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This year's meeting on Mathematical Optimization had been organized by John E. Dennis (Houston), Bernhard Korte (Bonn), and Klaus Ritter (Munich). Compared to earlier meetings, the size of the group - 35 participants from ten countries - was reduced. This helped to schedule fewer (27) but longer talks which was supported by the participants.

Strong interest in real world problems has marked one of the main trends of the meeting: VLSI-design, economic equilibrum problems, power plant coordination, or helicopter rotor design were found among the applications. Since many of these industrial applications are neither purely discrete nor purely continous, this trend increases the interaction between the two areas of optimization.

As for discrete optimization, a wide area ranging from structural results in graph theory to combinatorial algorithms for real world problems was covered. Many of these new developments and results reduce computing time drastically, making it possible to attack problems of truly huge size: An optimally solved weighted matching problem on $5,000,000$ nodes or an optimal geometric partitioning of several hundreds of thousands of cicuits in VLSI chip design are examples.

Within continuous optimization, a significant number of talks covered trust region and sequential quadratic programming (SQP) methods. Theoretical convergence results as well as new numerical experiences and improvements for these and other methods - lagrangian relaxation and derivative free algorithms, for example - were presented.

Several talks dealt with linear programming which lies at the intersection of discrete and continous optimization. New results and concepts leading to computational improvements - randomization, among others - were discussed.

Last but not least, the organizers and participants of the meeting would like to express their gratitude towards the Mathematical Research Institute and its staff for their friendly service.

## David Applegate

## Density Bounds for the $3 x+1$ Problem

The $3 x+1$ function $T(x)$ takes the values $(3 x+1) / 2$ if $x$ is odd and $x / 2$ if $x$ is even. Let $a$ be any integer with $a \not \equiv 0(\bmod 3)$. If $\pi_{a}(x)$ counts the number of $n$ with $|n| \leq x$ which eventually reach $a$ under interation by $T$, then for all sufficiently large $x, \pi_{a}(x) \geq x^{81}$. The proof is based on a system of difference inequalities of Krasikov, of the form

$$
\Phi_{k}^{m}(y) \geq \sum_{i \in I_{k}^{m}} \Phi_{k_{i}}^{m_{i}}\left(y-\alpha_{i}\right)
$$

where $m$ is a residue $\bmod 3^{\mathbf{k}}$, with $m \equiv 2(\bmod 3)$. These difference inequalities involve normal terms, for which $\alpha_{i}>0$, as well as "advanced" terms, for which $\alpha_{i}<0$. To obtain a strong bound from the system of difference inequalities, we replace some terms in the right had sides with their lower bound from other inequalities, and then use the fact that $\Phi_{k}^{m}$ is nondecreasing to truncate the remaining advanced terms to $y-\mu$ for some small $\mu>0$. This gives a system of difference inequalities with only normal terms, for which the optimal bound can be obtained from the optimum of a related non-linear program. However, we do not know the best combination of replacement and truncation, and thus $\pi_{a}(x) \geq x^{81}$ may not be the best bound available from this system.

This is joint work with Jeffrey C. Lagarias.

## Robert E. Bixby

## Strong Branching

In this talk we presented extensive computational results showing that a certain "branching rule" developed as part of work on the traveling salesman problem is very effective in treating more difficult instances of mixed integer programs (MIPs), linear programs in which some or all of the variables are restricted to take on integer values.

Consider a MIP instance $P$, suppose we are applying LP-based branch-and-bound to solve $P$, and let $x^{*}$ be a solution to the LP-relaxation $P^{\prime}$ of $P$ at some node in the branch-and-bound tree. Strong branching works as follows. Given positive integers $K$ and $M$, if possible select $K$ indices $j_{1}, \ldots, j_{k}$ such that $x_{j}^{*}$ is fractional for each $i$. Now starting from an optimal LP basis for $P$, perform up to $M$ dual simplex iterations on each of the $2 K$ problems obtained by appending in turn the constraints $x_{j_{1}} \leq\left\lfloor x_{j_{1}}^{*}\right\rfloor$, $x_{j_{1}} \geq\left\lceil x_{j_{1}}^{*}\right\rceil, x_{j_{2}} \leq\left\lfloor x_{j_{1}}^{*}\right\rfloor \ldots, x_{j_{k}} \leq\left\lfloor x_{j_{k}}^{*}\right\rfloor, x_{j_{k}} \geq\left\lceil x_{j_{k}}^{*}\right\rceil$. When $L_{1}, U_{1}, L_{2}, \ldots, L_{K}, U_{K}$ are the respective objective values, and assuming $P$ is a minimization problem, we select as branching variable $x_{j_{i}^{*}}$ where

$$
j_{i}^{*}=\underset{i}{\operatorname{argmax}}\left\{10 \cdot \min \left(L_{i}, U_{i}\right)+\max \left(L_{i}, U_{i}\right)\right\}
$$

This work is joint with V. Chvátal, W. Cook, and D. Applegate.

## Richard Byrd

## An Interior Point Method for Nonlinearily Constrained Optimization Using Trust Regions

We describe a method for constrained optimization involving extending primal and primal-dual interior point methods to the non-convex case. It applies sequential quadratic programming techniques to a sequence of barrier-problems, and uses trust regions to ensure robustness of the iteration and to the direct use of a possibly indefinite Hessian. Global convergence results without the assumption of full-rank constrained derivatives are presented.

## Vaček Chvátal

## Resolution Search

Brach-and-bound is one of the most popular ways of solving difficult problems such as integer and mixed-integer linear programming problems; wehn the branching is done on zero-one valued variables, the resulting variant of branch-and-bound is called implicit enumeration. Efficieny of brach-and-bound implicit enumeration algorithms depends heavily on the branching strategy used to select the next variable and its value. We propose an alternative to implicit enumeration. Our algorithm, which we call resolution search, seems to suffer less from inappropriate branching strategies than implicit enumeration does.

## Andrew R. Conn

## Two Step Algorithms for Nonlinear Optimization with Structured Applications

We propose extensions to trust-region and line-search algorithms in which the classical step is augmented with a second step that is required to yield a decrease in the value of the objective function. The classical convergence theory for trust-region and line-search algorithms is adapted to this class of two-step algorithms. It is shown that the algorithms are globally convergent to a stationary point under the "classical" assumptions.

The algorithms can be applied to any problem with variables whose contributions to the objective function are in a known functional form. In the nonlinear programming package lancelot they have been applied to update slack variables and variables introduced to solve minimax problems, leading to enhanced optimization efficiency. Numerical results are presented to indicate that these techniques can be very effective.
(Joint work with L. Vicente and C. Visweswariah)

William J. Cook

## Computing Perfect Matchings

We make several observations on the implementation of Edmond's blossom algorithm for solving minimum-weight perfect matching problems, and we present computational results for geometric problem instances ranging in size form 1000 nodes up to $5,000,000$ nodes. Our test set includes instances generated randomly as well as structured instances from Reinelt's TSPLIB and from VLSI design.

This is joint work with André Rohe (Bonn).

## William H. Cunningham <br> Optimal Path-Matchings

Given a graph $G$, a partition $\left\{T_{1}, T_{2}, R\right\}$ of $V(G)$ such that $T_{1}$ and $T_{2}$ are stable sets, and matroids $M_{1}$ on $T_{1}$ and $M_{2}$ on $T_{2}$, a basic path-matching is a set of vertexdisjoint paths joining a basis of $M_{1}$ to a basis of $M_{2}$, together with a perfect matching of the vertices of $R$ not in any path. We generalize to this setting results (existence theorems, polyhedral characterizations, polynomial-time algorithms) on (weighted) matching and (weighted) matroid intersection. We also describe some new applications. (Joint work with J. F. Geelen)

## John E. Dennis

## Some Optimization Problems form Engineering Design and Manufacturing

The purpose of this talk is to introduce to the academic research community some interesting and unusual nonlinear optimization problems of great importance in industry. The problems are characterized by:

- tens of decisions variables, though there may be hundreds of thousends of system variables determined by these decisions;
- objective function values that require hours or days of high performance computing time;
- no reliable derivatives are available in most cases.

Helicoptor rotor design and shot??? forming of sheet metal are given as examples. We also give a brief introduction to the effort by a Boeing/IBM/Rice collaboration to refine the usual engineering approach.

## Peter Deuflhard <br> Inexact Newton Multilevel FEM for Nonlinear Elliptic PDEs

The talk dealt with the multilevel solution of elliptic partial differential equations in a finite element setting: Uniform ellipticity of the PDE then goes with strict monotonicity of the derivative of some associated nonlinear convex functional. The approach therefore applies to PDEs that have an associated variation principle as most scientific PDEs have. A Newton multigrid method is advocated, wherein linear residuals are evaluated within the multigrid method for the computation of the Newton corrections. The globalization is performed by some damping of the ordinary Newton corrections. Affine conjugate local and global convergence results have been presented, which cover both the exact Newton method (neglecting approximation errors) and inexact Newton-Galerkin methods, wherein the inner iterations preserve some Galerkin orthogonality condition. The matching of the accuracies of discretization and iteration origin is derived on that theoretical basis. Illustrative numerical experiments with a NEWTON-CASCADE code are documented. One application of the new code ist the planning of a hyperthermia cancer therapy, where linear models do not yet cover the effect of systematic anticipation effects. Co-author is Martin Weiser.

## Roger Fletcher

## Numerical Experience with an SQP Filter Method for Nonlinear Programming

The "filter" technique is an new trust region approach to the globalization of the SQP method, which avoids the use of penalty parameters. A filter is a set of pairs ( $f_{i}, h_{i}$ ) of values of the objective function $f(\underline{x})$ and constraint function $h(c(x))$ at iterates $\underline{x}_{i}$ in the SQP algorithm. A pair $\left(f_{i}, h_{i}\right)$ is said to dominate another pair ( $f_{j}, h_{j}$ ) iff both $f_{i}<f_{j}$ and $h_{i}<h_{j}$. The filter is a list of pairs so that no pair dominates any other. If a new iterater $\underline{x}^{+}$is such that ( $f^{+}, h^{+}$) is dominated by any other point in the current filter, then $\underline{\underline{x}}^{+}$is rejected and the trust region is reduced. Otherwise $\underline{x}^{+}$becomes the new iterate, $\left(f^{+}, h^{+}\right)$is added to the filter, and any pairs in the filter dominated by ( $f^{+}, h^{+}$) are deleted.

Issues involving the incorporation of this idea into a trust region SQP algorithm include the possible use of second order connection steps and the need for a restoration phase if the QP subproblem becomes infeasible. Numerical experience is reported and is very encouraging with typical only say $10 \sim 20$ SQP iterations and moderate number of functions calls. The technique has proved very reliable and various problems have been solved which other techniques fail to solve. Heuristics are discussed which might enable global convergence of the algorithm to be proved.

## Jean Fonlupt <br> Critical Extreme Points and Some <br> Traveling Salesman Problem Polytope

We study the extreme points of the polytope $P(G)$, the relaxation of the traveling salesman polytope and also the relaxation of the 2-edge connected spanning subgraph polytope of a graph $G$. We introduce an ordering among the extreme points of $P(G)$ and we show that if $\bar{x}$ is a non-integer extreme point which is minimal for this ordering, $G$ and $\bar{x}$ can be reduced by means of simple operations (deletion of edges, contraction of edges) to a graph $G^{\prime}$ and an extreme point $\bar{x}^{\prime}$ of $P\left(G^{\prime}\right)$ which satisfies simple properties.

## Andreas Frank

## Covering Supermodular Functions by Graphs

In 1995, with T. Jordan [J. Comb. Th. Ser. B (1995)] we proved the following. Let $F$ be a crossing family of pairs of subsets. Then the maximum number of pairwise independent members of $F$ is equal to the minimum number of directed edges covering each pair in $F$.

This result implies a difficult theorem of Györi on intervals as well as a result on the minimum number of new edges necessary to be added to a specified digraph to increase its connectivity by 1 .

In the first part of this talk I outlined a constructive proof of the theorem above. The proof gives rise to a polynomial algorithm to compute the optima in question.

The second part was an account on a joint work with A. Beuczur. We proved the following: Let $p: 2^{V} \rightarrow \mathbf{Z}_{+}$be a symmetric crossing supermodular function. There exists an undirected graph $G=(V, E)$ with at most $\gamma$ edges so that $d_{G}(X) \geq \rho(X)$ for every $X \subset V$ if and only if $\sum(\rho(X): X \in P) \leq 2 \gamma$ holds for every sub-partition $P$ of $V$ and there is no partition $F$ of $V$ into $\gamma+2$ parts so that $\rho\left(\bigcup X: X \in F^{\prime}\right) \geq 1$ for every $F^{\prime} \subset F$. The motivation comes from hypergraph connectivity augmentation.

## Donald Goldfarb

## Polynomial-Time Algorithms for the Generalized Cirulation Problem

We present two new combinatorial algorithms to the generalized circulation problem. After an initial step in which all flow generating cycles are canceled and excesses are created, both algorithms bring these excesses to the sink via highest-gain augmenting paths. Scaling is applied to the fixed amount of flow that the algorithms attempt to send to the sink, and both node and arc excesses are used. The algorithms have worst-case complexities of $O\left(m^{2}(m+n \log n) \log B\right)$, where $n$ is the number of nodes,
$m$ is the number of arcs, and $B$ is the largest integer used to represent the gain factors and capacities in the network. This bound is better than the previous best bound for a combinatorial algorithm for the generalized cirulation problem, and if $m=O\left(n^{4 / 3-\varepsilon}\right), \varepsilon>0$, it is better than the previous best bound for any algorithm for this problem.

## Michael D. Grigoriadis

## Bounds for Resource Sharing, Multicommodity Flows and Matrix Games

Potential reduction based Lagrangian decomposition methods for structured minmax resource sharing problems with $K$ convex compact blocks and $M$ nonnegativevalued block-separable convex linking inequalities are considered. The exponential, the classical logarithmic barrier and a Karmarkar variant are used here as potential functions to relax the linking constraints. The minimization of the original piecewise convex function over the product $B$ of the blocks is thus approximated by the minimization of a smooth function over $B$ to a given relative accuracy $\varepsilon \in(0,1]$. The number of Lagrangian decomposition iterations is bounded by about $O\left(\varepsilon^{-2} K \ln M\right)$ above and by $\Omega(K(1+\ln M / K))$ below, for $K=O(M \log M)$ and when the exponential potential is used with "restricted" subproblems. In comparison, the bounds of roughly $O\left(\varepsilon^{-2} M \ln M\right)$ above and $\Omega(M)$ below are obtained for each of the two logarithmic potentials, using the original, "unrestricted" blocks. These methods form the basis for the general minimization model and for $O\left(\varepsilon^{-2} K N M\right)$ and $O\left(\varepsilon^{-2} K M^{2}\right)$ time algorithms (disregarding logarithmic factors) for minimum cost K-commodity flows in $N$-node, $M$-arc networks. Two parallel algorithms for ( $M, N$ ) matrix games are also developed. First, for a given accuracy $\varepsilon>0$, the exponential-potential algorithm can be specialized to run in $O\left(\epsilon^{-2} \log ^{2}(N+M)\right)$ deterministic time on an $N M$-processor EREW PRAM. Second, a modified fictitious play algorithm is shown to run in $O\left(\varepsilon^{-2} \log ^{2}(N+M)\right)$ expected time on $(N+M) / \log (N+M)$ such processors. The sequential variant of the latter algorithm runs in $O\left(\epsilon^{-2}(N+M) \log (N+M)\right)$ expected time, which is sublinear in the number of elements $M N$ of the problem. Significantly, any $1 / 2$-approximate deterministic algorithm is shown to run in $\Omega(M N)$ time. (Joint work with L. Khachiyan and with J. Villavicencio.)

## Matthias Heinkenschloss

## Optimization of Time-Dependent Problems

This talk is concerned with the solution of time-dependent (parabolic) optimal control problems. Since state variables are defined in time and space, the size of the problem does not allow the straightforward application of constrained optimization methods which treat states and controls as independent variables. As a remedy, the number of variables is reduced by using an inbetween formulation. The time interval is subdivided and only the states at the subinterval boundaries are viewed as variables.

The states inside the subintervals are functions of the values at the 'lower' interval boundary. This reduces the number of variables, but a straight-forward application of constrained optimization methods, such as sequential quardratic programming (SQP), makes little use of expensive information. We propose a new method which is motivated by a Gauß-Seidel-like iteration applied to the optimality system. The new method can be easily parallelized and it makes better use of expensive information. Moreover, each step in the Gauß-Seidel-like procedure can be performed using existing optimization methods. Convergence results in the quadratic case are presented and numerical results are discussed.

## Claude Lemarechal

Dual-Equivalent Convex and Non-Convex Problems, with Applications
Lagrangian relaxation is a possible approach for heuristic resolution of non-convex problems. It introduces a duality gap, and the aim of this talk is to give an expression for this duality gap. Specifically, it amounts to a convexification in the product of the three spaces containing respectively the variables, the objective, and the constraints. These results shed some light on a relaxation scheme, proposed by M. Guignard and S. Kim (Math. Prog. 1987); they also quantify the duality gap appearing in a specific application, highly non-linear: the unit-commitment problem.

Joint work with Arnaud Renaud, EDF.

## Thomas M. Liebling

"Laguerre News": Bistellar Flips in 2D and in 3D
Laguerre diagrams are generalizations of Voronoi diagrams where generating sites are a family $\tilde{S}$ of spheres in $\mathbf{R}^{d}$ (or $\Pi^{d}$ ) and rather than euclidean distances, power w.r. to these spheres is used as a discriminating function. For $d>2$, Laguerre diagrams are precisely the partitions of that space into convex (simple) cells. The corresponding dual Delaunay triangulations are in turn precisely the regular triangulations (supports of piecewise linear convex functions). Bistellar flips, insertions and deletions are elementary transformations that can be used to design an incremental algorithm that constructs a Laguerre partition (and corresponding Delaunay triangulation) in $O\left(n^{5}\right)$ operations, where $n$ is the cardinality of the generating set. It is also possible to use these transformations to maintain dynamically evolving diagrams. These ideas are applied to model foam and grain growth processes. A particular novel feature is the simulation of grain growth with several textures present.
(Based on work with Xue, Reghetti, Telley and Mocellin)

## Stephen M. Robinson

## Solution Methods for Variational Inequalities

Variational inequalities have been frequently used to model problems of equilibrum, of both deterministic and stochastic types. We discuss some current work on solution methods for such problems. Our main line of approach is through transformation of the variational inequality into an equivalent (generally nonsmooth) equation, which can then be solved by one of several methods. We present some results using a continuation procedure for the solution. Finally, we show how this approach can be expected to more general maximal monotone operator inclusions.

## Andreas Schulz .

Randomization Helps in LP-based Scheduling:
Improved Algorithms for the Average Weighted Completion Time
For quite a long time, there was only restricted success in deriving approximation algorithms for scheduling problems to minimize the average weighted completion time. Recent work, however, has led to the development of several techniques that yield constant worst-case bounds in a number of settings. Essentially, either preemptive schedules are converted into non-preemptive ones or, somewhat related, solutions to various LP's are rounded to feasible schedules. In two very recent papers Goemans as well as Cheberi, Motswani, Nataraja \& Stein have shown that randomness helps in improving the performance of either technique for single machine scheduling subject to release dates.

In this paper (joint work with Martin Skutella), we show that a combination of preemptive and LP relaxations together with a somewhat subtle use of randomness leads to a class of new approximation algorithms which give the best known performance guarantees for quite a few problems. This includes single machine, identical parallel or unrelated parallel machine scheduling subject to release dates.

## David F. Shanno <br> Computational Experience with Interior Point Methods for Complementarity Problems

The talk derives an interior point algorithm for mixed complemetarity problems based on classical logarithmic barrier methods. A globally convergent Newton's method which uses a backtracking line search is the basis of the algorithm. The convergence part uses a less restrictive condition than monotonicity, requiring only nonsingularity of a specific matrix of all iterates of the algorithm. This condition is trivially met for monotone problems. An example is given from an economic equilibrum problem to demonstrate that when monotonicity is not present, the method
can fail due to the matrix becoming singular. This is true for both linear and nonlinear complementarity. Current research concentrates on studying the behavior of the algorithm on nonconvex quadratic programming algorithms.

## Philippe Toint

## Convergence of a Derivative Free Algorithm

## for Unconstrained Optimization

We consider the unconstrained minimization problem in the case where derivatives of the objective function are unavailable. This occurs if the objective is given by a real measure or by applying a computercode whose source is unavailable. We derive a general algorithmic framework of the trust-region type for which reasonable global convergence properties can be obtained. The definition of the framework and the development within the convergence theory make extensive use of a new error bound for multivariate interpolation, itself based on the multivariate Newton fundamental polynomials. This work follows up earlier contributions by Powell and Conn + Toint.
(Joint work with Andy Conn and Katya Scheinberg)

## Michael Ulbrich <br> Global Convergence of Trust-Region Interior-Point Methods for Infinite-Dimensional Problems subject to Pointwise Bounds

We develop and analyze a trust-region interior-point method for problems of the form

$$
\min f(u) \text { s.t. } u \in L^{p}(\Omega), a(x) \leq u(x) \leq b(x)
$$

a.e. on $\Omega$.

The class of algorithms we present includes a function-space variant of a method introduced by Coleman an Li for finite-dimensional problems. They are based on a trustregion globalization of an interior-point Newton-like method for the non-differentiable equation given by the affine-scaling formulation of the first order necessary optimality conditions. We specify a class of affine-scaling functions for which each accumulation point of the generated sequence of iterates satisfies the first order necessary conditions if we impose a fraction of Cauchy-decrease condition on the trial steps. Moreover, for trial steps satisfying a fraction of optimal decrease condition, we give a characterization of subsequences whose accumulation points are second order optimal. Especially the latter assertion requires new proof techniques since the finite-dimensional theory is not transferable to our infinite-dimensional setting. Numerical results verify
the efficiency of the algorithm. This work is joint with Stefan Ulbrich and Matthias Heinkenschloss.

## Stefan Ulbrich <br> Superlinear Convergence of Affine-Scaling Interior Point Methods for Infinite-Dimensional Problems with Simple Bounds

We present a locally superlinear convergent algorithm for pointwise constrained problems of the form

$$
\min f(u) \text { s.t. } u \in B:=\left\{u \in L^{p}(\Omega) ; a(x) \leq u(x) \leq b(x), x \in \Omega\right\}
$$

with $2 \leq p<\infty$. The algorithm uses a Newton-like iteration to solve ${ }_{r}$ an affinescaling formulation of the first order necessary conditions. We show that a similar algorithm proposed by Coleman and Li for finite dimensional problems which uses a back-step stepwise-rule may converge slowly for the infinite dimensional problem. It turns out that a pointwise linear projection should be used instead and that a smoothing step, i.e. a Lipschitz continuous mapping $S: B \subset L^{p}(\Omega) \rightarrow L^{\infty}(\Omega)$ with the fixed-point property $S(\bar{u})=\bar{u}$ for KKT-points $\bar{u}$, is necessary to safeguard rapid convergence. A class of regularized problems is discussed for which a smoothing step can be constructed with the help of a fixed-point formulation of the KKT-condition. A convergence result for the convergence rate $1+\frac{1}{p+1}$ is presented. Numerical experiments with a heat conduction optimal control problem show the significantly improved convergence rate in comparison to the conventional method.

## Luis N. Vicente

## TRICE: Trust Region Interior-Point SQP Algorithms for the Solution of Optimal Control and Engineering Design Problems ${ }^{--}$

We will describe the development and implementation of a set of trust-region interior-point SQP algorithms for the solution of optimal control and engineering design problems.

The TRICE algorithms solve problems where the dominating equality constraints are of the form $c(y, u)=0$. In optimal control, $c(y, u)=0$ is the state equation, $y$ are the state variables, and $u$ are the control variables. The Trice algorithms are designed to efficently solve problems arising in optimal control, design, and parameter identification or other fields that share this common structure.

We will give a short description of the trice algorithms and briefly mention their convergence properties. The TRICE software will be discussed, with particular emphasis to the features that make TRICE tailored to the characteristics and structure of the problems.

Jens Vygen

## A Linear-Time Algorithm for a Geometric Partitioning Problem

The following combinatorial optimization problem is a crucial part of most algorithms for VLSI-placement (although it has not been explicitely formulated before):

Suppose the plane is divided by a horizontal and a vertical line into four regions $R_{0}, R_{1}, R_{2}, R_{3}$. Let $\kappa_{0}, \kappa_{1}, \kappa_{2}, \kappa_{3}$ be capacities of these regions. Let $C$ be a finite set, and for each $c \in C$ let coordinates $(x(c), y(c)) \in \mathbf{R}^{2}$ and a weight size(c) be given. We look for a partition $f: C \rightarrow\{0,1,2,3\}$ such that the capacity constraints $\sum_{c \in C, f(c)=i} \operatorname{size}(c) \leq \kappa_{i}$ are met and $\sum_{c \in C} \operatorname{size}(c) \cdot d\left((x(c), y(c)), R_{f(c)}\right)$ (the total movement) is minimized. Here $d$ stands for the $L_{1}$-distance. The problem is NPhard. However it is sufficient to consider its fractional relaxation, because there is an optimum fractional solution where the values are in fact integral except for three elements of $C$. The main result is: This fractional version of the problem can be solved optimally in linear time. The talk gives an outline of the (complicated) algorithm as well as some theorems based on it.

## Dominique de Werra <br> Multi-Constrained Chromatic Scheduling Problems

Practical experiments in timetabling and scheduling suggest the introduction of additional constraints in graph coloring models. A $k$-coloringof a graph $G=(V, E)$ is a partition of the node set $V$ into independent subsets $S_{1}, \ldots, S_{k}$. We say that a $k$-coloring is restricted if each node $v$ must get a color chosen in a set $\varphi(v)$ of feasible colors $(\varphi(v) \subseteq C=\{1, \ldots, k\})$. The problem $R C(G, \varphi, k)$ of finding a restriced $k$-coloring is generally NP-complete (even if $\varphi(v)=C \forall v$ ). When $G$ is the line-graph of a forest, one may produce in polynomial time a restricted $k$-coloring or a certificate of non-colorability by simple linear programming arguments based on a mathematical programming formulation with a totally unimodular matrix (see for instance D. de Werra, A. Hoffman, N.V.R. Mehodev, U. Peled, 1996).

In addition we also have to introduce global ???ments: Given a sequence $H=$ $\left(h_{1}, \ldots, h_{k}\right)$ a restricted $k$-coloring $S_{1}, \ldots, S_{k}$ is constrained if $\left|S_{i}\right| \leq h_{i}(i=1, \ldots, k)$. A necessary condition for the existence of a solution to the problem $C C(G, \varphi, k, M)$ of finding a constrained $k$-coloring is given. This condition is shown to be sufficient iff $G$ is $P_{3}$-free (union of cliques). Observe that $R C(G, \varphi, k)$ (and hence $C C(G, \varphi, k, H)$ ) is NP-complete for $P_{k}$-free graphs.

Several extensions of these ???ments are given and we show that network flow models can handle them.

For applications in robotics, the problem $C C(G, \varphi, k, H)$ where $G$ is a path does provide an adequate model; its status is still unknown except for some special cases.

## Stephen J. Wright

## Superlinear Convergence of a Stabilized SQP Method <br> to a Degenerate Solution

We describe a slight modification of the well-known sequential quadratic programming (SQP) method for nonlinear programming that converges superlinearly to a primal-dual solution even when the dual-solution is not unique and the Jacobian of the active constaints fails to have full rank at or near the solution. We analyze the local convergence properties under both exact and finite precision arithmetic, and describe some computational experience.

## Participants

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