MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 32/1997

Noncommutative Algebra and Representation Theory

17.08.-23.08.1997

This meeting was organized by G. Michler (Essen) and L. Small (San Diego). There were 43 participants from 10 countries (Belgium, Canada, Denmark, France, Germany, Israel, Norway, UK, Ukraine, and the US) and 31 lectures during the five day period.

The lectures were organized around the themes of: invariant theory and differential operators, representation theory of finite dimensional algebras, quantum groups, noncommutative algebraic geometry and classical ring theory.

Noncommutative algebraic geometry is emerging as a useful and deep subject with roots ranging from ring satisfying a polynomial identity to quantum statistical physics to familiar algebraic geometry. Applications of ring theoretic ideas were apparent in the presentations on quantum groups and invariant theory. Finite dimensional algebra theory and its influence were apparent in many of the lectures. Abstracts of many of these lectures are included in this report.

As is always the case at Oberwolfach, as much mathematics was taking place outside the lectures as in. The level and intensity of this meeting was great and illustrates the vigor and depth of contemporary noncommutative algebra and representation theory.

Jacques Alev

On the Invariants of A_1 under Finite Group Actions

Let G be a finite subgroup of $SL(2, \mathbb{C})$. G acts on $P = \mathbb{C}(x, y)$ and the rings P^G are the classical Kleinian singularities. G acts also on $A_1(\mathbb{C}) = \mathbb{C}[p, q]$, $pq - qp = 1$. In this talk we present the following results:

1. $Frac(A_1(\mathbb{C})^G) \approx D_1(\mathbb{C}) = Frac(A_1(\mathbb{C}))$, which can be considered as a noncommutative version of Noether's Problem in dimension 2.
2. $\dim_{\mathbb{C}}(H_0(A_1(\mathbb{C})^G) = s(G) - 1$, where $s(G)$ denotes the number of conjugacy classes of G; this allows us to separate "roughly" these algebras.
3. Endowed with the Bernstein filtration $H_0(A_1(\mathbb{C}))$ has the following property:

$$gr(H_0(A_1(\mathbb{C})^G)) \equiv H_0^{Pois}(P^G)$$

where the right hand side denotes the Poisson structure induced on P^G by the usual symplectic structure of the plane.

Amiram Braun (Haifa, Israel)

Noetherian P.I. Rings with Global Dimension Two

A structure theorem for rings as above will be presented as well as some of its consequences.

W. Crawley-Boevey (Leeds; UK)

A Conze Embedding for Deformed Preprojective Algebras

It was originally proved by M. P. Holland and the speaker that the deformed preprojective algebras associated to an extended Dynkin quiver could be embedded in a Weyl algebra or a polynomial ring. However, our proof used case-by-case analyses and computer calculations.

In the talk I shall discuss this result, applications of it, and sketch a direct proof which is a mixture of representations of quivers, ring theory and geometry.

Martin P. Holland (Sheffield, UK)

Quantization of the Marsden-Weinstein Reduction for Extended Dynkin Quivers

This talk is on joint work of W. Crawley-Boevey and the speaker. We study a family of noncommutative deformations of the coordinate ring of the Kleinian singularity K^2/Γ, where Γ is a finite subgroup of $SL_2(K)$ and K is an algebraically closed field of characteristic zero. The family is defined by $\mathcal{O}^\lambda = e(K\langle r, y\rangle * \Gamma/(xy - yx - \lambda)e$, for $\lambda \in Z(K\Gamma)$, where $K(\langle x, y\rangle * \Gamma$ is the skew group algebra for the action of Γ on the ring of noncommuting polynomials and e is the average of the group elements. If λ has trace zero on $K\Gamma$ then $\mathcal{O}^\lambda$ is commutative, and when $K = \mathbb{C}$ we show that as complex analytic spaces the Spec $\mathcal{O}^\lambda$ are exactly the fibres of the semiuniversal deformation of $\mathbb{C}^2/\Gamma$. On the other hand, if λ has nonzero trace on $K\Gamma$ we show that $\mathcal{O}^\lambda$ has only finitely many finite-dimensional simple modules. Using this we compute the ring-theoretic and homological properties of $\mathcal{O}^\lambda$. They are controlled by the incidence of λ with hyperplanes in $Z(KT)$ perpendicular to elements of the affine root system corresponding to the McKay graph of Γ. The main idea is to relate $\mathcal{O}^\lambda$ to a family of deformations of the preprojective algebra, and to study these using reflection functors.

Birge Huisgen-Zimmermann (Santa Barbara; USA)

Geometric Quotients of Varieties of Representations

(Joint work with Klaus Bongartz.) Let $A = KQ/I$ be a finite dimensional path algebra with relations, having underlying quiver Q and base field $K = K$. Moreover, let $n = rkK_0(A)$, and $\mathbb{S}$ a sequence of simple A-modules with $\dim \mathbb{S} = d \in \mathbb{Z}^k$. We consider the open subvariety $Rep\mathbb{S}$ of $Rep(A, d)$ consisting of those points in $Rep(A, d)$ which correspond to uniserial modules that have sequence $\mathbb{S}$ of consecutive composition factors; here $Rep(A, d)$ denotes the variety of bounded representations of Q of dimension vector d.

We prove that the geometric quotient $Rep\mathbb{S}/GL(d)$ "often" exists; in particular, each affine algebraic variety arises as such a quotient for suitable A and $\mathbb{S}$. In general, there is a "good approximation" to a geometric quotient $Rep\mathbb{S}/GL(d)$, which yields two geometric invariants capturing the complexity of the uniserial representation theory of A at $\mathbb{S}$. As applications we give (a) a characterization of the algebras of finite uniserial type, and (b) a description of the uniserial representation theory of tame algebras.

Peter Jørgensen (Copenhagen; Denmark)

Stanley's Theorem

Let A be a *commutative* connected $\mathbf{N}$-graded noetherian k-algebra. Suppose that A is a Cohen-Macaulay domain. Then Stanley's Theorem gives the following numerical criterion for A to be Gorenstein: A's Hilbert-series, $H_A(t)$, should satisfy

$$H_A(t) = \pm t^m H_A(t^{-1}),$$

for some integer m. In this equation, we treat $H_A(t)$ as a rational function.

The talk will give a generalization to the case where A is noncommutative. In particular, it will be shown that the theorem is true for noncommutative algebras with enough normal elements. This generalization is made possible by recent developments in the theory of balanced dualizing complexes.

Bernhard Keller (Paris; France)

On the Cyclic Homology of Exact Categories

The cyclic homology of an exact category was defined by R. McCarthy (*J. Pure Appl. Alg.*, 1994) using the methods of F. Waldhausen. McCarthy's theory enjoys a number of desirable properties; in particular, it admits a Chern character and, when applied to the category of finitely generated projective modules over an algebra, it agrees with the cyclic homology of the algebra. However, we show that it cannot be both invariant under derived equivalences and compatible with localizations (K-theory has these properties). This is our motivation for defining a new theory having all the good properties of McCarthy's construction *and* the two last mentioned ones. Thanks to these, we are able to compute the new theory for several nontrivial examples.

Lieven Le Bruyn (Antwerp; Belgium)

Smooth Noncommutative Varieties

Let K be a function field of transcendence degree d over an algebraically closed field of characteristic zero k and let Δ be a central simple K-algebra of p.i.-degree n. A model for Δ is a sheaf $\mathcal{A}$ of algebras over their central variety X, which is a normal projective variety of dimension d with function-field K. We call $\mathcal{A}$ a smooth (resp. regular) model if A is locally a smooth algebra in the sense of Schelter (that is, has the formally smooth lifting property in the category of algebras with trace of p.i.-degree n) (resp. locally an Auslander regular algebra). We want to investigate which Δ have a smooth and/or regular model.

At present, a local classification of Auslander regular algebras seems to be out of reach. For this reason we concentrate on the smooth algebras.

We have an étale local classification of smooth algebras. For fixed d (dimension) and n (p.i.-degree) there are only finitely many classes. The underlying combinatorics is based on a marked quiver Q, a dimension vector α of a simple representation of Q and a Morita setting given by an embedding of the base-change group $GL(\alpha) \hookrightarrow GL_n$.

In low dimensions we compile a list of occurring quiver-situations and study their corresponding rings of equivariant maps. Étale descent then gives us information on the Zariski local structure of smooth noncommutative models.

Helmut Lenzing (Paderborn; Germany)

Noncommutative Curves with an Exceptional Sequence

By definition, a smooth, projective, noncommutative curve $\mathbb{X}$ is given by an abelian category $coh(\mathbb{X})$ which is noetherian (i.e. objects satisfy the maximum condition on subobjects), hereditary (i.e. Ext^2 vanishes) and moreover has the form $\mathrm{mod}^H(R)/\ \mathrm{mod}_0^H(R)$ for some positively group graded algebra $\mathcal{R}$. We assume that $\mathcal{R}$ is a finite module over its center C which is requested to be affine over a field k.

Theorem. *Each such curve $\mathbb{X}$ which has a complete exceptional sequence has a tilting object T whose endomorphism ring $\Lambda = End(T)$ is a (finite dimensional) canonical algebra in the sense of Ringel and Crawley-Boevey.*

Corollary. *If k is algebraically closed, $\mathbb{X}$ is a weighted projected line.*

We discuss various classes of infinite dimensional algebras associated to curves with an exceptional sequence, the

- preprojective algebras of extended Dynkin quivers
- algebras of entire automorphic forms
- group-graded factorial domains of Krull-dimension two.

The next result shows that a recent theorem of A. Boudal and D. Orlov (1977) cannot be extended to the case of genus one.

Theorem (D. Kussin 1997). *There are two noncommutative curves $\mathbb{X}$ and $\mathbb{Y}$ with a complete exceptional sequence (and having genus one) which are nonisomorphic, i.e. $coh\mathbb{X} \ncong coh\mathbb{Y}$, but which have isomorphic bounded derived categories.*

Thierry Levasseur (Poitiers; France)

Invariant Differential Operators on the Tangent Space of Some Symmetric Spaces

(Joint work with Toby Stafford.) Let $\mathbf{g}$ be a semisimple complex Lie algebra with adjoint group G and $\Theta \in Aut(\mathbf{g})$ be a involution. Denote by $\mathbf{g} = \mathbf{k} \oplus \mathbf{p}$ the associated decomposition and let $K \subset G$ be the connected Lie subgroup such that Lie $(K) = \mathbf{k}$. Then K acts on $\delta(\mathbf{p}^*)$, $\mathcal{D}(\mathbf{p})$ (the algebra of differential operators on $\mathbf{p}$) and the differential of this action is denoted by $\tau : \mathbf{k} \to \mathcal{D}(\mathbf{p})$.
Assume that $(\mathbf{g}, \Theta)$ satisfies the following condition (introduced by J. Sekiguchi):

$$\forall \alpha \in \Sigma, \quad \dim \mathbf{g}^{\alpha} + \dim \mathbf{g}^{2\alpha} \leq 2$$

where Σ is the restricted root system of $(\mathbf{g}, \Theta)$. Then:

1. $\{D \in \mathcal{D}(\mathbf{p})\ D(S(\mathbf{p}^*)^K) = 0\} = \mathcal{D}(\mathbf{p}) = (\mathbf{k})$.
2. The algebra $R = \frac{\mathcal{D}(\mathbf{p})^K}{(\mathcal{D}(\mathbf{p})\subset(\mathbf{k}))^K}$ is simple and one has $GK \dim M \geq rk(\mathbf{g}, \Theta)$ for all finitely generated R-module $M \neq 0$.

If $\mathbf{g}_{\mathbb{R}} = \mathbf{k}_{\mathbb{R}} \oplus \mathbf{p}_{\mathbb{R}}$ is a real form of $\mathbf{g}, \Theta)$, these results imply in particular that there exist no nonzero singular invariant eigendistribution on $\mathbf{p}_{\mathbb{R}}$, a result first proved by J. Sekiguchi

Martin Lorenz

On K_0 of invariant rings

We give a description of the kernel of the induction map $K_0(R) \to K_0(S)$, where $R = S^G$ is the invariant ring of the action of a finite group G on a commutative ring S. The description is in terms of $H^1(G, GL(S))$. Applications include the calculation of the Picard group of multiplicative invariants as well as the construction of nontrivial torsion in K_0 of certain polycyclic group algebras.

Marie Paule Malliavin (Paris; France)

Algebras Associated to Some Categories of Representations of (Infinite) Lie Algebras or Groups

The group $S(\infty)$ of all permutations of the set of natural integers supports a topology for which it is possible to describe all the set of continuous unitary irreducible representations.

The (Hilbert) tensor product of two of these representations decomposes into a (Hilbert) direct sum of irreducible unitary representations (by Liebermann and Glohenski).

We (De Concini, P. Malliavin, M. P. Malliavin) attach to these data a commutative unital Banach algebra such that $\mathcal{S}(\infty)$ embeds in its Gelfand spectrum. It is proved that $\mathcal{M}$ support a probability measure.

More things are proved in "Integration invariante associee à un #-anneau de representations unitaires," C. R. Acad. Sci. Paris, t. 323, serie 1, p. 1265–1270, 1996.

John McConnell (Leeds; UK)

Locally Finite Dimensional Division Algebras

(Joint work with Lance Small.) Let D be a division algebra finite dimensional over its centre F. Then there exists an $n \in \mathbb{N}$ such that $\dim_F D = n^2$ (Wedderburn) and if K is any maximal commutative subfield of D then $\dim_F K = n$.

We construct a class of division algebras which are locally finite dimensional (i.e. each finitely generated subdivision algebra is finite dimensional over its centre). Let GK_{dim} denote the Gelfand-Kirillov (= "growth") dimension and $tr\deg$ denote the usual commutative transcendence degree. Let $m \in \mathbb{N}$. Then there is a division algebra D with $\mathit{Centre}\, D = \mathbb{Q}$ and $GK_{\mathrm{dim}}(D) = m$, which has a family of maximal commutative subfields $K_0, \ldots, K_m$ such that $tr\deg_{\mathbb{Q}} K_r = r$.

K. W. Roggenkamp (Stuttgart; Germany)

Derived Equivalences of Special Biserial Algebras

Lokal eingebettete Graphen, auch Brauergraphen genannt, sind direkte Verallgemeinerungen von Brauerbäumen. Analog zer Konstruktion von Brauerbaumalgebren von GABRIEL-RIEDTMANN für Bäume kann man zu einem lokal eingebetteten Graph eine Brauergraphalgebra mittels Köcher und Relationen definieren.

Gemeinsam mit KAUER wurde gezeigt, daß jede Brauergraphalgebra deriviert äquivalent zu einer Brauergraphalgebra ist, deren Graph ein verallgemeinerter Stern ist, also ein Graph, in dem alle Kanten inzident zu einer Ecke sind und, bis auf eventuell eine weitere Ecke, alle anderen Ecken Wertigkeit 1 haben.

Dies verallgemeinert ein Ergebnis von RICKARD, daß eine Brauerbaumalgebra deriviert äquivalent zu einer Brauerbaumalgebra ist, deren Baum ein Stern ist.

Markus Schmidmeier (Prague; Czech Republic)

Methods of Representation Theory for Artinian PI-rings

It is the aim of this talk to indicate how basic concepts from the representation theory of finite dimensional algebras can be used for the study of modules over artinian polynomial identity rings. In particular, we present an upper bound for the endolength of the transpose of an endofinite finite length module, and we also collect some information about the components of the AR-quiver of a hereditary artinian PI-ring with Euclidean diagram.

S. Paul Smith (Seattle, USA)

Lines in Quantum Spaces

(Joint work with J. Zhang.) After defining a quantum projective space, we define its linear subspaces, these being a noncommutative scheme in the sense of Artin and Zhang, namely a pair $(\mathcal{C}, \mathcal{O})$ consisting of a category and an object. We examine morphisms between linear subspaces of different quantum $\mathbb{P}^n$'s.

Michaela Vancliff (Eugene; USA)

Some Quantum $\mathbb{P}^3$s with One Point

In the late 80's, the notion of a quantum R^2 was defined by M. Artin, and a classification was given by Artin, Schelter, Tate and Van den Bergh in terms of geometric data. In this talk we will consider quantum $\mathbb{P}^3s$ and their point schemes. It is known that there exist quantum $\mathbb{P}^3s$ with infinite point sheme whose geometric data does not determine the algebra. This talk focuses on a family of quantum $\mathbb{P}^3s$ with (in some sense) the smallest point scheme possible.

James Zhang (Seattle; USA)

On Lower Transcendence Degree

We introduce a new definition of transcendence degree for division algebras not necessarily finite over their centers. Using this definition we answer several open questions.

Alexander Zimmermann (Amiens, France)

Derived Equivalences for Selfinjective Algebras and Gorenstein Orders

A theorem of Rickard gives a necessary and sufficient criterion for two rings A and B to have equivalent derived categories $D^b(A) \cong D^b(B)$. This is the case if and only if there is a complex T called "tilting complex" satisfying certain properties in $D^b(A)$ such that $End_{D^b(A)}(T) \cong B$.

For a certain T which occurs often only in practice and for A being a Gorenstein order joint work with S. Konig describes B in terms of a pullback of the endomorphism rings of the homology of T. In case A is a selfinjective algebra it is still possible to get a description of B in terms of the homology of T. For a Gorenstein order A with an additional hypotheses on T it is possible to realize explicitly an equivalence of the derived categories by constructing explicitly a complex X in $D^b(A \otimes B^{op})$ such that $X \overset{\mathbf{L}}{\underset{B}{\otimes}} - : D^b(B) \to D^b(A)$ is an equivalence of categories an X viewed as complex in $D^b(A)$ is isomorphic to T.

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