

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 29/1997

**Large Time Behavior in Dynamical Systems:  
Analysis and Numerics**

27.07.-02.08.1997

The meeting was organized by Klaus Böhmer (Marburg), Bernold Fiedler (Berlin), and John Guckenheimer (Ithaca). The 45 participants enjoyed - like so many others before - the warm hospitality and the stimulating atmosphere at the institute. In 28 talks a wide variety of numerical and analytical results was presented, including applications from molecular dynamics, optical fibers, pattern formation processes in chemistry and material science. Special emphasis was on the interaction of numerical and analytical aspects. New trends in numerical analysis of dynamical systems, symmetry breaking in partial differential equations, and theory and numerics of homoclinic bifurcations were topics of three evening discussion sections and several afternoon working sessions.

Numerical problems centered around the inherent instability of numerical computations over large times or for stiff, many-variable systems. Symplectic integrators, shadowing and backward analysis were investigated from this view point. In addition, significant interest arises, from both the analytical and the numerical side, in the direct computation of invariant measures and of particular dynamic objects like symbolic codings, periodic and homoclinic orbits, and their bifurcations. On the other hand, new dynamical phenomena in spatio-temporal pattern formation and near homoclinic orbits were reported, which in particular applications still provide challenging questions to both numerics and analysis community. But the bridge keeps growing.

## Vortragsauszüge:

### **Peter Ashwin: On some robust heteroclinic attractors in symmetric systems**

I discuss some heteroclinic-like attracting behaviour for which, although it is of infinite codimension in systems without symmetry, there are symmetric settings where it is generic. In particular I discuss new issues for homoclinic cycles with continua of connections (joint work with P. Chossast) and robust cycles connecting chaotic invariant sets (with A. Rucklidge).

In the former case, the connections corresponding to the most positive expanding eigenvalues are observed to determine the minimal attractors. In the latter case one can compute Liapunov exponents to determine the creation of an attracting cycle between chaotic sets at a blowout bifurcation leading to global connections. Finally I discuss recent work (with M. Field) on classifying a large class of heteroclinic-type attractors. We show that typically these can involve connections between non-trivial sub-heteroclinic cycles.

### **Wolf-Jürgen Beyn: Numerical coding of symbolic dynamics near homoclinic orbits**

Symbolic dynamical systems are closed sets of biinfinite sequences on  $N$  symbols which are invariant under the shift map  $\beta$ . If for a given map  $f \in \mathbb{R}^m$  one can find a closed invariant subset  $\Lambda$  on which  $f$  conjugates to the shift of a symbolic dynamical system this provides an efficient tool in studying the long-time dynamics of the map  $f$ . One of the best known instances where such a conjugacy  $\Phi$  exists is given by the Smale-Birkhoff theorem according to which any transversal homoclinic point generates a Cantor set  $\Lambda$  with an appropriate conjugacy. We develop a numerical method for evaluating  $\Phi$  in this situation at homoclinic and periodic sequences of symbols. The method makes use of the proof of the Smale-Birkhoff theorem given by Palmer (1988). To any symbolic sequence one associates pseudo orbits obtained by concatenating segments which consist either of the fixed point or a numerical approximation of the primary homoclinic.

It is shown that Newton's method for a suitable "discrete" boundary value problem converges when started at such orbits.

When applying parametric continuation to such systems several interesting phenomena occur, such as closed curves of homoclinic orbits with turning points that correspond to homoclinic tangencies and perturbed pitchfork bifurcations on branches of multi-humped homoclinics. (Joint work with J. M. Kleinkauf)

## Bjorn Birnir: Basic Attractors

Basic attractors are distinguished by the property that every point of the attractor has a basin that is not a shy set. A generalization of a theorem by Milnor, to infinite dimensions, gives a decomposition of compact attractors into a basic attractor and a remainder. The basic attractor is low-dimensional for a "generic" class of nonlinear PDE's, although the dimension of the full attractor (remainder) can be arbitrarily large. We present bifurcations of the basic attractor for dissipative nonlinear wave equations and give an example of a (strange) basic attractor that is globally extended in phase space. This attractor consists of two components with slow rotations and fast (chaotic) global excursion between them.

## Jack Carr: Coarsening

After a phase transformation process, such as nucleations and growth of particles from a supersaturated solution, the system reaches a stage in which the nuclei of one phase are surrounded by a supersaturated medium consisting of the other phase. Since smaller particles have (on a per volume basis) a larger interfacial energy than larger ones, the larger particles grow at the expense of the smaller ones. This process is known as coarsening and occurs in many systems. This phenomenology is described in a model of Lifshitz and Slyozov (LS). If  $F(R, t)$  is the distribution function of the particles with respect to their radii:

$$\frac{\partial F(R, t)}{\partial t} + \frac{\partial}{\partial R}[(R^\alpha u(t) - 1)F(R, t)] = 0 \quad (1)$$
$$u(t) = \int_0^\infty F(R, t) dR / \int_0^\infty R^\alpha F(R, t) dR$$

with  $\alpha = 1/3$ . Equation (1) has a one-parameter family of similarity solutions and the LS theory ( $\alpha = 1/3$ ) claims that there is a particular similarity solution which attracts all solutions. From a mathematical point of view, very little is known about the LS equations. Equations (1) also describe the large time behaviour of the Becker-Döring equations with  $0 < \alpha \leq 1$ . We study the  $\alpha = 1$  case which is simpler than the LS equations. We show that for each similarity solution, there is a large class of initial data which 'converges' to it. Also, there are solutions which do not converge. (Joint work with O. Penrose)

## Michael Dellnitz: The Approximation of Complicated Dynamical Behavior

Multilevel subdivision techniques are presented for the efficient numerical approximation of complicated dynamical behavior. Concretely we develop adaptive methods which allow to extract statistical information on the underlying dynamical system. This is done by an

approximation of natural invariant measures as well as (almost) cyclic dynamical components. We discuss issues concerning the implementation (e.g. parallelization strategies) and indicate potential applications of these methods. The results are illustrated by several numerical examples.

### **Peter Deuffhard: Essential Molecular Dynamics: Basic Concepts of a New Algorithm**

Mathematically speaking, MD systems are described by Hamiltonian differential equations. In the bulk of applications, individual trajectories are of no specific interest. Rather, time averages of physical observables as relaxation times of conformational changes need to be actually computed. Such information is contained in the natural invariant measure (infinite relaxation time) or in almost invariant sets ("large" finite relaxation times). The authors suggest the direct computation of these objects via eigenmodes of the associated Frobenius-Perron operator by means of an adaptive multilevel subdivision technique. In the advocated algorithmic approach ill-posed long-term trajectory computations are replaced by well-posed short-term subtrajectory computations. Monte-Carlo techniques are different and just structurally connected via the underlying Frobenius-Perron theory (these, however for the canonical ensemble prescribed). A simple four-well-potential problem is shown numerically to illustrate the new concepts. (Joint work with M. Dellnitz, O. Junge and Ch. Schütte)

### **Eusebius Doedel: On Computing Singular Solutions**

Extended systems for following singular solutions of systems of equations, including differential equations, have been introduced by many authors in the past 20 years. The simplest case is that of a fold, for which an extended system was first proposed by Seydel in 1977. There exist efficient solution techniques for solving the linear systems that arise in applying Newton's method to solving an extended system. However, these efficient solution techniques rely on making certain approximations in the Jacobian matrix. Thus, they are in fact *approximate* Newton techniques. The same can be said about the so called minimally extended systems. In this work we show how efficient solution techniques can be applied in general, thereby resulting in a proper Newton iteration. In addition to accelerated convergence, this modification also yields more accurate information on possible singularities of the extended system. Moreover, the algorithm is particularly simple to express. In addition to the model problem, the fold, we also treat the case of a period-doubling bifurcation.

**Heinrich Freistühler, Christian Fries: Nonlinear Asymptotic Stability of Viscous Shock Waves**

Consider a traveling wave solution  $u^*(t, x) = \phi^*(x - st)$  of a system

$$u_t + (f(u))_x = u_{xx}, \quad f : \mathbb{R}^n \rightarrow \mathbb{R}^n \quad (1)$$

of viscous conservation laws, i. e.,  $\phi^*$  satisfies

$$\phi' = f(\phi) - s\phi - q \text{ and } \phi(\pm\infty) = u^\pm \quad (2)$$

with some  $u^-, u^+, s, q$  such that  $f(u^\pm) - su^\pm = q$ .  $\phi^*$  is called *asymptotically stable* if there exist a Banach space  $(B, \|\cdot\|_B)$  and a  $\beta > 0$  such that the following holds for all  $\bar{u}_0 \in B$  with  $\|\bar{u}_0\| < \beta$ :

$$\begin{aligned} &\text{The solution } u \text{ to (1) with data } u(0, \cdot) \equiv \phi^* + \bar{u}_0 \text{ exists for all } t > 0, \text{ and} \\ &\limsup_{t \rightarrow \infty} \sup_{x \in \mathbb{R}} |u(t, x) - \phi(x - st)| = 0 \text{ with some solution } \phi \text{ of (2).} \end{aligned} \quad (3)$$

Goodman [ARMA 1985] has proved: If  $f$  has a simple eigenvalue  $\lambda$ ,  $\lambda$  is "convex" (i. e.,  $r \cdot \nabla \lambda \neq 0$  for  $r$  spanning  $\ker(f' - \lambda I)$ ),  $\phi^*$  is associated with  $\lambda$  (i. e.,  $\lambda(u^-) > s > \lambda(u^+)$ ), and the shock amplitude is small (i. e.,  $|u^+ - u^-| \ll 1$ ), then (3), with  $\phi \equiv \phi^*$ , holds for all  $\bar{u}_0 \in L^1$  with  $\int \bar{u}_0(x) dx = 0$  and  $\|U_0\|_{H^{2,2}} \ll 1, U_0(x) = \int_{-\infty}^x \bar{u}_0(y) dy$ .

Using an energy method with a weight that is somewhat similar to one Matsumura and Nishihara [CMP 1994] introduced for the scalar case, Fries [1997] proves the same as Goodman, but without the convexity assumption.

For the scalar case, with no convexity assumption, Freistühler and Serre [CPAM 1998] show asymptotic stability of viscous shock waves of arbitrary size, with  $B = L^1$  and  $\beta = +\infty$ . (Joint work with C. Fries)

**Gero Friesecke: Infinite-Dimensional Effects in Dissipative Systems: Metastability, Hysteresis and Failure of Convergence to Equilibrium**

We discuss systems where an infinite number of "modes" remain active as  $t \rightarrow \infty$ , e.g. as arising in pattern formation problems in materials science (solid-solid phase transitions). For the equation

$$u_t = (u_x^3 - u_x)_x + u_{xxt} - \alpha u, \quad \alpha > 0, x \in (0, 1)$$

minimizing sequences of the Lyapunov factor  $E = \int_0^1 \left( \frac{(u_x^2 - 1)^2}{4} + \alpha \frac{u^2}{2} \right) + \frac{u_t^2}{2}$  develop faster and faster oscillations, converging weakly but not strongly to zero in  $(u, u_t) \in H_0^1 \times L^2$ , while solutions to the dynamic equation converge strongly to a unique equilibrium. This is

a joint work with J.R. McLeod, e.g. (Archive Rat. Mech Analysis 1996). By contrast, for the nonlocal equation

$$u_t = (3\|u_x\|^2 - 1)u_{xx} + u_{xxt} - \alpha u$$

energy does get transported into higher and higher modes (Rall et al. 1990).

### **Martin Golubitsky: Hopf Bifurcation from Rotating Waves: Physical Space vs. Phase Space**

Rotating waves appear in a number of experiments including laminar premixed flames, BZ reactions and Taylor Couette. Hopf bifurcation leads to two frequency quasiperiodic motion in each case: nonuniformly rotating cells, meandering spiral waves, and modulated wavy vortices. Each of these quasiperiodic motions has distinctive characteristics. We discuss how symmetry properties in phase space lead to an understanding of the characteristic properties in physical space.

This lecture is based on joint research with Ian Melbourne and Victor LeBlanc.

### **John Guckenheimer: Computing Periodic Orbits with Automatic Differentiation**

This paper introduces a new family of algorithms for computing periodic orbits of vector fields. These global methods achieve high accuracy with relatively coarse discretizations of periodic orbits through the use of automatic differentiation. High degree Taylor series expansions of trajectories are computed at mesh points. On a fixed mesh, we construct closed curves that converge smoothly to periodic orbits as the degree of the Taylor series expansions increase. The algorithms have been implemented in Matlab together with the use of the automatic differentiation code ADOL-C. Numerical tests of our codes are compared with AUTO calculations using the Hodgkin-Huxley equations as a test problem.

### **Ale Jan Homburg: Homoclinic-doubling Cascades**

There are codimension-two homoclinic bifurcations of vector fields for which the homoclinic orbit undergoes a homoclinic-doubling bifurcation. This creates a doubled homoclinic orbit that rounds twice in a tubular neighborhood of the original homoclinic orbit. In the parameter plane of a two parameter family of vector fields unfolding such a homoclinic bifurcation, there is a curve of doubled homoclinic orbits branching, at the codimension-two point, from the curve of doubled homoclinic orbits. A cascade of successive homoclinic-doubling bifurcations arises if, each time following the curve of doubled homoclinic orbits,

somewhere on this curve a further homoclinic-doubling occurs so that a new curve of doubled homoclinic orbits branches. The homoclinic orbit, existing after  $n$  homoclinic-doubling bifurcations, forms a curve which gets arbitrarily long as  $n \rightarrow \infty$ .

We show that, in the space of two parameter families of three dimensional vector fields, there is an open set consisting of families that possess a cascade of homoclinic-doubling bifurcations. Similarities and differences with the familiar cascades of period-doubling bifurcations are discussed.

### **Jürgen Knobloch: Dynamics near Homoclinic Orbits in Discrete Systems**

We want to study the dynamics near homoclinic points in periodically forced systems by introducing an appropriate Poincaré-map. However, we consider the more general situation of a discrete system having a homoclinic orbit. To study particular orbits nearby the distinguished homoclinic orbit we extend the ideas of Lin's method to discrete systems. With that we detect homoclinic and periodic solutions (nearby the primary homoclinic orbit). We demonstrate the results on examples: we compute homoclinic and periodic orbits near a homoclinic point with quadratic tangencies and we determine homoclinic points of a Poincaré-map associated with a periodically forced system. Finally we give an idea how to detect  $n$ -periodic and  $n$ -homoclinic orbits.

### **Stig Larsson: A Posteriori Error Analysis and Shadowing for Finite Element Methods for Semilinear Parabolic Systems**

We study semilinear parabolic systems

$$\begin{cases} u_t + Au = f(u, Du), x \in \Omega \subseteq \mathbb{R}^d, t > 0, \\ u|_{\partial\Omega} = 0. \end{cases}$$

We first set the initial boundary value problem in a functional analytic setting, where the nonlinearity is controlled by the  $W_q^1(\Omega)$ -norm. The equations are discretized by the "discontinuous Galerkin method", which is a space-time finite element method. We prove an a posteriori bound of the discretization error in terms of a norm of the residual of the computed solution times a stability factor. We then prove a version of the shadowing lemma showing that a computed solution is shadowed by an exact solution.

### **Christian Lubich: Invariant Tori of (Perturbed) Hamiltonian Systems Under Symplectic Numerical Discretizations**

This talk is concerned with the long-time behaviour of symplectic numerical integrators

for Hamiltonian systems and dissipative perturbations thereof. The first part explains the notion of backward error analysis and gives a result on the almost-embedding, up to exponentially small terms, of the one-step map of symplectic numerical methods into the flow of a nearby modified Hamiltonian system.

In a second part, this result is used to show that KAM tori are “very sticky” under symplectic discretization, over a time span that is exponentially long in the time step. Finally, for dissipative perturbations of Hamiltonian systems, the result is applied to show that weakly attractive invariant tori persist under symplectic discretization, under a very mild restriction of the time step.

The talk is based on joint work with E. Hairer.

### Zhen Mei: Numerical Analysis of Periodic Solutions near Homoclinic Orbits of Large Systems

A numerical bifurcation function is discussed for locating periodic orbits near homoclinic orbits of large systems of autonomous differential equations, arising from, e.g. spatial discretization of reaction-diffusion equations or from modelling large networks. This requires effective approximation of the homoclinic orbit and the bounded solution of the adjoint variational equation. We exploit the truncated boundary value problems for approximating homoclinic orbits and a system of differential-algebraic equations for selecting the bounded solution of the adjoint variational equation. Furthermore, we make use of the Krylov subspace iteration method for numerical continuation of equilibrium and unstable eigenvectors of the linearized operator. As an example we examine periodic and quasi-periodic solutions of the Kuramoto-Sivashinsky equation near homoclinic cycles. Finite difference and Crank-Nicolson scheme are used for spatial and temporal discretizations.

### Peter Poláčik: Asymptotics of Positive Solutions of Time-Periodic Reaction-Diffusion Equations on $\mathbb{R}^N$

We study positive solutions of the following problem

$$\begin{aligned} u_t &= \Delta u + f(t, u), & x \in \mathbb{R}^N, & t > 0, \\ u(x, t) &\rightarrow 0, & |x| &\rightarrow \infty, t > 0. \end{aligned}$$

We assume that  $f$  is a  $C^1$  function in  $t$ , and that  $u \equiv 0$  is a linearly stable equilibrium. Our main result describes the spatio-temporal asymptotics of solutions whose trajectory is relatively compact in  $C_0(\mathbb{R}^N)$  (the space of continuous functions vanishing at  $\infty$ , with the sup norm). Each such solution is asymptotically  $\tau$ -periodic in time and asymptotically radially symmetric in space (about some point independent of  $t$ ). A complete description



of compact entire trajectories will also be given.

### **Tony Roberts: Initial Conditions are Long-Lasting**

Centre manifold theory and techniques are increasingly used to derive coarse, low-dimensional dynamical models of finely detailed, high-dimensional, physical dynamical systems. Often we use these coarse models to make forecasts. To do this we need to project the initial condition of the fine system onto the centre manifold,  $\mathcal{M}$ , to form an initial condition of the model. Unless this is done correctly, errors in the forecast may persist forever or even grow exponentially. The flow in the neighbourhood of  $\mathcal{M}$  determines the correct projection to ensure long-term fidelity between original and model. For apparently different reasons, a small time dependent forcing is also projected onto the model in exactly the same way.

### **Carlos Rocha: Realization of Meander Permutations by Boundary Value Problems and Attractors for Reaction-Diffusion Equations**

We consider Neumann boundary value problems of the form  $u_{xx} + f(x, u, u_x) = 0$  on the interval  $0 < x < 1$ , having exactly  $n$  solutions, where  $f$  is taken in a certain class of dissipative nonlinearities. To these problems we associate meanders in the phase space  $(u, u_x) \in \mathbb{R}^2$ . Meanders are connected oriented simple plane curves intersecting a fixed oriented line in  $n$  points. The permutations obtained by ordering the intersection points once along the base line and then along the meander are called meander permutations. The meander permutations associated to the second order ODE determine the global attractor of the infinite dimensional dynamical system generated by the semilinear parabolic differential equation  $u_t = u_{xx} + f(x, u, u_x)$  up to  $C^0$ -orbit equivalence. Therefore, these permutations provide topological invariants relevant for the classification of the (Morse-Smale) attractors for these dynamical systems. We present in this communication a characterization of the class of meander permutations that are realizable by the considered boundary value problems. This is a collaboration work with Bernold Fiedler.

### **Björn Sandstede: Analysis and Numerical Computation of Pulse Packets**

In this talk, the dynamics of localized solutions of PDEs on the real line is investigated. As an example, the PDE

$$u_t = -u_{xxxx} + f(u, u_x^2, u_{xx}), \quad x \in \mathbb{R}, t > 0, u(0) = u_0 \quad (1)$$

was considered. We assume that a localized stable equilibrium  $u_*$  of (1) exists. Then the

dynamics of pulse packets are of interest. These solutions are associated with initial values consisting of concentrated, widely spaced copies of  $u_*$ . It turns out that the pulse packets evolve in time according to the ODE

$$\begin{pmatrix} \dot{\tau} \\ T_j \end{pmatrix} = P\xi(T_j) \quad (2)$$

where  $\tau$  denotes the translation of the pulse packet, and the numbers  $T_j$  are the distances between consecutive copies of  $u_*$ . Also,  $P$  is a certain constant matrix. Surprisingly, the nonlinearity  $\xi(T_j)$  coincides with the bifurcation equations when applying Ljapunov-Schmidt reduction for determining the existence of equilibria of (1). Finally, an application to signal transmission in optical fibers with phase-sensitive amplification was given.

### Jesus Maria Sanz-Serna: A Method for the Numerical Integration of Oscillatory Differential Equations

Considered were methods to integrate Newton's equation  $M\ddot{x} = F_1(x) + F_2(x)$ , where  $F_2(x)$  is a slow hard-to-compute force and  $F_1(x)$  is a fast easy-to-compute force. Problems of that class appear in molecular dynamics, many-body problems, partial differential equations. An existing method (sometimes called the impulse method) was analyzed and shown to suffer from instabilities and order reduction. An alternative method was suggested and numerical comparisons were reported. (Joint work with B. Garcia-Archilla and R.D. Skeel)

### Arnd Scheel: Hopf Bifurcation to Spiral Waves

For a large class of reaction-diffusion systems on the plane, we show rigorously that typically  $m$ -armed spiral waves bifurcate from a homogeneous equilibrium when the latter undergoes a Hopf bifurcation. In particular, we construct a finite-dimensional manifold which contains the set of small rotating waves close to the homogeneous equilibrium. Examining the flow on this center-manifold in a very general example, we find different types of spiral waves, distinguished by their speed of rotation and their asymptotic shape at large distances of the tip. The relation to the special class of  $\lambda$ - $\omega$  systems and the validity of these systems as an approximation is discussed.

### Dmitry Turaev: An Example of a Wild Strange Attractor

We propose a new concept of a pseudohyperbolic attractor which possesses a weak analogue of a hyperbolic structure (the difference is that the linearized flow need not be expanding in the unstable subspace, we require only volume expansion). We give an

example of such an attractor by a modification of the geometrical Lorenz model. We prove that attractors of our example contain a wild hyperbolic set within. As a consequence, we prove that our attractors contain infinitely degenerate non-hyperbolic periodic orbits, coexisting periodic orbits with different dimensions of the unstable manifolds, etc.

### **Andre Vanderbauwhede: A Brief Course on Symplecticity**

In this lecture we give a short tutorial on symplectic structures and Hamiltonian systems. We review the main definitions and basic results, in particular the Poincaré lemma, the Darboux theorem, and the fact that the Poincaré return map near a periodic orbit in a Hamiltonian system is symplectic. We also briefly discuss complete integrability and symmetry reduction.

## Hans-Otto Walther: Smoothness of the Attractor of Almost all Solutions of a Delay Differential Equation

Consider the negative feedback equation

$$x'(t) = -\mu x(t) + f(x(t-1))$$

with  $\mu \geq 0$ , and with a  $C^1$ -function  $f : \mathbb{R} \rightarrow \mathbb{R}$  which satisfies  $f(0) = 0$  and  $f' < 0$ , and which is bounded from above or from below. There is an open and dense set of initial data in the state space  $C = C([-1, 0], \mathbb{R})$  so that the segments  $x_t = x(t + \cdot) \in C$ ,  $t \geq 0$ , of the corresponding solutions are eventually absorbed into the positively invariant set  $S \subset C$  of data with at most one sign change. An earlier result (*Mem. A.M.S. 544, 1995*) says that the global attractor  $A$  of the restricted solution semiflow  $[0, \infty) \times \bar{S} \rightarrow \bar{S}$  in the nontrivial case  $A \neq \{0\}$  can be written as a Lipschitz continuous graph which is homeomorphic to a closed disk and bordered by a periodic orbit; all solution curves  $t \mapsto x_t$  in  $A \setminus \{0\}$  wind around the stationary point 0.

We show that the attractor  $A$  is in fact  $C^1$ -smooth. The proof involves results on Floquet multipliers of periodic solutions, a-priori estimates for suitably chosen Poincaré maps, local invariant manifolds, and estimates of inclinations of tangent spaces (*to appear in Dissertationes Mathematicae*). (Joint work with Mustapha Yebdri)

## Thomas Wanner: Slow Motion in Higher-Order Systems and $\Gamma$ -Convergence in One Space Dimension

In certain singularly-perturbed parabolic equations in one space dimension one can observe extremely slow motion of transition layers, which is in fact exponentially slow with respect to the singular perturbation parameter  $\epsilon$ . I will present a new variational approach for establishing results of this type. Apart from covering known applications such as the Allen-Cahn and Cahn-Hilliard equations, this method also proved slow motion in the phase field model and other higher-order equations. Furthermore, it immediately implies  $\Gamma$ -convergence for certain energy functionals involving higher derivatives. (Joint work with W.D. Kalies, R.C.A.M. VanderVorst)

## Claudia Wulff: Center-Manifold Reduction near Relative Periodic Orbits of Non-Compact Group Actions

We study bifurcations of patterns arising in reaction-diffusion systems on the plane and in three-space. The systems possess Euclidean symmetry. They model various chemical and biological systems, e.g. the Belousov-Zhabotinsky reaction and catalysis on platinum surfaces. In order to analyze bifurcations we perform an equivariant center-manifold

reduction near relative periodic orbits of  $G$ -equivariant semiflows on Banach spaces. In contrast to previous results, the Lie group  $G$  may be non-compact and may act discontinuously on the underlying function space. As applications of this abstract result we investigate Hopf bifurcations from meandering spiral waves to invariant three-tori, periodic forcing of rigidly rotating and meandering spiral waves in the plane and periodic forcing of scroll waves in three-space. (Joint work with Björn Sandstede and Arnd Scheel)

Berichterstatter: Arnd Scheel

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