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Numerische Methoden der Approximationstheorie

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The conference was organized and directed by Prof. Dr. Dietrich Braess, Ruhr-Universität Bochum, Germany and Prof. Dr. Larry L. Schumaker, Vanderbilt University, Nashville, Tennessee. Many of the talks and discussions during this week focused on topics in the area of numerical solutions of differential equations by refinable functions, wavelets, and radial basis functions, where a special field of interest were preconditioning techniques. Another strong area of interest was in computer aided design and engineering methods.

Despite a larger number of volunteers, only a moderate number of talks had been scheduled, in accordance with the management of the institute. This was in order to leave sufficient time for discussions and joint work during the meeting. The participants acknowlegded this opportunity and made extensive use of it.

48 mathematicians from 9 countries attended the meeting and were generously accommodated by the mathematical institute. On behalf of the participants we would like to thank the director Prof. Dr. M. Kreck and his staff for their friendly hospitality and help which made this conference a success.

Abstracts of Presented Talks

(In the case of multiple authors, speakers are marked with an asterisk)

Minimal Norm Extensions in L^∞/H^∞

Laurent Baratchart, Juliette Leblond, Jonathan R. Partington*

Given a proper subset K of the unit circle $\mathbf{T}$ and $f \in L^\infty(K)$ we seek an extension $\psi \in L^\infty(\mathbf{T} - K)$ such that the concatenated function $f \vee \psi$ is as close as can be to the Hardy class H^∞ or to the meromorphic class $H^\infty + \mathcal{R}_N$, where $\mathcal{R}_N$ is the set of rational functions with at most N poles in the disk. This problem is in fact ill-posed in general (we discuss when) and for that reason we further constrain ψ to lie in some ball of $L^\infty(\mathbf{T} - K)$.

Such questions arise naturally from certain inverse problems, like deconvolution or Dirichlet-Neumann singularity detection, where partial values of a meromorphic function can be gathered on the boundary of the domain on analyticity with some measurement error. We then explain how the problem can be implicitly reduced to AAK meromorphic approximation, and we shall derive a constructive solution when the data have bounded derivative and K consists of finitely many arcs. This will entail discussing the generic character of multiplicity 1 for Hankel singular values so as to ensure the continuity of the implicit AAK approximation just mentioned, as well as estimating the modulus of continuity of certain outer factors near their singularity. We shall mention some open extension problems of this kind, and we shall finally present numerical experiments.

Large Scale Computations with Radial Basis Functions

Rick Beatson

Radial basis functions such as the thin–plate spline, the multiquadric and the sinc are popular for diverse applications such as titanium cranioplasty, modelling the earths magnetic field, and modelling rainfall distributions via GCV. There used to be numerical conditioning and computer resource difficulties in using globally supported rbfs for large problems. I will present experimental evidence that these problems are now essentially overcome. We may not yet know the "best" way to compute with rbf's but "good" methods are known. Namely, the combination of a good choice of basis, a fast matrix multiply, and a suitable iterative method enables fast stable computations of rbf's with many thousands of centers on modest workstations.

Large Besov Regularity for Elliptic Boundary Value Problems

Stephan Dahlke

We shall be concerned with the regularity of solutions to boundary value problems on Lipschitz domains Ω in $\mathbb{R}^d$ and its relationship with adaptive and other nonlinear methods for approximating these solutions. The smoothness spaces which determine the efficiency of such nonlinear approximation in

$L_p(\Omega)$ are the Besov spaces $B_\tau^\alpha(L_\tau(\Omega))$, $\tau := (\alpha/d + 1/p)^{-1}$. Thus, the regularity of the solution in this scale of Besov spaces is investigated with the aim of determining the largest α for which the solution is in $B_\tau^\alpha(L_\tau(\Omega))$. The deepest results are obtained for boundary value problems for the Laplace operator. Generalization to other problems will also be discussed. Especially, we shall consider real homogeneous elliptic operators of higher order. The proofs of the regularity theorems are combinations of recent results on the characterization of Besov spaces by wavelet expansions with concepts from classical harmonic analysis.

Multistep Approximation Algorithms: Improved Convergence Rates through Postconditioning with Smoothing Kernels

Greg Fasshauer, Joe Jerome*

First we show how certain widely used multistep approximation algorithms can be interpreted as instances of an approximate Newton method. It was shown in [1] that the convergence rates of approximate Newton methods (in the context of the numerical solution of PDEs) suffer from a "loss of derivatives", and that the subsequent linear rate of convergence can be improved to be superlinear using an adaptation of Nash-Moser iteration for numerical analysis purposes; the essence of the adaptation being a splitting of the inversion and the smoothing into two separate steps. In our talk we will show how these ideas apply to scattered data approximation, and we will show that radial basis functions serve as viable tools for the inversion, as well as for the smoothing operations. As a consequence we conclude that the rate of convergence of multistep algorithms such as the one presented in [2] can be improved by employing smoothing steps during the iteration. Numerical results are also supplied.

[1] Jerome, J. W., An adaptive Newton algorithm based on numerical inversion: regularization as postconditioner, Numer. Math. **47**, 1985, 123–138.

[2] Floater, M., and Iske, A., Multistep scattered data interpolation using compactly supported radial basis functions, preprint.

Refinable Subspaces of Refinable Spaces

Thomas A. Hogan

Refinable spaces are very important in the construction of wavelets. Several properties are necessary for the refinable functions to give rise to useful wavelets. Among these, are smoothness, approximation order, stability. Other desirable properties might be orthonormality and symmetry. It is easy enough to generate refinable spaces with high approximation order, orthonormality and symmetry, if one is willing to sacrifice continuity (*e.g.*, take sufficiently many truncated powers restricted to the unit interval, and run Gram-Schmidt on them). If one were to then take the space generated by these functions and intersect it with the space of continuous (or smoother) functions, it would still be refinable (and would provide the same approximation order). It would be great if a set of generators for this new space could be found, say with the same properties as the original set (orthonormality and symmetry). This is the basic idea behind my talk.

Numerical Computation of Smoothness of Multivariate Refinable Functions

Rong-Qing Jia

We are interested in multivariate refinable functions, which are solutions of refinement equations of the form

$$\phi = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\phi(M\cdot - \alpha),$$

where a is a finitely supported sequence on $\mathbb{Z}^s$, called the **refinement mask**, and M is an $s \times s$ integer matrix such that $\lim_{n\to\infty} M^{-n} = 0$, called a **dilation matrix**. Multivariate wavelets are generated from the corresponding refinable functions.

In this talk we will discuss efficient numerical algorithms to compute smoothness of multivariate refinable functions in Sobolev spaces. Our algorithms are based on a study of the subdivision operator and the transition operator associated to the refinement equation.

Simultaneous Polynomial Approximation in L_p, $0 < p \leq \infty$

Kirill Kopotun

Some problems in the area of simultaneous approximation of a function and its derivatives in the L_p-metric ($0 < p \leq \infty$) will be discussed.

Bivariate Spline Method for Navier-Stokes Equations

Ming-Jun Lai

Let D be a polygon and Q be a quadrangulation of D which consisting of convex quadrilaters. Adding the two diagonals of each quadrilateral of Q, we obtain a triangulation. Such a quadrangulation may be obtained by first triangulating D and then connecting the center of each triangle to the midpoint of its three edges. We consider the bivariate spline space S of smoothness r and degree 3r over such a triangulation T. We show the approximatin properties of this spline space S. Then we consider the Navier-Stokes' equations in stream function fomulation and apply the bivariate spline functions to solve those equations. We give a convergence analysis of the bivariate spline method. Some numerical experiments are shown.

Linking Discrete Orthogonality with Dilation and Translation for Incomplete Sigma-Pi Neural Networks of Hopfield-Type

Burkhard Lenze

In this talk, we show how to extend well-known discrete orthogonality results for complete sigma-pi neural networks on bipolar coded information in presence of dilation and translation of the signals. The approach leads to a whole family of functions being able to implement any given boolean function. Unfortunately, the complexity of such complete higher order neural network realizations increases exponentially with the dimension of the signal space. Therefore, in practise one often only considers incomplete situations accepting that not all but hopefully the most relevant information or boolean functions can be realized. At this point, the introduced dilation and translation parameters play an essential rôle because they can be tuned appropriately in order to fit the concrete representation problem as best as possible without any significant increase of complexity. In detail, we explain our approach in context of Hopfield-type neural networks including the presentation of a new learning algorithm for such generalized networks.

On the L^p Condition Number of the Multivariate Triangular Bernstein-Bézier Basis

Tom Lyche, Karl Scherer*

In this talk we give bounds for the L^p condition number

$$\kappa_{n,p}(\mathbb{R}^s) = \sup_{\mathbf{c}} \|\mathbf{c}\|_p / \Big\| \sum_{|\alpha|=n} c_\alpha \frac{n!}{\alpha!}\lambda^\alpha \Big\|_{L^p(\Sigma)} \sup_{\mathbf{c}\neq 0} \Big\| \sum_{|\alpha|=n} c_\alpha \frac{n!}{\alpha!}\lambda^\alpha \Big\|_{L^p(\Sigma)} / \|\mathbf{c}\|_p$$

of the triangular Bernstein basis $(n!\lambda^\alpha/\alpha!)$ of degree n, with respect to a simplex Σ in $\mathbb{R}^s$. This basis has gained increasing popularity through work in Computer Aided Geometric Design. In two space variables we give an upper bound which grows like 3^n when the degree n tends to infinity. Similar estimates for univariate B-splines has been given earlier by de Boor, Ciesielski, and the first author, and recently by Shadrin and the second author. This is a continuation of previous work by the authors for the L^∞ case.

On Discrete Tension Splines

*Paolo Costantini, Boris I. Kvasov, Carla Manni**

Let the data (x_i, f_i), $i = 0, \ldots, N+1$, be given, with $a = x_0 < x_1 < \ldots, x_{N+1} = b$. The classical continuous tension splines, widely studied during last years for their tension properties, are obtained as solution of a fourth order multipoint boundary problem involving some nonnegative tension parameters p_i :

$$s^{(4)} - (\frac{p_i}{h_i})^2 s^{(2)} = 0, \quad \text{in} \quad (x_i, x_{i+1}), \quad h_i = x_{i+1} - x_i,$$

$$s \in C^2[a,b], \quad s(x_i) = f_i$$

with specified end constrains.

For practical purposes it is often more interesting to know the values of the solution over a given tabulation of $[a,b]$ than its global analytic expression.

Here we study a natural discretization of the previous problem. We prove that the discretized problem has a unique solution, called *mesh solution*, and we study its properties. Of course it turns out that the mesh solution is not a tabulation of s but it can be extended on $[a,b]$ to a function, u, with properties very similar to those of s and which approaches s as the discretization step goes to zero. In particular it turns out that u can be computed via a tridiagonal system involving its second divided differences at the knots x_i.

In addition, as the tension parameters take the value 0, u reduces to the discrete interpolating cubic splines already studied in literature.

Due to these properties we will refer to u as *discrete tension spline* interpolating the data.

Recursive Techniques in the Wavelet Theory

Laura B. Montefusco

The notion of recursive matrix, introduced in [1], has revealed to be a powerful tool for a unified study of combinatorial problems related to double recurrences and, recently, it has been shown in [2] that it can be fruitfully used to represent and easily handle some linear operators that are widely used in filter theory and related fields. A natural setting for the application of recursive techniques seems to be the context of wavelet analysis. Indeed, the shift invariant property of wavelet analysis is closed related to the essence of the notion of recursive matrix, suggesting to undertake a systematic approach to general wavelet theory via recursive matrix techniques. In this talk we present some preliminary results of the forthcoming paper [3], where an elementary construction and characterization of general compactly supported wavelets is given, handling with equal ease the orthogonal and non-orthogonal case.

[1] Barnabei, M. and Brini, A. and Nicoletti, G., Recursive matrices and umbral calculus, J. Algebra, **75**, pp.546–573,(1982). [2] Barnabei, M and Montefusco, L. B., Recursive Properties of Toeplitz and Hurwitz Matrices, submitted to Linear Algebra and its Applications (1996). [3] Barnabei, M. and Guerrini, C. and Montefusco, L.B.", Umbral Methods for General Wavelet Construction in progress.

An Infinite Dimensional Extension of the Singer-Yamabe Theorem

Bernd Mulansky

Let X, Y be two linear topological spaces, $A \in L(X,Y)$ a continuous linear map from X into Y, $C \subset X$, $B \subset X$ a convex set dense in C, and $d \in A[C]$ an admissible data point. The Singer-Yamabe theorem states that $B \cap A^{-1}d$ is dense in $C \cap A^{-1}d$, assuming $C = X$ and Y is finite dimensional. This result has been generalized by Mulansky and Neamtu to arbitrary sets C, provided $d \in \text{int}(A[C])$ is an interior data point. In the talk we discuss an extension of the Singer-Yamabe theorem to the case of an infinite dimensional range space Y. Some applications to problems of shape preserving infinite interpolation, e.g., monotone extension of boundary data, are described.

Convergence of Subdivision Schemes

Mike Neamtu

Under the assumption that a given two-scale refinement equation possesses a continuous solution, necessary and sufficient conditions are derived for convergence of the corresponding univariate stationary subdivision scheme with a finitely supported mask. These conditions are expressed using the factorization of the subdivision mask and do not require the computation of a spectral radius of matrices or solving an eigenvalue problem. The main result is that the existence of a continuous solution of the refinement equation essentially implies convergence of subdivision. Namely, the solution can always be generated by employing a convergent subdivision corresponding to an appropriately chosen mask.

High Dimensional Numerical Integration

Erich Novak

High dimensional problems are difficult, there exists a curse of dimension. It was believed that only Monte Carlo methods or number theoretic methods can be used if the dimension d is large, say $d = 10$ (for many of the standard test examples) or $d = 360$ (in some recent applications from finance) or even $d = \infty$ (path integrals).

We prove (theoretical results) and demonstrate (by numerical examples) that a suitable method (based on polynomial interpolation using the construction of Smolyak) can be used with excellent results if the integrand is sufficiently smooth.

Our method is almost optimal (i.e., up to logarithmic factors) simultanously for each class

$$C_d^k = \{f : [0,1]^d \to \mathbf{R} \mid \|f^{(\alpha)}\|_\infty \leq 1, \ |\alpha| = k\}$$

and also for each class

$$F_d^k = \{f : [0,1]^d \to \mathbf{R} \mid \|f^{(\alpha)}\|_\infty \leq 1, \ \alpha_1 = \ldots = \alpha_d = k\}.$$

Our method (see Numer. Math. Vol 75 and further preprints) is also almost optimal with respect to its polynomial exactness.

Most work was done together with Klaus Ritter from Erlangen. I include remarks about tractability (i.e. behavior of the error bounds for a finite number of knots; not just the order of convergence). This part is based on joint work with Ian Sloan (Sidney) and Henry Woźniakowski (New York and Warsaw).

Regularity of Multivariate Refinable Functions with Infinite Masks: Computation and Applications

*Rudy Lorentz, Peter Oswald**

Optimal multiscale algorithms for operator equations require the construction of hierarchical Riesz bases in Sobolev spaces. The efficiency of associated multilevel solvers depends on Riesz bounds (condition numbers, iteration count) and the sparsity of discretization and intergrid transfer matrices (arithmetical complexity per iteration).

In the setting of a dyadic MRA on $\mathbb{R}^d$, the second requirement makes it plausible to investigate ad hoc choices of potential multiscale bases with small masks for scaling functions ϕ and prewavelet functions ψ^λ. To check the Riesz basis property of such systems, the essential step is to determine the exact regularity of the dual scaling function $\tilde{\phi}$ which is often of non-compact support. We discuss methods to efficiently compute such quantities, and present comparisons for box spline examples (mostly for the linear case which is closely connected to multilevel finite element preconditioners).

Validated Computations in Approximation Theory

Knut Petras

Tools for validated computation, such as interval arithmetic and automatic differentiation, allow the exact error estimation for many algorithms in numerical analysis and particularly in approximation theory. Usually one might think that the necessity to calculate guaranteed bounds implies that much more effort is necessary to solve the problem. However there are situations, in which it can be proved that the additionally collected information reduces the cost for calculating the approximation drastically. I give examples from quadrature theory and from uniform approximation of differentiable functions with unknown singularities as well as of piecewise analytic functions.

Rotational and Helical Surface Approximation for Reverse Engineering

Helmut Pottmann, Thomas Randrup*

We deal with a problem that arises in the context of reverse engineering of geometric models. Given a surface in 3-space or scattered points from a surface, we investigate the problem of deciding whether the data may be fitted well by a surface of revolution or a helical surface. Furthermore, we show how to compute an approximating surface and put special emphasis on basic shapes used in computer aided design. The algorithms apply methods of line geometriy to the set of surface normals in combination with techniques of numerical approximation.

Polynomial Frames and Bases

*H. N. Mhaskar, J. Prestin**

In this talk we investigate localization properties of polynomial frames and bases of wavelet type. Starting with trigonometric and Chebyshev polynomials we summarize how to find translation invariant bases and how one can construct corresponding wavelets.

For general orthogonal polynomials we consider Christoffel-Darboux kernels and some smoothed versions of it (with some weight function $g(k)$). By taking suitable differences of these kernels we obtain the wavelets. Depending on the smoothness function g we compute Riesz and frame bounds. Furthermore, for the Jacobi polynomial setting we study how these frame coefficients can be used to detect singularities of given functions. In particular, for truncated power functions exact asymptotic bounds for the decay of the frame coefficients are presented.

Finally, for some special cases we discuss whether these Riesz bases are also Schauder bases for the space of continuous functions.

Orthogonality of B-Splines and Applications

Ulrich Reif

We show that the cardinal B-Splines $B_{j,n}, j \in \mathbb{Z}$, of order n are an orthonormal system with respect to the weighted Sobolev norm

$$\|f\| := \left(\sum_{\mu=0}^{n-1} \omega_\mu \int_{\mathbb{R}} |\partial^\mu f(t)|^2 \, dt \right)^{1/2}$$

if and only if the weights ω_μ are chosen such that

$$1/\mathrm{sinc}^{2n}(y/2) = \sum_{\mu=0}^{n-1} \omega_\mu(n) y^{2\mu} + O(y^{2n}) .$$

Spline approximation $P : H^{n-1}(\mathbb{R}) \mapsto \mathrm{span}_j B_{j,n}$ with respect to this norm is a quasi-interpolant of maximal order. Compared with the standard L^2-approximation, the evaluation of P is significantly cheaper since no linear system has to be solved. Compared with standard quasi interpolants, P provides a best approximation with respect to a reasonable norm measuring the deviation of function values and certain derivatives. If approximation is subject to a set of linear constraints, then the solution is readily obtained by projecting the solution of the unconstrained problem on the feasible set. When applied to spline conversion problems, P yields explicit schemes for knot removal and degree reduction.

The L_2-Regularity of Refinable Functions

Amos Ron

The problem of finding the L_2-smoothness of a refinable function, or, more generally, a refinable vector of functions, from intrinsic properties of the corresponding mask is one of the cornerstones of wavelet theory, and is also of crucial significance in the analysis of uniform subdivision schemes. Tens of papers were written on the subject, and important advancements had been made. Nonetheless, even in the very special case of a singleton scaling function in one variable which is dyadically refinable, and has compact support, a complete characterization of the L_2-regularity problem was yet to be found. The most recent results on the matter are due to Reimenschneider and Shen, to Jia, and to Cohen, Gröchenig and Villemoes. In essence, all these authors provide lower bounds on the smoothness of a singleton compactly supported refinable function whose mask is finite (many variables, and general dilation matrices), and some of them show that under a stability assumption on the shifts of the refinable function, these lower bounds estimates are sharp. Alas, simple examples can be made to show that without such a stability assumption the above-mentioned lower bounds can be abysmal.

As of now, we finally know how to characterize completely the L_2-smoothness of refinable functions in terms of their corresponding transfer operator, or equivalently, in terms of the subdivision operator. The characterization is valid in any number of dimensions, for a singleton or vector-valued scaling function (and even for distributions), it does not assume any factorization of the mask, does not require any stability or related conditions of the underlying shifts, and applies to any dilation matrix. Stronger assertions can be made if one assumes the scaling functions to be compactly supported, but even these improvements do not require the mask to be finite. The characterization allows one to find *separately* the sharp smoothness class of each of scaling function in the vector.

The results that are alluded to above appear in an article joint with Zouwei Shen, from the National University of Singapore.

Vector Subdivision

Thomas Sauer

In contrast to the "classical" scalar subdivision schemes, vector subdivsion generates curves by iterating finitely supported bi–infinite vectors of $N \times N$ matrices on bi–infinite vectors of N vectors. The talk is concerned with the question of convergence of these schemes in L_p, $1 \leq p \leq \infty$, and with criteria for the regularity of the associated limit functions.

Radial Basis Function Approximations to Solutions of Partial Differential Equations

Robert Schaback

The first part of the talk reports on recent work giving a solid theoretical foundation to methods using Radial Basis Functions (RBFs) to solve PDEs. Together with C. Franke, collocation techniques were investigated, resulting in useful error bounds. For Rayleigh-Ritz techniques H. Wendland proved error bounds, while techniques based on homogenization (called dual reciprocity methods) are still in a preliminary stage. All of these approaches have the advantage of being meshless and easy to implement, but they still require too much regularity and cause quite a large numerical complexity, because they are nonstationary.

Thus the second part of the talk considers stationary techniques and compares RBF methods with the usual FEM techniques. On one side, FEM methods and thin plate spline RBFs allow a nice error analysis by application of Strang/Fix or Bramble/Hilbert techniques, while they have the problem of reducing computational complexity. On the other side, compactly supported RBFs have easy access to reduction of complexity, if used in a stationary setting, but they have problems to produce good error bounds. The techniques to overcome these are described: approximate approximation and multilevel methods. The latter differs from the multilevel preconditioning used in PDE contexts, and the differences are worked out in some detail.

Operator Algebra Techniques for the Study of Wavelet Bases and Frames

Joachim Stöckler

Affine frames in $L_2(R)$ are families of functions, which are complete and which are generated from a finite set of functions $\{\psi_i;\ 1 \leq i \leq n\}$ by dilation and shift. First we deal with the case where the functions ψ_i are related to a refinable function ϕ. The algebraic structure of the associated frame operators is investigated, and a representation in terms of generalized Laurent operators is found. The numerical computation of these operators acting on sequences in $\ell_2(Z \times Z)^n$ is performed by convolution and upsampling operations as in Mallat's pyramidal algorithm. A simple representation for the so-called lifting scheme is obtained in this way. Secondly, techniques from operator algebras, which were introduced for orthonormal wavelet bases by X. Dai and D. Larson [1], are generalized. This gives a characterization of all wavelet bases and affine frames by means of the local commutant with respect to dilation and translation by integers. It leads to a result, which states that under certain conditions on the Fourier transform of ψ the corresponding affine frame is already a Riesz basis.

[1] Dai, X. and D. Larson, Wandering vectors for unitary systems and orthogonal wavelets, to appear in Memoirs Amer. Math. Soc.

[2] Stöckler, J., Multivariate affine Frames, Habilitationsschrift, University of Duisburg, 1995.

Smoothing Spline ANOVA for 0-1 Data, and the Randomized GACV for Choosing the Smoothing Parameters

*Dong Xiang, Grace Wahba**

Let $x \in [0,1]^d, y \in \{0,1\}$, and $p(x)$ =probability $y = 1$ given x. Data $\{y_i, x(i), i = 1, \cdots n\}$ are observed and it is desired to estimate $p(x)$. Practical examples include risk factor estimation in demographic medical studies, for example $x = (x_1, x_2) = (blood\ pressure,\ cholesterol)$ and $y_i = 1$ or 0 according as the ith patient in a study with given $x(i)$ at the start had a heart attack before the end of the study or not. Let $f(x) = log p(x)/[1 - p(x)]$.

$f(x)$ is assumed to be an element of a reproducing kernel Hilbert space which is the tensor product of d univariate rk spaces, with ANOVA decomposition $f(x) = \mu + \sum_\alpha f_\alpha(x_\alpha) + \sum_{\alpha<\beta} f_{\alpha\beta}(x_\alpha, x_\beta) + \cdots$, corresponding to the expansion of $I = \prod_{\alpha=1}^{d}[\mathcal{E}_\alpha + (\mathcal{I} - \mathcal{E}_\alpha)]$ where $\mathcal{E}_\alpha$ is an averaging operator in the αth univariate rk space. f is fitted as the minimizer of $\mathcal{L}(y, f) + \sum_\alpha \lambda_\alpha J_\alpha(f_\alpha) + \sum_{\alpha<\beta} \lambda_{\alpha\beta} J_{\alpha\beta}(f_{\alpha\beta}) + \cdots$, where $\mathcal{L}(y, f)$ is the negative log likelihood of $y = (y_1, \cdots, y_n)$ given f, the $J_\alpha, J_{\alpha\beta}, \cdots$ are seminorms in the subspaces corresponding to the above decompositions and the sum is terminated in some manner. The randomized trace GACV method for choosing the smoothing parameters $\lambda_\alpha, \lambda_{\alpha\beta} \cdots$ is discussed in conjunction with approximate methods for solving the variational problem given very large data sets.

Related papers may be found in `http://www.stat.wisc.edu/~wahba`

Multilevel Interpolation and Approximation

*F. J. Narcowich, R. Schaback and J. D. Ward**

Interpolation by translates of a given radial basis function (RBF) has become a well-recognized means of fitting functions sampled at scattered sites in R^d. A major drawback of these methods is their inability to interpolate very large data sets in a numerically stable way while maintaining a good fit. To circumvent this problem, a multilevel interpolation (ML) method for scattered data was presented by Floater and Iske. Their approach involves m levels of interpolation where at the j^{th} level, the residual of the previous level is interpolated. On each level, the RBF is scaled to match the data density. In this talk, we will discuss some theoretical underpinnings to the ML method by establishing rates of approximation for a technique that deviates somewhat from the Floater-Iske setting. The final goal of the ML method will be to provide a numerically stable method for interpolating several thousand points rapidly.

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