensate solutions for the Chern-Simon-Higgs theory
We study the existence of condensate solutions for the Chern-Simon-Higgs model with the choice of a potential field where both the symmetric and asymmetric vacua occur as ground states. We show that if the Chern-Simons coupling parameter k is above a critical value, no such solutions can exist, while for $k > 0$ below this critical value there exist at least two condensate

solutions carrying the same quantized energy, as well as electric and magnetic charge. This multiplicity result accounts for the two vacuum states present in the model. In fact, as $k \to 0^+$ it is shown that the two solutions found "bifurcate" respectively from the asymmetric and symmetric vacuum states.

B. FIEDLER:

Meandering and drifting spirals

Hopf bifurcation of rotating spiral waves leads to meandering and drifting motions of the spiral tip. A center manifold reduction has recently been achieved, for the first time, for the noncompact symmetry group $G = SE(2)$ of rotations and translations in $\mathbb{R}^2$ (Sandstede, Scheel, Wulff). On $M = G \times_H V$, the dynamics takes the form

$$\dot{g} = g\,\alpha(v)\,, \quad \dot{v} = \varphi(v)\,.$$

Here v is in a local slice to the group orbit Gu_0, and $g \in G$. Equivariance with respect to h in the isotropy of u_0, that is $hu_0 = u_0$, takes the form

$$\varphi(hv) = h\varphi(v)\,, \quad \alpha(hv) = h\alpha(v)h^{-1}\,.$$

For trivial isotropy, $H = \{\mathrm{id}\}$, for example, a periodic solution $v(t)$ with frequency ω_{Hopf} gives rise to a cycloid tip motion with frequencies ω_{Hopf} and ω_0, the rotation frequency of u_0. Meandering occurs for $\omega_0 \notin \omega_{\mathrm{Hopf}} \cdot \mathbb{Z}$, whereas drift requires $\omega_0 \in \omega_{\mathrm{Hopf}} \cdot \mathbb{Z}$. (Based on joint work with B. Sandstede, A. Scheel and C. Wulff.)

J. YOU:

Perturbation of lower dimensional tori in nearly integrable Hamiltonian systems

In this talk, we prove the persistence of degenerate lower dimensional tori in integrable Hamiltonian systems under small analytic perturbations. That is

$$H = (\omega, y) + \frac{1}{2}\sum_{i=2}^{m} \Omega_i(\omega)(u_i^2 + v_i^2) + \frac{1}{2}v_1^2 - u_1^{2d} + P(x, y, u, v, \omega),$$

with an integer $d \geq 2$ possesses an n-dimensional torus for 'most' ω provided P is small and analytic. For the nondegenerate cases, both hyperbolic and elliptic, the persistence result has been given by many authors, for example, Graff, Elliason, Kuksin, Melnikov, Moser, Pöschel and Zehnder.

W. REICHEL:
Bifurcation for a p-Laplace equation in $\mathbb{R}^N$ by variational methods
We consider the nonlinear equation

$$\Delta_p u + \lambda u^{(p-1)} + s(x)u^{(\sigma+p-1)} = 0 \quad \text{in} \quad \mathbb{R}^N,$$
$$u \in W^{1,p}(\mathbb{R}^N), \quad N \geq 1,$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, $p > 1$, is the p-Laplacian and $t^{(q)} = |t|^q \operatorname{sign} t$ is the odd power function ($t \in \mathbb{R}, q > 0$). The problem is discussed under aspects of bifurcation from the trivial solution at $\lambda = 0$ in the $W^{1,p}$-norm. The nontrivial bifurcating sequence is obtained by solving the following constraint minimization problem for functions $u \in W^{1,p}(\mathbb{R}^N)$:

$$\text{Minimize} \int_{\mathbb{R}^N} \frac{|\nabla u|^p}{p}\, dx - \int_{\mathbb{R}^N} \frac{s(x)}{\sigma + p}|u|^{\sigma+p}\, dx \quad \text{over} \quad \int_{\mathbb{R}^N} |u|^p\, dx = \text{const.}$$

We assume

(H1) $s \in L^\infty(\mathbb{R}^N)$, $s(x) \geq A|x|^{-t}$ for $|x| \geq R_0$ and fixed $t \in [0, p)$, $A > 0$

(H2) $0 < \sigma < p(p-t)/N$

and one of the following conditions

(a) $s(x) \to 0$ for $|x| \to \infty$

(b) $s(x)$ radially symmetric and $N \geq 2$

(c) $s(x)$ even, s nonincreasing on $[0, \infty)$ and $N = 1$

(d) $s(x) \to s_\infty > 0$ for $|x| \to \infty$ and $\int_{|x|=\rho} s(\sigma)\, d\sigma \gneqq \int_{|x|=\rho} s_\infty\, d\sigma$ for all $\rho > 0$

In the cases (a)–(c), the problem has some compactness and the theorem is a straightforward exension of results of C.A. Stuart. The case (d) has a lack of compactness, which seems inaccessible by the standard method of concentration compactness. An application of Ekeland's variational principle for

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constraint minimization yields a minimizing sequence (v_m), which converges weakly in $W^{1,p}(\mathbb{R}^N)$ to v, and where furthermore $|\nabla v_m|^{p-2}\nabla v_m$ converges strongly in the dual space $W^{-1,p'}(\Omega)$ for every bounded $\Omega \subset \mathbb{R}^N$. From this, the convergence of $\nabla v_m \to \nabla v$ in $L^p_N(\Omega)$ follows by using the convexity of the map $a \to |a|^p$ for $a \in \mathbb{R}^N$ and Clarkson's inequalities. Having this extra compactness, the existence of a bifurcating sequence is standard.

S. MAIER-PAAPE:
On Neumann problems for semilinear elliptic equations with critical nonlinearity
In this talk we construct multi-peaked solutions of a semilinear elliptic Neumann problem with homogeneous and critical nonlinearity. The multi-peakedness of our solutions is forced by symmetries of the domain. In fact we obtain our solutions as local minimizers of an energy functional in a certain fixed-point space determined by the given symmetries. We are also able to give asymptotic properties on the shape of the solutions as well as on the energy.

One particular aspect of our investigations is to determine the exact symmetry of the solutions, i.e., the isotropy subgroup of our solutions. We find, for instance, all exceptional subgroups of $O(3)$ and the hypercube group in $O(n)$ as isotropy subgroups. (Based on joint work with K. Schmitt and Zhi-Qiang Wang.)

J.F. TOLAND:
The Peierls-Nabarro and Benjamin-Ono equations
An intimate connection between the Peierls-Nabarro equation in crystal-dislocation theory and the travelling-wave form of the Benjamin-Ono equation in hydrodynamics is uncovered. It is used to prove the essential uniqueness of Peierls' solution of the Peierls-Nabarro equation and to give, in closed form, all solutions of the analogous periodic problem. The latter problem is shown to be an example of global bifurcation with no secondary, symmetry-breaking, bifurcations for a nonlinear Neumann boundary-value problem or, equivalently, for an equation involving the conjugate operator, which is the Hilbert transform of functions on the unit circle.

H.J. KIELHÖFER:
Pattern formation of the stationary Cahn-Hilliard model
We investigate critical points of the free energy $E_\epsilon(u)$ of the Cahn-Hilliard

model over the unit square under the constraint of a mean value λ. We show that for any fixed λ in the so called spinodal region and to any mode of an infinite class there are critical points of $E_\varepsilon(u)$ having the characteristic symmetries of that mode provided $\varepsilon > 0$ is small enough. As ε tends to zero these critical points have singular limits forming characteristic patterns for each mode. Furthermore any singular limit is a stable critical point of $E_0(u)$. Our method consists of a global bifurcation analysis of critical points of the energy $E_\varepsilon(u)$ where the bifurcation parameter is the mean value λ.

J. MAWHIN:

Nonlinear differential equations with Floquet boundary conditions

The solutions of the first order complex-valued periodic boundary value problem

$$z'(t) = \beta(t)\,\theta(t)\,\bar{z}^p(t) + h(t, z(t)), \quad z(b) = z(a), \tag{1}$$

where $p > 1$, $\beta : [a,b] \to \mathbb{R}_+^*$, $h : [a,b] \times \mathbb{C} \to \mathbb{C}$ are continuous, $|z|^{-p} h(t,z) \to 0$ for $|z| \to \infty$ uniformly in $t \in [a,b]$, $\theta : [a,b] \to S^1$ is of class C^1 and $\theta(a) = \theta(b)$, can be reduced to the Floquet boundary value problem

$$\begin{aligned} w'(t) &= \beta(t)\bar{w}^p(t) - \frac{1}{p+1}\,\theta^{-1}(t)\,\theta'(t)\,w(t) + \theta^{-\frac{1}{p+1}}\,h(t, \theta^{\frac{1}{p+1}} w(t)) \\ w(b) &= e^{-\frac{2i\pi m}{p+1}}\,w(a), \end{aligned} \tag{2}$$

where m is the Poincaré index of θ. This is a special case of the Floquet boundary value problem

$$x'(t) = f(t, x(t)), \quad x(b) = Cx(a), \tag{3}$$

where $f : [a,b] \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous and $C \in GL_n(\mathbb{R})$. For problem (3), one can formulate a generalization of the method of bounding functions introduced by Gaines and the author in the periodic case. This extension requires the invariance of the boundary of the set associated to the bounding functions under the action of the group generated by C. The method can be successfully applied to problem (2) and hence to problem (1), or to the study of the solutions of the differential equation in (1) which satisfy the Floquet boundary conditions $z(b) = \alpha z(a)$, with α a $(p+1)^{th}$ root of unity, or which are bounded over $\mathbb{R}$, when $[a,b]$ is replaced by $\mathbb{R}$. When $h(t,0) = 0$ for all $t \in [a,b]$, the existence of nontrivial solutions of the Floquet boundary value problem for (1) is considered as well, leading to some type of bifurcation with respect to the discrete parameter m.

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