Mathematisches Forschungsinstitut Oberwolfach

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Algebraische Zahlentheorie

20.7. – 26.7.97

Die Tagung fand unter der Leitung von Jean-Benoît Bost (Bures/Yvette), Christopher Deninger (Münster), Gerhard Frey (Essen), Peter Schneider (Münster) und Anthony J. Scholl (Durham) statt.

Thema waren neuere Entwicklungen der algebraischen Zahlentheorie und ihr Zusammenspiel mit anderen Gebieten der reinen Mathematik, wie z.B. K-Theory und algebraische Geometrie. Es gab 20 ca. einstündige Vorträge, von denen im folgenden kurze Zusammenfassungen abgedruckt sind. Natürlich gab es auch noch genug Zeit für Gespräche und Diskussionen, die von den Teilnehmern konstruktiv und mit Freude genutzt wurde.

Eine gemeinsame Wanderung (bei schönem Wetter) und die ausgezeichnete Atmosphäre des Instituts rundeten die Tagung ab.

Abstracts

Grzegor Banaszak, Poznań
K-Theory and Homology of Complete and Henselian Rings. Wild Kernel for Higher K-Groups
(joint work with P. Zelewski, W. Gajda and P. Krasoń)

Let R be a local ring with maximal ideal $\mathfrak{p}$. If R is regular and essentially of finite type over a field or excellent d.v.r then we show that the map:

$$K_n(R_{\mathfrak{p}}^h) \longrightarrow K_n(R_{\mathfrak{p}})$$

is an embedding, where $R_{\mathfrak{p}}$ is the completion and $R_{\mathfrak{p}}^h$ is the henselisation.

In addition we show that

$$H_n(SL(R_{\mathfrak{p}}^h), \mathbb{Z}) \longrightarrow H_n(SL(R_{\mathfrak{p}}), \mathbb{Z})$$

is also an embedding with the same assumptions. We can drop in the case of homology the assumption R regular. We define a wild kernel $WK_n(F)$ for a number field F as the kernel of

$$K_n(F) \longrightarrow \prod_v K_n(F_v)$$

We conjecture that for all F, all $n \geq 0$, all $p > 2$

$$(\operatorname{div} K_n(F))_p \simeq WK_n(F)_p$$

We give an evidence for the conjecture and prove that it is essentially equivalent to the Quillen-Lichtenbaum conjecture.

Jean-Benoît Bost, Bures-sur-Yvette
Hermitian Vector Bundles and Stability

Let K be a number field, $\bar{E}$ an hermitian vector bundle over $\operatorname{Spec} \mathcal{O}_K$, $\widehat{deg}\,\bar{E}$ its Arakelov degree and $\hat{\mu}(E) := ([K:\mathbb{Q}]\, rk\, E)^{-1} \widehat{deg}\,\bar{E}$ its slope. One says that $\bar{E}$ is *semi-stable* when, for any subbundles $F \subset E$, $F \neq 0$, the following inequality holds:

$$\hat{\mu}(\bar{F}) \leq \hat{\mu}(\bar{E}).$$

The talk reported on various results concerning the following question and some variants of it:
Question 1: $\bar{E}, \bar{F}$ semi-stable hermitian vector bundles over $\operatorname{Spec} \mathcal{O}_K$

$$\stackrel{?}{\Rightarrow} \quad \bar{E} \otimes \bar{F} \text{ semi-stable}$$

In particular, the link between height inequalities and geometric semistability was emphasized, and the following question — a positive answer to it implies Question 1 — was considered:
Question 2: For any hermitian vector bundles $\bar{E}$ and $\bar{F}$ over $\operatorname{Spec} \mathcal{O}_K$, and any $\mathcal{O}_K$-submodule V in $E \otimes F$,

$$V_{\bar{K}} \subset E_{\bar{K}} \otimes F_{\bar{K}} \text{ geometrically semi-stable } \stackrel{?}{\Rightarrow} \hat{\mu}(\bar{V}) \leq \hat{\mu}(\bar{E}) + \hat{\mu}(\bar{F})$$

(i.e. $\bigwedge^{rkV} V_{\bar{K}} \subset \bigwedge^{rkV}(E_{\bar{K}} \otimes F_{\bar{K}})$ $SL(E_{\bar{K}}) \times SL(F_{\bar{K}})$-semi-stable).

Rob de Jeu, Durham
Towards regulator formulae for curves over number fields

We study the group $K_{2n-2}^{(n)}(F)$ $(n \geq 3)$ where F is the function field of a complete, smooth, geometrically irreducible curve C over a number field k, assuming the Beilinson–Soulé conjecture on weights. In particular, we compute the Beilinson regulator (expressed as an integral on $C \otimes_{\mathbb{Q}} C$) on a subgroup of $K_{2n-2}^{(n)}(F)$, using a cohomological complex $\widetilde{\mathcal{M}}^{\bullet}_{(n)}(F)$ in degrees 1 through n, given by

$$\widetilde{M}_{(n)}(F) \to \widetilde{M}_{(n-1)}(F) \otimes F^*_{\mathbb{Q}} \to \cdots \to \widetilde{M}_{(2)}(F) \otimes \bigwedge^{n-2} F^*_{\mathbb{Q}} \to \bigwedge^{n} F^*_{\mathbb{Q}}.$$

Here $\widetilde{M}_{(l)}(F)$ is a certain Q–vector space generated by symbols $[f]_l$ for $f \in F\backslash\{0,1\}$, $F_Q^\bullet = F^\bullet \otimes_Z Q$, and the differential is given by

$$\mathrm{d}\,([f]_l \otimes g_1 \wedge \cdots \wedge g_m) = [f]_{l-1} \otimes f \wedge g_1 \wedge \cdots \wedge g_m$$

for $l \geq 3$ and by

$$\mathrm{d}\,([f]_2 \otimes g_1 \wedge \cdots \wedge g_m) = (1-f) \wedge f \wedge g_1 \wedge \cdots \wedge g_m$$

for $l = 2$. There is a natural map $H^p(\widetilde{\mathcal{M}}^\bullet_{(n)}(F)) \to K'^{(n)}_{2n-p}(F)$, and it is the image of this map that we study for $p = 2$. For $n = 3$ and $n = 4$, we also study the boundary map of the image of $H^2(\widetilde{\mathcal{M}}^\bullet_{(n)}(F))$ in $K'^{(n)}_{2n-2}(F)$ in the localization sequence

$$0 \to K'^{(n)}_{2n-2}(C) \to K'^{(n)}_{2n-2}(F) \to \coprod_{x \in C^{(1)}} K'^{(n-1)}_{2n-3}(k(x))$$

and relate it to a map (coming from a map of complexes)

$$H^2(\widetilde{\mathcal{M}}^\bullet_{(n)}(F)) \to \oplus_{x\in C^{(1)}} H^1(\widetilde{\mathcal{M}}^\bullet_{(n-1)}(k(x))),$$

essentially given by mapping $[f]_{n-1} \otimes g$ to $\mathrm{ord}_x(g)[f(x)]_{n-1}$ for the x–component. Our complexes can be combined to a double complex similar to Goncharov's double complex. Putting our results together with his, we obtain a complete description of the image of $K_4^{(3)}(C)$ and $K_6^{(4)}(C)$ in $H^1_{\mathrm{dR}}(C \otimes_Q C, R(n))$ under the Beilinson regulator. This description is in fact independent of any conjectures.

Ehud de Shalit, Jerusalem
Modular symbols and p -adic periods of modular forms

In this talk we described our work (from 1995) on the relation between the p -adic period matrix of the modular curve $X_0(p)$ and modular symbols. Generalization of this work to level pN ($(p, N) = 1$) is in progress. The results may be applied to prove a relation between the Tate period of an elliptic curve over $\mathbb{Q}$ with prime conductor and its modular symbool, which was predicted by Mazur and Tate (a special case of the "refined conjecture" in their Duke J. paper from 1985).

Let m be the genus of $X_0(p)$ and S the set of supersingular elliptic curves in characteristic p. By the p -adic period matrix of $X_0(p)$ we understand a certain matrix of size $(m+1) \times m$ with values in $\mathbb{Q}_p$ describing a pairing on $\mathbb{Z}[S] \times \mathbb{Z}[S]_0$ which gives the p -adic uniformization of the generalized Jacobian $J_0^\sharp$ of $X_0(p)$ with respect to the two cusps. In the first part of our work we relate this matrix *modulo* r -th powers in $\mathbb{Q}_p$, where $r | p-1$, to the infinitesimal variation of the U_p -operator in the deformation of $J_0^\sharp[r]$ supplied by (a certain piece of) $J_1^\sharp[r]$.

In the second part of our work we relate this last quantity to an expression involving modular symbols (and therefore L -values). To that extent we develop a theory of "two-variable" theta elements, similar to the two variable p -adic L -function constructed by Greenberg and Stevens, but in the "refined setting".

Our work has been greatly influenced by the paper of Greenberg and Stevens (Invent. Math. 1993) where they prove the Mazur-Tate-Teitelbaum conjecture in weight 2.

Matthias Flach, Caltech
Equivariant Tamagawa Numbers of Motives

The talk reported about joint work with David Burns from King's College, London. We present a formulation of conjectures on leading coefficients of L-functions attached to motives over number fields *with coefficients* (due to Bloch-Kato, Fontaine, Perrin-Riou, ...) in terms of relative K-groups. In the case where M/K is a given motive and the coefficients arise by considering $M \otimes h^0(Spec\, L)$ where L/K is a finite abelian Galois extension with group G (so that the coefficient algebra is $E := \mathbb{Q}[G]$) the conjecture amounts to the vanishing of an invariant $T\Omega(L/K, M) \in K_0(\mathbb{Z}[G], \mathbb{R})$. Here $K_0(\mathbb{Z}[G], \mathbb{R})$ sits in a long exact sequence

$$
\begin{array}{ccccccccc}
K_1(\mathbb{Z}[G]) & \to & K_1(\mathbb{R}[G]) & \to & K_0(\mathbb{Z}[G], \mathbb{R}) & \to & K_0(\mathbb{Z}[G]) & \to & K_0(\mathbb{R}[G]) \\
 & & & & \cup & & & & \\
 & & & & K_0(\mathbb{Z}[G], \mathbb{Q}) & & & & \\
 & & & & \wr\vert & & & & \\
 & & & & \bigoplus_p K_0(\mathbb{Z}_p[G], \mathbb{Q}_p) & & & &
\end{array}
$$

So one might first try to prove $T\Omega(L/K, M) \in K_0(\mathbb{Z}[G], \mathbb{Q})$ and then $T\Omega(L/K, M)_p = 0 \; \forall$ prime numbers p. We discuss functoriality properties of $T\Omega(L/K, M)$ under passing to a subgroup or quotient of G, and under passage from M to $M^*(1)$. In particular one may study

$$\varprojlim_n T\Omega(K_n/K, M)_p \in \prod_{m=0}^{\infty} \mathbb{Q}_p(\zeta_{p^m})^\times / \mathbb{Z}_p[[T]]^\times$$

where $\mathbb{Z}_p[[T]]^\times$ is mapped into $\prod_{m=0}^{\infty} \mathbb{Q}_p(\zeta_{p^m})^\times$ via $f(T) \mapsto f(\zeta_{p^m} - 1)$ and K_n/K runs through a $\mathbb{Z}_p$-extension of K (i.e. $Gal(K_n/K) \simeq \mathbb{Z}/p^n\mathbb{Z}$, and $K \subset K_1 \subset K_2 \subset \ldots$). It is expected that $\varprojlim_n T\Omega(K_n/K, M)_p$ has an expression L_p^{alg}/L_p^{an} as a quotient of an algebraic and analytic p-adic L-function. This provides a link to classical Iwasawa theory and one might hope for a generalization of Iwasawa theory by studying this limit for other towers of extensions K_n/K.

Alexander Goncharov, MPI Bonn
On mixed elliptic motives

Let E be an elliptic curve over an arbitrary field k.
Problem: How to construct explicitly elliptic motivic complexes

$$\mathrm{RHom}_{\mathcal{MM}_k}(\mathbb{Q}(0), S^n\mathcal{H}(m))$$

where $\mathcal{H} = h^1(E)(1)$?

According to Beilinson's conjectures one should have

$$\mathrm{Ext}^i(\mathbb{Q}(0), S^n\mathcal{H}(m)) = \mathrm{gr}^{\gamma}_{m+n} K(E^{(n)})_{\mathrm{sym}}$$

where $E^{(n)} := \mathrm{Ker}\,(E^{n+1} \xrightarrow{\Sigma} E)$ and S_{n+1} acts on $E^{n+1}(n)$.

The talk reported on an explicit construction of the complex which should do the job for $\mathcal{H}(1)$, which is joint work with A. Levin.

Set $R_3^*(E) := \{(f) * (1-f)\} \subset I_E^4$, $f \in k(E)^*$ where I_E is the augmentation ideal in $\mathbb{Z}[E]$. Let $B_3^*(E) := \frac{I_E^4}{R_3^*(E)}$. We defined a map

$$B_3^*(E) \xrightarrow{\delta} k^* \otimes \mathcal{I} \quad (k = \bar{k})$$

Theorem: Assume $k = \bar{k}$. Then one has an exact mod-2-torsion sequence

$$\mathrm{Tor}(k^*, \mathcal{I}) \hookrightarrow \mathrm{gr}_2^{\gamma} K_2(E)_- \longrightarrow B_3^*(E) \xrightarrow{\delta} k^* \otimes \mathcal{I} \longrightarrow gr_2^{\gamma} K_1(E) \longrightarrow 0$$

This and results of Beilinson and Bloch implies
Theorem: E modular elliptic / $\mathbb{Q} \Longrightarrow \exists$ $\mathbb{Q}$-rat. divisor $P = \Sigma_i n_i P_i$ s.t.

$$q \cdot L(2, E) = \pi \cdot \mathcal{L}_{2,q}(P)$$

for some $q \in \mathbb{Q}^\times$ and we define $\mathcal{L}_{2,q}(z) := \Sigma_{n \in \mathbb{Z}} \mathcal{L}_2(zq^n)$,

$$\mathcal{L}_2(z) := \mathrm{Im}\,(-\int_0^z \log(1-t)\frac{dt}{t}) + \arg(1-t)\log|t|$$

The following conjecture explains the shape of the complex above:
Conjecture: $\exists$ a Lie coalgebra $\mathcal{L}^*(E)$ in the "small" $\otimes$-category of pure elliptic motives $\mathcal{P}^*(E)$ s.t.

$$H^i_{\mathcal{P}^*(E)}(\mathcal{L}^*(E))_{S^n\mathcal{H}(m)^\vee} = \mathrm{Ext}^i(\mathbb{Q}(0), S^n\mathcal{H}(m))$$

Here $\mathcal{P}^*(E) = \{\oplus S^n\mathcal{H}(m)\}$ and $S^{n_1}\mathcal{H}(m_1) \otimes' S^{n_2}\mathcal{H}(m_2) \stackrel{\mathrm{def}}{=} S^{n_1+n_2}\mathcal{H}(m_1+m_2)$.

Ralph Greenberg, Washington
Galois Theory for Selmer Groups

This lecture concerned the following general question. Let A be an abelian variety defined over a number field F. Let K/F be a Galois extension such that $\mathrm{Gal}(K/F)$ is a p-adic Lie group. What can one say about the kernels and cokernels of the natural maps

$$s_{K/F'} : \mathrm{Sel}_A(F')_p \to \mathrm{Sel}_A(K)_p^{\mathrm{Gal}(K/F')}$$

as F' varies over all finite extensions of F contained in K. Here $\mathrm{Sel}_A(M)_p$ denotes the p-primary subgroup of the Selmer group for A over an algebraic extension M of F. We concentrate on the case where A has good, ordinary reduction at all primes of F lying over p. In this case, the group $\mathrm{Sel}_A(M)_p$ has a very simple description solely in terms of the G_F-module $A[p^\infty]$. By exploiting this, one can give relatively straightforward proofs of earlier results—the case where K/F is a $\mathbb{Z}_p$-extension which was studied by B. Mazur (the kernels and cokernels are finite and of bounded order) and the case where $K = F(A[p^\infty])$ which was studied by M. Harris (the kernels and cokernels are finite). Without restrictive assumptions on K/F, one can give examples where $\ker(s_{K/F'})$ and/or $\mathrm{coker}(s_{K/F'})$ are infinite. Here is one rather general finiteness result: Assume that K/F is unramified outside a finite set of primes of F, that the p-primary subgroup of $A(K)$ is finite, and that, for every prime v of F lying over p, the Lie algebras of the decomposition and inertia subgroups of $\mathrm{Gal}(K/F)$ (for any prime of K lying over v) have the same derived Lie subalgebra. Then $\ker(s_{K/F'})$ and $\mathrm{coker}(s_{K/F'})$ are both finite for every F', $F \subseteq F' \subseteq K$, $[F' : F] < \infty$.

Annette Huber, Münster
Dirichlet Motives via Modular Curves

Anderson ($k = 0$) and Harder ($k \geq 1$) have given a construction of extensions

$$0 \to \mathbb{Q}_l[\chi] \to E_\chi \to \mathbb{Q}_l(-k-1) \to 0$$

of $G(\bar{\mathbb{Q}}/\mathbb{Q}(\mu_p))$-modules where χ is a Dirichlet character mod p with $\chi(-1) = (-1)^k$, using the sheaf $R^1\pi_*\mathbb{Q}_{l,E}$ on the modular curve $Y_1(p)$.

We show that these extensions are indeed induced by elements in

$$K_{2k+1}(\mathbb{Q}(\mu_N)) \otimes \mathbb{Q}$$

as suggested by conjectures on mixed motives. The Hodge theoretic regulator can be computed explicitly. Using results on the motivic polylog, this allows to give explicit formulae for the cohomology classes constructed by Harder.

Joachim Mahnkopf, Jerusalem
Values of twisted automorphic L-Functions

Let π be a cuspidal automorphic representation of $\mathrm{GL}_n(\mathbb{A})$, $\mathbb{A}$ the ring of Adeles of $\mathbb{Q}$ and fix a prime $p \in \mathbb{N}$. We assume that π_p is unramified. Let furthermore χ be an idele class character of finite order with infinity component $\chi_\infty = \mathrm{id}$ and conductor $f_\chi = p^e$ a p-power. We prove a formula for the (twisted) values of the automorphic L-Function $L(\pi \otimes \chi, s)$, which can be expressed using (de-Rham) cohomology classes of the symmetric space of $\mathrm{GL}_{n-1}(\mathbb{R})$. The proof exploites the decomposition of the Rankin-Selberg convolution on $\mathrm{GL}_n \times \mathrm{GL}_{n-1}$

$$L(\pi \times \sigma(\chi), s) = L(\pi \otimes \chi, s + k_1) \prod_{i>2} L(\pi, s + k_i)$$

where σ is the induced representation $\mathrm{Ind}(\chi||^{k_1}, ||^{k_2}, \dots, ||^{k_{n-1}})$, $k_i \in \mathbb{R}$ and we thus may use use the zeta integral of $L(\pi \times \sigma(\chi), s)$ to derive an integral formula for the values $L(\pi \otimes \chi, s + k_1)$. For the group GL_2 this coincides with a formula of A. Weil for the twist of modular forms. Assuming that π has non-trivial cohomology we derive in the case GL_3 the algebraicity of $L(\pi \otimes \chi, 1)/\Omega(\pi)$ for a certain period $\Omega(\pi) \in \mathbb{C}^*$ and also construct a distribution μ on $\mathbb{Z}_p^*$ which interpolates the L-Function $\int_{\mathbb{Z}_p^*} \chi_p \, d\mu = L(\pi \otimes \chi, 1)/\Omega(\pi)$.

Ernst Kani, Kingston
Diagonal Quotient Surfaces and a Question of Mazur

Let E be an elliptic curve over a number field K, N a *prime* and

$$\bar{\rho}_{E,N} = \bar{\rho}_{E/K,N} : G_K = Gal(\bar{K}/K) \to Aut(E[N]) \simeq Gl_2(\mathbb{Z}/N\mathbb{Z})$$

its associated Galois representation mod N. A question of Mazur (1978) may be generalized as follows.
Question: To what extent is (the isogeny class of) E/K determined by (the isomorphism class of) $\bar{\rho}_{E,N}$?

This question was studied by Kraus, Oesterlé, Halberstadt and others. Frey proposed in 1988
Conjecture 0: $\exists M = M(E, K)$ such that the set

$$S_{N,E}(K) := \{E'/K \ : \ E' \not\sim E, \ \bar{\rho}_{E,N} \cong \bar{\rho}_{E',N}\} = \emptyset, \quad \forall \, N \geq M.$$

Using the results of Wiles, he recently showed that this conjecture is (for $K = \mathbb{Q}$) equivalent to the *Asymptotic Fermat Conjecture.*

In 1994 H. Darmon formulated several conjectures which (partially) generalize Conjecture 0. In this talk the following stronger and more precise version of one of his conjectures was presented:

Conjecture 1: For every (prime) $N \geq 23$, the set

$$\mathcal{S}_N^*(K) := \{(E, E')_{/K} : \exists \text{ non-trivial } G_K\text{-isometry } \Psi : E[N] \xrightarrow{\sim} E'[N]\}$$

is finite. (Here: a G_K-isometry Ψ is called *trivial* if there exists a cyclic isogeny $f : E \to E'$ of degree $d \leq 27$, $d \neq 22, 23, 26$, such that $\Psi = k \cdot f|_{E[N]}$ for some $k \in \mathbb{Z}$.)

This conjecture has a nice interpretation in terms of the *Modular Diagonal Quotient Surfaces*

$$Z_{N,E} := \Delta_\varepsilon \backslash (X(N) \times X(N))$$

where $X(N)$ is the usual modular curve of level N,

$$\Delta_\varepsilon = \{(g, \alpha_\varepsilon(g)) : g \in G_N\} \leq G_N \times G_N, \; G_N = \mathrm{Sl}_2(\mathbb{Z}/N\mathbb{Z})/\pm 1$$

$$\alpha_\varepsilon : g \mapsto Q_\varepsilon \, g \, Q_\varepsilon^{-1}, \; \forall g \in G_N; \; Q_\varepsilon = \begin{pmatrix} \varepsilon & 0 \\ 0 & 1 \end{pmatrix}, \; \varepsilon \in (\mathbb{Z}/N\mathbb{Z})^\times.$$

Conjecture 2: For $N \geq 23$, every curve $C \subset Z_{N,\varepsilon}$ of genus ≤ 1 is a Hecke curve: $C = \bar{T}_{n,k}$ (induced by a Hecke correspondence).

It is not difficult to see that Conjecture 1 $\Rightarrow$ Conjecture 2. The converse is also true if one uses *Lang's Conjecture* (about rational points on surfaces of general type) because one has:

Theorem 1 (C.F. Hermann, E. Kani, W. Schanz): $Z_{N,\varepsilon}$ is of general type if $N \geq 13$.

Finally, some partial evidence for Conjecture 2 was discussed, such as its relation to an analogue of the *Minimality Conjecture* of Hirzebruch for Hilbert Modular Surfaces.

Bernhard Köck, Karlsruhe
Operations on Locally Free Classgroups

Let K be a number field and Γ a finite group. In order to construct annihilators of Stickelberger type for the locally free classgroup $Cl(\mathcal{O}_K\Gamma)$, Cassou-Noguès and Taylor have shown in 1985 that the Adams operations on the classical ring of virtual characters of Γ induce certain endomorphisms $\psi_K, k \geq 1$, on $Cl(\mathcal{O}_K\Gamma)$ in Fröhlich's Hom-description.

Recently, Burns and Chinburg have studied the question whether there is an algebraic description of ${\psi_K}^{CNT}$, for instance in terms of power operations on $\mathcal{O}_K\Gamma$-modules, and they have established a formula for $\psi_K(x)$ for certain elements x in $Cl(\mathcal{O}_K\Gamma)$ coming from a tame Galois extension of K.

We showed that ψ_k is a simply definable symmetric power operation on $Cl(\mathcal{O}_K\Gamma)$. For the proof of this result we use topological arguments based on the construction of power operations on higher K-theory. As an application of this result we interpreted the formula of Burns and Chinburg as an equivariant Adams-Riemann-Roch formula.

Klaus Künnemann, Münster
Arithmetic intersection pairings on abelian varieties

Let K be a number field and X_η a smooth projective variety over $\eta = \operatorname{Spec} K$. In order to describe the behavior of motivic L-functions associated with the variety X_η near the central point, Beilinson and Bloch have defined real-valued height pairings between Chow groups of homologically trivial cycles on X_η which extend the classical pairing of Néron and Tate. Therefore they have to assume that X_η has a regular model X which is proper over the ring of integers in K and that homologically trivial cycles on X_η admit suitable extensions to this model.

In the talk, we investigate these assumptions in the case of abelian varieties. We construct projective regular models for abelian varieties with semi-abelian reduction and prove that they have potentially semi-stable reduction. Using a non-archimedean analogue of the dd^c-lemma for differential forms, we show that that there is a well defined height pairing if an abelian variety has totally degenerate reduction at all places of bad reduction.

Andreas Langer, Münster
Zero-cycles on Hilbert-Blumenthal Surfaces

For a real quadratic field F of prime discriminant $q \equiv 1 \mod 4$ and class number 1 we consider the associated *Hilbert-Blumenthal surface* with respect to the full Hilbert modular subgroup $\mathrm{Sl}_2(\mathcal{O}_F)$ and show the Tate-conjecture in char. p for codim.-1-cycles at all good reduction primes p of the smooth projective model $S/\mathbb{Q}$, under the assumption that all Hilbert modular cusp forms of weight 2 are lifts of modular forms of level q and weight 2 under the Doi-Nagamma-lift. The idea is to reduce the Hirzebruch-Zagier cycles T_p mod p and let the Hecke algebra act on the two components of T_p mod p if p is split in F (for inert primes p there are no new cycles to consider), in order to obtain new cycles.

The result is then used to obtain finiteness results on the torsion subgroup in the Chow group of zero-cycles on S. For this one constructs new elements in $\mathrm{H}^1(S, \mathcal{K}_2)$ by defining modular units on T_p in the same way as Flach did by using the birational morphism $X_0(p) \to T_p$. One also has to use the finiteness of the Selmer group associated to the motive $\mathrm{H}^2(S)(2)$, which — in our situation — is derived from the finiteness of the Selmer group associated to the adjoint representation of the Galois representation ρ_f, where f is a modular form of level q and char. q. This is known from the works of Flach and Wiles.

Stefan Reiter, Heidelberg
Belyi-Triples and GAR-Realizations

We call a triple $(A, B, C), A, B, C \in GL_n(q), ABC = id, rk(A - id) = 1$ with $\langle A, B\rangle \leq GL_n(q)$ irreducible, a Belyi-triple.

We we can state the following

Theorem: For all $f, g \in \mathbb{F}_q[X]$ monic, $deg(f) = deg(g) = n, f(0)g(0) \neq 0, (f, g) = 1$, exits up to conjugation in $GL_n(q)$ one Belyi-triple s.t.

$$f_B = f \text{ and } f_{AB} = g,$$

where f_B resp. f_{AB} denotes the minimal polynomial of B resp. AB.
(This was independently proved by H. Völklein.)

Using classification theorems of primitive groups containing a homology or a transvection (see Wagner, Kantor and others) we can determine the group generated by a Belyi-triple. (E.g. let A be a transvection, $f_B = (X-1)^m(X-i)^m, f_{AB} = (X+1)^m(X+i)^m \in \mathbb{F}_p[X]$ for $p \equiv 5 \bmod 8, order(i) = 4$. Then $G/Z(G) \cong PCSp_{2m}(p)$ for $p \nmid m$.)

Using the rigidity criterion (Belyi, Matzat, Thompson) we find GA/GAR-realizations of classical groups of Lie-type (linear, unitary, orthogonal and symplectic groups) $G(p), p$ a prime, over $\mathbb{Q}$ under some conditions on the defining characteristic of these groups. (E.g: $PSp_{2m}(p)$ possesses GAR-realizations over $\mathbb{Q}$ for $(m, p) = 1$ and $p \not\equiv \pm 1 \bmod 24$.)

(The results for the linear groups have been already proved by Folkers and Malle.)

Jürgen Ritter, Augsburg
A local approach to Chinburg's root number conjecture

The concern of the talk is to introduce a refined Chinburg's multiplicative Ω-invariant that can be studied locally and so leads to local versions of Chinburg's root number conjecture, by which the root number class $W_{K/k}$ of a finite Galois extension K/k of number fields with group G determines $\Omega = \Omega(K/k) \in Cl(\mathbb{Z}G) \subset K_0(\mathbb{Z}G)$ via the Hom-description. In joint work with K. W. Gruenberg and A. Weiss new invariants Ω_φ depending on G-monomorphisms $\varphi : \Delta S \to E$ and Tate's canonical class $\tau_{K/k}$ have been defined, which belong to the Grothendieck group $K_0T(\mathbb{Z}G)$ of finitely generated torsion $\mathbb{Z}G$-modules of finite projective dimension and which all map to Ω under the natural homomorphism $K_0T(\mathbb{Z}G) \to K_0(\mathbb{Z}G)$. Above, S is a G-invariant finite set of primes of K which is large in the sense of Tate and so leads to a Tate sequence relating the S-unit group E in K to the augmentation submodule ΔS of $\mathbb{Z}S$ and having extension class $\tau_{K/k}$. Note that contrary to $K_0(\mathbb{Z}G)$ the group $K_0T(\mathbb{Z}G)$ admits a splitting into local components: $K_0T(\mathbb{Z}G) = \bigoplus_l K_0T(\mathbb{Z}_lG)$. It is shown that assuming Stark's conjecture, i.e., $A_\varphi \in \mathrm{Hom}_{G_{\mathbb{Q}}}(RG, \overline{\mathbb{Q}}^\times)$ (where RG is the $\overline{\mathbb{Q}}$-character ring of G), the deviation ω of Ω_φ from the element in $K_0T(\mathbb{Z}G)$ represented by $A_\varphi W_{K/k}$ is independent of φ and S and behaves well with respect to deflation and restriction of subgroups. We conjecture $\omega = 0$ or, what amounts to the same, $\omega^{(l)} = 0$ for all rational primes l where $\omega^{(l)} \in K_0T(\mathbb{Z}_lG)$ is the l-adic component of w. If this is true, then so is Chinburg's root number conjecture. In the case that K is absolutely abelian one knows the validity of the so-called Strong-Stark-conjecture of Chinburg (Ritter and Weiss in J AMS (1997)) and concludes that ω belongs to the torsion subgroup of $K_0T(\mathbb{Z}G)$; moreover, it is shown that in order to confirm $\omega^{(l)} = 0$ in this situation it suffices to assume that G modulo its Sylow l-subgroup is cyclic. A short account of all this is published in Jahresbericht der DMV (1997). Finally, an example is discussed which comes from joint work with A. Weiss. Here $K \subset \mathbb{Q}(\zeta_{p_1p_2})$ is of degree $l \neq 2$ over $k = \mathbb{Q}$; p_1, p_2, l are three distinct primes. The conjecture $\omega^{(l)} = 0$ now predicts congruences for l-adic L-values as well as a certain strengthening of results in connection with the Main Conjecture, and can be indeed confirmed as long as one of the primes p_i is not an lth power modulo the other one.

Takeshi Saito, Tokyo
Modular Forms and p-adic Hodge Theory

For a new form f, a 2-dimensional l-adic representation V_f of the absolute Galois group $G_{\mathbb{Q}} = Gal(\bar{\mathbb{Q}}/\mathbb{Q})$ is associated. It is known to be compatible with the local Langlands correspondence at $p \neq l$ in the following sense by Deligne-Langlands-Carayol. For $p \neq l$, the restriction to the inertia group I_p is quasi-unipotent and hence a representation of the Weil-Deligne group W_p' is associated to the restriction $V_f|_{G_p}$ to the decomposition group $G_p = Gal(\bar{\mathbb{Q}}/\mathbb{Q}_p)$. On the other hand, a cuspidal automorphic representation π_f is associated to f and it is decomposed as tensor product $\pi_f = \otimes\pi_{f,p}$. By local Langlands correspondence, an F-semisimple 2-dimensional representation $\sigma(\pi_{f,p})$ of W_p' is associated to the irreducible admissible representation $\pi_{f,p}$ of $Gl_2(\mathbb{Q}_p)$. The compatibility mentioned above is that the F-semisimplification of the representation of W_p' associated to $V_f|_{G_p}$ is isomorphic to $\sigma(\pi_{f,p})$.

The main result in the talk is that the compatibility remains true for $p = l$ by taking $D_{\mathrm{pst}}(V_f|_{G_p})$ as a representation of W_p'. Here D_{pst} is the function defined by Fontaine: $D_{\mathrm{pst}}(V) = \bigcup_{J \subset I_p \text{ open}} (B_{\mathrm{st}} \otimes V)^J$. It is proved as follows. First of all, we know that $D_{\mathrm{pst}}(V_f|_{G_p})$ is 2-dimensional and is described by using log cristalline cohomology of Hyodo-Kato by the C_{st}-conjecture proved by Tsuji. By the known result for $l \neq p$, it is enough to compare p and $l \neq p$. More precisely

1. $Tr(\sigma|D_{\mathrm{pst}}(V_{f,p})) = Tr(\sigma|V_{f,l})$ for $\sigma \in W_p$

2. N on D_{pst} is $0 \Longleftrightarrow N$ on $V_{f,l}$ is 0.

1. will be provéd by finally reducing to the Lefschetz trace formula which takes the same form for p (cristalline cohomology) and $l \neq p$ (l-adic etale cohomology). The reduction is made by the geometric construction of V_f using Kuga-Sato variety and Hecke operators (Deligne-Scholl) and an application of the weight spectral sequence (Rapoport-Zink for $l \neq p$ and Mokrane for p). It is proved also by studying the spectral sequence more closely.

Michael Spieß, Heidelberg
Zero Cycles on Products of Curves over p-adic Fields

Let k be a finite extension of $\mathbb{Q}_p$ and let $X_1, \ldots, X_d/k$ be smooth, projectives curves. We are interested in the structure of the Chow group of zero cycles $CH_0(X_1 \times \ldots \times X_d)$ of $X_1 \times \ldots \times X_d$. The talk reports on the proofs of the following two results:
Theorem 1: Let $d = 2$ and let X_1, X_2 be elliptic curves with good reduction. Then the prime-to-p torsion of $CH_0(X_1 \times X_2)$ is finite.
Theorem 2 (joint work with W. Raskind): Assume that the Jacobians of $X_1, \ldots,$ X_d have a mixture of multiplicative and good ordinary reduction. Then for any non-zero integer n, $CH_0(X_1 \times \ldots \times X_d)/n$ is finite.

Both proofs exploit the connection between Chow groups and algebraic K-theory.

Matthias Strauch, Bonn
Counting rational points and height zeta functions

Let X be a smooth projective variety over a number field F and $H^0(X, \omega_X) = 0$. Let $\mathbf{L} = (\mathcal{L}, (\|\cdot\|_v))$ be a metrized line bundle on X such that $\mathcal{L}$ lies in the interior of the cone of effective divisors $\Lambda_{\text{eff}}(X) \subset \text{Pic}(X)_{\mathbf{R}}$. Associated to $\mathcal{L}$ there is a height function $H_{\mathbf{L}} : X(F) \to \mathbf{R}_{>0}$. For appropriate subvarieties $U \subset X$ the number $N_U(\mathbf{L}, H) := \#\{x \in U(F) \mid H_{\mathbf{L}}(x) \leq H\}$ will always be finite. We are interested in the asymptotic behavior of this counting function. If $\Lambda_{\text{eff}}(X)$ is a finitely generated polyhedral cone Batyrev and Manin defined constants $a(\mathcal{L})$ and $b(\mathcal{L})$ depending on the position of $\mathcal{L}$ inside $\Lambda_{\text{eff}}(X)$. On the other hand, provided that the *height zeta function*

$$Z_U(\mathbf{L}, s) := \sum_{x \in U(F)} H_{\mathbf{L}}(x)^{-s}$$

has abscissa of convergence $a > 0$, meromorphic continuation to $\text{Re}(s) > a - \epsilon$ for some $\epsilon > 0$, a pole of order b at $s = a$ and no other poles in $\text{Re}(s) > a - \epsilon$

then we have

$$N_U(\mathbf{L}, H) \sim cH^a(\log H)^{b-1}$$

for some $c > 0$. In the talk we discussed three classes of varieties for which $a = a(L)$, $b = b(L)$ holds (as Batyrev and Tschinkel have shown, this is not universally true). These results were obtained by analyzing the corresponding height zeta function.

- (Franke) For flag varieties the height zeta function is an Eisenstein series $E_P^G(\lambda - \rho_P, e_G)$.
- (S.) Certain fibre bundles $Y = Q\backslash H \times^P G$, a fibration over $P\backslash G$ defined by a homomorphism $\eta : P \to H$. The height zeta function is an infinite sum of Eisenstein series.
- (S.-Tschinkel) Toric bundles over $P \to G$ with fiber a toric variety $X = \overline{T}$ defined by a homomorphism $\eta : P \to T$. In this case there is an integral representation

  $$Z(\varphi, \lambda) := \int_{(T(\mathbf{A})/T(F))^*} \hat{H}(\chi, \varphi) E_P^G(\lambda - \rho_P, (\chi \circ \eta)^{-1}) d\chi$$

  where $\hat{H}$ is the Fourier transform of the height function on $T(\mathbf{A})$.

Annette Werner, Münster
Local heights on Abelian varieties and rigid analytic uniformization

The goal of this talk is to express classical and p-adic local height pairings on an abelian variety A_K with split semistable reduction in terms of the corresponding pairings on the abelian part B_K of the Raynaud extension. Since B_K is an abelian variety with good reduction, this result provides a rather explicit step from the class of local height pairings on all abelian varieties with good reduction to the class of local height pairings on arbitrary abelian varieties.

We are interested in three kinds of local height pairings on A_K, where K is a non-archimedean local ground field: Néron's classical real-valued pairing, Schneider's norm-adapted p-adic height pairing and the p-adic height pairing in the ordinary reduction case defined by Mazur and Tate. We use an approach to height pairings via "splittings" of biextensions which is due to Mazur and Tate.

As an application of our "step" from the good reduction to the general case we calculate the difference between Schneider's p-adic height pairing and the p-adic Mazur-Tate pairing on A_K.

Jörg Wildeshaus, Münster

On the Generalized Eisenstein Symbol

Let $\mathcal{E}/B$ be an elliptic curve, N an integer ≥ 2 s.t. $N \in \mathcal{O}_B^*$, $k \geq 2$, and

$$\mathcal{E}^{(k-2)} := \ker(\Sigma : \mathcal{E}^{k-1} \longrightarrow \mathcal{E})$$

($\mathcal{S}_{k-1}$ acts naturally on $\mathcal{E}^{(k-2)}$).

The *Eisenstein symbol* Eis^{k-2} is a map

$$\mathrm{Eis}_N^{k-2} : Ch[\mathcal{E}[N](B) - 0] \longrightarrow \mathrm{H}_{\mathcal{M}}^{k-1}(\mathcal{E}^{(k-2)}, k-1)^{\mathrm{sgn}}$$

If $B = \mathrm{Spec}(K)$, K a number field, it is known that if $\mathcal{E}$ is a CM-curve (of Shimura type), then the image of Eis_N^{k-2}, for $N \gg 0$, generates a subspace of $\mathrm{H}_{\mathcal{M}}^{k-1}$ which ($\otimes\mathbb{R}$) maps surjectively to Deligne cohomology

$$\mathrm{H}_{\mathcal{D}}^{k-1}(\mathcal{E}^{k-2} \otimes_{\mathbb{Q}} \mathbb{R}, k-1)^{\mathrm{sgn}}$$

It is unreasonable to expect this to hold for arbitrary $\mathcal{E}$. One therefore tries to enlarge the source of Eis^{k-2} to a bigger subspace of $Ch[\mathcal{E}(B) - 0]$.

The *weak version of the elliptic Zagier conjecture* predicts the existence of certain quotients $\mathcal{B}l_k(\mathcal{E})$ of $Ch[\mathcal{E}(B) - 0]$, $k \geq 1$, of differentials

$$d_k : \mathcal{B}l_k(\mathcal{E}) \longrightarrow \mathcal{B}l_{k-1}(\mathcal{E}) \otimes \mathcal{E}(B), \qquad k \geq 2$$

and of maps

$$\mathrm{Eis}^{k-2} : \ker(d_k) \longrightarrow \mathrm{H}_{\mathcal{M}}^{k-1}(\mathcal{E}^{(k-2)}, k-1)^{\mathrm{sgn}}$$

(extending the Eisenstein symbol on torsion).

In the talk, it was reported how parts of these data can be constructed geometrically: one can define $\mathcal{B}l_k(\mathcal{E})$ and d_k, and a diagram

$$\begin{array}{ccc} \mathrm{H}_{\mathcal{M},\mathcal{E}(B)}^{k-1}(\mathcal{E}^{(k-2)}, k-1)^{sgn} & \subset & \mathrm{H}_{\mathcal{M}}^{k-1}(\mathcal{E}^{(k-2)}, k-1)^{sgn} \\ \downarrow \text{“}(\mathrm{Eis}^{k-2})^{-1}\text{”} & & \\ \ker(d_k) & & \end{array}$$

Berichterstatter: Stefan Wewers

Die Teilnehmer

Banaszak, G.	banaszak@math.amu.edu.pl
Bost, J.-B.	bost@ihes.fr
de Jeu, R.	rob.de-jeu@durham.ac.uk
Deninger, C.	chris@math.uni-muenster.de
de Shalit, E.	deshalit@math.huji.ac.il
Flach, M.	flach@cco.caltech.edu
Franke, J.	franke@rhein.iam.uni-bonn.de
Frey, G.	frey@exp-math.uni-essen.de
Gajda, G.	gajda@mpim-bonn.mpg.de
Goncharov, A.	sasha@mpim-bonn.mpg.de
Greenberg, R.	greenber@math.washington.edu
Huber-Klawitter, A.	huber@math.uni-muenster.de
Kani, E.	ernst.kani@queensu.ca
Kimura, K.	kimurak@math.tsukuba.ac.jp
Kings, G.	kings@math.uni-muenster.de
Kochubai, A. N.	ank@ank.kiev.ua
Köck, B.	Bernhard.Koeck@math.uni-karlsruhe.de
Kolster, M.	kolster@mcmaster.ca
Künnemann	kuenne@math.uni-muenster.de
Langer, A.	langer@mi.uni-koeln.de
Mahnkopf, J.	joachim@math.huji.ac.il
Matzat, B. H.	matzat@iwr.uni-heidelberg.de
Pop, F.	pop@math.uni-bonn.de
Reiter, S.	reiter@melpomene.iwr.uni-heidelberg.de
Ritter, J.	ritter@math.uni-augsburg.de
Roquette, P.	roquette@mathi.uni-heidelberg.de
Saito, T.	t-saito@ms.u-tokyo.ac.jp
Schappacher, N.	schappa@math.u-strasbg.fr
Schmidt, A.	schmidt@harmless.mathi.uni-heidelberg.de
Schmidt, C.-G.	cs@ma2s2.mathematik.uni-karlsruhe.de
Schneider. P.	pschnei@math.uni-muenster.de
Scholl, A. J.	a.j.scholl@durham.ac.uk
Spieß, M.	spiess@mathi.uni-heidelberg.de
Strauch, M.	mats@rhein.iam.uni-bonn.de
Werner, A.	werner@math.uni-muenster.de
Wewers, S.	wewers@exp-math.uni-essen.de
Wildeshaus, J.	wildesh@math.uni-muenster.de
Wingberg, K.	wingberg@harmless.mathi.uni-heidelberg.de
Xu, F.	feixu@math.uni-muenster.de

Tagungsteilnehmer

Prof.Dr. Grzegorz Banaszak
Instytut Matematyki
UAM
ul. Matejki 48/49

60-769 Poznan
POLAND

Prof.Dr. Gerhard Frey
Institut für Experimentelle Mathematik
Universität-Gesamthochschule Essen
Ellernstr. 29

45326 Essen

Prof.Dr. Jean-Benoit Bost
I.H.E.S.
35, Route de Chartres

F-91440 Bures-sur-Yvette

Prof.Dr. Wojciech Gajda
Instytut Matematyki
UAM
ul. Matejki 48/49

60-769 Poznan
POLAND

Prof.Dr. Christopher Deninger
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Prof.Dr. Wulf-Dieter Geyer
Mathematisches Institut
Universität Erlangen
Bismarckstr. 1 1/2

91054 Erlangen

Dr. Matthias Flach
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

69120 Heidelberg

Prof.Dr. Alexander Goncharov
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

53225 Bonn

Prof.Dr. Jens Franke
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

53115 Bonn

Prof.Dr. Ralph Greenberg
Dept. of Mathematics
Box 354350
University of Washington
C138 Padelford Hall

Seattle , WA 98195-4350
USA

Alexander Gurevich
Institut für Mathematik
Humboldt-Universität Berlin
Jägerstr. 10-11

10117 Berlin

Guido Kings
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Dr. Annette Huber-Klawitter
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Prof.Dr. Helmut Koch
Max-Planck-Arbeitsgruppe
"Algebraische Geometrie und
Zahlentheorie"
Jägerstr. 10-11

10117 Berlin

Dr. Rob de Jeu
Dept. of Mathematics
California Institute of Technology

Pasadena , CA 91125
USA

Prof.Dr. Anatoly N. Kochubei
Institute of Mathematics of the
National Academy of Sciences of the
Ukraine
Tereshchenkivskastr. 3

252601 Kiev 4
UKRAINE

Prof.Dr. Ernst Kani
Department of Mathematics and
Statistics
Queen's University

Kingston, Ontario K7L 3N6
CANADA

Dr. Bernhard Köck
Mathematisches Institut II
Universität Karlsruhe

76128 Karlsruhe

Prof.Dr. Kenichiro Kimura
Institute of Mathematics
University of Tsukuba
Tsukuba-shi

Ibaraki 305
JAPAN

Prof.Dr. Manfred Kolster
Department of Mathematics and
Statistics
Mc Master University
1280 Main Street West

Hamilton , Ont. L8S 4K1
CANADA

Dr. Klaus Künnemann
Mathematisches Institut
Universität zu Köln
Weyertal 86-90

50931 Köln

Stefan Reiter
Interdisziplinäres Zentrum
für Wissenschaftliches Rechnen
Universität Heidelberg
Im Neuenheimer Feld 368

69120 Heidelberg

Andreas Langer
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Prof.Dr. Jürgen Ritter
Institut für Mathematik
Universität Augsburg

86135 Augsburg

Dr. Joachim Mahnkopf
Institute of Mathematics
The Hebrew University

Jerusalem 91904
ISRAEL

Prof.Dr.Dr.h.c. Peter Roquette
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

69120 Heidelberg

Prof.Dr. B. Heinrich Matzat
Interdisziplinäres Zentrum
für Wissenschaftliches Rechnen
Universität Heidelberg
Im Neuenheimer Feld 368

69120 Heidelberg

Prof.Dr. Takeshi Saito
Dept. of Mathematics
College of General Education
University of Tokyo
Komba, Meguro

Tokyo 153
JAPAN

Dr. Florian Pop
Mathematisches Institut
Universität Bonn
Beringstr. 6

53115 Bonn

Prof.Dr. Norbert Schappacher
U.F.R. de Mathematique et
d'Informatique
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Dr. Alexander Schmidt
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

69120 Heidelberg

Dr. Michael Spieß
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

69120 Heidelberg

Prof.Dr. Claus-Günther Schmidt
Mathematisches Institut II
Universität Karlsruhe

76128 Karlsruhe

Matthias Strauch
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Prof.Dr. Peter Schneider
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Dr. Annette Werner
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Dr. Anthony J. Scholl
Dept. of Mathematical Sciences
The University of Durham
Science Laboratories
South Road

GB-Durham , DH1 3LE

Stefan Wewers
Institut für Experimentelle
Mathematik
Universität-Gesamthochschule Essen
Ellernstr. 29

45326 Essen

Prof.Dr. Ehud de Shalit
Institute of Mathematics
The Hebrew University
Givat-Ram

91904 Jerusalem
ISRAEL

Dr. Jörg Wildeshaus
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

Prof.Dr. Kay Wingberg
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

69120 Heidelberg

Prof.Dr. Fei Xu
Mathematisches Institut
Universität Münster
Einsteinstr. 62

48149 Münster

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