Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 20 / 1997

Discrete Geometry

25.5. – 31.5.1997

Unter der Leitung von L. Danzer (Dortmund), G. C. Shephard (Norwich) und E. Schulte (Boston) trafen sich 50 Teilnehmerinnen und Teilnehmer aus Belgien, Kanada, Deutschland, Frankreich, Großbritannien, Malaysia, Mexiko, Österreich, Russland, der Schweiz, Ungarn und den USA. Knapp ein Drittel der Teilnehmenden kam aus Deutschland. In den fünf Vormittagssitzungen wurden jeweils drei einstündige Vorträge gehalten, die Einsicht in ein komplexeres Themengebiet gaben oder aktuelle Entwicklungen zu einschlägigen Fragestellungen zusammenfaßten. Bei den vier Nachmittagssitzungen standen insgesamt siebzehn 20- bis 30-minütige Vorträge auf dem Programm, die sich speziellen Analysen und Ergebnissen widmeten.

Ein Themenschwerpunkt der Tagung war die Packungs- und Überdeckungsproblematik in ihren verschiedenen Ausprägungen, wie Dichten für endliche und unendliche Gitter- und allgemeine Packungen, endliche Überdeckungen sowie Kugelpackungen in hyperbolischen Räumen. Fast ebensoviel Raum nahm das aktuelle Gebiet der aperiodischen Strukturen ein, wobei neueste theoretische Ansätze und Ergebnisse mit Anwendungen in der Physik der Quasikristalle verbunden wurden. Ein dritter Schwerpunkt waren die abstrakten Polytope und die damit verbundenen Klassifizierungs- und Realisierungsfragestellungen. Weitere Vorträge kamen aus den Gebieten Gebäudetheorie, Rigidity, Gittertheorie und Konvexgeometrie sowie der Matroidtheorie und der algorithmischen Geometrie mit ihren Universalitätssätzen. – Es ist schwer zu sagen, welches die wichtigsten vorgetragenen Ergebnisse waren. Mindestens zwei können wohl als wegweisend und besonders zukunftsträchtig bezeichnet werden: Die Anwendung der *parametrisierten* Packungsdichte auf Fragen des Kristallwachstums von J. Wills und Coautoren, sowie die von J. Lagarias eingeführte Hierarchie der Modellmengen für Kristalle und Quasikristalle. Hier zeichnet sich endlich eine Ordnung in der Vielfalt der betrachteten Mengentypen ab. – Über die Lösung eines berühmten Problems, das auf der vorigen Tagung noch offen war, berichtete R. Connelly: Jede flexible, triangulierte und orientierbare 2-Mannigfaltigkeit im $\mathbb{E}^3$ (beliebiges Geschlecht, Selbstdurchdringungen zugelassen) hat konstantes Volumen (Sabitov 1995).

In einer Abendveranstaltung stellte J. Richter-Gebert das Programmpaket "Cinderellas Cafe" unter Ausnutzung der Präsentationsmöglichkeiten des Instituts vor. Dieses interaktive und im Onlinebetrieb nutzbare Programmpaket kann in der Lehre zur Darstellung geometrischer Sachverhalte sowie zum Training geometrischer Fertigkeiten eingesetzt

Gefördert durch DFG Deutsche Forschungsgemeinschaft

werden. Es ist aber auch auf hohem mathematischem Niveau nutzbar und ermöglicht es dem Forscher, komplexe geometrische Sachverhalte verschiedenster Art auf einfache Weise einzugeben und so einer computerunterstützten und visuellen Analyse zugänglich zu machen.

Besonders zu erwähnen sind sicherlich die "Problemsession" am Dienstagabend und die "Session on Solutions and Problems" am Ende der Tagung. Hier wurden insgesamt 26 Probleme unterschiedlichster Art vorgestellt, diskutiert und zum Teil gelöst. Bei diesen Sitzungen sowie in den selbstverständlich neben dem offiziellen Programm stattfindenden Diskussionen in kleineren und größeren Kreisen stellte sich die oben schon angedeutete Bandbreite der Teilnehmerinnen und Teilnehmer innerhalb der diskreten Geometrie als besonders anregend und fruchtbar heraus. Eine Auflistung der gestellten Probleme und der sofort oder bis vierzehn Tage nach der Tagung eingegangen Lösungen und Kommentare, findet sich in diesem Beicht im Anschluß an die Vortragsauszüge unter der Überschrift "Problems".

Berichterstatter: Gerrit van Ophuysen

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

VORTRAGSAUSZÜGE

Coincidence Structures and Colour Symmetries of Quasicrystals

Michael BAAKE (Tübingen)

Discrete point sets S (such as lattices or quasiperiodic Delone sets) may admit, beyond their symmetries, certain isometries R such that $S \cap RS$ is a subset of S of finite density. These are the so-called coincidence isometries which form a group under rather general assumptions. In the first part of this talk, the proper mathematical setting was developed, and a selection of examples in dimension 2 and 3 was solved. In many cases, due to the relation to algebraic number theory, one can fully characterize the group of coincidence isometries and one can determine a Dirichlet series generating function for the number of conicidence submodules of index m (for the module generated by S). In the second part of the talk, the closely related problem of similarity sublattices resp. submodules was considered and solved in the same spirit. This is helpful in understanding the possible colour symmetries of periodic and non-periodic discrete structures, as was briefly outlined.

Polyhedral fundametal domains for discrete subgroups of $\mathrm{PSL}(2,\mathbb{R})$

Ludwig BALKE (Bonn)

We consider the following situation: Let Γ be a discrete subgroup of $\mathrm{Isom}^+(\mathbb{H}^2)$, the group of orientation preserving isometries of the hyperbolic plane, Γ acts on $\mathrm{Isom}^+(\mathbb{H}^2)$ just by left translations. We want to construct a fundamental domain for this action. I describe the solution found by Thomas FISCHER in his Ph.D. thesis, 1991. Looking at the Poincaré disk model, $\mathrm{Isom}^+(\mathbb{H}^2)$ can be identified with $\mathrm{PSU}(1,1)$, where $S := \mathrm{SU}(1,1) = \left\{ \left(\begin{smallmatrix} a & b \\ \bar{a} & \bar{b} \end{smallmatrix}\right) \in \mathrm{M}(2\times 2,\mathbb{C}) \;\middle|\; a\bar{a} - b\bar{b} = 1 \right\} \subseteq \mathbb{C}^2 = \mathbb{R}^4$. In real coordinates, we have $S = \{x \in \mathbb{R}^4 \mid x_0^2 + x_1^2 - x_2^2 - x_3^2 = 1\}$. The full preimage of Γ is denoted by $\widetilde{\Gamma}$. We must assume, that Γ has elliptic elements and have to choose a point $u \in \mathbb{H}^2$ with non-trivial stabilizer. For $p \in S$, let $H_g := \{x \in \mathbb{R}^4 \mid x_0p_0 + x_1p_1 - x_2p_2 - x_3p_3 \leq 1\}$. Set

$$Q_x := \bigcap_{\substack{g\in\widetilde{\Gamma} \\ gu=x}} H_g \qquad \text{and} \qquad P := \bigcup_{x\in\Gamma u} Q_x.$$

Furthermore, let $\mathbb{R}^4_+ := \{x \in \mathbb{R}^4 \mid x_0^2 + x_1^2 - x_2^2 - x_3^2 > 0\}$ and let F_g denote the closure of the interior of $\partial P \cap \partial H_g \cap \partial\mathbb{R}^4_+$. Then, we have

Theorem: (Th. FISCHER)

F_g is a fundamental domain for the natural action of $\widetilde{\Gamma}$ on $\partial P \cap \mathbb{R}^4_+$. If, moreover, Γ

is cocompact, then F_g is a compact polyhedron. Projecting linearly from $\partial P \cap \mathbb{R}^4_+$ onto S yields the desired fundamental domain $\mathcal{F}_g$. The faces of $\mathcal{F}_g$ are totally geodesic in the geometry of S given by the bilinear form definings.

Concluding, I want to mention, that $\mathrm{Isom}(\mathbb{H}^2)$ acts on S via conjugation. This enables us to determine the symmetry group of the tiling of S by the fundamental domains $\mathcal{F}_g$, $g \in \widetilde{\Gamma}$.

On partial covering of convex regions by strips

András BEZDEK (Budapest)

Call the closed region between two parallel lines a strip. If K is a closed convex body in the plane, let w be the width of the narrowest strip, which covers K. In his memorable paper (1950) T. BANG solved the TARSKI plank problem (1932) by proving that should K be covered by a collection of strips, the sum of the widths of the strips must be at least w. Elegant generalizations, refinements of this result are known but they don't deal with the problem where K is partially covered by the strips and the task is to estimate the size of the uncovered pieces in term of their areas, widths, diameters and radii. It was shown in the talk that if K is the unit circular disk then the sum of the widths of the strips plus the sum of the diameters of the incircles of the uncoverd components is at least the width of K. The following new problem was discussed: Let $R(\rho, 1)$ be the closed ring determined by the concentric circles of radii $0 < \rho < 1$ and 1. It is conjectured that if $\rho < \frac{1}{2}$ and $R(\rho, 1)$ is covered by strips, then the sum of the widths of the strips is at least 2, i.e. the same as that, if the entire unit disk is to be covered. The case of 3 strips was proved.

On the status of the dodecahedral conjecture

Károly BEZDEK (Budapest)

KEPLER's conjecture (1611) says that the maximum density of packings in $\mathbb{E}^3$ with congruent spheres is $\frac{\pi}{\sqrt{18}} = 0.7404805\ldots$. In connection with this L. FEJES TÓTH (1943) conjectured that the volume of any Voronoi polyhedron of a packing with unit spheres in $\mathbb{E}^3$ is at least $5.550291\ldots$ the volume of a regular dodecahedron with inradius 1. This conjecture, called the dodecahedral conjecture implies that the density of any packing in $\mathbb{E}^3$ with congruent spheres is at most $0.75469\ldots$. Despite the recent efforts to prove the conjectures, both conjectures are unproven.

Let $\mathfrak{P}$ be an arbitrary packing of unit spheres $S_0, S_1, \ldots, S_m, \ldots$ in $\mathbb{E}^3$. If the centers of the unit spheres are denoted by $C_0, C_1, \ldots, C_m, \ldots$, then let

$$k = \mathrm{card}\{C_i \mid 1 \leq \tfrac{1}{2}\mathrm{dist}(C_0, C_i) \leq 1.0854\} \quad ;$$
$$l = \mathrm{card}\{C_i \mid 1 \leq \tfrac{1}{2}\mathrm{dist}(C_0, C_i) \leq 1.196\} \quad \text{and}$$
$$n = \mathrm{card}\{C_i \mid 1 \leq \tfrac{1}{2}\mathrm{dist}(C_0, C_i) \leq \sqrt{3}\tan\tfrac{\pi}{5} = 1.2584086\ldots\} \quad .$$

(One can prove that $k \leq 15$, $l \leq 19$ and $n \leq 21$.)

Conjecture: (The distance conjecture of 14 and 15 nonoverlapping unit spheres in $\mathbb{E}^3$.)
(Part 1) If $13 \leq k \leq 14$, then

$$\frac{1}{2}\left[\sum_{i=1}^{k} \operatorname{dist}(C_0, C_i)\right] \geq 12.0854 + 1.0854(k-12)$$

(Part 2) If $k = 12$ and $13 \leq l \leq 14$, then

$$\frac{1}{2}\left[\sum_{i=1}^{l} \operatorname{dist}(C_0, C_i)\right] \geq 12 + 1.196(l-12)$$

In the talk we give an outline of the proof of the following theorem.

Theorem: (1997) The above distance conjecture implies the dodecahedral conjecture.

On the random generation of oriented matroids

Jürgen Bokowski (Darmstadt)

The talk presented a new definition for oriented matroids via Petrie polygons, it described the algorithmic advantage of hyperline configurations. By using this concept, a decisive improvement for extending oriented matroids in a fast way was found. Applications were given:

- random generation of chirotopes
- generation of p_3-maximal line arrangements
- a possible solution of a long standing problem: Does there exist an orientable triangulated 2-manifold that is not geometrically embeddable in $\mathbb{R}^3$?

Main Reference: J. Bokowski, J.-P. Rondneft, T.-K. Strempel: All decompositions of the projective plane with Petrie Polygons of constant length. Discrete Comput. Geom. (1997).

Lattice points in large convex bodies

Károly Böröczky jr. (Budapest)

As already Gauss observed, the number of lattice points in a large convex body is close to its volume. Let K be a convex body in $\mathbb{R}^n$. The deep estimate

$$\#(rK \cap \mathbb{Z}^n) = V(K) \cdot r^n + O(r^{\frac{n(n-1)}{n+1}})$$

if ∂K has positive curvature was achieved through the work of Sierpiński, Hardy, Hlawka, etc.

Recently this estimate was improved. The smallest errorterm belongs to Huxley (1993) in the plane, and to Krätzel and Nowak in the higher dimensional spaces.

If K is any convex body then the optimal estimate is provided by a result of U. BETKE and myself.

A related problem is the number of lattice points in ∂K. ANDREWS (1963) proved that if ∂K is strictly convex then

$$\#(\partial K \cap \mathbb{Z}^n) \ll V(K)^{\frac{n-1}{n+1}} \ll S(K)^{\frac{n}{n+1}}.$$

BÁRÁNY and LARMAN (1997) showed that the estimate of ANDREWS is optimal. For general convex bodies, I could prove

$$\#(\partial K \cap \mathbb{Z}^n) < \delta(\partial K) + O(\operatorname{diam}(K)^{n-1-\frac{1}{n+2}})$$

where $\delta(K)$ is the so-called lattice surface area. Here the optimal exponent in the errorterm is at least $n - \frac{4}{3}$.

Realizations of Regular Polyhedra in $\mathbb{R}^4$

Javier BRACHO (México City)

Let $\mathcal{P}(d,n)$ be the set of similarity classes of non-degenerated faithfull realizations of d-dimensional incidence polytopes in $\mathbb{E}^n$.

The *opposite* of $P \in \mathcal{P}_{<\infty}(d,n)$ was described by projecting into projective space $\mathbb{P}^{n-1}$, taking there the opposite 1-skeleton (each edge goes the other way around) and then lifting once again to $\mathbb{E}^n$.

Theorem:

$$\mathcal{P}_{<\infty}(4,4) = \left\{ \begin{array}{c} 16 \\ \text{classical} \\ \text{polytopes} \end{array} \right\} \cup \text{ op} \left\{ \begin{array}{c} 16 \\ \text{classical} \\ \text{polytopes} \end{array} \right\} \cup \left\{ T\ ,\ \text{op}(T) \right\}^{*}$$

Conjecture:

$$\mathcal{P}_{<\infty}(n,n) = \{\Delta_n, C_n, O_n, \text{op}(\Delta_n), \text{op}(C_n), \text{op}(O_n)\}$$

The proof was outlined. It relies heavily on the classification of $\mathcal{P}(3,\mathbb{P}^3)$.

* T is a special case associated to the cube; its facets are toroidal maps $\{4,4|4\}$. This was pointed out to the author by Peter MCMULLEN.

A Universality Theorem for Realization Spaces of Maps

Ulrich BREHM (Dresden)

A universality theorem for maps in $\mathbb{R}^3$ is shown, stating essentially that every semialgebraic set can occur as a realization space of some map (with distinguished set of vertices); more precisely:

Theorem: (Universality Theorem for Maps).
Let $n, k \geq 0$ and G be a graph with vertex set $\{v_1, \ldots, v_5, w_1, \ldots, w_n\}$. Let $P \subseteq \mathbb{R}^{3n+k}$ be a semialgebraic set defined over $\mathbb{Q}$. Then there is a map $\mathfrak{M}$ (on some orientable 2-manifold)

- which contains only triangles and quadrangles and
- which contains G as an induced subgraph,

such that for each subfield $K \subseteq \mathbb{R}$ and each straight standard* embedding f of G in K^3 the following are equivalent:

1. f can be extended to a polyhedral embedding (i.e. the facets are strictly convex) of $\mathfrak{M}$ in K^3.
2. f can be extended to an nc-embedding (i.e. with planar facets) of $\mathfrak{M}$ in K^3.
3. There are $\lambda_1, \ldots, \lambda_k \in K$ such that $(f(w_1), \ldots, f(w_n), \lambda_1, \ldots, \lambda_k) \in P$.

(*: 'standard' means that $v_1, \ldots, v_5$ are mapped onto some fixed given projective base.)

Corollaries:

(1) For each strict subfield L of the field of real algebraic numbers there is a map $\mathfrak{M}$ which can be polyhedrally embedded in $\mathbb{R}^3$ but not in L^3.
(2) The realizability problem for maps in $\mathbb{R}^3$ is polynomial time equivalent to the 'Existential Therory of the Reals' and thus NP-hard.

Plenty of small regular thin geometries

Francis BUEKENHOUT (Bruxelles)

A thin geometry Γ is a connected labeled simplicial complex of some rank (the number of distinct labels) in which every rank 2 link is a polygon. It is regular provided its automorphism group G acts transitively on the set of maximal simplices. Together with M. DEHON, Ph. CARA and D. LEEMANS, we have developed computer programs in Magma, starting from a given group G and looking for all possible Γ up to isomorphism. We got hundreds of geometries of rank 3 and 4, none of higher rank, from a dozen of groups $\mathrm{PSL}(2, q)$, $q \leq 19$, the Mathieu group M_{11}, the Suzuki group $\mathrm{Sz}(8)$, etc.. The latter is particularly prolific with 181 rank 3, thin geometries, none of rank ≥ 4. A theory has been developed for the symmetric groups.

Inductively Minimal Geometries and trees

Philippe CARA (Bruxelles)

We consider finite incidence geometries with connected diagram and their automorphisms.

A pair consisting of a geometry of rank n and a group of automorphisms acting flag-transitively is called *minimal* if the order of the group is at most $(n+1)!$. Let (Γ, G) be such a pair. If we take the residue Γ_F of a flag F of Γ, the stabilizer G_F of F in G acts on Γ_F as a flag-transitive group of automorphisms. This yields a pair (Γ_F, G_F) for every flag of Γ. The pair (Γ, G) is called *inductively minimal* if (Γ_F, G_F) is minimal for every residue Γ_F whose diagram is connected.

For a given rank n, we are able to enumerate all inductively minimal pairs up to isomorphism. This is achieved by constructing a bijection with the class of trees of $n+1$ vertices.

The Bellows Conjecture – An Update

Robert CONNELLY (Ithaca, NY)

The Original Bellows Conjecture: Any flex of a triangulated 2-dimensional surface in $\mathbb{R}^3$ flexes with constant volume.

The Bellows Result (SABITOV – 1995): 12 times the volume bounded by a 2-dimensional (oriented) surface X is integral over the ring generated by the edge lenghts (squared).

Recall that λ is *integral* over the ring R if there is a polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, $a_{n-1}, \ldots, a_0 \in R$ such that $p(\lambda) = 0$.

A *flex* of a triangulation is a continuous motion of the vertices that preserves the edge lengths.

Generalizations:

1. The 2-dimensional manifold can be taken to be an oriented (possibly) singular simplicial 2-dimensional cycle.
2. (with Anke WALLS): The integrality result holds in $\mathbb{R}^4$ for 3-dimensional oriented cycles.

Remark: There is a "non-trivial" flex of a 3-dimensional oriented cycle (that is a singular 3-sphere) in $\mathbb{R}^4$. (Hint: This is obtained by taking the join of two flexing intersecting quadrilaterals (cyclic) in complementary copies of $\mathbb{R}^2$ in $\mathbb{R}^4$.)

Questions:

1. Does the integrality result extend to $\mathbb{R}^d$ for $d \geq 5$?
2. Is the 4-dimensional volume of a 4-simplex intergral over the ring generated by the (squared) areas of the ten 2-dimensional areas of the triangular faces?

Recognizing euclidean triangulated 3-orbifolds

Olaf DELGADO (Bielefeld)

Periodic tilings of a simply-connected space can be encoded combinatorially by so-called D-symbols or Delaney-symbols as introduced by A. DRESS. A D-symbol satisfying certain necessary local conditions can be interpreted as a triangulation of an abstract (topological) orbifold, i.e. a manifold with singularities encoding point stabilizers. Given a crystallographic group Γ, its action on $\mathbb{E}^3$, say, is completely encoded in the orbit space $\mathbb{E}^3/\Gamma$ as an orbifold. The D-symbol of a tiling is constructed basically as the image of the barycentric subdivision in the orbit space. Thus, the problem of determing whether a given D-symbol corresponds to a tiling of $\mathbb{E}^3$ can be reduced to determining whether a given triangulated orbifold is homeomorphic to a quotient $\mathbb{E}^3/\Gamma$ for some crystallographic group Γ.

The group corresponding to a given D-symbol can be calculated as a group given by generators and relators. By systematically searching for a candidate for the full translational subgroup of this group, the euclidicity problem can be reduced to the problem of recognizing a triangulated 3-torus. In joint work with D. H. HUSON, a general approach to the classification of periodic tilings with a certain pre-given topological/combinatorial type of tile is presented, and, using the machinery sketched above, applied to the classification of tile-transitive tilings by combinatorial cubes, octahedra and tetrahedra.

Embedding of Voronoi and Delone partitions into $\mathbb{Z}^n$

Michel DEZA (Paris)

Call the skeleton of a (Delone or Voronoi) partition of n-space *embeddable* if it is embeddable isometrically (or with doubled distances) into a cubic lattice. With M. I. SHTOGRIN we identify embeddable skeletons for partitions associated with:

1. irreducible root lattices,
2. parallelohedra tilings (incl. non-normalizable ones) of 3-space,
3. bilattices D-complex, Y-complex, h.c.p. and two generalizations A_n^+, D_n^+ of the diamond packing D_3^+.

Rigidity of Zonohedral Spheres

Nikolai DOLBILIN (Moscow)

Given a polyhedral abstract sphere S, $f : S \to \mathbb{E}^3$ is called a morphism if

1. $f|_F$ is isometric on all faces $F \subset S$,
2. $f(F)$ is a planar polygon,
3. f is an immersion of the edge-skeleton of S.

A morphism $f(S)$ is *rigid* if there is no non-trivial flex of it.

Theorem 1. A morphism of a sphere S with centrally symmetrical faces (zonohedron) is rigid.

Corollary. Quadrillages of a sphere admit rigid morphisms only.

Theorem 2. Given a sphere with positive curvature vertices only, if there is a morphism which flexes it, then the sphere has at least 8 $(2n+1)$-gons.

Corollary. A morphism of S with positive curvature vertices and at most 7 $(2n+1)$-gons is always rigid.

(jointly with STAN'KO, SHTOGRIN)

The (p, q)-problem

Jürgen ECKHOFF (Dortmund)

The "(p, q)-problem" in combinatorial geometry was introduced by HADWIGER and DEBRUNNER in 1957. In its "classical" form it asks for the smallest number of points needed to "pierce" all members of a family $\mathcal{K}$ of convex sets in $\mathbb{R}^d$ having the (p, q)-property. This property states that among any p members of $\mathcal{K}$, some q intersect. Here p and q are given integers satisfying $p \geq q \geq d + 1$. If $N(p, q; d)$ denotes the above minimum number, then HADWIGER and DEBRUNNER proved that $N(p, q; d) = p - q + 1$, provided $p(d - 1) < (q - 1)d$. To this day no other value of $N(p, q; d)$ has been determined. Even the existence of the remaining numbers had not been established for 35 years. Their finiteness was finally proved by ALOU and KLEITMAN in 1992.

In this surveying talk, we presented an outline of the ALOU–KLEITMAN proof which uses various ingredients from the combinatorial geometry of convex sets in a stunning way. We also described how the bounds obtained can be improved in special cases, such as for $N(4, 3; 2)$ (it turns out that $N(4, 3; 2) \leq 282$). A short review of general (p, q)-problems (which are studied in a much broader context today) was included.

Some problems on circle covering

Gábor FEJES TÓTH (Budapest)

We present the following two results on coverings with circles:

Let r_n be the maximum radius of a circular disc which can be covered by n closed unit circles. We have

$$r_8 = 1 + 2\cos\frac{2\pi}{7} \qquad \text{and} \qquad r_9 = 1 + \sqrt{2}.$$

Let $d(w)$ be the minimum density of unit circles covering a strip of width w, with respect to the strip. It is easily seen that $d(w) = \frac{\pi}{w\sqrt{4-w^2}}$ for $0 < w \leq \sqrt{3}$, however, for $w > \sqrt{3}$ the determination of $d(w)$ is a difficult task. We make a first step towards a solution by showing that $d(w) = \frac{3\pi}{w(2+\sqrt{4-w^2})}$ for $\sqrt{3} \leq w \leq w_0$ with some $w_0 > \sqrt{3}$.

On the area sum of a convex polygon and its polar reciprocal

August FLORIAN (Salzburg)

Let K be the unit circle centred at the origin O, and let P be a convex polygon inscribed in K. If P^* denotes the polar domain of P with respect to O, then

$$S(P) = a(P) + a(P^*) \geqq 6$$

with equality only if P is a square inscribed to K(J. ACZÉL an L. FUCHS, 1950).

If $S(P)$ is much greater than 6, what can be said about the deviation of P from a square? In this talk a stability theorem is proved that says, e.g.:

If $6 \leqq \delta \leqq 6.008$ and $S(P) \leqq \delta$, then there exists a square Q such that

$$\rho^H(P,Q) \leqq 8\sqrt{2(\delta-6)} \qquad (\rho^H \text{ denotes the Hausdorff distance}).$$

On the other hand, there is a pentagram $\overline{P}$ such that

$$\rho^H(\overline{P},Q) \geqq \frac{4}{\pi}(\cos\frac{\pi}{8} - \cos\frac{\pi}{4})\sqrt{\delta-6}$$

for every square Q.

Matching Rules & Substitution Tilings

Chaim GOODMAN-STRAUSS (Fayetteville,Arkansas)

A substitution species $\Sigma(T,\sigma)$ is a set of tilings in $\mathbb{E}^n$ such that every bounded configuration appears as the interior of some n-level supertile $\sigma^n(A)$, $A \in T$ — where T is a set of tiles and σ can be thought of as an "inflate and subdivide" procedure in that no tiling in the species is invariant under any infinite cyclic group acting on E^n.

Now, typically, our tiles T can tile in many ways other than in the tiling in $\Sigma(T,\sigma)$. Our problem becomes: Can we decorate our tiles and produce a new set of tiles T', such that every tiling with the tiles T' is essentially a tiling in $\Sigma(T,\sigma)$. In short, can $\Sigma(T,\sigma)$ be "enforced by matching rules".

We prove: (Theorem 1996) "Every substitution tiling $(*)$ $\Sigma(T,\sigma)$ in $E^{n>1}$ can be enforced by matching rules."

Where $(*)$ is an unfortunate technical, but mild condition:

$(*)$ "such that the tiles T admit a set of hereditary edges for which the tilings in $\Sigma(T,\sigma)$ are sibling edge to egde."

Fortunately, every known substitution tiling with polyhedral tiles T satisfies the condition so we can conjecture

Conjecture: "Every substitution tiling with polyhedral tiles T satisfies $(*)$."

As a corollary to the theorem: "Every substitution tiling $(*)$ in $E^{n>1}$ gives rise to an aperiodic set of tiles T'" — that is, a set of tiles that do admit a tiling of $\mathbb{E}^n$, but admit no tiling that is invariant under any ∞-cyclic group acting on $\mathbb{E}^n$.

Quotient Polytopes and the Flag Action

Michael HARTLEY (Klang,Malaysia)

It has long been known that every regular abstract polytope is a quotient of a universal polytope. In this talk I define an action, called the flag action, of a string Coxeter group on the set of flags of a polytope. We shall then see how the flag action may be used to show that any abstract polytope is a quotient of a universal polytope. These results will be applied to some cases of polytopes where facets are quotients of cubes.

Integral bases of polyhedral cones

Martin HENK (Berlin)

For a rational polyhedral cone $C = \mathrm{pos}\{a^1, \ldots, a^m\}$, $a^i \in \mathbb{Z}^d$, a subset $B(C) \subset C \cap \mathbb{Z}^d$ of minimal cardinality satisfying

$$C \cap \mathbb{Z}^d = \left\{ \sum_{i=1}^{l} n_i b^i \;:\; n_i \in \mathbb{N}, b^i \in B(C), l \in \mathbb{N} \right\}$$

is called an integral basis of C. We describe some geometrical properties of these bases as well as relations to Diophantine approximation problems and integer programming.

Generalized sphere packings in hyperbolic space

Ruth KELLERHALS (Göttingen)

For horoball packings $\mathbf{B}_\infty$ of extended hyperbolic space, the local density $\mathrm{ld}_n(B_\infty)$ for each element $B_\infty \in \mathbf{B}_\infty$ can be well-defined and estimated from above by the simplicial density function $\mathrm{d}_n(\infty)$ (Theorem of K. BÖRÖCZKY sen.). Here, $\mathrm{d}_n(\infty)$ is given by

$$\mathrm{d}_n(\infty) = (n+1)\frac{\mathrm{vol}_n(B_\infty \cap S^\infty_{\mathrm{reg}})}{\mathrm{vol}_n(S^\infty_{\mathrm{reg}})},$$

where $S^\infty_{\mathrm{reg}} \subset \overline{\mathbb{H}^n}$ denotes the ideal regular simplex formed by the "ankers" or base points of $n+1$ mutually tangent horoballs.

Formulae for $\mathrm{d}_n(\infty)$ and the euclidean simplicial density function d_{n-1} are presented, e.g.

$$\mathrm{d}_n(\infty) = \frac{n+1}{n-1} \cdot \frac{n}{2^{n-1}} \cdot \prod_{k=2}^{n-1} \left(\frac{k-1}{k+1}\right)^{\frac{n-k}{2}} \cdot \frac{1}{\nu_n}, \qquad \text{where } \nu_n = \mathrm{vol}_n(S^\infty_{\mathrm{reg}});$$

$$\mathrm{d}_2(\infty) = \frac{3}{\pi}.$$

Applications are discussed in connection with the monotonicity result $\mathrm{d}_n(r) \nearrow \mathrm{d}_n(\infty)$ for $n \gg 1$ and for hyperbolic manifolds M^n with m cusps (i.e. $M^n = M_{\mathrm{compact}} \cup C_1 \cup \cdots \cup C_m$, C_i diffeo. $N_i^{n-1} \times (0, \infty)$ with N_i euclidean compact mf.):

$$\mathrm{vol}_n(M^n) \geq m \frac{\mathrm{vol}_{n-2}(S^{n-2})}{2^{n-2}(n-1)^2} \cdot \frac{1}{\mathrm{d}_{n-1} \cdot \mathrm{d}_n(\infty)} \geq m \cdot \frac{2^n}{n(n+1)}.$$

Geometric Models for Quasicrystals

Jeffrey C. LAGARIAS (Florham Park, NJ)

Quasicrystals are physical materials which have long range order under translations as indicated by X-ray diffraction patterns with sharp spots, but which exhibit symmetries forbidden for crystals, e.g. 10-fold symmetry. This talk reviews point set models for crystals and quasicrystals (geometric crystallography) and then presents a hierachy of sets as possible models for the atomic structure of quasicrystals and related materials. A Delone set or (r, R)-set is a set X in $\mathbb{R}^n$ such that each ball of radius r contains at most one point of X (uniform discreteness) and each ball of radius R contains at least one point of X (relative denseness). A Delone set of finite type is a Delone set X such that $X - X$ is a discrete set. A Meyer set is a set X such that $X - X$ is a Delone set. The general class of Delone sets of finite type forms an "universal" class for most models of quasicrystalline materials, including random tiling models. Various characterizations of Delone sets of finite type are given. Finally, two notions of "perfect quasicrystal" are described: sets X with perfect local rules and linearly repetitive sets.

Regular Polytopes in Ordinary Space

Peter MCMULLEN (London)

The notion of regular polyhedron has been successively extended from Platonic (convex) polyhedron, through Kepler-Poinsot (star) polyhedron, Petrie-Coxeter polyhedron (sponge) to Grünbaum-Dress polyhedron. Thus the faces and vertex-figures are regular polygons, but not necessarily planar. DRESS completed the enumeration of regular polyhedra in $\mathbb{E}^3$; in this talk, a much briefer proof of his characterization was presented. The groups were also described; a key ingredient is the circuit criterion: the group of a regular polytope is determined by that of its vertex-figure and its edge circuits. A new notation for quotient polytopes was also introduced. Finally, it was shown that, in addition to the tiling $\{4, 3, 4\}$ of $\mathbb{E}^3$ by cubes, there are just 7 other discrete regular 4-apeirotopes in $\mathbb{E}^3$.

Applications of Topology To Geometric Transversal Theory

Luis MONTEJANO (México City)

The main purpose of the talk was to introduce the following two relevant concepts for the study of transversals of convex bodies

1. cohomological cycles of λ-planes in $\mathbb{R}^d$
2. Separoids.

In order to show the importance of (1) I will just state the most simple generalization of Hadwiger's theorem for transversals which of course, with this ideas can be easily generalized.

Theorem: Let $\mathcal{F}$ be an ordered family of four convex sets in $\mathbb{R}^3$ with the property that every three of them have a line transversal which meets the sets consistantly with the order, then there is an essential cycle of plane transversals to the whole family. Furthermore the converse is also true.

The idea of the proof uses heavily the notion of *separoid* which tries to capture the separation structure and is related with the concept of order type. A separoid is a finite set $\mathcal{F}$ together with a relation (|) on the subsets satisfying

$$S|T \Leftrightarrow T|S$$
$$S|T \Rightarrow S \cap T = \emptyset$$
$$S' \subset S \text{ and } S|T \Rightarrow S'|T.$$

Examples: Finite set of points or finite collection of convex sets in euclidean space with the usual separation structure.

The following two theorems for separoids are important for the study of transversals of convex sets.

Theorem: Let $\mathcal{F}$ be a family of convex sets in $\mathbb{R}^d$. $\mathcal{P}$ a finite family of points in $\mathbb{R}^n$, $d > n$, $\varphi: \mathcal{F} \to \mathcal{P}$ a bijection preserving the separation structure. Then the set of all vectors $v \in \mathbb{S}^{d-1}$ for which there is a hyperplane transversal to all members of $\mathcal{F}$ and perpendicular to v, is a homolocigal $(d-n-1)$-sphere.

Let us consider also the space $\Omega(r, d)$ of all embeddings of r points generating $\mathbb{R}^d$ up to affine equivalence. Then

Theorem: $\Omega(r, d)$ is a Grassmannean with a natural decomposition given by the Schubert cells in which the cells are the separoids.

Self-similarities and invariant densities for model sets

Robert V. Moody (Edmonton)

Model sets (also called cut and project sets) are generalizations of lattices. We introduce the notion of averaging operators on suitable spaces of functions on model sets, these averaging operators encoding information about entire classes of self-similarities with a given inflation factor. An averaging operator is a Hilbert-Schmidt operator or the space of continuous functions on the acceptance window of the model set. Its leading eigenvalue $(= 1)$ gives rise to an invariant density on the model set. There is a strong connection with the theory of continous refinement operators and this leads to a description of the invariant density as an infinte convolution product. We derive some properties by an invariant density, inducting an infinite product expansion for the amplitude function.

Canonical Theorems for Convex Sets

János Pach (New York)

I present various structure theorems for families of convex sets, including the following result of Solymosi and myself. Let $\mathcal{F}$ be a family of pairwise disjoint compact convex sets in the plane, none of which is contained in the convex hull of two others, and let r be a positive integer. We show that $\mathcal{F}$ has r disjoint $\lfloor c_r n \rfloor$-membered subfamilies $\mathcal{F}_i$

Gefördert durch DFG Deutsche Forschungsgemeinschaft

$(1 \le i \le r)$ such that no matter how we pick one element C_i from each $\mathcal{F}_i$, they are in convex position, i.e., every C_i appears on the boundary of the convex hull of $\bigcup_{i=1}^r C_i$. (Here c_r is a positive constant depending only on r.) This generalizes some results of ERDÖS–SZEKERES, BISZTRICZKY–G. FEJES TÓTH, BÁRÁNY-VALTR and others.

We can also prove that if $\mathcal{F}$ is a family of n compact convex sets in the plane, no r of which pairwise intersect, then $\mathcal{F}$ has two disjoint $\lfloor c_r n \rfloor$-membered subfamilies such that no member of the first one intersects any member of the second. We do not know if under the same assumption $\mathcal{F}$ has $\lfloor c_r n \rfloor$ pairwise disjoint members $(r \ge 3)$.

Symmetries of Tilings

Charles RADIN (Austin)

This concerned tilings of Euclidean spaces such as the kite&dart and pinwheel tilings of the plane. The key feature of such tilings emphasized in this talk was their hierarchical structure. Two themes were discussed: their statistical rotational symmetry; and the problem of distinguishing such tilings using only global features. One solution to the latter problem involved use of a "stable manifold" under the hierarchical map.

NP-hard problems in combinatorial Geometry

Jürgen RICHTER-GEBERT (Zürich)

Matroids and oriented matroids are important objects of combinatorial geometry. While matroids model incidence relations in linear vector spaces, oriented matroids in addition model relative position information. If $E = \{1, \dots, n\}$ is a finite set of labels and $n \in \mathbb{N}$ is an integer any matroid on E of rank d can be given by a map $\mu : E^d \to \{0, 1\}$, while an oriented matroid is a map $\chi : E^d \to \{-1, 0, +1\}$. Any oriented matroid gives rise to an underlying matroid $\mu_\chi = |\chi|$. The orientability problem asks for the opposite:"Given a matroid μ. Is there an oriented matroid χ with $|\chi| = \mu$?"

It is proved that this decision problem is NP-complete. The proof is done by encoding 3-satisfyability into pseudoline arrangements with prescribed incidence properties.

Extremal lattices

Rudolf SCHARLAU (Dortmund)

An overview on some recent notions and problems for lattices in euclidean n-space is given. Starting from common properties of certain dense lattices (COXETER-TODD, BARNES-WALL, LEECH, QUEBBEMANN in dimensions 12, 16, 24, 32, and others). H.-G. QUEBBEMANN has introduced the notion of a modular lattice of level l. For small levels $l \in \{1, 2, 3, 5, 6, 7, 11, 14, 15, 23\}$, the notion of "extremality" ('large minimum') is defined, using the theory of modular forms. This poses an existence- and uniqueness-problem for a finite list of parameters (n, l), which is briefly discussed. Work of B. VENKOV (mostly unpublished) relates this analytic extremality to the classical notion of a lattice being extreme, that is perfect and eutactic. This is achieved using theta series with harmonic coefficients and spherical designs.

On Constructing PL-homeomorphisms, isomorphic triangulations and pairwise disjoint paths in the plane

Rephael WENGER (Columbus, Ohio)

Let P and Q be homeomorphic polygons, possibly with holes. I described algorithms for constructing piecewise-linear homeomorphisms from P to Q using $O(n)$ vertices where n is the original number of vertices of P and Q.

Sphere Packings and Crystal Growth

Jörg M. WILLS (Siegen)

Let K and C be convex bodies in Euclidean d-space $\mathbb{E}^d$, $d \geq 2$ and let V denote the volume.

For a finite set $C_n = \{c_1, \ldots, c_n\} \subset \mathbb{E}^d$ let $C_n + K$ denote a finite packing of translates of K, if $\operatorname{int}((c_i + K) \cap (c_j + K)) = \emptyset$ for $i \neq j$. Finally let $\rho > 0$. Then

$$\delta(K, C_n, \rho C) = \frac{nV(K)}{V(\operatorname{conv} C_n + \rho C)}$$

is the parametric density of the packing $C_n + K$ with respect to ρ and C. ρ and C control the influence of the boundary region of the finite packing, ρ its intensity and C its isotropy. A similar definition can be given for lattice packings.

This definition has been introduced by the author in 1992, and it turns out that this definition is good and flexible enough to developa joint theory of finite packings and coverings.

Here we show for lattice packings of spheres that for large n and suitably chosen ρ and C one obtains Wulff-shapes, i.e. the shape of real crystals. The C is responsible for unisotropies by chemical bounds.

Even extreme shapes of crystals (e.g. whiskers) can be realized via parametric density and density deviation.

Triangulations of Lattice Simplices

Günter M. ZIEGLER (Berlin)

This is a survey of both classical and recent work about unimodular triangulations of lattice simplices: triangulations with only integral vertices into simplices of unit volume. (this topic relates e.g. to the geometry of numbers, toric varieties, Gröbner bases, ...)

Some, but not all lattice simplices have unimodular triangulations. We describe a large class of "Watanabe simplices" that do, and a large class of "elementary simplices" that don't.

A classical theorem of KUNDERSEN ET AL. (1977) shows that for every lattice simplex some large (?) dilatation has a regular unimodular triangulation. An open problem is to bound the dilatation constant: is there some c_d that depends only on the dimension? We show that $c = 4$ suffices for 3-dimensional elementary lattice simplices. A complete solution even for 3-dimensional tetrahedra is still not available.

Problems

1. For the background on regular polytopes, consult papers by McMullen and Schulte (or wait for "Abstract Regular Polytopes").

   a) Let P be a discrete realization of a regular apeirotope, which has a full group of translational symmetries. Suppose that P is blended (not pure, so that the group is affinely reducible), and that one component of the blend is discrete. Must the other component also be discrete?

   b) Let $\Gamma = \langle \rho_0, \dots, \rho_{n-1} \rangle$ be a string C-group, and let $\Delta = \langle \sigma_0, \dots, \sigma_{m-1} \rangle$ for some $1 < m < n-1$ be such that $\rho_j \mapsto \sigma_j$ for $j = 0, \dots, m-1$ induces an isomorphism. The mix $\Gamma \diamond \Delta \subseteq \Gamma \times \Delta$ has generators (ρ_j, σ_j) for $j = 0, \dots, n-1$, with $\sigma_m = \cdots = \sigma_{n-1} = \epsilon$. Is $\Gamma \diamond \Delta$ a C-group? (This is true for $m = 1$ or $n - 1$.)

Peter McMullen

2. **Preliminary:** Let $p_i \geq 2$ and let W be a string Coxeter group of type $\{p_1, \dots, p_{d-1}\}$, i.e. the group $\langle s_0, \dots, s_{d-1} \rangle$ with relations

$$\begin{aligned} s_i^2 &= 1 && \text{for all } i \\ (s_i s_j)^2 &= 1 && \text{for all } i \neq j, j \pm 1 \\ (s_{i-1} s_i)^{p_i} &= 1 && \text{for all } i \text{ such that } p_i \text{ finite.} \end{aligned}$$

Diagram: 

[Let $\mathfrak{M}$ be the universal polytope based on W.]

Let A be a subgroup of W satisfying

$$\begin{aligned} &(1) \quad s_k H_k \cap w^{-1} A w = \emptyset && \text{for all } k, \text{ and} \\ &(2) \quad Aw(W_{<j} \cap W_{>i}) H_k = Aw W_{<j} H_k \cap Aw W_{>i} H_k && \text{whenever } i < k < j, \end{aligned}$$

where $W_{<j}$, $W_{>i}$ and H_k are the parabolic subgroups:

$$\begin{aligned} W_{<j} &= \langle s_l : l < j \rangle \\ W_{>i} &= \langle s_l : l > i \rangle \\ \text{and} \quad H_k &= \langle s_l : l \neq k \rangle . \end{aligned}$$

[That is, $\mathfrak{M}/A$ is a well-defined quotient polytope.]

Problem: ?? Then ?? (Proof (preferably!) or counterexample required)

$$AwW_{>i} \cap AwW_{<i+1} = Aw \qquad \text{for all } w \in W \text{ and for all } i$$

and therefore

$$\bigcap_{\text{all } i} AwH_i = Aw.$$

Motivation: It can be shown that any polytope $\mathfrak{P}$ is isomorphic to a quotient $\mathfrak{M}/A$ where A is a stabilizer of a given flag of $\mathfrak{P}$ under a certain action of W. Classifying polytopes can thus be done by classifying quotients. It can be further shown that if A' is the stabilizer of a certain flag of a quotient $\mathfrak{M}/A$, that $AwH_i = A'wH_i$ for all w and i. If it can be shown that $A = A'$, all other results about these quotient will become much more powerful.

Michael Hartley

3. Find the minimum number $n = n(d)$ with the property, that the unit d-cube can be covered by n smaller cubes.

Known: $n(d) \leq d + 1$ (perhaps the best possible).

Wlodzimierz Kuperberg and Gábor Fejes Tóth

4. Cut the d-cube in two parts with a hyperplane through the cube's center, so that

a) the diameter
b) the circumradius

of each part is minimum.

Wlodzimierz Kuperberg

Solution: a) Let a hyperplane P cut an edge $\overline{AB}$ of the unit cube in a point C, so that vertices A and B are on opposite sides of the hyperplane and let $-A$, $-B$ and $-C$ be the centrally symmetrical images of the points. Then $\overline{(-B)(-A)}$ is cut by the hyperplane in $-C$, and the points A and $-B$ are on one side of it. Consider a rectangle $\square(AB(-A)(-B))$. It has edges length $\sqrt{n-1}$ and 1 and is cut by P into two equal parts with diameters not less than $\sqrt{(n-1) + (1/2)^2}$. Since the diameter of one part of the cube is at least the diameter of the corresponding part of the rectangle, it cannot be less than $\sqrt{(n-1) + (1/2)^2}$. However, this value is always attained when the cutting hyperplane is perpendicular to some edge. So we have

$$\text{min diam} = \sqrt{(n-1) + (1/2)^2}.$$

b) The circumdiameter is not less than the diameter. Let us take a cut perpendicular to an edge of the cube, minimizing the diameter. However, in this case the circumdiameter is equal to the diameter of the parts. Hence

$$\text{min circumdiam} = \text{min diam} = \sqrt{n - 1 + (1/2)^2}.$$

Nikolai Dolbilin and Igor Sharygin

5. A *separoid* is a finite set $\mathcal{F}$ together with a relation (|) between certain subsets of $\mathcal{F}$, satisfying

1) $S \mid T \quad \Leftrightarrow \quad T \mid S$
2) $S \mid T \quad \Rightarrow \quad T \cap S = \emptyset$
3) $S' \subset S$ and $S \mid T \quad \Rightarrow \quad S' \mid T$

Fact: Every separoid can be realized with convex sets.

Problem: Characterize which separoids can be realized with points.

Luis Montejano and Javier Bracho

6. Given a lattice $L \subset \mathbb{E}^2$ and an L-polygon P general, $G(P) := \mathrm{card}(L \cap P)$ $t \in \mathbb{E}^2 \setminus L$, χ: Euler-char.
Hadwiger and W. (1976) in Crelle Journal 28o, p.61-69 have shown

$$G(P) - G(P+t) \geq \chi(P) \qquad \text{and “=” occurs for all } \chi \in \mathbb{Z}.$$

Problem: Analogue in $\mathbb{E}^d, d \geq 3$?

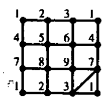
Jörg M. Wills

7. a) Given a 3-polytope $P = \mathrm{conv}(v_1, \ldots, v_n)$,

$$\mathcal{R}_k(P) := \left\{ (w_1, \ldots, w_n) \in \mathbb{R}^{3n} \;\middle|\; w_i = v_i \text{ for } i \leq k \text{ and } \begin{array}{l} \mathrm{conv}(w_1, \ldots, w_n) \\ \text{is combinatorically} \\ \text{isomorphic to } P \end{array} \right\}$$

Problem: How complicated can $\mathcal{R}_k(P)$ be ?

b) Decision problem **TOR**:
Given: a cell-decomposition C of the Torus.
Decide: is C realizable (i.e. flat embeddable, no intersect.).
Problem: Is **TOR** NP-hard ?



Jürgen Richter-Gebert

8. Let G be a simple graph with vertex set $V = \{v_0, \ldots, v_4, w_1, \ldots, w_n\}$. Let $R(G) := \{(f(w_1), \ldots, f(w_n)) \mid f : V \to \mathbb{R}^3 \text{ is a standard embedding of } G\}$ the realization space;
standard: $f(v_0) = (0,0,0)$, $f(v_1) = (1,0,0)$, $f(v_2) = (0,1,0)$, $f(v_3) = (0,0,1)$, $f(v_4) = (1,1,1)$.
Let $R(G,g) := \{f \in R(G) \mid f \text{ is topologically isotopic to } g\}$.
Problem: How complicated can be $R(G,g)$? Characterize up to stable equivalence the set of all sets of the form $R(G,g)$. Does there hold a kind of universality theorem?

Ulrich Brehm

9. The following is a problem of C. ZONG:

Let P be a polytope formed by intersecting $2n$ closed halfspaces containing the unit ball $\mathbb{B}^n$ of $\mathbb{R}^n$. Must P have a vertex at least $\sqrt{n}$ from the origin? This is true for $n \leq 4$ or if P has at most 2^n vertices.

David Larman

10. Is there a tetrahedron $T \subset \mathbb{E}^3$ and a similarity s, such that sT can be tiled by congruent copies of T, other than the three obvious examples ($\frac{1}{48}$ of the cube, $\frac{1}{24}$ of the cube, $\frac{1}{24}$ of the rhombic dodecahedron circumscribed to the cube)?

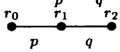
Chaim Goodman-Strauss

11. a) Given $p, q \in \{3, 4, \dots\}$. Does there exist a regular $\{p, q\}$-tiling of some compact surface S?

b) Find $f : \{3, 4, \dots\}^2 \mapsto \mathbb{Z}$ so that $f(p, q)$ is $\chi(S)$, where S is a compact surface of minimal genus admitting a regular $\{p, q\}$-tiling.

Chaim Goodman-Strauss

Solution to part a): Let $\frac{1}{p} + \frac{1}{q} \leq \frac{1}{2}$. Are there regular maps of type $\{p, q\}$?

Yes! Define W: 

W is residueally finite.
$w_1, \dots, w_n \in W \setminus \{1\} \quad \Rightarrow \quad \exists$ hom. φ from W onto a finite group, such that $\varphi(w_i) \neq 1 \;\forall i$.
$\Rightarrow$ Normal subgroups

$$N \cap \underbrace{\langle r_0, r_1 \rangle}_{2p} \cdot \underbrace{\langle r_1, r_2 \rangle}_{2q} = \emptyset$$

$W/N = \langle \rho_0, \rho_1, \rho_2 \rangle$ $(\rho_i = r_i N)$ is the automorphism group of a map with $\rho_i^2 = 1 = (\rho_0\rho_1)^p = (\rho_1\rho_2)^q = (\rho_0\rho_2)^2$. EDMUND, EWALD and KOLKANI proved that.

Egon Schulte

12. Let S be the boundary of a convex body in $\mathbb{R}^3$. A *geodesic circle* of radius r on S is a subset C of S such that the geodesic distance between each point of C and a given point of S, the center of C, is at most r.

a) Consider a packing (covering) of at least three geodesic circles of equal radius on the boundary of a convex body such that each of them is a topological disc. Can the density of the circles be arbitrarily close to 1? Can it be 1? What happens in higher dimensions?

b) The boundary of a convex body with rotational symmetry can be tiled with two circles. Does there exist a convex body with this property, which does not have rotational symmetry?

Gábor Fejes Tóth

Remark on (a): **Packing of geodesic discs on convex surfaces**

Let δ_n be the maximal possible density for a packing of $n \geq 2$ geodesic discs of equal radius on a convex surface.

Conjecture: $$\delta_n = \frac{n \cdot \pi}{2(n-2)\sqrt{3} + 2\pi}.$$

Theorem: $$\delta_n \geq \frac{n \cdot \pi}{2(n-2)\sqrt{3} + 2\pi}.$$

Proof: For $n = 2$ obvious. For $n = 3$ we give one surface where equality is attained, for $n \geq 4$ a large class of surfaces. Let $n \geq 4$ and (V, E) be the graph of a triangulation of the sphere $\mathbb{S}^2$ with n vertices such that the valency of each vertex is ≤ 6, where $V = \{1, \ldots, n\}$, $E \subseteq \binom{V}{2}$.

Let $\mathbf{A}$ be a symmetric $(n \times n)$-Matrix with $a_{ij} = a_{ji} \in [1, 4]$ if $\{i, j\} \in E$ and $a_{ij} = 0$ otherwise, such that $\sum_{j=1}^{n} a_{ij} = 6$ for each $i = 1, \ldots, n$ $\quad (*)$

(i.e. each edge gets a weight between 1 and 4 such that the sum of the weights of the edges meeting at a vertex is equal to 6 for each vertex). With each such matrix we associate the following convex metric on $\mathbb{S}^2$, which is isometric to a convex surface by the theorem of Pogorelov. Let $\alpha_{ij} := a_{ij} \cdot \frac{\pi}{3}$, $\beta_{ij} := (a_{ij} - 1) \cdot \frac{\pi}{3}$. Note that $\beta_{ij} \geq 0$. Let $j_1, j_2, \ldots, j_{v(i)}$ be the neighbours of the vertex i in the canonical cyclic order of the embedded graph (V, E) on $\mathbb{S}^2$, where $v(i)$ denotes the valency of i.

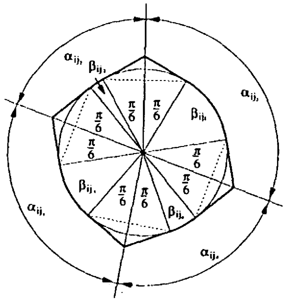


Figure 1

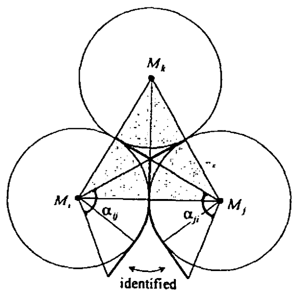


Figure 2

With the vertex i we associate the following set M_i: M_i is a unit disc with caps having an angle of $\frac{2\pi}{3}$ at the cone point and with angles $\alpha_{ij_1}, \ldots, \alpha_{ij_{v(i)}}$ between the cone points seen from the centre of the disc. If $\{i, j\} \in E$ then the parts of the boundaries of M_i and M_j corresponding to $\alpha_{ij} = \alpha_{ji}$ (see Fig. 1 and Fig. 2) are identified. A triangle of the triangulation corresponds to the intersection of the three corresponding sets M_i, which is a common cone point (see Fig. 2). (V, E) is a triangulation of the sphere, thus we have by EULER's formula $n - e + \frac{2}{3}e = 2$, thus $e = 3n - 6$. We have $\frac{2}{3}e = 2n - 4$ shaded triangles, each having area $\sqrt{3}$ and sectors of unit discs having total area

$$\sum_{\substack{\{i,j\}\in E\\ i<j}} \beta_{ij} = \frac{\pi}{3}\left(\left(\sum_{\substack{\{i,j\}\in E\\ i<j}} a_{ij}\right) - e\right) = \frac{\pi}{3}(3n - 3n + 6) = 2\pi.$$

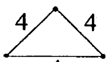
Thus the total area of the convex surface is $(2n-4)\cdot\sqrt{3} + 2\pi$ and the total area of the geodesic unit discs is $n\cdot\pi$. The set of the n unit discs contained in the M_i's forms the wanted packing with density

$$\frac{n\cdot\pi}{2(n-2)\sqrt{3}+2\pi}.$$

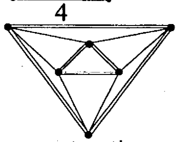
The M_i's are the Dirichlet-Voronoi domains of the centres.

It remains to construct for each $n \geq 4$ a triangulation of $\mathbb{S}^2$ with edge weights satisfying $(*)$.

For $n \equiv 0(\text{mod}3)$ we can take the weighted graph G_{3m}.

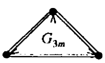
$n = 3$: 

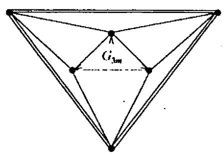
(special case, with the same construction as for a triangulation).

$n = 6$: 

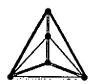
where ══ indicates edge weight 2, all other edges having edge weight 1.

Recursive construction:

replace 

by $G_{3(m+1)}$ = 

For $n \equiv 1(\text{mod}3)$ and $n \equiv 2(\text{mod}3)$ replace one or both of the two triangles 

by . In the cases $n = 4$, $n = 5$ we get  and .

This finishes the proof. □

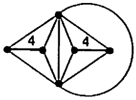
Remarks:

(1) The curvature measure is concentrated on those parts of the boundaries of the circles which are also on the boundary of the M_i's. Everywhere else the metric is Euclidean.

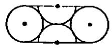
(2) The set of possible weights for a triangulation satisfying $(*)$ is a polytope or empty. For the edge graph of the icosahedron this polytope is 18-dimensional, on the other hand there are also many triangulations allowing only one weight matrix satisfying $(*)$.

(3) For some graphs and weights the convex surface degenerates to a planar convex set being regarded as two-sided. It is easy to characterize those graphs

where this can happen. On the other hand for describing the degenerated case one has to consider also embedded graphs with multiple edges, such as



corresponding to



regarded as a two-sided surface with 6 circles. In the theorem and conjectures the convex surfaces shall be non-degenerate, i.e. the boundary of some compact convex set in $\mathbb{R}^3$ with nonempty interior.

Conjecture: Each packing of $n \geq 3$ geodesic unit discs on a convex surface with density $\frac{n \cdot \pi}{2(n-2)\sqrt{3}+2\pi}$ is congruent to one of the packings constructed in the proof, i.e. for $n \geq 4$ there is a triangulation of $\mathbb{S}^2$ together with a weight matrix $\boldsymbol{A}$ satisfying $(*)$ such that the associated packing (and surface) is congruent to the given one.

In Figures 3, 4 and 5 we show some views of the convex surfaces for $n = 3, 9, 5$.

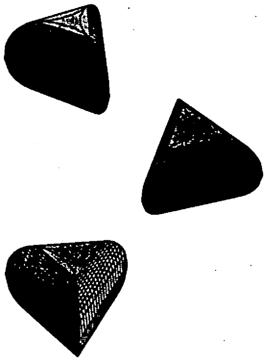


Figure 3

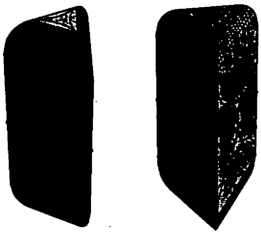


Figure 4

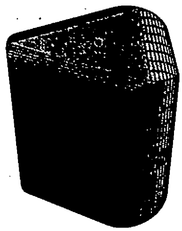


Figure 5

Ulrich Brehm

Solution to part (b): Consider the ellipsoid $\mathcal{E}$ in $\mathbb{E}^d$ $(d \geq 3)$ defined by

$$\mathcal{E} := \{x \mid |x - f_1| + |f_2 - x| = 2\}$$

with focuses f_1 and f_2 $(|f_2 - f_1| < 2)$. Let $\mathcal{M}$ be a $(d-2)$-manifold in $\mathcal{E}$ such that the union $\mathcal{U}$ of the cones

$$K_i := \left\{\overline{xf_i} \mid x \in \mathcal{M}\right\}, \qquad i = 1, 2$$

is the boundary of a convex body K (e.g.: $\mathcal{M} := \mathcal{E} \cap \mathcal{H}$, where $\mathcal{H}$ is a hyperplane separating f_1 from f_2 not orthogonal to $f_2 - f_1$.). In $\mathcal{U}$ the shortest paths joining f_1 and f_2 are pecisely the paths $\overline{f_1 x} \cup \overline{x f_2}$ with x in $\mathcal{M}$. So, if $0 < \rho < 2$, the two geodesic balls $C(f_1, \rho)$ and $C(f_2, 2 - \rho)$ form a tiling of $\mathcal{U}$.

Ludwig Danzer (and for $d = 3$ also Peter Schmitt, Ulrich Brehm et al.)

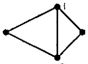
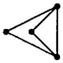
13. Give a useful and precise definition of a *hierarchical* tiling in d dimensions; at least for $d = 2$.

You may employ the idea of substitution, but neither translations nor similarities. The definition shall be applicable as well to $\mathbb{E}^d$ as to $\mathbb{H}^d$.

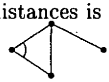
Ludwig Danzer

14. Example: Given two sets of 3 points, on a 2-sphere any set of 3 angles *or* distances which match are sufficient to force local congruence (i.e. $p, q \in \mathbb{S}^2$, same 3 data and $\|p - q\| < \epsilon \Rightarrow p$ congruent to q).

Example: 4 points on a sphere.

5 distances  ,  not local, no problem.

So 5 distances is fine.

Error:  4 distances and 1 angle, but 4 data on one triangle!

Given $|V|$ points in generic position on the 2-sphere, and $|L|$ lines defined by pairs of points, with incidences I. A *necessary condition for independence* of the constraints – angles A, distances D

$$|D'| + |A'| + |I'| \leq 2|V'| + 2|L'| - 3 \quad \text{for } \textit{all} \text{ subsets with } |V| + |L| \geq 2 \qquad \text{(i)}$$

To be a minimal set forcing local congruence, add

$$|D| + |A| + |I| = 2|V| + 2|L| - 3 \qquad \text{(ii)}$$

Conjecture: (i) and (ii) are sufficient for a minimal set forcing local congruence.

Comments.

a) If $|A| = 0$, distances only (drop L), then this is true (Larman's theorem for plane rigidity applies to the sphere).
b) If we take general configurations of points and lines, and try to characterize even independent incidences, this is probably NP-hard.
c) Nothing like this is true on the plane for angles alone. (This problem is also very hard.)
d) The counts $|D| + |A| + |I| \leq 2|V| + |L| - 3$ etc. define a submodular function $f: D \cup A \cup I \to \mathbb{N}$ which defines a matroid on pairs of points (D) pairs of lines (A) and pairs of points and lines (I). This generates a nice $O(|V|^2)$ algorithm.
e) This formulation takes care of "degeneracies"

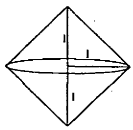
cycle of angles

$|D| + |A| + |I| = 4 + 4 > 2 + 8 - 3 = 2|V| + 2|L| - 3$

$\therefore$ Forbidden!

Walter J. Whiteley

15. What are the packing properties (in $\mathbb{E}^3$) of the double cone determined by the circle $x^2 + y^2 = 1, z = 0$ (as the base) and the two points $(0, 0, 1), (0, 0, -1)$? Is (one of?) the densest packing obtained by putting the base circles into the square faces of the cubic lattice (and the points into the centres of the cubes)?



Peter Schmitt

16. a) For fixed k and n sufficiently large ($k \geq 2$) is it true that the volume of an n-simplex is integral over the ring generated by the "areas" of the k-dimensional faces? Is it true for $k = 2$, $n = 4$? (For $k = 1$, $n \geq 1$ it is well-known.)

b) Is it true that if the areas of the k-faces of an n-simplex are fixed, then there exists (k small, n large) a one-parameter family of n-simplices σ_t^n, $0 \leq t \leq 1$ such that each σ_t^n is *not* congruent to σ_s^n for $t \neq s$? Does vol(σ_t^n) change? Is it true for $k = 2$, $n = 4$? (Peter McMullen claims to have such an example for $k = n - 2$, n large and odd.)

Robert Connelly

17. Let $\Lambda = \{x_i | i \in \mathbb{Z}, x_i \in \mathbb{R}^1\}$ be a Delone set with parameters (r, R) and $f : \Lambda \to \mathbb{Z}$, $f(x_i) := i$ strictly increasing. As $2r < |x_i, x_{i+1}| < 2R$ it is

$$2r =: c < \frac{|x_i - x_j|}{|f(x_i) - f(x_j)|} < C := 2R.$$

Hence f is a Lipschitz map.

Conjecture: This is true for $\mathbb{E}^d$, $d \geq 2$

Nikolai Dolbilin

Comment: M. BAAKE presented a preprint "Seperated nets in Euclidean space and Jacobians of biLipschitz maps" (1997) by D. BURAGO and B. KLEINER in which it is proved that:

There exists a seperated ($\equiv$ a Delone) set in the Euclidean plane which is not biLipschitz equivalent to the integer lattice. So the conjecture fails for all $n \geq 2$.

It turned out that this question was first posed by M. CROMOV - "Asymptotic invariants for infinite groups." In NIBLO and ROLLER (eds.), Geometric group theory. London Math. Soc.; 1993.

Nikolai Dolbilin

18. Suppose $\mathcal{T}$ is a tiling of $\mathbb{S}^2$ with n congruent regions each of diameter smaller than $\frac{\pi}{3}$. Can n be greater than 120?

Wlodzimierz Kuperberg

A historical comment: A rather similar question was posed by B. DELONE after his talk to a representative of the Soviet Space Agency (a soviet analogue of the NASA) in the early 1960's. The representative visited him to possibly get a consultation on the following problem. Assume one needs to build a *large enough* semispherical construction ... on the moon (1). Since the construction was supposed to be erected by robots it would be desired to have deviation from being *congruent* to each other *as little as possible* (2). On the other hand, the limited size of a rocket gives a *strong upper bound* for the size of the pieces (3).

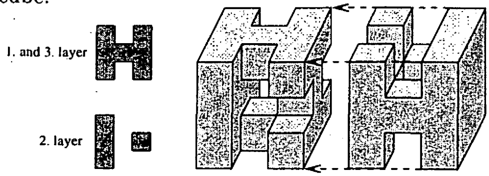
These three requirements inspired DELONE to the question: is it possible to tile a sphere by congruent copies whose diameters are smaller than the diameter of the fundamental domain of the symmetry group of the icosahedron?

Nikolai Dolbilin

19. Is there a connected non-simply connected rep-tile?

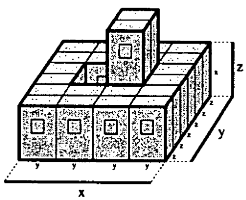
Chaim Goodman-Strauss

Example with $12^3 = 1728$ tiles: Build the figured torus with unit cubes. Two such tiles can be assembled to a $3 \times 3 \times 4$-block, $4 \cdot 4 \cdot 3$ of which rebuild the basic cube as $12 \times 12 \times 12$-cube.



Michael Hartley

Example with 24 tiles: Again two tiles fit together building a rectangular box, twelve of which give a bigger tile (See the figure). The edgelengths are $x = \sqrt[3]{24}$ and $y = \sqrt[3]{9}$ if z is choosen to be of unit length. It should be not to difficult to give in this manner examples for tiles with even higher genus ("just glue two or more tiles together at their xz-face").



Gerrit van Ophuysen

20. a) Charakterize all neighborly regular polytopes!
Expample: $\{3,5\}_5$.

b) An abstract regular polytope P is called centrally symmetric, if there exists an involution in the center of $\Gamma(P)$ which does not fix any vertex. In how many ways can a centrally symmetric regular polytope be centrally symmetric?

Egon Schulte

21. This is a question not a problem:

Fenchel's Conjecture: In a finite Fuchsian group you can find a subgroup without torsion.

What is the first reference? (Known: J.Nielsen, Mat. Tidsskift B 1948.)

Hans-Christoph Im Hof

22. For a covering C of the plane by closed unit circles, let $D(C)$ denote the part of the plane covered at least twice by the members of C. Let $d(p,q)$ denote the distance between two points p and q of the plane, and for $p,q \in D(C)$ let $l(p,q)$ denote the length of the shortest path connecting p to q inside $D(C)$. Determine (give bounds for) the value

$$\sup_{C} \sup_{p,q \in D(C)} \frac{l(p,q)}{d(p,q)}.$$

Gábor Fejes Tóth

23. Consider an n-gon in $\mathbb{E}^2$, n odd. Let the midpoints of every edge move parallel to the edge, then the center of gravity stays at its place and the area is fixed.
Problem: What is the situation in $\mathbb{E}^3$?

Luis Montejano

24. The carpenter's rule problem from Joe Mitchell:

Consider a polygonal arc A embedded in the Euclidean plane. Can one continuously unfold A into a straight line segment keeping each edge of A at the same constant length and at no time having any self-intersections?

Robert Connelly

25. Given n points $p_1, \ldots, p_n$ in the plane and given k, decide whether there exist k lines isolating the given points. What is the complexity of this decision problem? (Is it NP-complete?)

Komei Fukuda

26. Strange Unfoldings of Convex Polytopes

An unfolding of a convex polytope P in $\mathbb{R}^3$ is a planar embedding of its boundary obtained by cutting the edges of some spanning tree T of the graph of P and flattening the boundary along the remaining edges. Two natural (but naive) questions are

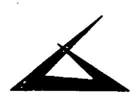
(a) Is every unfolding of a convex polytope non-selfoverlapping?

(b) Is every unfolding of a convex polytope unambiguous?

Here an unfolding is defined to be unambiguous if the original polytope is uniquely constructible from it.

Both questions have negative answers. There are many constructions known for the negative answer of (a), but Makoto NAMIKI (namiki@waka.c.u-tokyo.ac.jp) constructed the smallest example, a skinny tetrahedron, which admits a selfoverlapping unfolding. Note that it has a non-selfoverlapping unfolding as well.

For the question (b), Tomomi MATSUI (tomomi@misojiro.t.u-tokyo.ac.jp) constructed a polytope with 6 facets and 5 vertices which admits an ambiguous unfolding.These two examples can be found in the UnfoldPolytope package for Mathematica by NAMIKI and FUKUDA (1992).



Selfoverlapping unfolding

Geometrically ambiguous unfolding

Consequently more intelligent questions are

(a') Does every convex polytope admit a non-selfoverlapping unfolding?

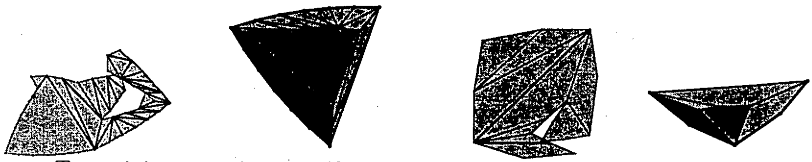
(b') Does every convex polytope admit an unambiguous unfolding?

As far as I know, these questions are still open. I conjectured at the Dagstuhl meeting on Computational Geometry (February 1997) that

(1) Any minimum-length spanning tree of a convex polytope induces a non-selfoverlapping unfolding.

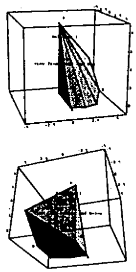
The positive answer to this would resolve the question (a') positively as well.

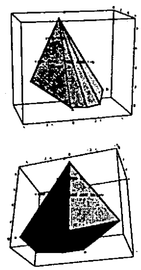
Recently Günter ROTE (rote@opt.math.tu-graz.ac.at) has constructed counterexamples to this conjecture. The smallest among them has 7 facets and 9 vertices (1997). ROTE constructed also a polytope which admits a combinatorially ambiguous unfolding (1997). One can construct two combinatorially different polytopes from such an unfolding: MATSUI's example mentioned above gives rise to two geometrically different polytopes which are combinatorially equivalent.



Two minimum-perimeter selfoverlapping unfoldings and their polytopes

Note that a question (related to (a')) on the existence of an unfolding without overlaps, where it is allowed to cut any place in the boundary, was answered positively by ARONOV and O'ROUKE (1991). The key idea was to cut through geodesic paths from a fixed vertex to all other vertices. In fact this result motivates us to pose another open problem.





Combinatorially ambiguous unfolding and its two polytopes

(2) Does a shortest-path spanning tree of a convex polytope induce a non-selfoverlapping unfolding?

Here a shortest-path spanning tree is a tree composed of shortest paths from a fixed vertex to all other vertices.

Komei Fukuda

Remark: Recently, Mr. Wolfram SCHLICKENRIEDER (schlicke@math.tu-berlin.de) has reported that he found several examples that answered the question above negatively. We shall post some example(s) in the www page http://www.ifor.math.ethz.ch/staff/fukuda/unfold_home/unfold_open.html as soon as we verify his claim.

Komei Fukuda

List of Participants

PD Dr. Michael BAAKE
Institut für Theoretische Physik
Universität Tübingen
Auf der Morgenstelle 14
D-72076 Tübingen
GERMANY
phone: ..49 - 7071 - 2976099
fax: ..49 - 7071 - 295604
e-mail: michael.baake@uni-tuebingen.de
Mail should be sent to office

privat:
Ursrainer Ring 98
72076 Tübingen
GERMANY
phone: ..49 - 7071 - 66869

Dr. Ludwig BALKE
Mathematisches Institut
Universität Bonn
Wegelerstr. 10
D-53115 Bonn
GERMANY
phone: ..49 - 228 - 73 - 3158
e-mail: Balke@math.uni-bonn.de
Mail should be sent to office

privat:
Schenkstr.27
91052 Erlangen
GERMANY
phone: ..49 - 9131 - 16850

Prof. Imre BÁRÁNY
Mathematical Institute of the Hungarian
Academy of Sciences
P.O. Box 127
Reáltanoda u. 13-15
H-1364 Budapest
HUNGARY
phone: ..36 - 1 - 1182875
fax: ..36 - 1 - 1771166
e-mail: barany@math-inst.hu
Mail should be sent to office

privat:
14 Blaha L.
1165 Budapest
HUNGARY
phone: ..36 - 1 - 4033034

Prof. András Bezdek
Dept. of Mathematics
Auburn University
& Hungarian Academy of Science
218 Parker hall
Auburn, AL 36849-5310
USA
phone: ..1 - 334 - 844 - 6562
fax: ..1 - 334 - 844 - 6555
e-mail: bezdean@mail.auburn.edu
Mail should be sent to office

privat:
537 Sherwood dr.
Auburn, Al 36830
USA
phone: ..1 - 334 - 887 - 8794

Prof. Károly Bezdek
Dept. of Geometry
Institute of Mathematics II
Eötvös Lorand University
Rakoczi ut 5
H-1088 Budapest
HUNGARY
phone: ..36 - 1 - 2669833
fax: ..36 - 26362723
e-mail: kbezdek@ludens.elte.hu
Mail should be sent to office

privat:
H-2083 Solymàr
Toldi utca 23/c
HUNGARY
phone: ..36 - 26362723
fax: ..36 - 26362723

Prof. Jürgen Bokowski
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7
D-64289 Darmstadt
GERMANY
phone: ..49 - 6151 - 16 - 2489 (4690)
fax: ..49 - 6151 - 16 - 4011
e-mail: bokowski@mathematik.th-darmstadt.de
Mail should be sent to office

privat:
Am Elfengrund 31a
D-64297 Darmstadt
GERMANY
phone: ..49 - 6151 - 52461

Prof. Károly Böröczky jr.
Mathematical Institute of the Hungarian
Academy of Sciences
P.O. Box 127
Reáltanoda u. 13-15
H-1364 Budapest
HUNGARY
phone: ..36 - 1 - 1173050
fax: ..36 - 1 - 1177166
e-mail: carlos@math-inst.hu
Mail should be sent to office

privat:
1112 Budapest
Sasadi út 161
HUNGARY
phone: ..36 - 1 - 3190032

Prof. Javier BRACHO
Instituto de Matematicas
U.N.A.M.
Circuito Exterior
Ciudad Unversitaria
04510 México, D.F.
MEXICO
phone: ..52 - 5 - 662 - 4523
e-mail: jbracho@math.unam.mx
Mail should be sent to office

privat:
Cuauhtemoc pte. 38-B
M.Hidalgo, Tlalpan
México DF 14410
MEXICO
phone: ..52 - 5 - 606 - 0043

Prof. Ulrich BREHM
Institut für Geometrie
Technische Universität Dresden
01062 Dresden
GERMANY
phone: ..49 - 351 - 463 - 4168
fax: ..49 - 351 - 463 - 6027
e-mail: brehm@math.tu-dresden.de
Mail should be sent to office

privat:
Drosselweg 4
D-14195 Berlin
GERMANY
phone: ..49 - 30 - 8328866
fax: ..49 - 30 - 8328586

Prof. Francis BUEKENHOUT
Dépt. de Mathématiques
Universite Libre de Bruxelles
CP 216 Campus Plaine
Bd. du Triomphe
B-1050 Bruxelles
BELGIQUE
phone: ..32 - 2 - 6505871
fax: ..32 - 2 - 6505867
e-mail: fbueken@ulb.ac.be
Mail should be sent to office

privat:
26 Drève Des Deux Bois
B-1490 Court-St-Etienne
BELGIQUE
phone: ..32 - 10 - 613510

Prof. Heidi L. BURGIEL
Department of Mathematics and
Computer Science
m/c 249
University of Illinois at Chicago
851 S. Morgan St.
Chicago, IL 60607-7045
USA
phone: ..1 - 312 - 996 - 3041
fax: ..1 - 312 - 996 - 1491
e-mail: burgiel@math.uic.edu
Mail should be sent to office

privat:
3246 N.Cilfton Ave., 1E
Chicago, IL 60657
USA
phone: ..1 - 773 - 871 - 7119

Philippe CARA
Dept. of Mathematics – Fac. Wetn.
Vrije Universiteit Brussel
Pleinlaan 2
B-1050 Bruxelles
BELGIQUE
phone: ..32 - 2 - 629 - 3349
fax: ..32 - 2 - 629 - 3495
e-mail: pcara@vub.ac.be
Mail should be sent to office

privat:
Lange Eikstraat 4
B-1970 Wezembeek-Oppem
BELGIQUE
phone: ..32 - 2 - 731 - 8248

Prof. Robert CONNELLY
Dept. of Mathematics
Cornell University
White Hall
Ithaca, NY 14853-7901
USA
phone: ..1 - 607 - 255 - 4301
fax: ..1 - 607 - 255 - 7149
e-mail: connelly@math.cornell.edu
Mail should be sent to office

privat:
220 Willard Way
Ithaca, NY 14850
USA
phone: ..1 - 607 - 272 - 3802

Prof. Ludwig W. DANZER
Inst. f. Math.
Universität Dortmund
D-44221 Dortmund
GERMANY
phone: ..49 - 231 - 755 - 3067 (3066)
fax: ..49 - 231 - 755 - 5307
e-mail: danzer@math.uni-dortmund.de
Mail should be sent to office

privat:
Stortsweg 9
D-44227 Dortmund
GERMANY
phone: ..49 - 231 - 753367

Dr. Olaf DELGADO FRIEDRICHS
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25
33615 Bielefeld
GERMANY
phone: ..49 - 521 - 106 - 4765
fax: ..49 - 521 - 106 - 6007
e-mail: delgado@mathematik.uni-bielefeld.de
Mail should be sent to office

privat:
Ehlentruper Weg 77a
33604 Bielefeld
GERMANY
phone: ..49 - 521 - 286454

Prof. Michel M. Deza
École Normale Supérieure, Dept. Math.
45 rue d'Ulm
75005 Paris
FRANCE
phone: 331 - 44 - 32 - 20 - 31
e-mail: deza@dmi.ens.fr
Mail should be sent to home

privat:
17, passage de l' Industrie
F-75010 Paris
FRANCE
phone: 331 - 47 - 70 - 36 - 73

Prof. Nikolai Dolbilin
Steklov Mathematical Institute
Academy of Sciences
Gubkin ul. 8
Moscow 117 966 GSP-1
RUSSIA
phone: ..7 - 095 - 1351490
fax: ..7 - 095 - 1350555
e-mail: nikolai@dolbilin.mian.su
Mail should be sent to office

privat:
Leninski prosp 123 - 1 - 320
RUSSIA
phone: ..7 - 095 - 4131159

Prof. Jürgen Eckhoff
Fachbereich Mathematik
Universität Dortmund
44221 Dortmund
GERMANY
phone: ..49 - 231 - 755 - 3177 (3066)
fax: ..49 - 231 - 755 - 5307
e-mail: eckhoff@mathematik.uni-dortmund.de
Mail should be sent to office

privat:
Kirchhörder Str. 239
44229 Dortmund
GERMANY
phone: ..49 - 231 - 734946

Prof. Gábor Fejes Tóth
Mathematical Institute of the Hungarian
Academy of Sciences
P.O. Box 127
H-1364 Budapest
HUNGARY
phone: ..36 - 1 - 1173050
fax: ..36 - 1 - 1177166
e-mail: gfejes@matgh-inst.hu
Mail should be sent to office

privat:
Bimbó út 214/b
H-1026 Budapest
HUNGARY
phone: ..36 - 1 - 1762228

Prof. August FLORIAN
Institut für Mathematik
Universität Salzburg
Heilbrunnerstr. 34
A-5020 Salzburg
AUSTRIA
phone: ..43 - 662 - 8044 - 5306
fax: ..43- 662 - 8044 - 5330
e-mail: August.Florian@mh.Sbg.ac.at
Mail should be sent to office

privat:
Lederwaschg. 20
A-5020 Salzburg
AUSTRIA
phone: ..43 - 662 - 628423

Prof. Komei FUKUDA
IFOR
ETH Zentrum
CH-8092 Zürich
SWITZERLAND
phone: ..41 - 1 - 632 - 4023
fax: ..41 - 1 - 632 - 1025
e-mail: fukuda@ifor.math.ethz.ch
Mail should be sent to office

privat:
Av. du Mont d'Or 9
CH-1007Lausanne
SWITZERLAND
phone: ..41 - 21 - 6177687

Prof. Chaim GOODMAN-STRAUSS
Dept. of Mathematics, SE 307
University of Arkansas
Fayetteville, AR 72701
USA
phone: ..1 - 501 - 575 - 6332
e-mail: cgstraus@comp.uark.edu
Mail should be sent to office

privat:
524 W Prospect
Fayetteville, AR 72701
USA

Prof. Peter M. GRUBER
Abteilung für Analysis
Technische Universität Wien
Wiedner Hauptstr. 8-10/1140
A-1040 Wien
AUSTRIA
phone: ..43 - 222 - 58801 - 5363
e-mail: pmgruber@pop.tuwien.ac.at
Mail should be sent to office

privat:
Postweg 5
A-2371 Hinterbrühl
AUSTRIA
phone: ..43 - 2236 - 26493

Prof. Michael HARTLEY
Sepang Inst. of Technology
Level 5, Klang Parade
2112 Jalan Meru
41050 Klang, Selangor
MALAYSIA
phone: ..6 (03) 343 - 0628 ext. 296
fax: ..6 (03) 343 - 0240
e-mail: hartleym@sit.edu.my
Mail should be sent to office

privat:
61 Jalan Kelicap 41
Klang, 41050, Selangor
MALAYSIA
phone: ..6 (03) 344 5571

PD Dr. Martin HENK
Konrad-Zuse-Zentrum für Informationstechnik
Takustr. 7
D-14195 Berlin-Dahlem
GERMANY
phone: ..49 - 30 - 84185 - 0
fax: ..49 - 30 -84185 - 125
e-mail: henk@zib.de
Mail should be sent to office

privat:
Brandenburgische Str. 37
10707 Berlin
GERMANY
phone: ..49 - 30 - 8914950

Prof. Hans-Christoph IM HOF
Mathematisches Institut
Universität Basel
Rheinsprung 21
CH-4051 Basel
SWITZERLAND
phone: ..41 - 61 - 267 - 2697
fax: ..41 - 61 - 267 - 2695
e-mail: imhof@math.unibas.ch

Dr. Daniel HUSON
Forschungsschwerpunkt Mathematisierung
Universität Bielefeld
Postfach 100131
33501 Bielefeld
GERMANY
phone: ..49 - 521 - 106 - 4765
fax: ..49 - 521 - 106 - 6007
e-mail: huson@mathematik.uni-bielefeld.de
Mail should be sent to office

privat:
Teutoburger Str. 41
33604 Bielefeld
GERMANY
phone: ..49 - 521 - 177196

PD Dr. Ruth KELLERHALS
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5
D-37073 Göttingen
GERMANY
phone: ..49 - 551 - 39 - 7766 (7760)
fax: ..49 - 551 - 39 - 2985
e-mail: ruth@cfgauss.uni-math.gwdg.de
Mail should be sent to office

privat:
Hinterm Knick 18
37083 Göttingen
GERMANY
phone: ..49 - 551 - 795797

Prof. Wolfgang KÜHNEL
Mathematisches Institut B
Universität Stuttgart
D-70550 Stuttgart
GERMANY
phone: ..49 - 711 - 685 - 7042 / 7043
fax: ..49 - 711 - 685 - 5304
e-mail: kuehnel@mathematik.uni-stuttgart.de

Prof. Wlodzimierz KUPERBERG
Dept. of Mathematics
Auburn University
218 Parker Hall
Auburn, AL 36849-5310
USA
phone: ..1 - 334 - 844 - 6594
fax: ..1 - 334 - 844 - 6555
e-mail: kuperwl@mail.auburn.edu
Mail should be sent to office

privat:
767 Heard Ave
Auburn, AL 36830
USA
phone: ..1 - 334 - 887 - 6607

Dr. Jeffrey C. LAGARIAS
AT & T Labs-Research
Room C235
180 Park Avenue
Florham Park, NJ 07932 - 0971
USA
phone: ..1 - 973 - 360 - 8416
e-mail: jcl@research.att.comm
Mail should be sent to office

privat:
133 Summit Ave #24
Summit, NJ 07901
USA
phone: ..1 - 908 - 277 - 6017

Prof. David G. LARMAN
Department of Mathematics
University College London
Gower Street
GB-London, WC1E 6BT
UBITED KINGDOM
phone: ..44 - 171 - 387 - 7050
e-mail: dgl@ucl.ac.uk
Mail should be sent to office

privat:
64, Hillview Road, Hatch End
Pinner, Middx HA5 4PE
UNITED KINGDOM
phone: ..44 - 181 - 421 - 3815

Prof. Carl W. LEE
Dept. of Mathematics
University of Kentucky
715 POT
Lexington, KY 40506-0027
USA
phone: ..1 - 606 - 257 - 3336
fax: ..1 - 606 - 257 - 4078
e-mail: lee@ms.uky.edu
Mail should be sent to office

privat:
649 Squires Rd
Lexington, KY 40515
USA
phone: ..1 - 606 - 263 - 1820

Prof. Peter MCMULLEN
Department of Mathematics
University College London
Gower Street
GB-London, WC1E 6BT
ENGLAND
phone: ..44 - 171 - 387 - 7050 ext 2849
e-mail: p.mcmullen@ucl.ac.uk
Mail should be sent to office

privat:
34 Tenby Avenue
Harrow Middlesex , HA3 8RX
ENGLAND
phone: ..44 - 181 - 907 - 2260

Prof. Barry MONSON
Department of Mathematics
University of New Brunswick
P.O.Box 4400
Fredericton, N.B. E3B 5A3
CANADA
phone: ..1 - 506 - 453 - 4768
fax: ..1 - 506 - 453 - 4705
e-mail: bmonson@unb.ca
Mail should be sent to office

privat:
4 Shamrock Tr.
Fredericton N.B.
CANADA
phone: ..1 - 506 - 455 - 2038

Prof. Luis MONTEJANO PEIMBERT
Instituto de Matematicas
U.N.A.M.
Circuito Exterior
Ciudad Universitaria
04510 México, D.F.
MEXICO
phone: ..52 - 5 - 622 - 4525
e-mail: luis@miroslava.math.unam.mx
Mail should be sent to office

privat:
Prol. Ayuntamiento # 162
Col Romero de Terreros,
Coyoacan
México, D.F. 04310
MEXICO
phone: ..52 - 5 - 6588893

Prof. Robert V. MOODY
Dept. of Mathematical Sciences
University of Alberta
Edmonton, AB T6G 2G1
CANADA
phone: ..1 - 403 - 492 - 3613
e-mail: rvm@miles.math.ualberta.ca
Mail should be sent to office

privat:
9712 - 87 Ave
Edmonton T6E 2N4
CANADA

Gerrit VAN OPHUYSEN
Fachbereich Mathematik
Universität Dortmund
44221 Dortmund
GERMANY
phone: ..49 - 231 - 755 - 3066
fax: ..49 - 231 - 755 - 5307
e-mail: Gerrit.vanOphuysen@mathematik.uni-dortmund.de
Mail should be sent to office

privat:
Kaiserstraße 98
44135 Dortmund
GERMANY
phone: ..49 - 231 - 524686
fax: ..49 - 231 - 524696

Prof. János PACH
Courant Institute of Mathematical Sciences
New York University
& Hungarian Academy of Sciences
251, Mercer Street
New York, NY 10012-1110
USA
phone: ..1 - 212 - 998 - 3184
fax: ..1 - 212 - 995 - 4121
e-mail: pach@cims.nyu.edu
Mail should be sent to office

privat:
Németvölgyi út 72/c
H-1124 Budapest
HUNGARY
phone: ..36 - 1 - 395 - 4237
fax: ..36 - 1 - 1177 - 166

Prof. Charles Radin
Dept. of Mathematics
Universtiy of Texas at Austin
Austin, TX 78712-1082
USA
phone: ..1 - 512 - 471 - 0174
fax: ..1 - 512 - 471 - 9038
e-mail: radin@math.utexas.edu
Mail should be sent to office

privat:
8705 Oakmountain Circle
Austin, TX 78759
USA
phone: ..1 - 512 - 345 - 0462

Prof. Jürgen Richter-Gebert
Dept. Inf.
ETH-Zentrum
CH-8092 Zürich
SWITZERLAND
phone: ..41 - 1 - 6327391
fax: ..41 - 1 6321172
e-mail: richter@inf.ethz.ch

privat:
Frohburgstr. 101
CH-8006 Zürich
SWITZERLAND
phone: ..41 - 1 - 3645068

Prof. Rudolf Scharlau
Fachbereich Mathematik
Lehrstuhl II
Universität Dortmund
D-44221 Dortmund
GERMANY
phone: ..49 - 231 - 755 - 3065 (3066)
fax: ..49 - 231 - 755 - 5307
e-mail: Rudolf.Scharlau@Mathematik.Uni-Dortmund.DE

PD Dr. Peter Schmitt
Institut für Mathematik
Universität Wien
Strudlhofgasse 4
A-1090 Wien
AUSTRIA
phone: ..43 - 1 - 40181 - 4012 (31367 - 4000)
fax: ..43 - 1 - 31367 - 4040
e-mail: Peter.Schmitt@univie.ac.at
Mail should be sent to home

privat:
Adolf Gstöttnerg. 6/37
A-1200 Wien
AUSTRIA
phone: ..43 - 1 - 3324408

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof. Egon Schulte
Dept. of Mathematics
Northeastern University
567 Lake Hall
Boston, MA 02115
USA
phone: ..1 - 617 - 373 - 5511
fax: ..1 - 617 - 373 - 5658
e-mail: schulte@neu.edu
Mail should be sent to office

privat:
963 Hancock Street, Apt. 3D
Quincy, MA 02170
USA
phone: ..1 - 617 - 786 - 9952

Prof. Marjorie Senechal
Smith College
Department of Mathematics
Northampton, MA 01063-0001
USA
phone: ..1 - 413 - 585 - 3862
fax: ..1 - 413 - 585 - 3786
e-mail: senechal@minkowski.smith.edu
Mail should be sent to office

privat:
82 Washington Ave
Northampton, MA 01060
USA
phone: ..1 - 413 - 586 - 9690
fax: ..1 - 413 - 584 - 3402

Prof. Geoffrey C. Shephard
School of Mathematics
University of East Anglia
University Plain
Norwich, Norfolk, NR4 7TJ
GREAT BRITAIN

e-mail: g.shephard@uea.ac.uk
Mail should be sent to home

privat:
130 Norwich Rd.
Stoke Holy Cross
Norwich NR14 8QJ
GREAT BRITAIN
phone: ..44 - 1508 - 492471

Prof. Asia Ivic Weiss
Dept. of Mathematics & Statistics
York University
4700 Keele Street
North York, Ont. M3J 1P3
CANADA
phone: ..1 - 416 - 736 - 5250
fax: ..1 - 416 - 736 - 5757
e-mail: asia.weiss@mathstat.yorku.ca
Mail should be sent to office

privat:
804 Euclid Ave
Toronto, Ont M6G 2V4
CANADA
phone: ..1 - 416 - 535 - 5776

Prof. Rephael S. WENGER
Dept. of Computer and Information Science
The Ohio State University
2015 Neil Ave.
Columbus, OH 43210
USA
phone: ..1 - 614 - 292 - 6253
e-mail: wenger@cis.ohio-state.edu
Mail should be sent to office

privat:
2490 Floribunda Drive
Columbus, OH 43209
USA
phone: ..1 - 614 - 338 - 0353

Prof. Walter J. WHITELEY
Dept. of Mathematics & Statistics
York University
4700 Keele Street
North York, Ont. M3J 1P3
CANADA
phone: ..1 - 416 - 736 - 2100 ext 33971
fax: ..1 - 416 - 736 - 5757
e-mail: whiteley@mathstat.yorku.ca
Mail should be sent to office

privat:
191 Mullen Dr.
Thornhill, Ont L4J 2V8
CANADA
phone: ..1 - 905 - 886 - 8266

Prof. Jörg M. WILLS
FB 6 – Mathematik
Universität Siegen
D-57068 Siegen
GERMANY
phone: ..49 - 271 - 740 - 3253
fax: ..49 - 271 - 740 - 2646
e-mail: wills@hrz.uni-siegen.d400.de
Mail should be sent to office

privat:
Eichlingsborn 6
D-57076 Siegen
GERMANY
phone: ..49 - 271 - 75486

Prof. Günter M. ZIEGLER
Fachbereich Mathematik
Sekr. 6 – 1
Technische Universität Berlin
Straße des 17. Juni 136
D-10623 Berlin
GERMANY
phone: ..49 - 30 - 314 - 25730
fax: ..49 - 30 - 314 - 25191
e-mail: ziegler@math.tu-berlin.de
Mail should be sent to office

privat:
Dreysestr. 16
D-10559 Berlin
GERMANY
phone: ..49 - 30 - 3975088

Gefördert durch
DFG Deutsche Forschungsgemeinschaft