MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 31/1997

Stability for Classical and Non-Newtonian Fluids

10.08. - 16.08.1997

The conference on stability for classical and non-Newtonian fluids was organized by G. P. Galdi (University of Pittsburgh), K. R. Rajagopal (Texas A. & M. University, College Station), and Wolf von Wahl (University of Bayreuth). 27 participants from 10 countries represented a broad spectrum reaching from classical problems of stability (as the Taylor-Couette problem) to the numerical treatment of the equations for second grade fluids.

The program of the conference consisted of 19 talks of different length. As a consequence there was much time left which was used for intense discussions. Particular topics were new classes of perturbations and numerical existence proofs for certain steady flows being not known previously.

Organizers and participants thank the institute and its staff for the hospitality and the unique atmosphere.

F. H. BUSSE

Tertiary and Quaternary Solutions in Systems of Fluid Flow with and without Bifurcations

The subject of this presentation is part of the general theme of understanding the transition from simple to complex forms of fluid flows with increasing control parameter such as the Reynolds number Re. In the standard situation the transition can be understood and analyzed in terms of sequences of bifurcations. Restricting the attention to fluid systems that are homogeneous in two spatial dimensions and steady in time one can often follow a sequence of supercritical bifurcations each of which breaks one or more symmetries of the preceeding solution. In Taylor-Couette system and the Raleigh-Bénard layer represent the best analyzed examples of such systems. Fluid systems with plane parallel shear flow as basic state often exhibit delayed bifurcations or no bifurcation at all such as in the case of plane Couette flow or pipe flow. In this case bifurcation theory is still applicable if the problem is imbedded in a larger manifold of problems. We may consider, for instance, a horizontal plane Couette flow layer which is heated from below. There are two control parameters, the Reynolds number Re and the Rayleigh number Ra in this case. The basic state of plane Couette flow with linear temperature profile of pure conduction becomes unstable to longitudinal convection rolls when Ra exceeds 1708 independent of Re. But the secondary bifurcation to wavy rolls strongly depends on Re. For small values of Re the bifurcation to steady wavy rolls is supercritical. But for larger values of Re it becomes subcritical, such that solutions in the form of wavy rolls exist for Ra = 0. These three-dimensional steady solutions have been investigated as function of Re and the wavenumber parameters α_x and α_y. Besides these steady spatially periodic solutions and the various time dependent solutions that bifurcate from them there exist solitary solutions which have been found by Cherhabili and Ehrenstein (Eur. J. Mech. B/ Fluids **14**, 677, 1995) at much higher values of Re. They correspond to steady two-dimensional isolated transverse rolls. Similar solitary roll solutions have been found in the Ekman-Couette problem in which a rotation about an axis normal to the plates is added to the plane Couette problem (Hoffmann, Busse, and Chen, Subm. to J. Fluid Mech.). But these two types of solitary solutions do not seem to be connected in any way.

Susan FRIEDLANDER

Nonlinear instability in the Euler equation

This is joint work with M. Vishik and W. Strauss.

We discuss a result for general nonlinear evolution equations which shows that, under certain rather general conditions on the nonlinearity and under a gap condition on the spectrum of the linearized operator, linear instability implies nonlinear instability. This abstract theorem is applied to the Euler equations governing the motion of an inviscid incompressible fluid. In particular, we show that any 2-D space periodic flow without a stagnation point or any 2-D shear flow that is linearly unstable is also nonlinearly unstable. We describe the complete unstable spectrum of shear flow with a sinusoidal profile.

Isom H. HERRON

A unified solution of several classical hydrodynamic stability problems

The long-standing problems of the linear stability of plane Couette flow and circular pipe flow (to axisymmetric disturbances) are solved by operator theory: It is shown simply that both are stable for all Reynolds numbers and wave numbers. The proof is based on the infrequently used von Neumann extension of a semi-bounded symmetric operator, and the universally used Fredholm alternative theorem. It is made clear how the method will apply in other problems with a similar structure.

Yoshiyuki KAGEI

Asymptotic stability of steady flows in infinite layer of viscous incompressible fluid in critical cases of stability

It is known that the motionless state in the Bénard problem is unconditionally stable up to the critical value of the control parameter. We show that in the critical case the motionless state is asymptotically stable and, moreover, the unconditional stability holds for 2-D perturbations. We also consider the stability problem of the plane Couette flow in a rotating fluid layer for 2-D perturbations which represents the axisymmetric Taylor-Couette problem in the small-gap approximation. It is shown that the basic flow is unconditionally stable in the critical case of stability. We also consider the case where the rotating layer is heated from below.

Ralf KAISER

Mathematical problems in dynamo theory

Dynamo theory is concerned with the maintenance or amplification of magnetic fields by the motion of a conducting fluid (dynamo process). The exclusion of the dynamo process for certain classes of velocity or magnetic fields is formulated in so-called anti-dynamo theorems. Three (from a physical point of view) simple situations are presented, where anti-dynamo theorems are supposedly valid but not yet (fully) proved.

Hervé LE MEUR

Theoretical aspects of the Orr-Sommerfeld methods. Limitations.

I give a mathematical presentation for deriving the Orr-Sommerfeld equations for stability analysis. This enables me to stress:

1) The range of validity of this method.

2) An assumption commonly made by physicists.

I give a necessary and sufficient condition for making the assumption and finally, I partly justify it.

Maria Carmela LOMBARDO

Zero viscosity limit of Stokes equations in a 2-D channel

The time dependent Stokes equations in a 2-D channel are considered. We decompose the solution of these equations in an inviscid solution, two boundary layer solutions and a correction term. Bounds on these solutions in the appropriate Sobolev spaces are obtained in terms of the norms of the initial and boundary data. The correction term is shown to be of the order of the square root of the viscosity.

R. MEYER-SPASCHE

Secondary bifurcations of convection rolls and Taylor vortex flows

Numerical investigations of stationary Taylor vortex flows in wide gaps ($\mu = 0$) showed that

- nonlinear interactions of flows of different periods dominate the flow patterns observed and affect the existence of solutions;
- the Eckhaus and short-wave length instability are strongly related to secondary bifurcations.

The broken symmetry of the Taylor apparatus turns many bifurcation points into fold points and thus obscures the picture.

The bifurcation structure of convection rolls in a related but more symmetric configuration is much easier to understand. Reproducing a result on Taylor vortex flows for convection rolls by perturbation methods allowed to guess the general pattern of interaction for a large family of flows with different numbers of rolls.

Vinicio MOAURO

Non-integrability and chaotic behavior in the four point vortices problem

In the vortex model for planar Eulerian incompressible fluids the field of velocities depends on the intensities of the vortices and on their motion. This motion is governed by a Hamiltonian system with n degrees of freedom, where n is the number of point-vortices. It is possible to prove, by using Melnikov method, that for n=4 the solutions of the aforesaid Hamiltonian system present a chaotic behavior.

Jiři NEUSTUPA

Stability of steady solutions of parabolic systems in exterior domains

The question of stability of a steady solution of a nonlinear parabolic system (including the Navier-Stokes equations) in an exterior domain Ω can be transformed to the question of stability of the zero solution of the operator equation

$$\frac{du}{dt} = Lu + N(t, u)$$

in a suitable Hilbert space H. L is a linear operator in H and N(t,.) is a nonlinear operator in H. Operator L has an essential spectrum which touches the imaginary axis. We show how this difficulty can be overcome and we apply general results from preceeding works to a concrete parabolic system. We also discuss possibilities of applications to the Navier-Stokes equations.

Niko SAUER

Stability of flows in regions with permeable boundaries

We consider a model for fluid flow through a permeable boundary induced by stress differences over the boundary. It is assumed that only normal flow over the boundary is possible. The equations are

$$\begin{aligned} D_t(pv) &= T(p,v) \text{ in the region} \\ \nabla \cdot v &= 0 \\ \gamma_0 v &= -\eta(n,t)n \text{ at the boundary.} \end{aligned}$$

The velocity η is also an unknown function and satisfies an equation of the form

$$\sigma \frac{\partial \eta}{\partial t} = n \cdot Tn + l(t).$$

Under the additional assumption that at the boundary $w \wedge n = 2\nabla\eta$, with $w = \nabla \times v$, the vorticity, we can prove stability of the rest state for fluids of grade 2 provided the mean curvature of the boundary is always positive.

Bruno SCARPELLINI

Ljapounov instability of rolls in the Benard problem

We consider equilibrium solutions u_0, τ_0, p_0 of the Boussinesq-equations $(k = (0,0,1))$

$$(*) \partial_t u = \nu \Delta u - \nabla \pi - g(1-\alpha(T-T_0))k - (u\nabla)u$$

$$\partial_t T = \chi \Delta T - (u\nabla)T, \ \text{div } u = 0,$$

$$T(x,y,\frac{1}{2}) = T_1 < T_0 = T(x,y,-\frac{1}{2}).$$

We assume u_0, τ_0, p_0 to be L_1, L_2-periodic in x, y.

Def. 1: u_0, τ_0, p_0 is linearly $L^2(\Omega)$-instable if the linearization of the righthand side of $(*)$ around u_0, τ_0, p_0, viewed as an operator acting on $L^2(\Omega)$-functions, has a λ with Re $\lambda > 0$ in its spectrum.

Def. 2: u_0, τ_0, p_0 is E(ckhaus)-instable if there is an integer $N > 0$ such that the linearization of $(*)$ around u_0, τ_0, p_0, viewed as an operator NL_1, NL_2-periodic functions, has λ with Re $\lambda > 0$ in its spectrum.

Theorem: If u_0, τ_0, p_0 is either a roll or else rectangular and subject to a certain symmetry (S) then Def. 1 and Def. 2 are equivalent. (S) reads

$$u = (u_1, u_2, u_3) : u_1(-x,y,z) = -u_1(x,y,z),$$
$$u_2(-x,y,z) = u_2(x,y,z), \ u_3(-x,y,z) = u_3(x,y,z),$$
$$T(-x,y,z) = T(x,y,z).$$

Burkhard J. SCHMITT

The influence of mean values and nonperiodic pressure in the Oberbeck-Boussinesq equations

When considering the Oberbeck-Boussinesq equations in an infinite layer it is mostly assumed that the pressure p is periodic in the plane, whereas the equations only require ∇p to be periodic. We study the influence of the general admissible form of the pressure on the velocity field $\underline{u}$ below the onset of convection. - Under stress-free boundary conditions and periodic pressure the mean value of $\underline{u}$ over the layer is constant in time. If this constant $\underline{c}$ is not 0 there is in most cases no longer an exchange of stability on the onset for the linearization around $\underline{c}$. We study its spectrum on the onset.

Guido SCHNEIDER

Nonlinear stability of Taylor vortices in infinite cylinders

We consider the Taylor-Couette problem in an infinitely extended cylindrical domain in case when the Couette flow is weakly unstable and a family of spatial periodic equilibria, called the Taylor vortices, has bifurcated from this trivial ground state. We show that the non-Eckhaus-unstable Taylor vortices are nonlinearly stable with respect to small spatially localized perturbations. The main difficulty in showing this result stems from the fact that on unbounded cylindrical domains the Taylor vortices are only linearly marginal stable with continuous spectrum up to the imaginary axis. Bloch-wave representations of the solutions and renormalization theory allow us to show that the nonlinear problem behaves asymptotically as the linearized one which is under a diffusive regime. This result answers a question which was open for more than three decades, the nonlinear stability of non-Eckhaus-unstable solutions for a hydrodynamical problem in an infinitely extended cylindrical domain.

Adélia SEQUEIRA

A Decoupled FEM Approach for the Simulation of Steady Flows of Second-Grade Fluids

For a large class of non-Newtonian viscoelastic fluid models, and in particular for second-grade fluids, the equations of motion are nonlinear systems of PDE's of mixed type: elliptic-hyperbolic, in the steady state, and elliptic-parabolic in the unsteady state. Efficient numerical methods for these mixed systems are not yet well developed. In fact, they must be well adapted to the mathematical properties of the models and they should not introduce numerical instabilities in well-posed problems.

The existence and uniqueness of weak and regular steady solutions for second-grade fluids in a bounded domain have been proved using an appropriate splitting of the problem into an (elliptic) Stokes system and a (hyperbolic) transport equation, along with a fixed point argument (see Coscia & Galdi 1994, Videman 1997).

Here we present a stable finite element approximation of this problem, based on the mathematical analysis developed for the exact model. The method consists of using a decoupled fixed point iterative scheme for solving alternatively the Stokes system by a Hood-Taylor FEM and the transport equation by an upwinding technique of discontinuous Galerkin type, introduced by Lesaint and Raviart (1974). Proving the convergence of the algorithm, we show the local existence and uniqueness of a solution for the discretized problem and we give error estimates.

Similar techniques have been developed by Najib & Sandri (1995) for an Oldroyd type model.

Y. SHIBATA

$L_p - L_q$ decay estimate of Stokes semigroup in 2 dimensional exterior domain

Here, I would like to report my joint work with Wakako Dan, Tsukuba Univ., concerning the $L_p - L_q$ decay estimate of Stokes semigroup in 2 dimensional exterior domain and its application to the Navier-Stokes equation.

First, let $\{e^{-tA}\}_{t\geq 0}$ be the Stokes semigroup in the 2 dimensional exterior domain Ω. Namely, with some pressure p, $u = e^{-tA}a$ solve the Stokes eqn:

$$\begin{cases} u_t - \Delta u + \nabla p = 0, \ \nabla \cdot u = 0 \text{ in } \Omega. \ t > 0 \\ u|_{\partial\Omega} = 0, \ u|_{t=0} = a \\ \lim\limits_{|x|\to\infty} u(t,x) = 0. \end{cases}$$

Then, we have the following theorem.
Theorem 1. ($L_q - L_q$ estimate).

- $\|e^{-tA}a\|_{L_p(\Omega)} \leq C_{p,q} t^{-(\frac{1}{q}-\frac{1}{p})} \|a\|_{L_q(\Omega)}$
for $1 < q \leq p \leq \infty, \ 1 < q < \infty$;

- $\|\nabla e^{-tA}a\|_{L_p(\Omega)} \leq C_{p.q} t^{-(\frac{1}{q}-\frac{1}{p})-\frac{1}{2}} \|a\|_{L_q(\Omega)}$
  for $1 < q \leq p \leq 2$ and

for any $a \in J_q(\Omega) = \overline{\{u \in C_0^\infty(\Omega) | \nabla \cdot u = 0\}}^{L_q(\Omega)}$ and $t > 1$.
Now, we consider the Navier-Stokes equation:

$$\begin{cases} u_t - \Delta u + (u \cdot \nabla)u + \nabla p = 0, \;\; \nabla \cdot u = 0 \text{ in } \Omega, \;\; t > 0 \\ u|_{\partial\Omega} = 0, \;\; u|_{t=0} = a \\ \lim_{|x| \to \infty} u(t.x) = 0. \end{cases}$$

We know the unique-existence of Leray-Hopf solution for any data $a \in J_2(\Omega)$. Moreover, Masuda proved that

$$\|u(t)\|_{L_2(\Omega)} \to 0 \text{ as } t \to \infty.$$

Kozono and Ozawa proved that

$$u \in C^0([0.\infty).J_2(\Omega)) \cap C^1((0.\infty); J_2(\Omega))$$

$$u(t) \in \mathcal{D}(A) \; \forall t \in (0, \infty). \; Au \in C^0((0, \infty); \; L_2(\Omega))$$

and

$$(D) \begin{cases} \|u(t)\|_{L_p(\Omega)} = o(t^{-(\frac{1}{2}-\frac{1}{p})}), \;\; 2 \leq p < \infty \\ \|u(t)\|_{L_\infty(\Omega)} = o(t^{-\frac{1}{2}}\sqrt{log\ t}) \\ \|\nabla u(t)\|_{L_2(\Omega)} = o(t^{-\frac{1}{2}}) \end{cases}$$

as $t \to \infty$.

Applying Theorem 1, we can improve (D) as follows:

$(D)'$ $\qquad \|u(t)\|_{L_\infty(\Omega)} = o(t^{-\frac{1}{2}}).$

Henri TASSO

Lyapunov stability in magnetohydrodynamics

Lyapunov methods for linear stability in magnetohydrodynamics (MHD) as well as for nonlinear stability of MHD flows are reviewed. Recent work on nonlinear stability of force-free-fields, Trkal flows and "MHD generalized" Trkal flows is explained in detail, and an explicit sufficient condition for unconditional stability is given. Finally, nonlinear stability of magnetized parallel flows is proved for two-dimensional perturbations perpendicular to the flow direction. This analysis suggests a conjecture on linear stability of magnetized Couette flows for three-dimensional perturbations and for all Reynolds numbers.

Elisabetta TORMATORE

Stochastic equations for a viscous gas

We are interested in the stochastic equation for a viscous gas. In the one-dimensional case the system which we consider is

$$dv = (\mu(\rho v_x)_x - p_x)\,dt + dW,$$

$$\rho_t + \rho^2 v_x = 0$$

with the boundary and initial conditions

$$v|_{x=0,x=1} = 0$$

$$v|_{t=0} = v_0;\ \rho|_{t=0} = \rho_0$$

where W is a Wiener process in the Hilbert space $H_0^1(0,1)$.

We prove the existence and uniqueness of global solutions, a result which generalizes that of Kazhikhov (1995) for the corresponding deterministic system.

In the bi-dimensional case, the system we are interested in is

$$\rho dv = [-\rho(v\cdot\nabla)v + \mu\Delta v + \nabla(\lambda\nabla\cdot v) - \nabla p]\,dt + \rho\, dW$$

$$\rho_t + \nabla\cdot(\rho v) = 0.$$

We are going to discuss some remarks on the system, which seem to be useful for its resolution.

Hisao Fujita YASHIMA

Gleichungen der kugelsymmetrischen Bewegung eines viskosen Gases und asymptotisches Verhalten ihrer Lösung

In diesem Vortrag versuchen wir erstens, das Gleichungssystem darzustellen, das die kugelsymmetrische Bewegung eines viskosen Gases beschreiben soll, und dann legen wir die Existenz (und die Eindeutigkeit) der globalen Lösung dieses Gleichungssystems dar.

Im zweiten Teil des Vortrags wollen wir über asymptotisches Verhalten der Lösung diskutieren. Wir folgen den Beweisführungen von Jiang Song, der einige grundsätzliche Resultate über das asymptotische Verhalten der Lösung des Gleichungssystems für ein wärmeleitendes viskoses Gas bewiesen hat. Wir diskutieren auch die Resultate von Matušu-Nečasová, Okada und Makino für ein vereinfachtes Gleichungssystem, in welchem der Druck bloß durch die Dichte bestimmt wird.

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