MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 40/1997

Nonlinear Systems, Solitons and Geometry

19.10. bis 25.10.1997

This conference was the fourth in a series being held in Oberwolfach. It was organized by Professors M. Ablowitz (Boulder), B. Fuchssteiner (Paderborn), M. Kruskal (Princeton) and V. Matveev (St. Petersburg).
The participants presented their most recent work in the meeting. This and the marvelous surrounding again created a lively scientific atmosphere with many stimulating discussions which certainly will influence future directions and will contribute to further progress in the field of integrable non-linear systems and their applications.
The lecture program covered a broad range of topics in this diverse area. In particular, many contributions dealt with the remarkable relation between integrability and classical geometry.

VORTRAGSAUSZÜGE

M.J. ABLOWITZ: *On "Reflectionless" Potentials of the Time-Dependent Schrödinger Equation and Solutions of the Kadomtsev-Petviashvili Equation*

It was shown some years ago that the time-dependent Schrödinger equation has a class of real nonsingular rational potentials which are analogues of the reflectionless potentials of the time independent Schrödinger equation. These potentials give rise to the lump solutions of the Kadomtsev-Petviashvili-I (KP-I) equation. It turns out that there is a much larger class of such potentials. They have a definite spectral interpretation and are related to a topological number; i.e. an index or charge. These potentials are real and non-singular and are related to an interesting class of solutions of the KP-I equation.

A.I. BOBENKO: *Discrete elastic curves and spinning top*

Elastic curves are extremals of the functional

$$\mathcal{L} = \int (k^2 + \alpha\, \tau^2) dx, \quad \alpha = const ,$$

where k, τ are the curvature and the torsion of the framed curve. The Kirchhoff kinetic analogue theorem claims that the frame of an elastic curve describes the rotation of a symmetric spinning top (one should treat the arclength parameter of the curve as the time variable).

In the talk all these notions are discretized preserving the integrability. In particular, the discrete evolution $T : \mathbb{Z} \to S^2$ of the axis of the discrete spinning top is given by

$$2\,T_n \times \left(\frac{T_{n+1}}{1 + \langle T_n, T_{n+1} \rangle} + \frac{T_{n-1}}{1 + \langle T_n, T_{n-1} \rangle} \right) + c \left(\frac{T_{n+1} + T_n}{1 + \langle T_n, T_{n+1} \rangle} - \frac{T_n + T_{n-1}}{1 + \langle T_n, T_{n-1} \rangle} \right) = T_n \times a$$

with some constants $c \in \mathbb{R}$, $a \in \mathbb{R}^3$. T_n is simultaneously the edge of of the corresponding discrete elastic curve. The corresponding Lagrangians are derived. The construction is based on the Ablowitz-Ladik hierarchy of the smoke-ring evolution of discrete curves by Doliwa and Santini.

M. BOITI (JOINT WORK WITH F. PEMPINELLI, A. POGREBKOV AND WITH THE PARTICIPATION OF B. PRINARI IN THE FRAMEWORK OF A PH. D. THESIS) *Solving the Kadomtsev-Petviashvili equation with initial data not vanishing at large distances*

We consider, in the framework of the inverse scattering method, the solution of the Kadomtsev-Petviashvili equation in its version called KP-I. The spectral theory is extended, by using a new approach called resolvent approach, to the case in which the initial data $u(x,y)$ are not vanishing along a finite number of directions at large distances in the plane. Special attention is paid to the solutions describing N solitons on a generic background. In order to explore the analytic properties of the Jost solutions and the structural properties of the spectral data both solutions built using Bäcklund transformations and the dressing method are considered. In the first case, if only one soliton is present, all quantities, including the so called extended resolvent, are explicitly computed by using the Darboux transformations. In the second case it is shown that the Jost solution can have in addition to the traditional cut on the real axis of the complex spectral plane a cut connecting the discrete value of the spectrum and its complex conjugate.

[1] M. Boiti, F. Pempinelli and A. Pogrebkov, Journal of Math. Phys. 35 (1994) 4683.

[2] M. Boiti, F. Pempinelli and A. Pogrebkov, Physica D 87 (1995) 123.

[3] M. Boiti, F. Pempinelli and A. Pogrebkov, Inverse Problems 13 (1997) L7.

L. BORDAG (JOINT WORK WITH M.V. BABICH): *Projective differential geometrical structure of the Painlevé equations*

The necessary and sufficient conditions that an equation of the form $y'' = f(x,y,y')$ can be reduced to one of the Painlevé equations under a general point transformation are obtained. A constructive procedure to check these conditions is found. The theory of invariants plays a leading part in this investigation. The reduction of all six Painlevé equations to the form $y'' = f(x,y)$ is obtained.
Following Cartan the space of the normal projective connection which is uniquely associated with a class of equivalent equations is considered. The specific structure of the spaces under investigation allow to immerse them into RP^3. Each such immersion generates a triple of two-dimensional manifolds in RP^3. Those surfaces corresponding to the Painlevé equations are presented.

M. BORDEMANN (JOINT WORK WITH J. HOPPE): *Integrable Hypersurface Motions in Riemannian Manifolds*

Let (Σ, ϱ) be an oriented compact manifold of dimension $M \geq 1$ and $(\mathcal{N}, \eta)$ an orientable Riemannian manifold of dimension $M+1$ with Riemannian metric η. We consider hypersurface motions (of codimension 1) described by one parameter families of immersions $x_t : \Sigma \to \mathcal{N}$ satisfying the first order partial differential equation

$$\frac{\partial x_t}{\partial t} = \alpha\Big(\frac{\sqrt{g[x_t]}}{\varrho}\Big)\, n[x_t] \qquad (*)$$

where $n[x_t]$ is the surface normal, α is a diffeomorphism of an open interval of the positive real line onto an open interval of the positive real line, and $\sqrt{g[x_t]}$ is the Riemannian volume of the pulled-back metric $x_t^*\eta$. Assuming that for short times there is an open neighbourhood $\mathcal{N}_\epsilon$ of $x_0(\Sigma)$ in $\mathcal{N}$ foliated by $x_t(\Sigma)$ we derive a second order partial differential equation for the corresponding time function $\tau : \mathcal{N}_\epsilon \to \mathbb{R}$ which equals t on $x_t(\Sigma)$. For $\alpha(z) = z$ one finds that τ is harmonic

$$\Delta\tau = 0 \,, \qquad (**)$$

hence equation $(*)$ is linerizable in the sense $(**)$ for this particular case. A reconstruction of a solution x_t to $(*)$ from a given time function (obeying its PDE) is possible (for short times, assuming a nice foliation) by considering the flow of $\nabla\tau/\eta(\nabla\tau, \nabla\tau)$, $\nabla\tau$ being the gradient of τ. If $(\mathcal{N}, \eta)$ is flat $\mathbb{R}^{M+1}$, then $(*)$ for $\alpha(z) = z$ takes the form

$$\frac{\partial x^i}{\partial t} = \frac{1}{M!} \sum_{i_1,..,i_M=1}^{M+1} \sum_{r_1,..,r_M=1}^{M} \epsilon_{i i_1 \ldots i_M} \epsilon^{r_1 \ldots r_M} \frac{\partial x^{i_1}}{\partial \varphi^{r_1}} \cdots \frac{\partial x^{i_M}}{\partial \varphi^{r_M}}$$

($1 \le i \le M+1; \varphi^1, \ldots, \varphi^M$ being coordinates on Σ) and is integrable.

F. CALOGERO: *Some Recent Results On Integrable Dynamical Systems*

Some recent results on integrable dynamical systems have been reviewed. The presentation has focused i) on certain solvable dynamical systems *in the plane* which display a very rich phenomenology, and ii) on certain integrable Hamiltonian systems "of Ruijsenaars type" whose trajectories are *all* completely periodic.

[1] F. Calogero: "A solvable n-body problem in the plane. I", *J. Math. Phys.* **37**, 1735–1759 (1996); F. Calogero: "Motion of strings in the plane: a solvable model", *J. Math. Phys.* **38**, 821–829 (1997); F. Calogero: "Three solvable many-body problems in the plane", *Acta Appl. Math.* (in press).

[2] F. Calogero: "A class of integrable Hamiltonian systems are (perhaps) all completely periodic", *J. Math. Phys.* (in press); F. Calogero: "Tricks of the trade: relating and deriving solvable and integrable dynamical systems", to be published in the *Proceedings* of the International Workshop on "Calogero-Moser-Sutherland Models" held at the Université de Montréal in March, 1997; F. Calogero and J.-P. Francoise: "Solution of certain integrable dynamical systems with completely periodic trajectories", *Commun. Math. Phys.* (submitted to).

P.A. CLARKSON: *Symmetry reductions and exact solutions of generalized Camassa-Holm equations*

In this talk I shall discuss symmetry reductions and exact solutions of the generalized Camassa-Holm equations

$$u_t - \epsilon u_{xxt} + 2\kappa u_x = u u_{xxx} + \alpha u u_x + \beta u_x u_{xx} \,, \qquad (1)$$

$$u_{tt} - \epsilon u_{xxtt} = u u_{xxxx} + \alpha u_x u_{xxx} + \beta u_{xx}^2 + (\gamma u^2 + \kappa u)_{xx} \qquad (2)$$

with α, β, γ, κ and ϵ constants.
Three special cases of equation (1) have appeared in the literature, which all possess unusual travelling wave solutions. The Fornberg-Whitham equation (for $\epsilon = 1$, $\alpha = -1$, $\beta = 3$, $\kappa = 1/2$), admits a wave of greatest height, as a peaked limiting form of the travelling wave solution; the Rosenau-Hyman equation ($\epsilon = 0$, $\alpha = 1$, $\beta = 3$, $\kappa = 0$), admits a "compacton" solitary wave solution; and the Fuchssteiner-Fokas-Camassa-Holm equation ($\epsilon = 1$, $\alpha = -3$, $\beta = 2$), has a "peakon" solitary wave solution.
Equation (2) is a "Boussinesq–type" equation which arises as a model of vibrations of an anharmonic mass–spring chain and admits both "compacton" and conventional solitons.
A catalogue of symmetry reductions and exact solutions for equations (1) and (2) is obtained using the classical Lie method and the nonclassical method due to Bluman and Cole. In particular we obtain several reductions using the nonclassical method which are *not* obtainable through the classical method.

A. CONSTANTIN: *Some Aspects of a Shallow Water Equation*

We consider the periodic problem for the Camassa-Holm-Fokas-Fuchssteiner equation modelling waves on shallow water. We discuss integrability related questions as well as some structural properties of the model disclosed by global existence and blow-up results of this quasilinear hyperbolic PDE.

A. DOLIWA: *Geometric constructions and integrable multidimensional lattices*

In this lecture I review recent results of the geometric theory of the discrete multidimensional integrable systems from the point of view of the linear constructibility of the corresponding lattices. Plan of the talk:

1. Linear constructibility scheme of the Multidimensional Quadrilateral Lattices (MQLs).
2. Quadratic reductions of the MQLs.
3. Line congruences and transformations of the lattices.

E.V. FERAPONTOV: *Surfaces in Lie sphere geometry and the stationary Davey-Stewartson hierarchy*

We introduce two basic invariant forms which define generic surfaces in 3-space uniquely up to Lie sphere equivalence. Two particularly interesting classes of surfaces associated with these invariants are considered, namely, the Lie minimal surfaces and the diagonally-cyclidic surfaces. For diagonally-cyclidic surfaces we derive the stationary modified Veselov-Novikov equation, whose role in the theory of these surfaces is similar to that of Calapso's equation in the theory of isothermic surfaces. Since Calapso's equation itself turns out to be related to the stationary Davey-Stewartson equation, these results shed some

new light on differential geometry of the stationary Davey-Stewartson hierarchy. Diagonally-cyclidic surfaces are the natural Lie sphere analogues of the isothermally-asymptotic surfaces in projective differential geometry for which we also derive the stationary modified Veselov-Novikov equation with the different real reduction.
Parallels between invariants of surfaces in Lie sphere geometry and reciprocal invariants of hydrodynamic type systems are drawn in the conclusion.

A.P. FORDY: *Commuting Hamiltonians and systems of hydrodynamic type*

In this seminar I shall exhibit a surprising relationship between separable Hamiltonians and integrable, linearly degenerate systems of hydrodynamic type. This gives a new way of obtaining the general solution of the latter. Our formulae also lead to interesting canonical transformations between large classes of Stäckel systems.
I shall then consider a class of non-homogeneous systems of hydrodynamic type:

$$q^i_t = v^i_j(q) q^j_x + \varphi^i(q), \quad i = 1, \dots, n,$$

which can be related to quadratic Hamiltonians with electromagnetic terms. Whilst it is unlikely that these systems are *generally* integrable, they do possess intriguing properties, such as having a higher conservation law and a $2n$-parameter family of exact solutions. In fact these systems coincide with those possessing a conservation law with the density $\mathcal{L}$ being a quadratic expression in the first derivatives:

$$\mathcal{L} = \frac{1}{2} \sum_{i,j=1}^{n} g_{ij} q^i_x q^j_x + \sum_{k=1}^{n} A_k q^k_x - h.$$

The corresponding $2n$-parameter family of exact solutions are just the stationary points of this integral. There are several examples, some of which have important applications.
These results are joint work with E.V. Ferapontov [1,2].

[1] E.V. Ferapontov and A.P. Fordy. Separable Hamiltonians and integrable systems of hydrodynamic type. *J. Geom. and Phys.*, 21:169–82, 1997.

[2] E.V. Ferapontov and A.P. Fordy. Nonhomogeneous systems of hydrodynamic type related to velocity dependent quadratic Hamiltonians. *Physica D*, 1997.

B. FUCHSSTEINER: *The Camassa-Holm Equation*

By compatibility, splitting of the KdV-recursion operator leads to the hereditary recursion operator $\Phi = \Phi_1 \Phi_2^{-1}$, $\Phi_1 = DuD^{-1} + u + 2k$, $\Phi_2 = D^2 - I$, of the so called "factored KdV" $u_t = \Phi(u) u_x$. This equation is a Bäcklund transformation $u = v - v_{xx}$ away from the Camassa-Holm equation. It is shown how this factorization leads to the isospectral formulation in a straightforward way. Furthermore, how intertwining of this isospectral formulation gives rise to the hodograph link to the (-1)-st generalization of the KdV. This hodograph link then allows for master symmetries and a $(2+1)$-dimensional generalization. An abundance of open problems is presented along the lines of this derivation of the Camassa-Holm equation.

B. GRAMMATICOS (COLLABORATIVE WORK WITH A. RAMANI): *Discrete Painlevé Equations: A Self-Dual Approach*

The discrete forms of the Painlevé equations can be derived systematically using the method of singularity confinement which plays the role of the Painlevé property for discrete systems. The application of this method results in difference or q-equations involving their full complement of free parameters. A detailed analysis of these most general forms of the discrete (difference) Painlevé equations (d-P) reveals the property of self-duality: the same equation describes the evolution along the discrete independent variable and along the parameters (the latter induced by the Schlesinger's of the d-P). An explanation of the self duality can be sought in the relation of the difference Painlevé equations ($q - P_{VI}$ being a notable exception). Self-duality can be used to construct the basis for a geometrical classification of discrete Painlevé equations.

P.G. GRINEVICH (COLLABORATIVE WORK WITH M.U. SCHMIDT): *Conformal invariant functionals of immersions of tori into* $\mathbb{R}^3$.

We show, that higher analogues of the Willmore functional, defined on immersions $M^2 \to \mathbb{R}^3$, where M^2 is a two-dimensional torus, are invariant under conformal transformations of $\mathbb{R}^3$. This hypothesis was formulated recently by I. A. Taimanov.

Higher analogues of the Willmore functional are defined in terms of the modified Novikov-Veselov hierarchy, associated with the zero-energy scattering problem for the two-dimensional Dirac operator.

J. HIETARINTA: *Singularity Confinement, Degree Growth, and the Identification of Integrable Maps*

One important problem in the study of difference equations is to identify the integrable ones. For differential equations we have the Painlevé test, which works very well (although passing the test is not rigorously equivalent to integrability). For difference equations an analogue, the "singularity confinement test", was provided in [1]. The idea is as follows: From certain initial values one may end up in an ill-defined situation, such as $\infty - \infty, 0 \cdot \infty$ etc. One should then study the behavior around these initial values, and if one can continue past the "singularity" in a finite number of steps, without losing information, then the system is said to pass the test.

We point out that singularity confinement is not sufficient for integrability: sometimes even chaotic maps pass the test. As a particular example we have $x_{n+1}+$

$x_{n-1} = x_n^{-2} + x_n$, which shows numerical chaos. Rational maps are best studied as polynomial maps in projective space. Introducing $x_n = u_n/f_n, x_{n-1} = v_n/f_n$ we can write the above map as $(u, v, f) \to (u^3 + f^3 - u^2 v, u^3, f u^2)$. The degree growth of such maps is usually exponential, but the conjecture is that for integrable maps the growth is only polynomial due to cancellation of common factors (projectivization) [2]. For the above map we have

$$\lim_{n\to\infty} \frac{1}{n} \log(\text{degree of } n^{th} \text{ iteration}) = \log\left(\frac{3+\sqrt{5}}{2}\right) .$$

The conclusion is that the singularity confinement test should be accompanied with a study of degree growth before predictions about integrability are made. This work was done in collaboration with C.M. Viallet.

[1] B. Grammaticos, A. Ramani and V. Papageorgiou, Phys. Rev. Lett. **67**, 1825 (1991)

[2] G. Falqui and C.M. Viallet, Commun. Math. Phys. **154**, 111 (1993)

N. JOSHI (COLLABORATIVE WORK WITH M.D. KRUSKAL): *A Natural Sum of Divergent Asymptotic Series*

Divergent asymptotic series commonly occur in the analysis of solutions of differential equations near an irregular singular point. Often such series hide exponentially small terms which are essential for identifying the solution uniquely. Here we show how a new extension of conventional asymptotics allows a unique identification of such a solution.

A.V. KITAEV: *Isomonodromy approach to periodic solutions of the Ernst equation*

We present a construction of periodic solutions of the Ernst equation in terms of isomonodromy deformations of a linear 2×2 matrix ODE of the spectral parameter. Its coefficients are meromorphic functions of the spectral parameter. It is shown that in the static case this construction reduces to the periodic analogue of the Schwarzschild solution constructed by R.C. Myers (1987) and D. Korotkin & H. Nicolai (1994).

B.G. KONOPELCHENKO: *Induced surfaces, their integrable dynamics and application*

A new method of analysis of surfaces and their properties is discussed. The method is based on a construction of surfaces in $\mathbb{R}^3$ via the two-dimensional linear equations and generation of their deformations via 2 + 1-dimensional nonlinear integrable equations. This method allows us to analyse surfaces and deformations using explicit formulae taking advantage of the knowledge of 2 + 1-dimensional integrable equations. Two basic examples are considered.

The first is given by a generalized Weierstraß formula which allows to construct any surface in $\mathbb{R}^3$ both locally and globally starting with a system of two linear equations (the Davey-Stewartson-II linear problem). The corresponding formulae define conformal immersions of surfaces in $\mathbb{R}^3$. Integrable deformations in this case are generated by the modified Veselov-Novikov equation. A characteristic feature of such deformations is that they preserve the Willmore functional (total squared mean curvature) as well as an infinite set of other functionals. The Clifford torus is a stationary point of these deformations. Lelieuvre's formula in affine geometry is the second example. Deformations of affine conormal of a surface via the Nizhnik-Veselov-Novikov equation generate integrable deformations of affine surfaces. These deformations preserve the total affine mean curvature. Affine spheres are stationary points of such deformations. A particular class of deformations is governed by the Korteweg-de Vries equation. Applications of the method of induced surfaces and their integrable deformations in string theory, the theory of liquid biological membranes and other fields are discussed.

M. LAKSHMANAN: *Geometrical Interpretation of* $(2+1)$ *Dimensional Integrable Nonlinear Evolution Equations and Localized Solutions*

A large class of $(1+1)$ dimensional equations of AKNS type as well as spin equations can be associated with moving space curves or with surfaces in E^3, thereby giving a simple but effective geometrical interpretation of these equations. In this lecture, it will be pointed out that by extending the formalism an important class of $2+1$ dimensional integrable nonlinear evolution equations can also be interpreted as equations of motion of moving space curves but endowed with an extra spatial variable or equivalently in terms of moving surfaces (in orthogonal coordinates). Topological conserved quantities naturally follow as geometrical invariants. Underlying evolution equations are shown to be equivalent to a triad of linear equations. Geometrical equivalence between a class of $2+1$ dimensional spin equations such as Myrzakulov equations and Ishimori equation with Zakharov-Strachan and Davey-Stewartson equations, respectively, will be brought out. Special localized solutions of some of these systems will also be reported.

S.B. LEBLE: *Elementary and binary generalized gauge transformations at differential rings geometry*

Elementary gauge transformations (with spectral parameter) of covariant Zakharov-Shabat operators on differential rings and modules factorize Darboux transformations. The form of such transformations is introduced and investigated in general and for Schlesinger versions. Reductions are discussed. Binary transformations that correspond to the iteration of elementary ones with special choices of solutions of the Zakharov-Shabat problem and its conjugate are introduced. They widen the class of possible reductions. We exhibit an infinitesimal version of the transform. The geometrical meaning of the constructions via Darboux surfaces of the corresponding Lie group and generalizations for rings is discussed. A generalization of Bianchi-Lie transformation formulas is derived. Applications to the spectral theory of operators and to soliton theory are outlined. The example of N-wave interaction equation and its generalization to rings is discussed. Soliton solutions and its infinitesimal deformations are analyzed. It widens the class of initial conditions and allows to check stability.

Yi Li (in collaboration with P. Olver): *Convergence of solitary-wave solutions of a perturbed bi-Hamiltonian system*

This lecture is concerned with the bi-Hamiltonian system

$$U_t + \nu U_{xxt} = \alpha U_x + \beta U_{xxx} + 3\gamma U U_x + \gamma\nu (U U_{xxx} + 2 U_x U_{xx}) \qquad (1)$$

as a model to generalize the Korteweg-de Vries equation. However, different from the KdV equation, the perturbed system possesses not only analytic travelling wave solutions, but also weak, non-analytic solitary-wave solutions, called peakons and compactons. To understand singularity formations of these weak solutions, we investigated the dynamical system governing travelling wave solutions, showing that the nonlinear dispersion term UU_{xxx} has generated a singularity in the dynamical system, at which the compacton occurs. In addition, we extended real-analytic solitary-wave solutions to functions defined on the complex plane, showing that they are analytic functions except for countably many branch points and branch lines. We demonstrated that these analytic functions converge to the functions whose restrictions to the real line are peakons or compactons. Moreover, branch points of the analytic functions also approach singularities of peakons or compactons in the process of convergence. This fact has also been used to explain why peakons and compactons are weak solutions of the system (1).

W.X. Ma: *Graded Symmetry Algebras of Evolution Equations in 2+1 Dimensions*

Integrable hierarchies of evolution equations in $2+1$ dimensions are presented from Gelfand-Dikij spectral problems, which include the KP hierarchy and the modified KP hierarchy as examples. Associated with each such hierarchy, there exist infinitely many hierarchies of master symmetries which constitute a graded Lie algebra. On the other hand, starting from graded Lie algebras, time-dependent evolution equations, which may involve a group of arbitrary time functions, are analyzed and their graded symmetry algebras are given. The basic tool adopted is the Lax operator method, which may also be applied to higher dimensional cases.

F. Magri (collaborative work with G. Falqui and G. Tondo): *Separation of variables for bi-Hamiltonian systems: a concrete example*

The aim of this talk is to emphasize the role of the Hamiltonian structures in the problem of constructing separable coordinates for a given Hamiltonian flow.

We shall consider a specific example: the stationary flows of the Boussinesq hierarchy. They are defined as follows. In the space of third–order differential operators $Q = \partial_x^3 + u(x)\partial_x + v(x)$, where $u(x)$ and $v(x)$ are two arbitrary periodic functions of the space coordinate x, one considers the submanifold M of the operators obeying the non linear constraint $[Q, (Q^{\frac{4}{3}})_+] = 0$, where $(Q^{\frac{4}{3}})_+$ is the differential part of the fourth power of the cubic root of Q (in the algebra of formal pseudo–differential operators). One can prove that M is finite–dimensional: $\dim M = 8$. On M one considers the first three non trivial flows of the Boussinesq hierarchy, defined by the equations $\partial Q/\partial t_j = [Q, (Q^{\frac{j}{3}})_+]$ for $j = 1, 2, 5$.
Our aim is to prove that these equations can be solved by separation of variables and that the separable coordinates can be explicitly computed by carefully using the bi-Hamiltonian structures of these equations. In fact, on the manifold M, one can construct a linear pencil of Poisson brackets $\{F, G\}_\lambda = \{F, G\}_0 + \lambda\{F, G\}_1$ with the following properties:

1. it has two Casimir functions $H(\lambda) = H_1\lambda + H_2$ and $K(\lambda) = H_3\lambda^2 + H_4\lambda + H_5$;
2. the functions H_1 and H_3 are Casimir functions of $\{F, G\}_1$, the functions H_2 and H_5 are Casimir functions of $\{F, G\}_0$;
3. the functions H_2,H_4 and H_5 are the Hamiltonian functions, with respect to the first Poisson bracket $\{F, G\}_1$, of the three vector fields under scrutiny;
4. the functions H_1, H_3, H_4 are their Hamiltonian functions with respect to the second Poisson bracket $\{F, G\}_0$;
5. all these functions are in involution with respect to all brackets of the Poisson pencil.

This bi-Hamiltonian structure of the Boussinesq flows can be used to integrate the equations of motion according to the following scheme.
First one considers a level surface S of the functions H_1 and H_3. It is a six–dimensional symplectic leaf of the Poisson bracket $\{F, G\}_0$. The given vector fields are tangent to S, so they can be restricted to this manifold. Then one observes that both Poisson brackets $\{F, G\}_0$ and $\{F, G\}_1$ naturally reduce to S (by a Marsden–Ratiu reduction procedure). So one can conclude that S is a Poisson–Nijenhuis manifold, i.e., S is a manifold endowed with a symplectic form ω (induced by $\{F, G\}_0$) and with a compatible Nijenhuis tensor N (induced by $\{F, G\}_1$).
The final step is to construct the Darboux coordinates defined on S. It is a general result [1] that on any Poisson–Nijenhuis manifold of dimension $2n$, there exist canonical coordinates $(\lambda_j; \mu_j)$ such that $\omega = \sum_{j=1}^n d\mu_j \wedge d\lambda_j$ and $N^* d\lambda_j = \lambda_j d\lambda_j$, $N^* d\mu_j = \lambda_j d\mu_j$. The coordinates λ_j are the roots of the minimal polynomial of N; the coordinates μ_j are the values, at the points $\lambda = \lambda_j$, of a "conjugate" polynomial $\mu = f_0\lambda^n + f_2\lambda^{n-1} + \ldots + f_n$, where f_i, $i = 0, \ldots, n$, are suitable chosen functions. In this case, we have explicitly computed the Darboux coordinates for the stationary Boussinesq flows, and we

have shown that the Hamilton–Jacobi equation associated with the Hamiltonian functions (H_2, H_4, H_5) can be solved by separation of variables in the Darboux coordinates.

In our opinion these flows provide a good example of the tight connection between separable coordinates and the bi-Hamiltonian structure of soliton equations.

[1] F. Magri, T. Marsico, *Some developments of the concepts of Poisson manifolds in the sense of A. Lichnerowicz,* Gravitation, Electromagnetism and Geometrical Structures, (G. Ferrarese, ed.), Pitagora editrice, Bologna 1996, pp. 207–222.

V.B. MATVEEV: *Darboux Transformations in Associative Rings and Functional-Difference Equations*

We formulate and prove some general covariance theorems for a certain class of functional-difference or functional-differential multi-dimensional equations. These theorems are not yet sufficiently explored. They include as a special case the previous results of the author concerning the extension of the original results by Darboux and Crum to the case of nonabelian partial derivative, differential-difference and difference-difference evolution equations. We briefly describe the application to the Hirota-like bilinear lattice equations and some other nonlinear integrable problems.

A.V. MIKHAILOV: *Towards classification of 2+1 dimensional integrable equations: the symmetry approach*

During the last 18 years the Symmetry Approach suitable for $(1 + 1)$ dimensional nonlinear partial differential equations and difference differential equations has been created and developed. It proved to be a powerful tool for testing the integrability and solving the classification problem for integrable equations. In the current work we try to extend the above theory to the $2 + 1$ dimensional case. The main feature of integrable quations in $2 + 1$ dimension is that the equations themselves, their higher symmetries and conservation laws are nonlocal, and that is the main obstacle for a straightforward extension of the $1 + 1$ dimensional approach. To overcome this problem, a new concept of quasi-local functions which are a natural generalization of local functions is introduced. All known integrable equation and its hierarchies of symmetries and conservation laws can be described in terms of quasi-local functions. This observation will be exploited for an extension of the Symmetry Approach to multi-dimensions, creating integrability tests, and a classification of the most important types of equations. Some results have already been obtained this way. A few first integrability conditions for scalar equations based on the concept of quasi-local functions have been found. A few first classification results for Davey-Stewartson type equations, based on existence of one extra symmetry, have been recently obtained.

T. MIWA: *Solvable lattice models and representation theory of infinite dimensional algebras*

I explain two topics from the semi-infinite construction of the highest weight modules of the affine Lie algebra $\widehat{sl}_2$ and its q-deformation. One is on the identification

$$\cdots \otimes \mathbb{C}^3 \otimes \mathbb{C}^2 \otimes \mathbb{C}^3 \otimes \mathbb{C}^2$$

with

$$\oplus_{i,j=0,1} V(\Lambda_i) \otimes V(\Lambda_j).$$

Here the latter is the tensor product of the level one highest weight modules. This is used for solving the integrable quantum spin chain with mixed spins $(1/2, 1)$.

The other is on polynomial identities related to the higher level characters for $\widehat{sl}_2$. In one side of the identities we count the number of bases of given weight and degree, and in the other side, the number of symmetric polynomials with certain conditions.

In both problems the energy functions in the crystal base theory play an essential role.

P. VAN MOERBEKE: *Random Matrices And Soliton Equations*

The spectrum of very large random matrices provide a model for excitation spectra of heavy nuclei at high excitations (Wigner). The analysis of nuclear experimental data has shown "spectral rigidity" and "level repulsion". The distribution of the spectrum of large random matrices have come up in the spacing of the zeros of the Riemann ζ-function. It is also believed that the quantum version of chaotic dynamical systems leads to the same "spectral rigidity", typical of random matrices.

What is the connection of random matrices with integrable systems? Is this connection really useful? Introduce an appropriate time $t = (t_1, t_2, \ldots)$-dependence in the probability distribution

$$c_N \, e^{-TrV(M)+\sum t_i TrM^i} \, dM\,,$$

where dM is a Haar measure on the space of $N \times N$ Hermitian matrices M_N. The probability that N eigenvalues belong to the disjoint union $E = \cup_1^r [A_{2i-1}, A_{2i}] \subset \mathbb{R}$ is a ratio of functions

$$P(\text{spectrum } M_N \in E) = \frac{\tau_N(E,t)}{\tau_N(t)}$$

which satisfy

(i) $\tau_N(t)$ and $\tau_N(E,t)$ are solutions of the KP-Equation.

(ii) $\tau = (\tau_N(H))_{N\geq 0}$ is a τ-vector of the Toda lattice on tridiagonal symmetric matrices.

(iii) Given $-V' = \frac{h}{f} = \frac{\sum_{i\geq 0} b_i \zeta^i}{\sum_{i\geq 0} a_i \zeta^i}$ rational, $\tau(t)$ and $\tau(E,t)$ satisfy the following Virasoro constraints

$$\sum_i \left(a_i \mathbb{J}^{(2)}_{i+m} - b_i \mathbb{J}^{(2)}_{i+m+1}\right) \tau = 0 \,,$$

$$\left(\sum_i \left(a_i \mathbb{J}^{(2)}_{i+m} - b_i \mathbb{J}^{(2)}_{i+m+1}\right) - \sum_i A_i^{m+1} f(A_i) \frac{\partial}{\partial A_i}\right) \tau(E,t) = 0 \,,$$

where, in the last expression, the time-part and the boundary-part decouple.

Given some extra-conditions on V', it is possible to express the t-partials in terms of A-partials of τ. Putting these expressions into the KP equation leads to a "non-commutative" KP hierarchy. In the special case where $E = (A, \infty)$, the first equation of the hierarchy is one of the Painlevé equations.
The methods can be extended to capture symmetric and symplectic ensemble, and also coupled random matrices.

O.I. Mokhov: *On the equations of associativity and compatible Poisson structures of hydrodynamic type*

The problem of classifications of compatible Poisson structures of hydrodynamic type is studied. It is shown that in the two-component case such compatible pairs can be completely described by a 4-component homogeneous nondiagonalizable system of hydrodynamic type, which has two double eigenvalues and four eigenvectors at any point. Two Riemann invariants of this system are found and it is proved that the system has no other Riemann invariants. An integrable two-component reduction of the system is constructed. For an arbitrary number of components N the theory of compatible deformations of two Frobenius algebras is developed. The equations of the compatible deformations correspond to some natural special case of the equations for compatible Poisson structures of hydrodynamic type. For $N = 2$ these equations of deformations are completely integrated.

F. Nijhoff: *Elliptic Solutions of Lattice KP Systems and Integrable Discrete Many-Body Systems*

In a recent paper [F.W. Nijhoff, O. Ragnisco and V. Kuznetsov, Commun. Math. Phys., 1996] a discrete version of the Ruijsenaars' model (the relativistic generalization of the Calogero-Moser model) was derived and its integrability was proven as a multivalued Lagrangian map. This discrete model, which in its generic form is given in terms of the Weierstraß σ-function, was integrated in the rational and hyperbolic limits, and in the the latter case a connection with soliton solutions of the lattice KP equation was pointed out. More recently, Krichever and collaborators have shown how the discrete Ruijsenaars' model in the elliptic case arises from pole expansions of the lattice KP equation. In the present talk another connection between lattice KP systems (including the lattice modified KP equations, Hirota-Miwa equation, etc.) and the Ruijsenaars' model is explained. The connection arises via linear integral equations with arbitrary contours and measures and an elliptic kernel given in terms of the Lamé function $\Phi_y(x) \sim \sigma(x+y)/(\sigma(x)\sigma(y))$. Such integral equations had been studied in the past in the rational case to study solutions of integrable partial difference equations of KP type. In the elliptic case special reductions (from special choices of contours and measures) yield solutions of elliptic soliton type and they are given in terms of the Ruijsenaars' Lax matrix. The Ruijsenaars' flow can be shown to be compatible with the lattice KP flows. Finally, the general problem of the existence of a "discrete" integration scheme for the elliptic Ruijsenaars' model is discussed.

W. Oevel: *QR-Factorization and Elementary Dressing Transformations of the Toda Hierarchy*

QR-factorization of matrices is known to provide transformations leaving the (generalized) Toda hierarchy invariant. We consider the numerical algorithm computing the orthogonal factor Q as a product of Householder (reflection) matrices. It is shown that for Lax operators of upper Hessenberg type the partial transformations induced by the Householder matrices also provide invariances of the Toda flows. In contrast to the complete QR-step, the partial transformations are only elementary: they introduce an additional entry into the Hessenberg matrices.

F. Pempinelli: *Finite-Dimensional Systems Integrable via Inverse Scattering*

The discrete spectral problem of Ablowitz-Ladik is considered in the case in which the potential has a finite support of length L. The spectral transform is explicitly computed and a recurrence relation on the length L for computing it in L algebraic step is given. This spectral transform can be used to generate via the scattering method a finite dimensional version of the dynamical systems associated to the Ablowitz-Ladik spectral problem. A special case in which the potential can be constrained to evolve in time on a semi-line is proposed. In this case the evolution of the spectral transform is governed by a Riccati equation. The truncated soliton (i.e. the potential obtained by putting to zero the one soliton outside an interval of length L) is examined in detail. The sufficient and necessary condition for having a soliton contained in the truncated soliton solution is derived. Finally, the continuous counterpart of these finite-dimensional systems is considered. The spectral transform via a Riccati equation and the special case of the truncated soliton is studied.

O. Ragnisco: *Integrable Hamiltonian systems from co-algebras*

I present an algorithm to construct N-body (classical and quantum) completely integrable Hamiltonian systems from representations of co-algebras with Casimir elements. In particular, this construction shows that q-deformations can be viewed as structures generating integrable deformations of Hamiltonian systems with co-algebra symmetry. To illustrate the method, I consider as a first example a canonical realization of $so(2,1)$ that yields the Gaudin-Calogero system. Then, the Drinfeld-Jimbo q-deformation of $so(2,1)$ is shown to yield an integrable deformation of the previous model; finally, a deformation of the co-product of the $(1+1)$-Poincaré algebra, compatible with the algebra itself, is shown to provide a simple, but non-trivial Hamiltonian of the Ruijsenaars-Schneider type.

P.C. SABATIER: *On Multidimensional Darboux Transformations*

In contrast to known results on one-dimensional Darboux transformations, known results on multidimensional Darboux transformations do not seem convenient for disentangling the sets of solutions in global inverse problems. In the present paper, on one hand it is given a wide generalization of vectorial Darboux transformation and a more limited generalization of binary transformations. On the other hand, reduction of multidimensional transformations in special examples is described to show the limits of their use in Inverse Theory, to generate one dimensional transformations, and to give a better understanding of how the latter ones work.

P.M. SANTINI (JOINT WORK WITH A. DOLIWA, J. CIESLINSKY AND S. MANAKOV): *Discrete Geometry and Integrability in Multidimensions*

We present the discretizations of some classical notions of Differential Geometry and their connection with integrable difference equations in multidimensions. We show that the discrete analogue of an N dimensional manifold in $\mathbb{R}^M$, $N \leq M$, parametrized by conjugate coordinates is what we call an N-dimensional quadrilateral lattice, i.e. an N-dimensional lattice $\vec{x} : \mathbb{Z}^N \to \mathbb{R}^M$ whose elementary quadrilaterals are planar. We also prove that the N dimensional quadrilateral lattice can be uniquely constructed assigning $N(N-1)$ arbitrary functions of 2 discrete variables on $N(N-1)/2$ intersecting boundary surfaces [1]. We also show that the discrete analogue of an N-dimensional orthogonal net is what we call a circular (spherical) lattice, i.e., an N-dimensional lattice whose elementary quadrilaterals are inscribed in circles. We prove that the circular lattice is a true reduction of the quadrilateral lattice, i.e. the circularity constraint, once imposed on the $N(N-1)/2$ boundary quadrilateral surfaces, is preserved in the quadrilateral construction [2]. We finally show that the above lattices can be solved via the $\bar{\partial}$ (dbar) dressing method; in particular, we identify the linear constraint on the $\bar{\partial}$-data of the quadrilatal lattice, which allow one to solve the circular lattice [3].

[1] A. Doliwa and P.M. Santini, "Multidimensional quadrilateral lattices are integrable", Phys. Letters A 233, 365 (1997).

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mensional quadrilateral lattice I: The multidimensional circular lattice", Preprint ROME1-1172/97

W.K. Schief (in collaboration with B.G. Konopelchenko): *Three-dimensional integrable lattices in Euclidean spaces: conjugacy and orthogonality*

It is shown that the discrete Darboux system descriptive of conjugate lattices in Euclidean spaces admits constraints on the eigenfunctions which may be interpreted as discrete orthogonality conditions on the lattices. Thus, it turns out that the elementary quadrilaterals of orthogonal lattices are cyclic. Orthogonal lattices on lines, planes and spheres are discussed and the underlying integrable systems in one, two and three dimensions are derived explicitly. A discrete analogue of Bianchis Ribaucour transformation is mentioned and particular orthogonal lattices such as discrete Egorov coordinate systems are given.

E. Sklyanin (in collaboration with V. Kuznetsov): *Some remarks on Baecklund transformations for many-body systems*

Using the n-particle periodic Toda lattice and Ruijsenaars' relativistic generalization of the elliptic Calogero-Moser system as examples, we revise the basic properties of the Baecklund transformations (BT's) from the Hamiltonian point of view. The analogy between BT and Baxter's quantum Q-operator pointed out by Pasquier and Gaudin is exploited to produce a conjugated variable μ for the parameter λ of the BT such that μ belongs to the spectrum of the Lax operator $L(\lambda)$. As a consequence, the generating function of the composition of n BT's gives rise also to another canonical transformation separating variables for the model. For the Toda lattice the dual BT parametrized by μ is introduced.

Y.B. Suris: *Integrable discretizations for lattice systems: local equations of motion*

The approach to the problem of integrable discretization based on the notion of r–matrix hierarchies is developed. One of its basic features is the coincidence of Lax matrices of discretized systems with the Lax matrices of the underlying continuous time systems. A common feature of the discretizations obtained in this approach is non–locality. We demonstrate how to overcome this drawback. Namely, we introduce the notion of localizing changes of variables and construct such changes of variables for a large number of examples, including the Toda and the relativistic Toda lattices, the Volterra and the relativistic Volterra lattice, the second flows of the Toda and of the Volterra hierarchies, the modified Volterra lattice, the Belov–Chaltikian lattice, the Bogoyavlensky lattices, the Bruschi–Ragnisco lattice, and a novel class of constrained lattice KP systems. Pulling back the differential equations of motion under the localizing changes of variables, we find also (sometimes novel) integrable one–parameter deformations of integrable lattice systems. Poisson properties of the localizing changes of variables are also studied: they produce interesting one–parameter deformations of the known Poisson algebras.

S.P. TSAREV: *On superposition principles and completeness of (2+1)-dimensional Baecklund transformations*

The usual superposition formulas for Baecklund transformations of (2+1)-dimensional integrable systems include quadratures unlike the well known case of (1+1)-dimensional integrable systems where the fourth solution is found with algebraic operations. We show how one can find an analogous extended formula of nonlinear superposition for some (2+1)-dimensional integrable systems. Also we solve positively the problem of (local) density of solutions of such (2+1)-dimensional integrable systems obtainable from a given initial solution with consecutive Baecklund transformations in the space of all solutions of the systems in question.

A.P. VESELOV: *Integrable Gradient Flows and Morse Theory*

Examples of Morse functions with integrable gradient flows on some classical Riemannian manifolds are considered. This gives an explicit cell decomposition and geometric realization of the homology for such a manifold. As another application of the integrable Morse functions we give an elementary proof of Vassiliev's theorem on the flag join of Grassmannians.

[1] A.P. Veselov and I.A. Dynnikov, "Integrable gradient flows and Morse theory", St. Petersburg Math. Journal, vol.8, n.3 (1997), 429–446.

M. WADATI: *Collapse of the Bose-Einstein Condensate under Magnetic Trap*

We consider the Bose-Einstein condensate with attractive interparticle interactions (more precisely, negative S-wave scattering lengths) under a magnetic trap. By using a model equation, axially symmetric nonlinear Schrödinger equation with harmonic potential terms, we investigate the time-evolution of the wave function. We prove that the singularity of wave functions emerges in a finite time even when the total energy of the system is positive. We present a formula for a critical number of atoms above which the collapse of the condensate occurs. This number can be the same as the one in the recent experiment.

[1] T. Tsurumi and M. Wadati, J. Phys. Soc. Jpn. **66** (1997) 3031.

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V.E. ZAKHAROV: *Dressing Method in Differential Geometry*

The dressing method is one of the most powerful tools in the theory of solitons. The advanced version of this method is based on the "non-local $\bar{\partial}$-problem". Let $\chi = \chi(\lambda, \bar{\lambda}, x), x \in \mathbb{C}^n$, be an $N \times N$ quasi-analytical matrix function of $\lambda, \bar{\lambda}$, satisfying the equation:

$$\frac{\partial \chi}{\partial \bar{\lambda}}(\lambda, \bar{\lambda}, x) = \int \chi(\mu, \bar{\mu}, x)\, R(\mu, \bar{\mu}, \lambda, \bar{\lambda}, x)\, d\mu\, d\bar{\mu}\,. \tag{1}$$

An integrable system can be constructed after introducing the set of commuting differential operators

$$D_i\chi = \frac{\partial\chi}{\partial x_i} + \chi\, I_i(\lambda) \ , \quad [D_i, D_j] = 0 \ , \tag{2}$$

$i = 1, \ldots, n$, $n \geq 3$. Here $I_i(\lambda)$ are given commuting rational matrix functions. The integral operator $\hat{R}$ commutes with all D_i:

$$[\hat{R}, D_i] = 0 \ . \tag{3}$$

If $I_i(\lambda)$ are polynomials, condition (3) imposes an infinite number of differential constraints on the coefficients of the asymptotic expansion

$$\chi = 1 + \frac{Q_1}{\lambda} + \frac{Q_2}{\lambda^2} + \ldots \ , \quad \lambda \to \infty \ . \tag{4}$$

They can be interpreted as an integrable system with a set of conservation laws. The simplest choice $I_1 = \lambda, I_2 = \lambda^2, I_3 = 4\lambda^3, N = 1$ leads to the KP-II equation for $U = 2\partial Q_1/\partial x_1$.

The dressing method can be efficiently applied to solve problems of Classical Differential Geometry. The easiest problem is the classification of conjugated nets in n-dimensional Euclidean space. It is given by solving the system of Laplace equations

$$\varphi_{i,j} = Q_{ij}\varphi_j + Q_{ji}\varphi_i \ , \tag{5}$$

whose compatibility conditions lead to the Darboux system:

$$Q_{ij,k} = Q_{ik}Q_{kj} \ , \tag{6}$$

which can be integrated by posing

$$I_i = diag(\overbrace{\underbrace{0, \ldots, 1}_{i}, 0, \ldots, 0}^{n}) \ . \tag{7}$$

Now $Q_{ij} = Q_{1ij}$. The "dressing" matrix R has the form:

$$R_{ij}(\mu, \bar{\mu}, \lambda, \bar{\lambda}, x) = e^{\mu x_i - \lambda x_j}\, R_{0ij}(\mu, \bar{\mu}, \lambda, \bar{\lambda}) \ . \tag{8}$$

Solutions of the Laplace equations (5) are given by the solution of the $\bar{\partial}$-problem dual to (1) having a simple pole at $\lambda = 0$.

The next important problem is the classification of n-orthogonal conjugated nets. Orthogonality imposes on Q_{ij} following additional constraints

$$\frac{\partial Q_{ij}}{\partial x_j} + \frac{\partial Q_{ji}}{\partial x_i} + \sum_{k \neq i \neq j} Q_{ik}Q_{jk} = 0 \ , \tag{9}$$

which is satisfied, if R_0 obeys the "differential reduction"

$$R_0^{tr}(-\mu, -\lambda) = \frac{\mu}{\lambda}\, R_0(\lambda, \mu) \ . \tag{10}$$

Formulae (1–4) and (7–10) give the solution of the classical problem of classification of n orthogonal curvilinear coordinate systems making it possible to find all "rotation coefficients" $\beta_{ij} = Q_{ji}$. To find the Lamé coefficients H_i ($Q_{ij} = H_j^{-1}\,\partial H_i/\partial x_j$) one has to consider the matrix function

$$\varphi_{ij}(\lambda,\bar{\lambda},x) = \chi_{ij}(\lambda,\bar{\lambda},x)\,e^{-\lambda x_j}\ .$$

Each column of φ at any given λ is a set of H_i.
Integration of the Lamé-Darboux system (6),(9) makes it possible to solve another classical problem of Differential Geometry: the integration of the Gauss-Codazzi equations. They arise if $n = 3$ and $\partial Q_{ij}/\partial x_j \equiv 0$. Then $Q_{1j} = 0$, and the Lamé-Darboux system degenerates to three equation for four rotation coefficients $Q_{21}, Q_{31}, Q_{32}, Q_{23}$, equivalent to the Gauss-Codazzi system. This procedure makes it possible to embed the theory of surfaces in 3-d space into the theory of solitons.

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