MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Differentialgeometrie im Großen

08.–14.06.1997

Die Tagung "Differentialgeometrie im Großen" sollte dem Gedankenaustausch unter Differentialgeometern dienen und zog dadurch viele Teilnehmer an, aus deren neueren Forschungsergebnissen die Tagungsleiter W. Ballmann (Bonn), J.-P. Bourguignon (Bures-sur-Yvette) und W. Ziller (Philadelphia) ein interessantes Vortragsprogramm zusammenstellen konnten.

Die ersten Tage standen im Zeichen dreier Themenschwerpunkte: auf dem Gebiet der Einstein- und Kähler-Einsteinmetriken wurden interessante Existenz- und Regularitätsresultate vorgestellt, Beispiele für Gromov-Hausdorff-Grenzübergänge oder allgemeine Konvergenzfragen unter Krümmungsvoraussetzungen untersucht, und es wurden neuere Klassifikationsergebnisse möglicher Holonomien erzielt und entsprechende Beispiele konstruiert. Zur Einführung in jedes dieser Schwerpunktgebiete wurde ein Überblicksvortrag gehalten. Daneben gab es Vorträge zu den unterschiedlichsten Einzelergebnissen.

Insgesamt bot sich viel Anreiz zu angeregten Diskussionen, die allen Teilnehmern neue mathematische Impulse gaben.

Vortragsauszüge:

Lionel BÉRARD BERGERY:

Holonomy of pseudo-Riemannian manifolds

This was a survey talk on the holonomy of pseudo-Riemannian manifolds, i. e. manifolds M equipped with a quadratic form of signature (p,q), $p+q=n=\dim M$. Here, the canonical Levi-Civitá connection D gives rise to a parallel transport along curves inside M. The holonomy group at a point m is generated by all these parallel transports along loops based at m. This is a Lie subgroup of $O(T_mM, g_m)$, but not all subgroups of $O(p,q)$ are possible holonomies. In Riemannian geometry there is a precise description of possible holonomy groups: any manifold is (at least locally) a product $\mathbb{R}^{d_0} \times M_1 \times \ldots \times M_r$, where the product is Riemannian, $\mathbb{R}^{d_0}$ is flat and each M_i has an *irreducible* holonomy H_i, i. e. H_i acts irreducibly on T_mM_i. Then the list of possible irreducible holonomy groups in the non-symmetric case is quite short (due to M. Berger) and examples have been given for each of them (R. Bryant). In the pseudo-Riemannian case, the picture is more complicated, since the decomposition in products leaves at the end "indecomposable" holonomy, that is the case where H acts on $T_m M$ in such a way that there exists no (non-trivial) *non-degenerate* invariant subspace (but there may exist invariant degenerate subspaces). As in the Riemannian case, there is a list of possible groups in the *irreducible* case (Berger) (and a classification of symmetric spaces), but there is no such list in the non-irreducible indecomposable case. A possible list is given in the Lorentz case (through joint work with A. Ikemakhen in Marrakech), in the $(2,2)$ case and more work has been done in the $(n-2,2)$ case (Ikemakhen). The (n,n) case has some special features, since it is possible in that case to get two supplementary totally isotropic invariant subspaces. In this last case, the metric may be given by a potential, and the description of holonomy is clear. Finally, we give the relationship between the holonomy problem and the Ricci curvature, with special attention to the case where the Ricci operator is not diagonalizable with respect to the metric.

Olivier BIQUARD:

Einstein Metrics with Cusps

We give a rigidity theorem for Einstein metrics on finite volume quotients of 2–$\dim_{\mathbb{C}}$ complex hyperbolic space, generalizing LeBrun's theorem in the compact case: on such a finite volume quotient, any Einstein metric g, complete, bounded curvature, with diameter of the "horocycles" going to zero and mean curvature of these horocycles bounded below, differs (up to a constant) from the standard quotient metric by a diffeomorphism.
The method consists in producing a nontrivial solution to the Seiberg–Witten equations for the metric g (on a non-compact manifold): this is achieved by approximating g by a sequence of smooth metrics on a compactification and by studying the convergence of solutions of the Seiberg–Witten equations on the compactification for these approximating metrics.

Christoph BÖHM:

Examples of Einstein metrics on spheres

We prove: S^5, S^6, S^7, S^8 and S^9 carry infinitely many cohomogeneity one Einstein metrics with positive scalar curvature. We obtain a sequence of Einstein metrics, denoted by g_i, which converges for $i \to \infty$ to a "metric" g_∞, which is smooth outside the singular orbits (totally geodesic submanifolds of codimension ≥ 3). More precisely: $\text{inj}(g_i)|_{Q_{1,2}} \to \infty$ and $K(g_i)|_{Q_{1,2}} \to \infty$ on the singular orbits Q_1 and Q_2.
One gets similar results for low-dimensional products of spheres, e. g. for $S^2 \times S^3$, $S^2 \times S^5$ and $S^2 \times S^7$. Hence, these manifolds carry infinitely many homogeneous and inhomogeneous Einstein metrics with positive scalar curvature.

Robert BRYANT:

Finsler n–spheres of constant curvature

I review the basic constructions of Finsler geometry: a *Finsler structure* Σ on a smooth n–manifold M is a (smooth) hypersurface $\Sigma \subset TM \xrightarrow{\pi} M$, transverse to

the fibers of π with the property that each fiber $\Sigma_x = \Sigma \cap T_xM$ is a strictly convex hypersurface enclosing $0_x \in T_xM$. If $\Sigma = -\Sigma$, we say that Σ is *symmetric.* A Finsler structure defines a notion of length for oriented immersions $\gamma : [a,b] \to M$ and the locally length minimizing curves in M are the geodesics. There is a unique vector field E on Σ that generates the geodesic flow. I recall how Σ carries a canonically defined metric ds^2 and contact structure ω (for which E is the Reeb vector field).
I review the construction, for any $u \in \Sigma$, of a canonical splitting $T_{\pi(u)}M = \mathbb{R}u \oplus u^{\perp}$ together with a canonical metric on $u^{\perp}$, together with a parallel translation along geodesics for the bundle $(\dot{\gamma})^{\perp}$, where $\gamma : [a,b] \to M$ is a unit speed geodesic.
The Jacobi equation takes the form $\ddot{v} + R(\dot{\gamma}(s))v = 0$, where $R(u) : u^{\perp} \to u^{\perp}$ is the *flag curvature operator*, a symmetric linear map. A Finsler structure is said to have constant flag curvature, if $R(u) \equiv c\, id_{u^{\perp}}$ for some constant c. One problem is to describe the Finsler structures of constant flag curvature (a Riemannian Finsler structure with constant flag curvature is of constant sectional curvature).
For surfaces ($n = 2$), one can be very explicit. I review Hilbert's construction of a Finsler structure with $c = -1$ on any strictly convex domain in $\mathbb{R}^n$ and Akbar–Zadeh's 1988 result that a compact Finsler surface with $R(u) \equiv -id_{u^{\perp}}$ is necessarily Riemannian. I then review my earlier construction of non–Riemannian Finsler structures with $R(u) \equiv id_{u^{\perp}}$ on S^2 and its relation with Guillemin's space of Zollfrei metrics. Finally I state and prove

Theorem: If $\Sigma \subseteq TM$ is a Finsler structure with constant flag curvature $c = +1$, then $ds^2 - \omega^2$ defines a Kähler metric on the space $\Lambda = \Sigma/E$ of geodesics of M. When $c = -1$, there is a canonical pseudo-metric $d\tilde{s}^2 - \omega^2$ (got by reversing the sign of ds^2 on the appropriate horizontal space) on Λ that is of reduced holonomy $GL(n-1,\mathbb{R})$ (as described in Bérard–Bergery's talk).

Theorem: There is, up to projective equivalence, an n–parameter family of distinct Finsler structures of constant flag curvature $c = +1$ on S^n whose geodesics are the great circles of the standard metric.

Dima BURAGO:

Geometric approach to semi–dispersing billiards

The talk is based on a joint work with S. Ferleger and A. Kononenko. Let M be a manifold with sectional curvature $K_\sigma \leq K_0$ and injectivity radius $r_{inj} \geq r_0$ and B_i, $i = 1, \dots, k$ a collection of smooth convex bodies $B_i \subset M$. We consider a billiard system in the complement $M \setminus \bigcup B_i$, where a particle moves along geodesics and collides elastically with the B_i's. We assume that this system is non–degenerate in the sense that every point is contained in an open cone with vertex at this point. We prove that:

Theorem 1: There exists a c such that every trajectory of length ≤ 1 experiences no more than c collisions.

Theorem 2: Assume $K_\sigma \leq 0$ and $\bigcap B_i \neq \emptyset$. Then there exists a c such that every trajectory experiences no more than c collisions.

Theorem 3: Assume M is compact and $K_\sigma \leq 0$. Then the topological entropy h_{top} is finite.

As the main examples these theorems cover systems of balls in a box or empty space ($\mathbb{R}^n$ or H^n). The proofs are based on gluing Alexandrov spaces of curvature bounded above and distance-type estimations in these spaces. This problem is also closely related to the problem of existence of gluing rules, which allow to glue a compact space without boundary out of copies of a given block, e. g. a tetrahedron.

Eugenio CALABI:

Homeometric embeddings of some homogeneous spaces in Hilbert space

Given two metric spaces (X,d) and (Y,d'), a topological embedding $f: X \to Y$ is called a *homeometric embedding*, if there exists a monotone function $\varphi: \mathbb{R}^+ \to \mathbb{R}^+, \varphi(0) = 0$ (distortion function) such that for any two points $x_1, x_2 \in X$: $d'(f(x_1), f(x_2)) = \varphi(d(x_1,x_2))$. Example: for $X = \mathbb{R}$, $Y =$ (Hilbert space) $= L_2(\mathbb{R})$, $\varphi(r) = \sqrt{(r)}$: $f(x) = \{t \mapsto \frac{1}{2}(sign(x-t) + sign(t))\}$. This property on an embedding is interesting in the case where the ambient space (Y,d') is a (real) Hilbert space.
A classical theorem by I. Schoenberg states that *every* metric space (X,d) (assumed to be separable) admits a homeometric embedding in a separable Hilbert space H with distortion function $\varphi(t) = t^\alpha$ if and only if $0 < \alpha < \frac{1}{2}$. On the other hand, if (X,d) is a Euclidean space and a distance function, a distortion function $\varphi(t) = t^\alpha$ is compatible with a homeometric embedding $f: X \to H$, if and only if $0 < \alpha < 1$, while a Euclidean n-sphere ($S^n, d =$ geodesic distance) admits a homeometric embedding with distortion function $\varphi(t) = t^\alpha$, if and only if $0 < \alpha \leq \frac{1}{2}$.
Consider the case where (X,d) is a Riemannian manifold with distance defined as the geodesic distance in X, and assume the existence of a homeometric embedding of X in H with distortion function $\varphi(t) = \sqrt{t}$ (the "critical value" of the exponent α). In this case (X,d) must satisfy some strong conditions touching on possible closed geodesics. Some more examples of explicit homeometric embeddings $f: X \to H$ can be constructed analytically, if X is an irreducible symmetric space of non-compact type.

Tobias COLDING:

Regularity results for spaces with a lower Ricci curvature bound

In this talk we will discuss a theory of regularity and singularities of spaces which occur as Gromov–Hausdorff limits of manifolds with a uniform lower Ricci curvature bound. We will also point out the similarities to the well-known regularity and singularity theory of minimal submanifolds and harmonic maps. We will also discuss relations of this theory with geometric measure theory. Most of the work in this talk is joint work with Jeff Cheeger.

Jens HEBER:

Non–compact Homogeneous Einstein spaces

An *Einstein solvmanifold* is a 1–connected solvable Lie group S (hence diffeomorphic to $\mathbb{R}^n$), endowed with a left invariant Einstein metric Q_0. We call (S, Q_0) of *standard type*, if (on the Lie algebra level), $[\mathfrak{s},\mathfrak{s}]^\perp$ is Abelian. All known examples of non-compact homogeneous Einstein spaces are of this form.
We exhibit structural and uniqueness results for standard Einstein solvmanifolds and describe their role in the moduli space. We prove:

1. Solvable Lie groups fall into three *disjoint* classes: Groups with an (essentially unique) left invariant standard Einstein metric (subject to many algebraic restrictions), groups with no left invariant Einstein metric (we provide many classes of examples), and groups with a nonstandard left invariant Einstein metric (no example known).

2. Consider the moduli space $\mathcal{M}^n$ of n-dimensional Einstein solvmanifolds (with C^∞-topology) and the subspace $\mathcal{M}^n_{st}$ of standard spaces. We prove: $\mathcal{M}^n_{st} \subset \mathcal{M}^n$ is *open* (union of finitely many compact path connected components) and real semi-algebraic. For each component $\mathcal{M}' \subset \mathcal{M}^n_{st}$, we give an explicit representation $G^* \to GL(V)$ (G^* real algebraic, reductive, $\dim V < \infty$), and a union $\tilde{\mathcal{M}} \subset V$ of closed G^*-orbits such that $\mathcal{M}'$ is homeomorphic to $G^*\backslash\tilde{\mathcal{M}}$. We thus compute the dimension of $\mathcal{M}^n$ to be 0 near $\mathbb{R}H^n$, $\mathbb{C}H^k$, $\mathbb{H}H^2$ and $8m^2 - 6m - 8$ near $\mathbb{H}H^{m+1}$ for $m \geq 2$, 84 near $\mathbb{C}aH^2$.

Dominique HULIN:

Kähler–Einstein Metrics and Projective Embeddings

We are mainly interested in compact Kählerian manifolds with either $c_1(M) < 0$ or $c_1(M) = 0$. Since the work of Aubin–Calabi–Yau it is known that such a manifold carries a Kähler–Einstein metric. But only few explicit examples of such metrics are available as yet.
When M has ample canonical bundle (or more generally when M is projective) one can wonder whether one of the Einstein metrics carried by M can be obtained by a complex embedding into a projective space equipped with its Fubini-Study metric. We show that this never happens: namely that a complex submanifold of $\mathbb{P}^N$ which is Einstein for the induced metric must be Fano. This result contrasts with an asymptotic theorem by G. Tian and T. Bouche which asserts that the Einstein metric on a compact M with $c_1(M) < 0$ (which is unique up to dilatation) is a limit of metrics induced by embeddings into projective spaces (whose dimensions and holomorphic sectional curvature are unbounded).
In the same spirit, we show that any germ of a complex submanifold of the projective space, which is Einstein for the induced metric, actually extends to a complete complex submanifold of $\mathbb{R}^N$, which is immersed without self-intersections, and that the value of the Einstein constant is a nonzero rational number (provided the Fubini–Study metric is normalized so to have say holomorphic sectional curvature 1).

Thalia JEFFRES:

Kähler–Einstein Cone Metrics

We look for a Kähler-Einstein cone metric in a situation which falls roughly into the negative first Chern class case.
Namely, suppose M is a compact complex manifold containing D, a divisor with one smooth irreducible component. We assume: $K_M + \alpha D$ is ample. Let V be a smooth volume form and s a defining section of $[D]$ and α a real number $0 < \alpha < 1$. The singular Kähler cone potential is

$$\hat{V} = \frac{V}{||s||^{2\alpha}(1 - \varepsilon||s||^{2(1-\alpha)})^2},$$

$\omega = i\partial\bar{\partial} \log \hat{V}$ is a current on M and a cone metric on $\Omega = M \setminus D$. Searching for a metric $\omega + \partial\bar{\partial}u$ which is Kähler-Einstein gives the usual Monge-Ampere equation:

$$\frac{\det(g_{i\bar{j}} + \partial_i\partial_{\bar{j}}u)}{\det g_{i\bar{j}}} = e^{f+u} \quad \text{on } \Omega.$$

Analytic features are that Ω is non-compact and the background metric is itself singular. This leads to the following in the application of the continuity method described by Yau:
1. In the openness step, the horizontal operator is degenerate elliptic, and
2. in the closeness step, the maximum principle must be applied to functions which may achieve a maximum in a cusp.
The first is dealt with by using weighted Hölder spaces and the theory of "edge" elliptic operators, and the second by replacing u by $u + F$, with F a controllable function so that $u + F$ achieves a maximum in a smooth manner over the interior Ω.

François LABOURIE:

Projective structures and affine differential geometry

A flat real projective structure on a surface S is an atlas modelled on $\mathbb{R}P^2$ with coordinate changes in $PSL(3,\mathbb{R})$. The most simple examples are the following:
i) hyperbolic surfaces,
ii) convex surfaces in A^3, the affine 3–space.
For i), one uses the Klein model of hyperbolic space, for ii), one uses as a developing map the map $S \to \mathbb{R}P^2$, $s \mapsto T_sS$. A *convex* flat real projective structure (or $\mathbb{R}P^2$–structure) is such that the developing map takes values in a convex set in $\mathbb{R}P^2$.
In this talk we address the following question: is there a better surface (convex surface in A^3) which represent a given $\mathbb{R}P^2$–structure? Combining several results, one shows that convex $\mathbb{R}P^2$–structures are represented by affine spheres; for non-convex structures, one has to add a parameter, in this case a complex structure on S, and then there is a one to one correspondence between non-convex $\mathbb{R}P^2$–structures and affine surfaces with constant mean affine curvature 1; furthermore, to each representation of $\pi_1(S)$ in $Aff(3)$, there exists a unique affine surface with constant affine Gauss curvature 1. This latter result gives information about the action of $\pi_1(S)$ on A^3.

Joseph LANDSBERG:

Algebraic geometry and local differential geometry

This talk is based on two related themes:

I. *How many derivatives does one need to take to see if a manifold is built out of linear spaces ?*
Classically it was known that to see if a surface in $\mathbb{R}^3$ is ruled by lines one needs to take 3 derivatives at a general point. I discuss several generalizations of this theorem, in particular, to see if an n–fold hypersurface is ruled by lines one needs to take $n+1$ derivatives.

II. *Local and global differential geometry of dual varieties (with B. Ilic)*
I explain the local differential geometry of varieties $X^n \subset \mathbb{C}P^{n+a}$ with degenerate dual varieties. Ironically this project led us to study and solve an open question in linear algebra, namely: Given a linear system $A \subset S^2\mathbb{C}^n$ of quadrics of constant rank r (i. e. for all $q \in A$: $\operatorname{rank} q = r$). How is $\dim A$ bounded in terms of n and r?

I. and **II.** are related, because varieties with degenerate dual varieties are built out of linear spaces and the results of **I.** provide structure theorems for **II.** in some extremal cases.

Claude LEBRUN:

4–dimensional Einstein manifolds

Since the work of Hitchin and Thorpe, it has been known that a 4–manifold M^4 (always assumed smooth, compact and without boundary) admits an Einstein metric g only if its Euler characteristic $\chi(M)$ and its signature $\tau(M)$ satisfy the inequality $2\chi \geq 3|\tau|$. Moreover, if $2\chi = 3|\tau|$, any Einstein metric is locally hyper–Kähler. As an example of the latter, any Einstein metric on the 4–torus T^4 must be flat.
In this lecture, a survey was given of various recent results concerning existence and uniqueness of Einstein metrics. The main results are:

Theorem A: (Besson–Courtois–Gallot) If $M = \mathcal{H}^4/\Gamma$ is a compact quotient of hyperbolic 4-space, then the only Einstein metric on M, up to diffeomorphism and rescaling, is the hyperbolic metric.

Theorem B: (LeBrun) If $M^4 = \mathbb{C}\mathcal{H}^2/\Gamma$ is a compact quotient of complex hyperbolic 2–space, then the only Einstein metric on M, up to diffeomorphism and rescaling, is the complex–hyperbolic metric.

Theorem C: (LeBrun) There are infinitely many compact smooth simply connected M^4 which satisfy $2\chi > 3|\tau|$, but which carry no Einstein metric.

Theorem D: (Sambusetti) If (a, b) is a pair of integers with $a \equiv b \bmod 2$, there is a 4–manifold M^4 (with $\pi_1(M)$ of exponential growth) such that $\chi(M) = a$, $\tau(M) = b$, and such that M carries no Einstein metric.

Theorems A and D follow from entropy estimates, whereas Theorems B and C are proved by Seiberg–Witten theory.

Bernhard LEEB:

Characterizing symmetric spaces and Euclidean buildings by their geometry at infinity

Let X be a locally compact Hadamard space (CAT(0) space) which is geodesically complete, i. e. every geodesic segment can be extended to a complete geodesic.

Main Theorem: If the Tits boundary $\partial_{Tits}X$ of X is a thick irreducible spherical building of dimension ≥ 1, then the following dichotomy occurs: X is a Riemannian symmetric space iff geodesics do not branch. If geodesics branch, then X is a Euclidean building.

Addendum: If X and X' are as above, then every Tits isometry $\partial_{Tits}X \to \partial_{Tits}X'$ which is cone topology continuous is induced by a homothety $X \to X'$.

These results can be applied to extend the Mostov and Prasad Rigidity Theorems as Gromov had done before in the smooth case:

Application: Let X be a Riemannian symmetric space or thick Euclidean building, irreducible and of rank ≥ 2. Let X' be a locally compact and geodesically complete Hadamard space. Suppose that a finitely generated group Γ acts properly discontinuously and cocompactly on X and X'. Then there is a Γ-equivariant homothety $X \to X'$.

Example: On a compact quotient of an irreducible symmetric space of higher rank there exists no piecewise Euclidean (singular) metric of non-positive curvature.

Xiaobo LIU:

The homogeneity of infinite dimensional isoparametric submanifolds

This talk is based on a joint work with Ernst Heintze. The major result presented in this talk is the following theorem: Every irreducible, complete, connected, full, isoparametric submanifold in an infinite dimensional Hilbert space with codimension at least 2 is extrinsically homogeneous.
This result extends a similar theorem of Thorbergsson on the homogeneity of finite dimensional isoparametric submanifolds to infinite dimensions. Our method also provide a new proof to Thorbergsson's theorem which simplifies previous proofs given by Thorbergsson and Olmos respectively.

Peter PETERSEN:

Comparison theory with integral curvature bounds

In this talk we explain how some of the classical comparison estimates for manifolds with lower sectional or Ricci curvature bound extend in an integral sense to situations where one only has integral sectional or Ricci curvature bounds. This can then be used to establish generalizations of Heintze–Karcher volume comparison, relative volume comparison, Cheng–Yau gradient estimates, Colding's L^2–Toponogov, Abresch–Gromoll excess estimate, etc. With this foundational work one can thus establish several new optimal pinching and compactness results.

Xiaochun RONG:

Collapsed manifolds with pinched positive sectional curvature

Let M^n be a manifold of sectional curvature $0 < \delta \leq K_{M^n} \leq 1$, let X be an Alexandrov space of curvature ≥ -1. Suppose the Gromov–Hausdorff distance between M^n and X is less than $\varepsilon(n, \delta) > 0$. Our main results are presented in the talk:

A: If X has the lowest possible dimension $\frac{n-1}{2}$, then M^n is diffeomorphic to a lens space $S^n/\mathbb{Z}_q$, such that

$$\frac{C(n,\delta)}{\mathrm{vol}(M^n)} \leq q \leq \frac{\mathrm{vol}(S^n_\delta)}{\mathrm{vol}(M^n)}$$

B: If X has nonempty boundary, then M^n is diffeomorphic to a lens space provided ε depends also on the Hausdorff measure of X.

Moreover, a universal lower bound for the Hausdorff measure in terms of only n and δ is given for which all examples of M^n in B are included.

Lorenz SCHWACHHÖFER

The classification of holonomies of torsion-free connections

In this talk, we present the recently completed classification of irreducibly acting holonomies of torsion-free connections (joint work with S. Merkulov). The list of possible holonomy groups correspond almost completely to the isotropy groups of symmetric spaces, as has been pointed out by W. Ziller. More precisely, we get the following classification theorem for complex irreducible holonomy groups:

Theorem: Let $H^{\mathbb{C}} \subseteq \mathrm{Aut}(\mathbb{C}^n)$ be a semi-simple irreducible connected Lie subgroup, let $K \subset H^{\mathbb{C}}$ be its maximal compact subgroup. Then:

1. If there exists an irreducible real symmetric space G/K, then $H^{\mathbb{C}}$ occurs as a holonomy.
2. If there exists an irreducible hermitean symmetric space $G/(U(1)\cdot K)$, then $H^{\mathbb{C}}$ and $\mathbb{C}^* \cdot H^{\mathbb{C}}$ occur as holonomy.
3. If there exists an irreducible quaternionic symmetric space $G/(Sp(1)\cdot K)$, then $H^{\mathbb{C}}$ occurs as a holonomy.
4. The above are all irreducible holonomy groups, except
   (a) $G_2^{\mathbb{C}} \subseteq \mathrm{Aut}(\mathbb{C}^7)$,
   (b) $Spin(7,\mathbb{C}) \subseteq \mathrm{Aut}(\mathbb{C}^8)$,
   (c) $\mathbb{C}^* \cdot Sp(2) \subseteq \mathrm{Aut}(\mathbb{C}^4)$.

From here, we can also deduce the possible real holonomy groups. We also describe the method to prove the existence of connections with one of the holonomies as in 3. above. This method is based on a quadratic deformation of a linear Poisson structure, and taking local symplectic realizations (joint work with Chi, Merkulov).

Gang TIAN:

Kähler–Einstein metrics with positive scalar curvature

This is a survey talk on Kähler–Einstein manifolds. We start with the definition of Kähler–Einstein metrics and Calabi's problem. We first give all known and major results on existence and uniqueness of Kähler–Einstein metrics in compact cases; particularly, Yau's solution of Calabi's conjecture on Ricci-flat metrics. Then we discuss non-compact cases. After describing what necessary algebraic conditions should be for complete Kähler–Einstein metrics, we discuss major existence theorems due to Cheng–Yau, R. Kobayashi, Tian–Yau, etc. We also give a brief discussion why it is also interesting to study Kähler–Einstein cone metrics. Finally, we discuss our recent work on Kähler–Einstein metrics with positive scalar curvature and stability of underlying manifolds in the sense of Chow–Mumford. This includes a theorem relating the existence with the properness of a certain functional, which is the Lagrangian of complex Monge-Ampere equations considered. Then we state a result which identifies the properness of the functional restricted to the space of induced metrics with the stability of Chow–Mumford. We end up with a counterexample to a folklore conjecture: there is a Kähler–Einstein metric on any compact Kähler manifold with positive first Chern class and without any nontrivial holomorphic vector fields.

Domingo TOLEDO:

Monodromy and complex hyperbolic manifolds

We review classical constructions of Picard (and reworked by Deligne–Mostow) of complex hyperbolic surfaces arising from suitable cyclic branched covers of $\mathbb{P}^1$. Then we present a recent joint work with D. Allerck and J. Carlson on a similar construction for cyclic branched covers of $\mathbb{P}^3$, branched over a non–singular cubic surface. By studying the monodromy and period map of this family of cyclic cubic 3–folds, we prove the following theorem:

Theorem: Let M be the moduli space of stable cubic surfaces in $\mathbb{P}^3$. Then there is a natural isomorphism $M \cong \ _{\Gamma}\backslash \mathbb{C}H^4$, where $\mathbb{C}H^4$ is the complex hyperbolic 4-space

$$\mathbb{C}H^4 = \{l \in \mathbb{P}(\mathbb{C}^5) : h|_l > 0\}$$

where h is the pseudo–hermitean form $||z_0||^2 - ||z_1||^2 - ||z_2||^2 - ||z_3||^2 - ||z_4||^2$ and Γ is the following discrete subgroup of $PU(h)$. Let $E = \mathbb{Z}[\frac{-1+i\sqrt{3}}{2}]$ be the ring of Eisenstein integers, then $\Gamma = Aut(E^5, h)$.

Gregor WEINGART:

The first eigenvalue of the Dirac operator on quaternionic Kähler manifolds

Quaternionic Kähler manifolds are Riemannian manifolds with holonomy group $Sp(1)Sp(m) := Sp(1) \times Sp(m)/\mathbb{Z}_2 \subset SO(\mathbb{H}^m)$ together with a reduction of the bundle of orthonormal frames to a principal $Sp(1)Sp(m)$–bundle. If M^{4m} is quaternionic Kähler, spin and with positive scalar curvature $\kappa > 0$, then the first eigenvalue λ of the Dirac operator on spinors satisfies

$$\lambda^2 \geq \frac{\kappa}{4}\frac{m+3}{m+2}$$

Thus for all holonomy groups allowing positive scalar curvature the appropriate bounds on the first eigenvalues are known:

holonomy group	*eigenvalue bound*	*limiting manifolds*
SO_m	$\lambda^2 \geq \frac{\kappa}{4}\frac{m}{m-1}$	S^m and others
U_m, m odd	$\lambda^2 \geq \frac{\kappa}{4}\frac{m+1}{m}$	$\mathbb{C}P^m$ and others
U_m, m even	$\lambda^2 \geq \frac{\kappa}{4}\frac{m}{m-1}$	$\mathbb{C}P^{m-1} \times T^2$ and others
$Sp(1)Sp(m)$	$\lambda^2 \geq \frac{\kappa}{4}\frac{m+3}{m+2}$	$\mathbb{H}P^m$ only?

The proof presented in the talk works for all three cases, without conceptual changes, turning from 1st order differential operators (like Dirac and twistor operators) to their hermitean squares (A^*A), which are 2nd order DO related to genuine 2nd order DO on M by several "Weitzenböck" formulas. They are derived using representation theory of the holonomy group by decomposing $TM \otimes TM \otimes S$ using the associativity

$$(\mathrm{Sym}^2 TM \oplus \Lambda^2 TM) \otimes S \quad \cong \quad (TM \otimes TM) \otimes S \quad \cong \quad TM \otimes (TM \otimes S)$$

genuine 2nd order DO and curvature terms — hermitean squares of 1st order DO

and apply it to the special section $\nabla^2 \psi \in \Gamma(TM \otimes TM \otimes S)$, where $\psi \in \Gamma(S)$.

Takao YAMAGUCHI:

The convergence of 3–manifolds under a lower curvature bound

The study of convergence or collapsing theory of Riemannian manifolds is completely open except in the cases when the limit Alexandrov space has the possible maximal or minimal dimension. In the case when the limit is a point, the complete understanding includes the classification of all non-negatively curved manifolds, which is quite far at this stage.
Therefore the focus here is on the three–dimensional case. In the joint work with T. Shioya, we have clarified the collapsing phenomena of closed (orientable) 3–manifolds with a lower sectional curvature bound and an upper diameter bound. Let M_i^3, $i = 1, 2, \ldots$ be such a sequence converging to an Alexandrov space X. We have the fibration theorem in the case when X is a Riemannian manifold without boundary.
So our main concern are the cases $\dim X = 1, 2$ and X has singularities. The problem is to get the topology of a small neighborhood in M_i^3 near the (essential) singular point of X. This can be done by a rescaling argument with critical point theory and by classifying the complete open non-negatively curved Alexandrov 3–spaces.

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