

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

**Random Graphs and Combinatorial Structures**  
28/9/97 – 3/10/97

Organizers

- A. D. Barbour (Zürich)
- B. Bollobás (Cambridge and Memphis)
- I. Wegener (Dortmund)

Although probabilistic combinatorics goes back almost forty years, to the work of Erdős and Rényi on random graphs and Broadbent and Hammersley on percolation, over the past ten years the subject has undergone dramatic changes with the influx of powerful new methods and the emergence of new areas of application, especially to computer science. The purpose of this meeting was to bring together people from combinatorics, probability and computer science, all of whom are concerned with probabilistic combinatorial structures. Among the themes explored in the talks and discussions were new aspects of: the Stein-Chen method, the inequalities of Janson and Suen, Kolmogorov complexity, sharp thresholds for monotone properties, pseudorandomness, probabilistic graph colouring, randomized algorithms, average case analysis of algorithms, learning pattern languages, Ramsey numbers, polynomial time approximation schemes, first-passage percolation, rapid mixing and phase transitions.

A selection of talks will be published in a special issue of *Combinatorics, Probability and Computing*.

Due to unfortunate circumstances, Andrew Barbour and Ingo Wegener were unable to attend the meeting; Martin Dietzfelbinger and Friedhelm Meyer auf der Heide kindly stepped in at short notice to help with the running of the meeting.

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### Diameters and Isoperimetric Inequalities in Sequence Spaces

Rudolf Ahlswede, Bielefeld

We survey a number of results obtained with L.K. Khachatrian, N. Cai, I. Althofer, and Z. Zhong. These results fall into four classes.

I. Contributions to problems concerning the *diameter*:

- a complete solution of the binary constant weight worst case diameter problem of Erdős, Ko and Rado from 1938,
- an exact solution of the worst case diameter problem in Hamming spaces over arbitrary alphabets,
- an asymptotically optimal solution of the average case diameter problem in Hamming spaces,
- an asymptotically optimal solution of the average case diameter problem for arbitrary "sum-type" cost functions (including distances, Hamming, Lee, Taxi... ) in sequence spaces.

II. Contributions to *edge-isoperimetric* problems:

- derivation by information theoretical methods of asymptotically optimal bounds for a class of problems, including all cartesian sum graphs and all sequence spaces with "sum-type" distance functions. In a special case we have an exact result,
- the lexicographic order is shown to be a solution of an edge-isoperimetric problem for any power (cartesian sum) of graphs exactly, if it is the solution for one and two factors. An edge-isoperimetric theorem for powers of every complete bipartite graph is a consequence.

III. Contributions to *vertex-isoperimetric* problems:

- a novel information theoretic result we call the "Inherently Typical Subset-Lemma" implies a rate-wise asymptotically optimal vertex-isoperimetric theorem. It is now already for the non-binary Hamming case (the binary case is settled exactly by Harper's well-known theorem).
- for the space  $(\bigcup_{n=0}^{\infty} \{0,1\}^n, \theta)$ , where  $\theta(x^n, y^n)$  counts the minimal number of insertions and deletions necessary to transform one word into the other, we establish an exact vertex-isoperimetric theorem.

IV. A novel *edge-diameter* theorem for specified Hamming diameter sets in  $\{0,1\}^n$  with maximal number of pairs of numbers with distance 1, whose proof is based on a new "pushing-pulling" technique.

It seems that an earlier method of generating sets, which originated in combinatorial number theory and led to a solution of the problem of Erdős, Ko and Rado mentioned above, cannot be applied here.

### Phase Transitions and Hard Constraints (II)

Graham Brightwell, London

This follows on from the talk by Peter Winkler - see below.

The situation can be extended as follows: now the *edges* of the constraint graph  $H$  have activities  $\lambda_{ij}$ , and the Gibbs condition states that the conditional probability that a finite 'patch' of sites in the regular free  $\tau$  takes on a certain configuration (i.e. the map to  $H$  has a given restriction  $e$ ), is proportional to the product of the activities on the edges  $\lambda_{\phi(a)\phi(b)}$ . Every set of activities arises from some reversible Markov chain, with state space  $V(H)$ , run on  $T_r$ . We show that, provided  $H$  has two non-incident edges, there are values of the  $\lambda_{ij}$  such that more than one reversible Markov chain - and hence more than one simple invariant Gibbs measure - corresponds to the set of activities.

We also consider aspects of "memory" when running  $\tau$ -branching random walks according to a Markov chain  $M$ . When  $r > 1/(\lambda_2)^2$ , where  $\lambda_2$  is the second largest eigenvalue of the transition matrix of  $M$ , just the number of colours at the distance- $n$  leaves of  $T_r$  gives information about the root colour: when  $r < 1/|\lambda_2|^2$ , it does not suffice. For certain constraint graphs, in particular for  $K_3$  with no loops, a large branching of the tree suffices to enable us to reconstruct the root colour with confidence, with high probability. This illustrates a dichotomy between two types of constraint graphs, depending on whether the set  $\text{Hom}(T_r, H)$  of homomorphisms from  $T_r$  to  $H$  is connected under simple-point recolourings.

Joint work with Peter Winkler.

## Compound Poisson Approximation on Groups

Louis Chen, Singapore

Let  $X$  be a measurable abelian group, that is, an abelian group such that the group operation is a measurable map of  $X^2$  to  $X$ . Let  $\pi$  be the compound Poisson distribution  $e^{\lambda(\mu - \delta_0)}$ , where  $\lambda > 0$ ,  $\mu$  is a probability measure on  $B(X)$  with no atom at the identity 0, and  $\delta_0$  is the Dirac measure at 0. That is,  $\pi = L(Y_1 + \dots + Y_N)$  where  $Y_1, Y_2, \dots$  are independent  $X$ -valued random variables with common distribution  $\mu$ ,  $N$  is a Poisson random variable with mean  $\lambda$ , and  $N$  is independent of  $Y_1, Y_2, \dots$ .

For appropriate choices of  $X$  and  $\mu$ , a Poisson distribution on  $Z$ , a compound Poisson distribution on  $Z$ , a multivariable Poisson distribution on  $Z^d$  and a Poisson point process can all be obtained as a compound Poisson distribution on a group. Thus the compound Poisson approximation unifies various Poisson-related approximations.

Let  $Z = Y_1 + \dots + Y_N$ . By using the  $L^2$  space approach, a Stein identity of  $L(Z)$  is found to be

$$E\{\lambda \int f(Z+t)d\mu(t) - E(N|Z)f(Z)\} = 0$$

for a suitable choice of  $f$ . The conditional expectation  $E(N|Z)$  is difficult to compute in general, but if  $N$  is a function of  $Z$ , then  $E(N|Z)$  will have an explicit form.

This is possible for the following special case. Let  $K_0$  be a subgroup of  $X$  and let  $K_1, K_2, \dots$  be distinct cosets of  $K_0$  such that  $K_r = K_{r-1} + K_1$  for  $r = 1, 2, \dots$ . If  $\text{supp}(\mu) \subset K_1$ , then  $Z$  takes values in  $\bigcup_{r=0}^{\infty} K_r$ , and  $N = \psi(Z)$  where  $\psi(Z) = r$  if  $\xi \in K_r$ ,  $r = 0, 1, 2, \dots$ . The above Stein identity for  $L(Z)$  then takes the explicit form:

$$E\{\lambda \int f(Z+t)d\mu(t) - \psi(Z)f(Z)\} = 0$$

and the Stein equation

$$\lambda \int f(w+t)d\mu(t) - \psi(w)f(w) = h(w) - Eh(z)$$

can be solved analytically.

This provides a framework for compound Poisson approximation on groups in this special setting. A special case of this setting is the multivariable Poisson approximation on  $Z^d$  for  $1 \leq d \leq \infty$ . As an application, we obtain a compound Poisson approximation result bridging two extreme cases for certain sums of independent group-valued random variables.

### Probability vs. Paradoxity

Walter Deuber, Bielefeld

Tarski's alternative states the following. Let  $G$  be a pseudogroup of transformations of a set  $X$ . Then either there is a  $G$ -invariant finitely additive probability measure on the subsets of  $X$ , or else there is a paradoxical  $G$ -decomposition of  $X$ .

We discuss the situation for the pseudogroup  $W(X)$  of all bounded variations of the identity. This is joint work with M. Simonovits and V.T. Sós.

### Approximately Counting Colourings and Independent Sets

Martin Dyer, Leeds

We review some recent results obtained jointly with Russ Buble and Catherine Greenhill on approximate counting of  $k$ -colourings and independence sets in graphs with maximum degree  $\Delta$ . For the case of independent sets of size  $s$ , this can be done provided  $s \leq \frac{n}{2}(\Delta + 1)$ . For all independent sets, provided  $\Delta \leq 4$ . For independence sets such that size  $s$  has probability proportional to  $X^s$  ( $0 < \lambda \leq 1$ ) provided  $\lambda \leq \frac{2}{\Delta-2}$ . For colourings we describe attempts to achieve  $k < 2\Delta$ . This has been achieved only for  $\Delta = 3$  or  $\Delta = 4$  and triangle-free.

### Total Path Length for Recursive Trees

Bob Dobrow, Kirksville, Missouri

Total path length, or search cost, for a rooted tree is defined as the sum of all root-to-node distances. Let  $T_n$  be the total path length for a random recursive tree of order  $n$ . Mahmoud showed in 1991 that  $W_n := (T_n - E[T_n])/n$  converges in distribution to a nondegenerate limiting random variable  $W$ . Here we give two recurrence relations for the moments of  $W_n$  and  $W$  and show that  $W_n$  converges to  $W$  in  $L^p$ . We confirm the conjecture that the distribution of  $W$  is not normal. We also show that

$$W \stackrel{d}{=} V(1+W) + (1-V)W^* - \mathcal{E}(V),$$

where  $V$  is a uniform  $(0, 1)$  random variable,  $W^*$  is an independent copy of  $W$ , and  $\mathcal{E}(x) := -x \ln x - (1-x) \ln(1-x)$  is the binary entropy function. Finally, we derive an approximation

for the distribution of  $W$  using a Pearson curve density estimator. Simulations exhibit a high degree of accuracy in the estimation.

The results were obtained jointly with James Fill.

### **Compound Poisson Approximation for Dissociated Summands**

**Peter Eichenbacher, Bielefeld**

In joint work with M. Roos of Zürich, we consider an arbitrary finite collection of indices  $\Gamma$  and for each  $\alpha \in \Gamma$  let  $J_\alpha$  be  $\beta - 1$  valued, possibly dependent random variables and  $W := \sum_{\alpha \in \Gamma} J_\alpha$ . The Poisson distribution provides good description of rare events. If “clumps” of 1s tend to occur, because of the dependence between events, it could be hoped that approximation by a compound Poisson distribution would improve results.

We developed an estimation of the total variation distance of the law of  $W$  and a compound Poisson distribution for the class of dissociated  $J_\alpha$ .

Therefore we apply the Stein-Chen method for compound Poisson distributions introduced by Barbour, Chen and Loh '92. We apply our results for counting  $k$ -runs in a Bernoulli sequence, and colouring at random fixed graphs. We improve the general Theorem using asymptotic expansion techniques and apply it to a “DNA-breakage” model.

### **Can Stochastic Monotonicity Be Realized?**

**James A. Fill, Baltimore**

With Motoya Machida we studied the following problem which arose in the comparison of the Markov chain Monte Carlo perfect sampling algorithms of Propp and Wilson and of Fill. Consider a system  $\mathcal{P} := (P_a : a \in A)$  of probability measures on a common finite partially ordered set (poset)  $S$ , indexed by a (possibly different) finite poset  $A$ . We say that  $\mathcal{P}$  is *stochastically monotone* if  $P_a \leq P_b$  stochastically (meaning  $P_a(U) \leq P_b(U)$  for every up-set  $U$  in  $S$ ) whenever  $a \leq b$ . We say that  $\mathcal{P}$  is *realizably monotone* if there exists a system  $(X_a : a \in A)$  of  $S$ -valued random variables defined on some common probability space such that (i)  $X_a$  has the distribution  $P_a$  for every  $a \in A$  and (ii)  $X_a \leq X_b$  (for all sample points) whenever  $a \leq b$ .

It is easy to see that stochastic monotonicity is a necessary condition for realizable monotonicity. It is perhaps surprising that the condition is not always sufficient. When  $A = S$ , we show that the two notions are equivalent if and only if the Hasse diagram (regarded as an undirected graph) for  $A$  is acyclic. We show also that the notions agree for a given  $A$  and arbitrary  $S$  if and only if the Hasse diagram for  $A$  is acyclic, and that they agree for a given  $S$  and arbitrary  $A$  if and only if the Hasse diagram for  $S$  is a disjoint union of paths.

### **Necessary and Sufficient Conditions for Sharp Thresholds for Graph Properties** **Ehud Friedgut, Jerusalem**

In this talk we present some theorems about graph properties that “emerge slowly” in the building of a random graph. Stated roughly, if the transition interval (the probabilities for which one expects the property to appear with probability bounds from 0 and 1) is

large in comparison to the critical probability then the property can be approximated by the property of having a subgraph from a given list.

Another theorem is that such “coarse” thresholds only take place when the critical probability is close to a rational power of  $h$ .

## A Quick Approximation to Matrices and Its Applications Alan Frieze, Pittsburgh

The aim of the lecture is to present results obtained jointly with Ravi Kannan. We give algorithms to find the following simply described approximation to a given matrix. Given an  $m \times n$  matrix  $A$  with entries between  $-1$  and  $1$ , say, and an error parameter  $\epsilon$  between  $0$  and  $1$ , we find (implicitly) a matrix  $D$  which is the sum of  $O(1/\epsilon^2)$  simple rank  $1$  matrices such that the sum of entries of any one of the  $2^{m+n}$  submatrices of  $A - D$  is at most  $\epsilon mn$  in absolute value. The time taken by our algorithms depends only on  $\epsilon$  and allowed probability of failure (not on  $m$  and  $n$ ).

We draw on two lines of research to develop these algorithms: one is built around the fundamental regularity lemma of Szemerédi in graph theory and the constructive version due to Alon, Duke, Lefmann, Rödl and Yuster, and the second one is from the papers of Arora, Karger and Karpiński, Fernandez de La Vega and, most directly, of Goldwasser, Goldreich and Ron, who developed algorithms for a set of graph problems.

Our matrix approximation implies a great many results, including the above algorithms, the regularity lemma and several other results.

We generalize our approximations to multi-dimensional arrays and from that derive approximation algorithms for all dense Max-SNP problems as well as a constructive version of the regularity lemma for hypergraphs.

## Probabilistically Checkable Proofs and Inapproximability Johan Håstad, Stockholm

By designing an efficient probabilistically checkable proof for an arbitrary  $NP$ -statement we get  $p - \epsilon$  inapproximability result for linear equations mod  $p$ . The result applies to equations with only  $3$  variables in each equation. By reduction we get optimal inapproximability results also for Max- $k$ -Sat for  $k \geq 3$ . We also get improved, but not optimal, results for Max-2-Sat, Max-Cut, Max-Di-Cut, and Vertex Cover.

## On a Random Sphere of Influence Graph Pawel Hitczenko, Raleigh

A *random sphere of influence graph* is constructed as follows. Consider points distributed uniformly and independently in the unit cube of dimension  $d$ . Around each point  $x_i$  draw a sphere (of influence) with radius equal to the distance to the point closest to it, and draw an edge between two points if their spheres of influence intersect.

Asymptotics for the expected number of edges is found, and a sharp concentration of the total number of edges around its expected value is established by extending Azuma's inequality. Also, bounds on the tail of the  $k$ th largest radius are obtained, and used to

prove a bound on the variance of the size of a graph. Most of the talk is based on joint work with T.K. Chalker, A.P. Godbole, J. Radcliff, and O.G. Ruehr.

### An Algorithm For Heilbronn's Problem

Thomas Hofmeister, Dortmund

Heilbronn conjectured that among any  $n$  points in the unit square there are three which form a triangle of area  $O(1/n^2)$ . Komlós, Pintz and Szemerédi proved by a probabilistic argument that this conjecture is false. To be precise, they proved that for every  $n$  there is a configuration of  $n$  points in the unit square such that all triangles have area  $\Omega(\log n/n^2)$ . In this talk, we give a polynomial-time algorithm which for every  $n$  constructs such a configuration of  $n$  points.

We then consider a generalization of this problem due to Schmidt: what is the minimal area of the convex hull of  $k$  of the points? We obtain the following result. For every  $k \geq 4$ , there is a polynomial-time algorithm which on input  $n$  computes  $n$  points in the unit square such that the convex hull of any  $k$  of them has area  $\Omega(1/n^{(k-1)/(k-2)})$ . Schmidt proved the existence of such a configuration for  $k = 4$ .

This is joint work with Claudia Bertram-Kretzberg and Hanno Lefmann from Dortmund.

### New Versions of Suen's Inequality

Svante Janson, Uppsala

We give several new versions of Suen's inequality for the probability that none of a number of dependent events occur. A typical result is that if  $\{B_i\}$  is a collection of events with a dependency graph  $\Gamma$ , then

$$P(\bigwedge \bar{B}_i) \leq e^{-\mu + \Delta c^{2\delta}}$$

where  $\mu = \sum P(B_i)$ ,  $\Delta = \sum_{ij \in E(\Gamma)} P(B_i \wedge B_j)$ ,  $\delta = \sup_i \sum_{j: ij \in E(\Gamma)} P(B_j)$ . Another version, useful when  $\Delta > \mu$ , is

$$P(\bigwedge \bar{B}_i) \leq e^{-\mu^* / \max(8\Delta, 4\mu, 6\delta)}$$

### The Swendsen-Wang Process Does Not Always Mix Rapidly

Mark Jerrum, Edinburgh

The Swendsen-Wang process provides one possible dynamics for the  $q$ -state Potts model (equivalently, the random cluster model) in the ferromagnetic case. It is widely employed in computer simulations, and appears to converge rapidly to equilibrium in situations of practical interest. Nevertheless, a simple example demonstrates that the Swendsen-Wang process may take exponential time (in the size of system) to approach close to the stationary distribution. Absence of rapid mixing is related to the phenomenon of first-order phase transition, which is exhibited by the Potts model when  $q \geq 3$ . The

question of whether the Swendsen-Wang process is rapidly mixing when  $q = 2$  (the Ising model) is open.

This is joint work with Vivek Gore.

## Polynomial Time Approximation Schemes For Some Dense Instances of $NP$ -Hard Optimization Problems

Marek Karpinski, Bonn

We give an overview of a general method for designing polynomial time approximation schemes (or approximation algorithms) for dense instances of many  $NP$ -hard optimization problems including: MAX-CUT, SEPARATOR, BISECTION, MAX-3-SAT, SETCOVER, STEINER TREE, VERTEX COVER, and BANDWIDTH. The unified method begins with the idea of random sampling and the exhaustive placement, and then develops into a new approximation technique for smooth polynomial integer programs. Some other approximation techniques have been developed recently for problems like STEINER TREE, VERTEX COVER, and BANDWIDTH.

## Checking Pseudorandomness of Graphs

Yoshiharu Kohayakawa, Sao Paulo

A simple idea is shown to be applicable in two contexts: (i) in the theory of pseudorandom or quasirandom graphs, as developed by Thomason and Chung, Graham and Wilson, and (ii) on the algorithmic aspects of Szemerédi's regularity lemma as developed by Alon, Duke, Lefmann, Rödl and Yuster.

Roughly speaking, the idea can be stated as follows. Consider, for simplicity, an  $\lfloor n/2 \rfloor$ -regular graph  $G = G^n$  on  $n$  vertices. Then, a necessary and sufficient condition for  $G$  to be a quasirandom graph is that the codegree

$$d(x, y) = |\Gamma(x) \cap \Gamma(y)|$$

of the vertices  $x$  and  $y$  should be approximately equal to  $n/4$  for all pairs  $x, y$  with  $x$  adjacent to  $y$  in a fixed Ramanujan graph  $J$  with  $V(J) = V(G)$ . The graph  $J$  may be taken to be linear-sized, i.e.  $r$ -regular with  $r = o(1)$ .

This idea gives rise to a *deterministic*  $o(n^2)$  time algorithm for checking quasirandomness of graphs  $G = G^n$ , and also gives a  $o(n^2)$  time *deterministic* algorithm for constructing a Szemerédi partition of a given graph  $G = G^n$ .

The results were obtained jointly with V. Rödl.

## Contrast-Optimal $(k, n)$ Secret Sharing Schemes in Visual Cryptography

Matthias Krause, Dortmund

In joint work with Thomas Hofmeister and Hans U. Simon, we study the  $k$  out of  $n$  secret sharing schemes introduced by M. Naor and A. Shamir in 1995.

A sender wishing to transmit a secret message distributes  $n$  transparencies to  $n$  players. If any  $k$  player show their transparencies together, the secret becomes visible. For any



set of  $k - 1$  players absolutely no information can be gained from their transparencies. The important measure of the quality of a scheme is given by the contrast, the relative difference in the grey levels of superpixels corresponding to white and black pixels of the original menage. Naor and Shamir could construct for all  $k$  ( $k, k$ )-scheme of contrast  $2^{-(k-1)}$  which they could prove to be optimal. Using an approach from Coding Theory we can construct  $(2, n)$ -schemes which are optimal w.r.t. to contrast and pixel expansion for all  $n$ . Using a linear programming approach we give an efficient algorithm which computes a contrast optimal  $(k, n)$ -scheme for all  $k \geq n$  from  $N$ .

By means of Approximation Theory we show that the largest possible contrast a  $(k, n)$ -scheme can achieve belongs to the interval

$$\left[ \frac{1}{4^{k-1}}, \frac{1}{4^{k-1}} \frac{n^{k-n}}{(n-1)(n-2)\dots(n-k+1)} \right],$$

and it equals the upper bound if and only if  $k = 2$  or  $3$ .

### Sparse 0-1-Matrices

Hanno Lefmann, Dortmund

In this talk we consider the problem to determine the maximal number  $N(m, k, r)$  of columns in a 0-1-matrix with  $m$  rows, and exactly  $r$  ones per column, such that any  $k$  columns are linearly independent over  $\mathbb{Z}_2$ . For  $k \geq 4$ ,  $k$  even and  $r \geq 2$  the lower bound  $N(m, k, r) = \Omega(m^{\frac{kr}{2(k-1)}})$  is known from work of Pudlák, Savický and Lefmann. Here we sketch a proof that for  $\gcd(k-1, r) = 1$  this lower bound can be improved to  $N(m, k, r) = \Omega(m^{\frac{kr}{2(k-1)}} (\log m)^{1/(k-1)})$ . Furthermore, we construct a polynomial-time algorithm achieving this lower bound. There is still much to be done. For example, for fixed values of  $k$  and  $r$ , we still do not know the correct order of  $N(m, k, r)$ . Also, for applications of the results, explicit constructions are desired.

This is joint work with Claudia Bertram-Kretzberg and Thomas Hofmeister from Dortmund.

### A Remark About Dijkstra's Algorithm on Random Digraphs

Kurt Mehlhorn, Saarbrücken

We present joint work with Andreas Crauser and Uli Meyer.

In the standard implementation of Dijkstra's algorithm *one* vertex is removed from the priority queue in each iteration of the algorithm. This is a bottleneck for parallel implementations and for implementations using external memory. We show that if the edge destinations are random and the edge weights are uniform then in each iteration about  $\sqrt{q}$  vertices can be removed from the queue, where  $q$  is the current size of the queue. This implies that about  $\sqrt{n}$  iterations suffice for Dijkstra's algorithm.

### Graph Colouring with the Probabilistic Method

Mike Molloy, Toronto

In this talk we discuss a simple probabilistic technique and survey several of its applications. The technique is to colour a graph using the following procedure:

1. assign a random colour to each vertex.
2. uncolour every vertex which receives the same colour as one of its neighbours.

Clearly, this will produce a proper partial colouring of the graph. The key step is to prove that with positive probability, this partial colouring will satisfy certain properties which ensure that the colouring can be completed successfully to an appropriate colouring of the entire graph. The aspects of the technique that vary from application to application are: a) exactly what properties we require the partial colouring to have; b) the manner in which we complete the colouring. Two examples of this are the following.

a) We almost always require that each vertex has many neighbours which retain their colours. If the graph has maximum degree  $\Delta$  and we use  $\Delta$  colours, then it is straightforward to compute that the expected number of such neighbours is at least  $\frac{\Delta}{c}$ . Using a standard concentration of probability tool such as Azuma's Inequality or Talagrand's Inequality, we can show that the probability that this number is much smaller than  $\frac{\Delta}{c}$  is exponentially small in  $\Delta$ . Furthermore, this number is independent of the corresponding numbers for all vertices of distance greater than 4. Thus, it is a straightforward application of the Lovász Local Lemma to show that, with positive probability, the subgraph induced by the uncoloured vertices has maximum degree not much bigger than  $\Delta(1 - \frac{1}{c})$ .

b) The most common way to complete the colouring is to repeat the procedure on the uncoloured vertices. It is important that any vertex which retains its colour does not lose it during any subsequent iteration. By ensuring that a significant portion of each neighbourhood gets coloured during each iteration, we can guarantee that after  $O(\log \Delta)$  iterations, the degree of the subgraph induced by the uncoloured vertices is extremely small, and this allows us to complete the colouring in a straightforward manner.

This technique works best when each vertex has relatively few edges within its neighbourhood. For example, this is the case with triangle-free graphs and line graphs. For denser graphs we require a variation of the procedure.

We survey applications of the technique due to Johansson, Kahn, Kim, Reed, and Molloy and Reed, including bounds on the chromatic number of a triangle-free graph, the list chromatic index of a graph, and the total chromatic number of a graph.

## A Large Deviation Principle for the Giant Component in a Sparse Random Graph

Neil O'Connell

Let  $X_n$  denote the size (in vertices) of the largest connected component in the random graph  $G(n, \frac{c}{n})$ , where  $c > 0$ . Erdős and Rényi, in one of their early seminal papers on random graphs (~1960) proved that if  $c \leq 1$ ,  $X_n/n$  converges in probability to 0; if  $c > 1$ ,  $X_n/n$  converges in probability to the positive solution  $a_c$  to the equation  $a = 1 - e^{-ac}$ .

We prove a large deviation principle for the sequence  $X_n/n$ : there exists a function  $I_c : [0, 1] \rightarrow \mathbf{R}_+$  such that

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log P(X_n > xn) = -I_c(x)$$

for  $x > a_c$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log P(X_n < xn) = -I_c(x)$$

for  $x < a_c$ .

(This is also valid for  $c \leq 1$ , setting  $a_c \equiv 0$  in this case.)

We also obtain an explicit expression for the rate function  $I_c$ . If  $c \leq 1$ , there are no surprises, the rate function looks more or less as expected, but for  $c > 1$ , the rate function has a most unusual form.

In particular, if  $c > 1$ ,  $I_c$  is not convex. This means that the usual generating function techniques, based on exponential change of measure, do not apply here. We prove this LDP using a more general change of measure technique where, instead of exponential tilting, we simply vary the parameter  $c$ . The idea is to use the information contained in the Erdős-Rényi laws of large numbers, which hold for each value of  $c$ .

## Induced Ramsey Numbers

Hans Jürgen Prömel, Berlin

We investigate the induced Ramsey number  $r_{\text{ind}}(G, H)$  of pairs of graphs  $(G, H)$ . This number is defined to be the smallest order of a graph  $\Gamma$  with the property that, whenever its edges are coloured red and blue, either there is a red induced copy of  $G$ , or else a blue induced copy of  $H$ . We show that, for any  $G$  and  $H$  with  $k = |V(G)| \leq t = |V(H)|$ , we have

$$r_{\text{ind}}(G, H) \leq t^{ck \log q},$$

where  $q = \chi(H)$  is the chromatic number of  $H$  and  $c$  is a universal constant. Furthermore, we also investigate  $r_{\text{ind}}(G, H)$  under some conditions on  $G$ . For instance, we prove a bound which is polynomial in both  $k$  and  $t$  in the case when  $G$  is a tree. In our proofs we make use of random graphs based on projective planes.

The results were obtained jointly with Y. Kohayakawa from Sao Paulo and V. Rödl from Atlanta.

## A Randomized Algorithm for $k$ -SAT

Pavel Pudlak, Prague

We present a simple algorithm which for a satisfiable  $k$ -CNF produces a satisfying truth assignment in expected time  $2^{n(1-\frac{1}{k})+O(\log n)}$ , where  $n$  is the number of variables.

## Learning Pattern Languages Fast On Average

Rüdiger Reischuk, Lübeck

Patterns are a simple and natural way to generate formal languages. The learning model we consider is exact learning in the limit from positive data (Gold 1967). A sequence of sample strings from the unknown pattern language  $L(\pi)$  is presented to the learner, and after each new sample he has to compute a hypothesis pattern  $\pi'$  such that these hypotheses eventually converge to  $\pi$ .

In contrast to most previous work, our goal is to minimize the total learning time, not just the so-called update time for computing the next hypothesis. A new algorithm for learning one-variable pattern languages is proposed and analyzed with respect to its average-case behaviour. The main technical tool is a careful analysis of the combinatorics of words generated by a 1-variable pattern. For the expectation it is shown that for almost all meaningful distributions defining how the pattern variable is replaced by a string to generate random samples of the target pattern language, this algorithm converges within a constant number of rounds with a total learning time that is linear in the pattern length. Thus, the algorithm is average-case optimal in a strong sense.

This is joint work with Thomas Zengmann, Fukuoka.

## Colourings Generated by Monotone Properties Oliver Riordan, Cambridge

We give an account of some joint work with Béla Bollobás. Let  $Q$  be a (monotone decreasing) class of graphs and let  $G \in Q$ . Colour the edges of  $G$  *black*, and an edge  $e$  of  $G^c$  *blue* if  $G \cup \{e\}$  belongs to  $Q$ , and *red* otherwise. This red-blue-black colouring of the edges of  $K_n$  was recently introduced by de la Vina and Fajtlowicz, and studied for a number of properties. The black subgraph, which is restricted by the property  $Q$ , is usually 'small', i.e. contains no large complete subgraphs. The red-blue colouring is then generated by  $Q$  and the blank edges, and it is this we wish to study. The general questions we are interested in is how 'close to random' this red-blue colouring can be, using the order of the largest monochromatic complete subgraph as a measure.

In their paper, de la Vina and Fajtlowicz answered this question for the property of  $l$ -colourability, showing that there is always a monochromatic clique of order  $l^{-1/2}n^{1/2}$ . They conjectured with Erdős that for  $K_r$ -free graphs, there must be such a clique of order  $n^{1/r}$ . We disprove this conjecture for all  $r \geq 4$ , using random graphs in an unexpectedly complicated way. In fact, for  $r \geq 5$  we show that the largest monochromatic clique can have order  $O(\log n)$  which is best possible by Ramsey's theorem. Since the case ' $K_r$ -free' might be rather special, Erdős asked the same questions for graphs with no 4-cycle, strongly expecting a monochromatic clique of order  $n^\epsilon$ . We disprove this also, obtaining a bound of  $O((\log n)^2)$ , and it is this result which forms the main part of the talk.

Although our proofs are rather complicated, the central idea is fairly simple. We connect an algorithm which performs a series of tasks on the graph, asking if a certain set of edges is present. To analyse the behaviour of this algorithm we use the following straightforward but useful lemma.

Let  $U_1, U_2$  be *independent* up-sets, and  $D$  a down-set in the weighted cube. Then  $\mathbb{P}(U_1|U_2 \cap D) \leq \mathbb{P}(U_1|U_2) = \mathbb{P}(U_1)$ . We note that the independence condition is, perhaps surprisingly, necessary - without it there is a counter-example in two dimensions!

## 1-Factors in $\epsilon$ -Regular Graphs and the Blow-up Lemma Andrej Rucinski, Poznań

Recently Komlós, Sárközy and Szemerédi proved the so-called Blow-Up Lemma: a powerful theorem which allows one to embed a graph of bounded degree into another graph

with the same number of vertices and sufficiently regular structure. This result, even before its explicit formulation, has had numerous applications in graph theory, especially when combined with the Regularity Lemma of Szemerédi.

In the talk we present an alternative proof of the Blow-Up Lemma, obtained jointly with V. Rödl, which relies on simple facts about random 1-factors in  $\epsilon$ -regular graphs.

### The Number of Boolean Functions Computed by Formulas of a Given Size P. Savicky, Prague???

In joint work with A.R. Woods, we give estimates for the number  $B(n, L)$  of distinct functions computed by propositional formulas of size  $L$  in  $n$  variables, constructed using only literals and  $\wedge, \vee$  connectives.

Writing  $B(n, L) = b(n, L)^L$ , we find that if  $L$  and  $\alpha(n)$  go to infinity as  $n \rightarrow \infty$  and  $L \leq 2^n/n^{\alpha(n)}$ , then  $b(n, L) \sim en$ , where  $e = 2/(\ln 4 - 1)$ . For all  $L$  up to maximum complexity, we have  $b(n, L) \geq n/4 \log 2$ . The last result contains an improvement on the bound on maximum complexity given by Luby.

### Construction of Expander Graphs Using Kolmogorov Complexity Uwe Schöning, Ulm

We consider as basic model a bipartite graph with a set  $L$  of  $n$  left vertices and a set  $R$  of  $\lambda n$  right vertices, with each vertex  $x \in L$  having degree  $d$  and with each vertex  $y \in R$  having degree  $d/\lambda$ . This bipartite graph is an  $(\alpha, \beta)$ -expander if every subset  $S$  of  $L$  of size  $\alpha n$  has more than  $\beta \lambda n$  neighbours. Such expanders can be shown to exist (for a subtle choice of  $\alpha, \beta, d$ ) by a probabilistic construction. As an alternative, here we propose an approach using Kolmogorov complexity. The "interconnection pattern" between the left and right vertices is given by some permutation  $\pi$  of  $[dn]$ . Choose  $\pi$  such that  $C(\pi|n, d, \lambda) \geq \log((dn)!) - h$  where  $C$  denotes the conditional Kolmogorov complexity. If the graph  $G = G\pi$  defined is not an expander, then  $\pi$  could be more compactly described than by  $\log((dn)!) - h$  bits. This can be done by splitting  $\pi$  into 2 bijections  $\pi_1, \pi_2$  where  $\pi_1 : A \rightarrow B; A, B \subseteq [dn]$ . For a "split" permutation  $\pi$  we have  $C(\pi|d, \lambda, n, A, B) \leq \log((dn)!) - h(|A|/n) \cdot n + \theta(\log n)$ , where  $h$  is the binary entropy function. Thus we gain  $h(\alpha)n$  bits of entropy. This approach leads to a theorem that says that an expander as described exists if  $d < \frac{h(\alpha) + h(\beta)\lambda}{h(\alpha) - h(\alpha/\beta)\beta}$ . As a further application, we reconsider the construction of superconcentrators and improve the best known density of 36 to about 34.

### Random Graphs Chromatic Decompositions via Squeezing Janson's inequality Eli Shamir, Jerusalem

Consider spaces of random graphs, for example the spaces  $\mathcal{G}_{n,p}$  of random graphs on  $n$ -vertices. Also,  $\{X_i\}_{i \in I}$  is a collection of  $r$ -sets in  $G$ ,  $\{Y_\ell\}_{\ell \in L}$  is a collection of  $(r-1)$  sets in  $G$ ,  $C_\ell =$  is an event defined by the edges and non-edges in  $Y_\ell$ , and  $B_i =$  is the event that  $C_\ell$  occurs for some  $(r-1)$ -set  $Y_{\ell'} \subseteq X_i$ .

Let us assume that the collection  $\{C_\ell\}_{\ell \in L}$  (and hence  $\{B_i\}_{i \in I}$ ) satisfies the conditions

needed for Janson's inequality. The squeezed form we prove is

$$Pr(\bigwedge_{i \in I} \bar{B}_i) \leq \prod Pr(\bar{B}_i) e^{-\frac{1}{2} \hat{\Delta}} \leq e^{-\Gamma + \frac{1}{2} \hat{\Delta}}$$

and

$$Pr(\bigwedge_{i \in I} \bar{B}_i) \leq e^{-\frac{\mu^2}{2\hat{\Delta}}}$$

where  $\mu = \sum_{i \in S} Pr(B_i)$ ,  $\hat{\Delta} = \sum_i \sum_{\ell \sim i} Pr(B_i \wedge C_\ell)$  and  $\ell \sim i$  if  $|X_i \cap Y_\ell| \geq 2$ .

The main advantage of this inequality is that in some applications  $r\hat{\Delta}$  is much smaller than  $\Delta = \sum_i \sum_{s \sim i} Pr(B_i \wedge B_s)$ , the quantity appearing in the (usual) Janson inequalities. Note that

$$\bigwedge_{i \in L} \bar{B}_i = \bigwedge_{\ell \in L} \bar{C}_\ell.$$

We also present an application of this result to the  $Q$ -chromatic numbers of random graphs. For a collection  $Q$  of graphs, the  $Q$ -chromatic number  $\chi_Q(G)$  of a graph  $G$  is the minimal value of  $k$  such that  $V(G) = \cup_{i=1}^k V_i$ , with no  $G[V_i]$  containing a member of  $Q$ . Note that if  $Q$  has only one member then  $\chi_Q(G)$  is precisely the chromatic number  $\chi(G)$ .

In the book of Alon and Spencer,  $\chi(G)$  is computed (with high probability) for random graphs  $G_{n,1/2}$  with the aid of Janson's inequality. Using the squeezed form, we compute  $\chi_Q(G)$  for sparse random graphs  $G_{n,p}$ ,  $p(n) \rightarrow 0$ . All we have to assume is that  $\chi_Q(G) \gg \sqrt{n}$ .

## Hereditarily Extended Properties and Quasi-Random Graphs

### Miklós Simonovits, Budapest

In joint work with Vera T. Sós, we investigate graph properties  $P$  which do not imply quasi-randomness (in the Thomason-Chung-Graham-Wilson sense) but are such that if every subgraph of a graph  $G$  has  $P$  then  $G$  is quasi-random. Some important instances are closely connected to counting subgraphs of a given type. We encounter different phenomena depending on whether our subgraphs are assumed to be induced or not. A typical result is the following.

Let  $L_\gamma$  be a fixed sample graph and let  $(G_n)$  be a sequence of graphs. If for some fixed  $\gamma$  and every induced  $F_n \subseteq G_n$  we have

$$N(L_\gamma \subseteq F_n) = \gamma \cdot h^\nu + o(n^\nu)$$

as  $n \rightarrow \infty$ , then  $(G_n)$  is  $p$ -quasi-random, where  $N(L_n \subseteq F_n)$  is the number of (not necessarily induced) copies of  $L_\gamma$  in  $G_n$  and  $p = p(\gamma)$ .

## Generating $d$ -Regular Graphs Quickly

### Angelika Steger, Munich

In this talk we consider the following problem. Given  $n, d$  generate a (random)  $d$ -regular graph on  $n$  vertices. A first solution to this problem is implicit in the work of Bollobás '80, who provided a proof for the asymptotic number of labelled  $d$ -regular graphs

on  $n$  vertices based on a probabilistic argument. This approach immediately gives rise to a uniform generation algorithm of expected running time  $o(e^{d^2/4}nd)$ . Subsequently, this bound has been improved by various authors. The currently best bound was obtained in a paper by McKay & Wormald '90. We improve this work by showing that the original algorithm based on Bollobás approach can be modified in order to obtain a fast and easy to implement algorithm which generates  $d$ -regular graphs almost uniformly. The analysis of our algorithm is heavily based on various concentration inequalities for random graphs.

This is joint work with Nick Wormald.

## **Inequalities for Means of First-Passage Times in Percolation Theory**

**John C. Wierman, Baltimore**

First-passage percolation theory was introduced by Hammersley and Welsh in 1963. The setting is an infinite graph, such as the square lattice, in which every edge has a non-negative random travel time. The travel time of a path is the sum of travel times of the edges in the path. The object of study is the first-passage time between two points, i.e. the shortest travel time over any path between the two points. Asymptotic properties of first-passage times have been studied for many years and much has been learned. However, little is known about first-passage times between points that are close to each other. In fact, a counterexample disproves the natural conjecture that an expected first-passage time is a nondecreasing function of distance between the points. The talk introduces an inequality between sums of pairs of first-passage times, which can be used to establish some monotonicity, convexity, and concavity properties of means of first-passage times. These results are joint work with Sven Erick Alm of Uppsala University.

## **Peter Winkler, ???**

we model physical systems with "hard" constraints by random homomorphisms from an infinite graph  $G$  to a fixed finite constraint graph  $H$ , whose nodes are equipped with positive real "activities"  $\lambda_1, \dots, \lambda_n$ . When  $G$  is the infinite, regular  $k$ -branching tree  $T_k$ , the simple, invariant Gibbs measures on  $\text{Hom}(G, H)$  correspond to node-weighted branching random walks on  $H$ .

We characterize the graphs  $H$  which exhibit multiple phases, that is, those  $H$  for which there are different node-weighted random walks yielding the same activities  $\lambda_1, \dots, \lambda_n$ .

Joint work with Graham Brightwell, London.

## **The Degree Sequence of a Random Graph**

**Nick Wormald, Melbourne**

This is joint work with Brendan McKay of the Australian National University in Canberra. A model of the degree sequence of a random graph in  $\mathcal{G}(n, p)$  was obtained by us recently. This model relates the degree sequence to the sequence of integers obtained from independent binomial distributions subject to having even sum and averaged to a certain effect. This makes many previous results on the degree sequence much easier to obtain, to extend greatly, to make more precise, and to explain in terms of the difference

between the degree sequence model and the truly independent binomial model. One of the main steps is to obtain the difference between independent binomials and the conditional space in which the sum is even. In many interesting cases we show that the difference is negligible. The results depend on asymptotic enumeration results for graphs with a given degree sequence, for which there is a gap when the average degree is between  $(o\sqrt{n})$  and  $cn/\log n$ . We have a conjectured asymptotic formula for this range which implies that our model for degree sequence of  $G \in \mathcal{G}(n, p)$  is correct when  $o(1/\sqrt{n}) < p < c/\log n$ . For other values of  $p$  we have proved the formula, and hence the model, correct.



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