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Combinatorial Convexity and Algebraic Geometry

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The meeting was organized by G. Ewald (Bochum), P. McMullen (London) and T. Oda (Sendai). 46 scientists from Germany (#16), USA (#13), Japan (#7), France (#3), England (#2), Poland (#2), Sweden (#2) and Switzerland (#1) participated. During the five days of the conference, 28 main talks were given and several study groups were organized. The following keywords sketch the main topics:

- Intersection (co)homology of toric varieties
- Complexes of modules and homology of complexes
- Coherent subdivisions and triangulations, resolution of singularities
- Combinatorics of posets, weights, upper bound theorems
- Rational and non-rational polytopes and fans
- Lattice-free polytopes, lattice points in convex bodies
- Zeta functions, hypergeometric functions
- Binomial ideals, A-graded ideals
- Automorphism groups of toric varieties
- Minimal models of divisors of toric varieties
- Toric quiver varieties, quotients of toric varieties
- Strange duality and polar duality
- Mirror Symmetry, reflexive polytopes
- McKay correspondence

The organizers and participants thank the 'Mathematisches Forschungsinstitut Oberwolfach' for making this conference possible.

The abstracts are listed in the order the talks were given. At the end, abstracts of informal talks are enclosed.

Submitted abstracts

Intersection cohomology of toric varieties
Mark McConnell

Let Σ be a fan, and $X = X(\Sigma)$ the associated toric variety. We explicitly construct a complex of finite–dimensional real vector spaces whose cohomology is the intersection cohomology of X. We use any perversity $\overline{p}$; as special cases, this includes ordinary cohomology and homology. If Σ comes from a convex polytope P, then our complex computes $h_i = \dim IH^{i}_{\overline{m}}(X)$ (middle perversity), where (h_i) is the generalized h–vector of $\wp$. Our construction works whether or not Σ and P are rational; if they are not, it defines IH^*–groups for "virtual toric varieties" . For rational Σ, the computations can be carried out using the author's programs *Sheafhom*.

Resolutions and the homology of chessboard and matching complexes
Victor Reiner

The chessboard and matching complexes are two families of simplicial complexes whose topology has been studied by several different researchers. We show how to compute the rational homology of these complexes (and some generalizations) using results on the minimal free resolutions of certain specific affine semigroup rings. These results combine a folklore theorem about resolutions of semigroup rings with resolutions of determinantal ideals due to Lascoux and Jozefiak, Pragacz, Weyman, and some more general resolutions of modules that we must compute. (Joint with Joel Roberts.)

Local–global intersection homology
Jonathan Fine

This paper defines new intersection homology groups. The basic idea is this. Ordinary homology is locally trivial. Intersection homology is not. It may have significant local cycles. A local-global cycle is defined to be a family of such local cycles that is, at the same time, a global cycle. The motivating problem is the numerical characterisation of the flag vectors of convex polytopes. Central is a study of the cycles on a cone and a cylinder, in terms of those on the base. This leads to the topological definition of local-global intersection homology, and a formula for the expected Betti numbers of toric varieties. Various related questions are also discussed.

Toric varieties and complexes of modules

Masanori Ishida

For a finite fan Δ in a real space $N_{\mathbf{R}}$, we define the additive category GEM(Δ) whose object is a collection of modules over exterior algebras. For a perversity p, which is defined to be a map $\mathrm{p} : \Delta\backslash\{0\} \to \mathbf{Z}$, we define a complex $\mathrm{ic}_{\mathrm{p}}(\Delta)^{\bullet}$ of objects in GEM(Δ) which we call the *intersection complex of the fan* Δ. Let $Z(\Delta)$ be the associated toric variety defined over $\mathbf{C}$. By a natural functor from modules on Δ to differential complexes of coherent sheaves on $Z(\Delta)$, we get the intersection complex of the toric variety consisting of coherent sheaves.

Minimal models of divisors of toric varieties

Shihoko Ishii

1. I construct a projective minimal model of a Δ–regular divisor whose adjoint divisor has non–negative Kodaira dimension on a non–singular toric variety. The model has the abundance. The proof of the existence also gives the algorithm to construct a minimal model.
2. As a relative version of 1., I construct a minimal model of a germ of a hypersurface on an affine toric variety defined by a non–degenerate function.

Generalized McKay correspondence for higher dimensional singularities

Yukari Ito

Let G be a finite subgroup of $SL(n, \mathbf{C})$. If we have a crepant resolution $\pi : X \to \mathbf{C}^n/G$ (i.e. $K_X = \mathcal{O}_X$), then there is a conjecture, called McKay correspondence, which is a relation between the Grothendieck group (or (co)homology group) of X and the representations (or conjugacy classes) of G.
If $n = 3$, then we have a crepant resolution, but the construction depends on the classification of the finite subgroup of $SL(3, \mathbf{C})$. In case $n = 2$, we can construct a minimal resolution using the Hilbert scheme of points of $\mathbf{C}^2$ without the classification. When we consider the same for $n = 3$, there are some difficulties. But if G is abelian, then we can see the problems with toric geometry.
I would like to talk on more precise formulation of the conjecture when X is a certain variety associated with the Hilbert scheme of points in $\mathbf{C}^n$ and on the proof for an abelian subgroup G of $SL(3, \mathbf{C})$. These new results are due to a joint work with H. Nakajima.

On crepant resolutions of Gorenstein toric singularities

Dimitrios I. Dais

In my talk I present two necessary criteria for the existence of crepant resolutions of Gorenstein toric singularities in dimensions ≥ 4.

Integral and reflexive simplices
Heinke Wagner

To any integral n–simplex $\Delta = \text{conv}\{v_o, \ldots, v_n\} \subseteq \mathbf{R}^n$ with $0 \in \text{int}\Delta$ a system of weights $Q = (q_0, \ldots, q_n) \in \mathbf{N}^{n+1}$ can be associated, s.t. $\sum_{i=0}^{n} q_i v_i = 0$. We describe an algorithm that allows to recover a simplex Δ from its weight system. This is done by associating to Q a system of $n+1$ lattices $\Gamma_{Q,q_i} \subseteq \mathbf{Z}^n$ in which the positive orthant defines the rings of affine toric varieties that cover the weighted projective space (w.p.s.) $\mathbf{P}(Q)$ of type Q. Dualizing and representing everything with respect to the same lattice we obtain a simplex with weights Q, and by the way, the fan of the w.p.s. $\mathbf{P}(Q)$. After giving a characterization of the property of being reflexive in terms of Q, we describe an algorithm for the classification of reflexive simplices of a given dimension n up to an isomorphism. In the language of toric varieties this is equivalent to a classification algorithm for projective toric Fano varieties with at most Gorenstein singularities and cyclic Picard group. For $n = 3$ there are 27 reflexive pairs corresponding to 48 non–isomorphic such simplices/varieties.

An Euler–MacLaurin formula for rational polytopes
Michel Brion

We express the sum of values of a polynomial function at all lattice points of a rational convex polytope, in terms of the integral of the function over the deformed polytope (where all facets are translated independently). This generalizes work of Khovanskii, Kantor and Pukhlikov, and gives some information on the coefficients of the Ehrhart polynomial of a lattice polytope. As another application, we express the Todd class of a complete, simplicial toric variety in terms of classes of invariant divisors, refining a result of Pommersheim. Finally, we recover a multidimensional version of the Euler–MacLaurin summation formula, announced by Cappell and Shaneson. These results appear in the Crelle Journal (1997) and the Journal of the American Mathematical Society (two papers, 1997). (Joint work with Michèle Vergne.)

Lattice points in convex bodies, special values of the zeta function of a totally real number field, and toric varieties
James Pommersheim

Like the Riemann zeta function, the zeta function of a number field can be extended to a meromorphic function on $\mathbf{C}$ with a simple pole at $s = 1$. For a totally real field, the values of the zeta function at negative integers are known to be rational numbers related, via the Lichtenbaum conjectures, to the global behaviour of the number field. In this talk, we present a new formula for these numbers in the case of a real quadratic field. To do this, we use invariants arising in the theory of toric varieties. In particular, we use the Todd operator appearing in a theorem of Brion–Vergne (generalizing the theorem of Khovanskii–Kantor–Pukhlikov). (Joint work with S. Garoufalidis.)

An upper bound theorem for rational polytopes

Margaret Bayer

The "toric" h-vector of a rational polytope is the sequence of middle perversity intersection homology Betti numbers of the associated toric variety. We prove the upper bound inequality, $h_i - h_{i-1} \leq \binom{n-d+i-2}{i}$, for rational d-polytopes with n vertices. The proof uses a result of Braden and MacPherson on h-vectors of quotient polytopes, the Lawrence polytope construction, and the upper bound theorem for simplicial polytopes.
In addition, the talk will describe briefly some results obtained in joint work with Richard Ehrenborg. These include explicit formulas relating the toric h-vectors, the flag h-vectors and the cd-indices of polytopes (or Eulerian posets). The formulas can be proved in several ways, using previous results by Fine, Bayer-Klapper, and Ehrenborg-Readdy. As a consequence we show that Kalai's duality result, $g_{n/2}(P) = g_{n/2}(P^*)$, is the only equation relating the h-vectors of a polytope and its dual. Another corollary is a parity result on toric h-vectors of zonotopes (or oriented matroids).

Weights on almost simple polytopes

Carl Lee

A polytope is almost simple if it becomes simple upon the truncation of its vertices. We discuss some weight-spaces on such polytopes whose dimensions match the components of the toric h-vector. (Joint work with Sue Foege.)

Polytopal linear groups

Winfried Bruns

By a polytopal linear group we mean the group of graded automorphisms of a polytopal semigroup ring $k[S_P]$. Here k is a field, P is lattice polytope in $\mathbf{Z}^n$, and S_P is the semigroup generated by the elements $(x, 1) \in \mathbf{Z}^{n+1}$.
Generalizing classical results for $\mathrm{GL}_m(k)$ and $\mathrm{Aut}(\mathbf{P}^{m-1}) \cong \mathrm{GL}_m(k)/k^*$ we show:
(a) $\Gamma_k(P) = \mathrm{gr.aut}_k(k[S_P])$ is generated by torus actions, elementary transformations, and symmetries of P; moreover, the first two classes of automorphisms generate the connected component $\Gamma_k(P)^0$ of unity. The elementary transformations are defined by so-called column vectors.
(b) There exists a lattice polytope Q with the same normal fan as P such that the natural antihomomorphism $\Gamma_k(Q) \to \mathrm{Aut}(X)$, $X = \mathrm{Proj}(k[S_P])$, is an antiisomorphism. (Here we assume that P is "very ample".)
Theorem (b) is a description of the automorphism groups of all projective toric varieties. For this class it generalizes theorems of Demazure and Cox.
We report on joint work with J. Gubeladze (Tbilisi).

Quadratic binomial ideals with no quadratic Gröbner bases
Takayuki Hibi

Let K be a field and $K[\mathbf{t}] = K[t_1, t_2, \ldots, t_d]$ the polynomial ring in d variables over K with each $\deg t_i = 1$. Let $\mathcal{A} = \{f_1, f_2, \ldots, f_n\}$ be a finite set of monomials belonging to $K[\mathbf{t}]$ having the same degree and write $K[\mathcal{A}]$ for the affine semigroup ring which is generated by $f_1, f_2, \ldots, f_n$. Let $K[\mathbf{x}] = K[x_1, x_2, \ldots, x_n]$ denote the polynomial ring in n variables over K. The *toric ideal* of $\mathcal{A}$ is the kernel $I_{\mathcal{A}} \subset K[\mathbf{x}]$ of the surjective homomorphism defined by $x_i \mapsto f_i \in K[\mathcal{A}]$. We then know the hierarchy

$I_{\mathcal{A}}$ has a quadratic Gröbner basis

$\Rightarrow$ $K[\mathcal{A}]$ is Koszul, i.e., $[\mathrm{Tor}_i^{K[\mathcal{A}]}(K,K)]_j = 0$ for all $i \neq j$

$\Rightarrow$ $I_{\mathcal{A}}$ is generated by quadratic binomials.

However, for a long time, it is unknown if the converse of each $\Rightarrow$ is true. Quite recently, by [1] and, independently, by [3], the converse of each $\Rightarrow$ turned out to be false. The purpose of the present talk is to report some results and examples discussed in [2] and [3].

[1] J.-E. Roos and B. Sturmfels, Monomial curves defined by quadrics, in preparation.

[2] H. Ohsugi and T. Hibi, Koszul bipartite graphs, *Adv. in Appl. Math*, to appear.

[3] H. Ohsugi and T. Hibi, Toric ideals generated by quadratic binomials, preprint (October, 1997).

Invariant valuations in convex geometry
Daniel Klain

A valuation on the set of all compact convex subsets of Euclidean space is a real-valued function that satisfies the inclusion–exclusion principle for pairs of convex sets whose union is convex. Important examples include Euclidean volume, surface area, the Euler characteristic, and certain mixed volumes. Valuations that are invariant under rigid motions (and other groups of motions) are especially important in both convexity and integral geometry. This talk will give a brief overview of valuations, their applications, and a characterization of several types of valuations, including those invariant under rigid motions (Hadwiger's characterization theorem). The talk will conclude with some open questions.

Some coherent unimodular triangulations
Günter Ziegler

The combinatorics, existence and construction of coherent (a.k.a. projective, regular) unimodular (i.e. basic, minimal) triangulations ("CUTs") are at the

core of the resolution of singularities in toric geometry.
In this lecture we discuss two results outside toric geometry which also reduce to CUTs: *All abelian complete-intersection quotient singularities have projective crepant resolutions* (Dais–Henk–Z 1997), and the Semistable Reduction Theorem (Kempf et al. 1977).
Our survey of methods includes the Hyperplane Lemma, hypersimplex triangulations (Santos 1997), lattice joins and expansion, and the classification of elementary simplices according to width.
Strange examples in this context include a 9–dimensional 0/1 polytope which has a UT but no CUT (Ohsugi–Hibi, Firla–Z 1997), and a 3–dimensional lattice tetrahedron that is covered by a binary cycle supported on unimodular simplices, but has no UT (Firla–Z 1997).

The antiprism fan of a convex polytope

Anders Björner

Let P be a d–dimensional convex polytope in $\mathbf{R}^d$ and P^* its polar dual with respect to the origin (interior to both). The map $x \to (x, 1)$ places a copy of P in the hyperplane with last coordinate +1 in $\mathbf{R}^{d+1}$, and $x \to (x, -1)$ places a copy of P^* in the parallel hyperplane at -1. From now on we identify P and P^* with these copies.
Construct a d–dimensional polyhedral cell complex C_P sitting in $(d + 1)$–space (possibly with self–intersections) as follows. Take as maximal cells P, P^* and all polytopes of the form $F \oplus F^*$, where F is a proper face of P and F^* the dual face of P^*. Here "$\oplus$" denotes the join operator, which is legal since F and F^* sit in skew subspaces of complementary dimensions. Thus, all maximal cells $F \oplus F^*$ are d–polytopes.
The following facts are shown:

1. C_P is a shellable polyhedral sphere embedded in $\mathbf{R}^{d+1}$ without self–intersections and with the origin in its interior.

2. C_P is the boundary of a $(d + 1)$–polytope if and only if for each proper face F of P the perpendicular in d–space from the origin to the affine span of F hits aff(F) in the relative interior of F. Every 3–polytope can be realized so that this condition is satisfied, but the question is open in higher dimensions.

3. C_P is star–convex with respect to the origin in $\mathbf{R}^{d+1}$, i.e., any ray emanating from the origin intersects the body of C_P in exactly one point. This has the consequence that C_P spans a complete fan X_P centered at the origin of $(d+1)$–space. If the polytope P is rational then so is the fan X_P.

4. The face lattice of C_P (and of X_P) is isomorphic to the poset of all intervals of L_P (the face lattice of P) ordered by reverse inclusion (include the empty interval to get a top element).

5. The toric h–polynomial of X_P is related to the g–polynomials of P, P^*, and all their faces F, F^* via the following summation over the face lattice of P:

$$h_{X_P}(x) = \sum_{F \in L_P} x^{\dim F+1} g_F(1/x) g_{F^*}(x)$$

Enumeration in ranked posets

Gabor Hetyei

We show that the closure of the convex cone generated by all flag f-vectors of graded posets is polyhedral. In particular, we give the facet inequalities to the polar cone of all nonnegative chain-enumeration functionals on this class of posets. These are in one-one correspondence with antichains of intervals on the set of ranks and thus are counted by Catalan numbers. Finding all extreme rays seems to be an extremely hard problem in the field of combinatorial optimiziation. Furthermore, we prove that the convolution operation introduced by Kalai assigns extreme rays to pairs of extreme rays in most cases.
The labeling-technique used in the proof of our main theorem may be replaced with any labeling from a larger class which we call chain-edge labelings with the first atom property. Such labelings allow to decompose the order complex of any graded partially ordered set in a "shelling-like" manner: the intersection of the lastly added cell with the previous part will always be homotopy equivalent to a ball or to a sphere. A special subclass of these labelings is the class of lexicographically shellable posets studied by Björner and Wachs.
This is a joint work with Louis J. Billera.

Valuation algebras and binomial ideals

Bernard Teissier

Given a field k and a local noetherian excellent integral k–algebra R, to each valuation ν of the field of fractions K of R which is positive on R and trivial on k, denoting by Φ the (totally ordered) group of values of ν, I associate to (R, ν) the *valuation algebra*

$$\mathcal{A}_\nu(R) = \bigoplus_{\phi \in \Phi} \mathcal{P}_\phi(R) v^{-\phi} \subset R[v^\Phi],$$

where $\mathcal{P}_\phi(R) = \{x \in R/\nu(x) \geq \phi\}$ and $R[v^\Phi]$ is the group algebra with coefficients in R. Setting $\mathcal{P}^+_\phi(R) = \{x \in R/\nu(x) > \phi\}$, the main interest of this algebra is that it realizes a specialization, parametrized by $\operatorname{Spec} k[v^{\Phi_+}]$, of R to the graded ring

$$\operatorname{gr}_\nu R = \bigoplus_{\phi \in \Phi_+} \mathcal{P}_\phi(R)/\mathcal{P}^+_\phi(R).$$

If the valuation ring of ν has residue field k, $\operatorname{gr}_\nu R$ is a toric variety (defined by binomials in countably many variables), and its resolution should contain all the combinatorics associated with the local uniformization of (R, ν).

Witnessing irregularity and incoherence

Louis J. Billera

We give a necessary and sufficient condition for a polyhedral subdivision Δ (fan Σ) to be incoherent (equivalently, irregular, i.e., to fail to have a strictly convex lifting). In particular, Δ (Σ), having vertices (rays) $\{v_1, \ldots, v_n\} \subseteq \mathbf{R}^d$, is incoherent if and only if there exist affine (linear) relations $x^\sigma \in \mathbf{R}^n$ for each maximal cell σ in Δ (Σ) such that

(i) $x_i^\sigma \geq 0$ if $v_i \notin \sigma \, \forall \sigma$ and $\exists \, \overline{\sigma}, v_{\overline{i}} \notin \overline{\sigma}$ with $x_{\overline{i}}^{\overline{\sigma}} > 0$,

(ii) $\sum_\sigma x^\sigma = 0$, the sum over all maximal cells.

A family of relations satisfying (i) and (ii) is called a *witness* to the incoherence of Δ (Σ). We apply this to Ewald's whirls and Björner's antiprism fan. (Joint work with R. Hastings.)

A–graded ideals

Irena Peeva

Let A be a finite subset of $\mathbf{N}^d$ and J be the toric ideal defined by A. An ideal M is called A–graded if it has the same multigraded Hilbert function as J. The study of A–graded ideals was initiated by Arnold, who showed that the structure of such ideals is encoded in continued fractions in the case when J defines a monomial curve in $\mathbf{A}^3$. Arnold, Korkina, Post and Roelofs proved that if J defines a monomial curve in $\mathbf{A}^3$ then any A–graded ideal is isomorphic to an initial ideal of the toric ideal J. Later, Sturmfels studied arbitrary A–graded ideals and related their structure to subdivisions of the convex envelope of A. He conjectured the following generalization of Arnold–Korkina–Post–Roelofs' result: if the toric ideal J has codimension 2 then any A–graded ideal is isomorphic to an initial ideal of J. We prove this conjecture. This is a joint work with Vesselin Gasharov.

Toric quiver varieties

Klaus Altmann

Let Q be a quiver consisting of a set of vertices Q_0 and a set of arrows Q_1, respectively. Whenever you are given a so-called weight $\theta \in \mathbf{Z}^{Q_0}$ satisfying $\sum_i \theta_i = 0$, there is a lattice polytope, the flow polytope $\Delta(Q,\theta) := \mathbf{R}_{\geq 0}^{Q_1} \cap \pi^{-1}(\theta)$ with π denoting the incidence map $\pi : \mathbf{Z}^{Q_1} \to \mathbf{Z}^{Q_0}$. If $\theta = \theta^c := \pi(\underline{1})$ is the canonical weight, then $\Delta(Q, \theta^c)$ is reflexive.
Whenever θ is in general position (avoiding certain walls), then the projective toric variety associated to $\Delta(Q,\theta)$ is the fine moduli space of θ-stable representations of Q with dimension vector $\underline{1}$. Our main theorem is that, for the canonical weight, the direct summands of the universal bundle T form a strong exceptional sequence on this moduli space.
This is joint work with Lutz Hille (Hamburg).

Strange duality and polar duality

Wolfgang Ebeling

Arnold's strange duality between the 14 exceptional unimodal singularities can be considered as part of a 2–dimensional version of Mirror Symmetry. We consider a relation between this duality and a polar duality between the Newton polytopes of the singularities which was observed by M. Kobayashi. We show that this relation continues to hold for the extension of Arnold's strange duality found by C. T. C. Wall and the speaker. By a method of Ehlers-Varchenko, the characteristic polynomial of the monodromy of a hypersurface singularity can be computed from the Newton diagram. This method is generalized to the isolated complete intersection singularities embraced in the extended duality. We use this to explain the duality of characteristic polynomials of the monodromy discovered by K. Saito for Arnold's original strange duality and extended by the speaker to the other cases.

Flops of hypergeometric functions on toric varieties

Nobuki Takayama

We will discuss about a correspondence between polyhedral geometry of a given set of points A and the A–hypergeometric system introduced by Gelfand, Kapranov and Zelevinsky. Natural domain of definition of the A–hypergeometric systen is the toric variety defined by the secondary fan of A, of which face lattice is isomorphic to the face lattice of the regular polyhedral subdivisions of A. For each maximal dimensional cone C_ω of the secondary fan, we can associate a vector of vol(A) hypergeometric series $\Phi(C_\omega; z)$. This vector of hypergeometric series naturally appears in studies of mirror maps for Calabi–Yau threefolds in toric variety (Hosono, Lian and Yau).
It is known that $\Phi(C_\omega; z)$ is the analytic continuation of $\Phi(C_{\omega'}; z)$. We will discuss about the problem of finding the analytic continuation of $\Phi(C_\omega; z)$ to the cone $C_{\omega'}$; we will determine the constant matrix $M(C_\omega, C_{\omega'})$ such that

$$\gamma^* \Phi(C_\omega; z) = M(C_\omega, C_{\omega'}) \Phi(C_{\omega'}; z)$$

where $\gamma^* \Phi(C_\omega; z)$ denotes an analytic continuation of $\Phi(C_\omega; z)$. Our explicit formula of $M(C_\omega, C_{\omega'})$ is expressed in terms of flops (modifications) of triangulations of A.

Algebraic shifting increases relative homology

Art Duval

We show that algebraic shifting applied separately to a simplicial complex and subcomplex increases relative homology Betti numbers in each dimension. In other words, for simplicial complexes $L \subseteq K$, we show $\beta_i(\Delta(K), \Delta(L)) \geq \beta_i(K, L)$, where Δ denotes the algebraic shifting operation, and β_i denotes relative Betti numbers in dimension i.

This may have applications for characterizing jointly the f-vectors, Betti numbers, and relative Betti numbers of a pair of simplicial complexes.

Equivariant intersection homology of compact toric varieties

Karl–Heinz Fieseler

The intersection homology groups $IH^\bullet(X)$ of a compact toric variety X (with respect to the middle perversity and rational coefficients) can be computed by induction on $\dim X$. In order to get a better understanding of the intersection (co)homology groups it is thus necessary to investigate the different steps in the induction procedure. This is most easily done by replacing usual intersection homology groups $IH^\bullet(X)$ with equivariant intersection homology $IH^\bullet_T(X)$. Both theories provide the same information, since

$$IH^\bullet_T(X) \cong H^\bullet(BT) \otimes IH^\bullet(X),$$

when $BT = (\mathbf{P}_\infty)^n$, $n = \dim X$, is the classifying space of the torus T, but the equivariant theory has a much simpler behaviour than usual intersection homology under the intermediate steps of the computation, where also non–compact toric varieties have to be considered and the above Künneth formula does not hold any longer.

Quotients of toric varieties

Annette A'Campo–Neuen

We consider quotients for the action of a subtorus H of the big torus on a toric variety X. Since categorical quotients with respect to the category of algebraic varieties do not exist in general we introduce the following notion of quotient which seems more appropriate in this setting: We call an H-invariant toric morphism $p : X \to Y$ a toric quotient if every H-invariant toric morphism $f : X \to Z$ factors through p.
Our main result is the following theorem: A toric quotient always exists and there is an explicit algorithm for constructing the quotient from the combinatorial data corresponding to the pair consisting of the subtorus and the toric variety.
We also discuss relations to the so-called good quotients. Finally, we give an example of a $\mathbf{C}^*$-action on a smooth open subvariety of $\mathbf{C}^4$ such that the toric quotient is $\mathbf{C}^3$, but the toric quotient map is not surjective. That implies that in this case a categorical quotient for the category of algebraic varieties cannot exist.
(Joint work with Jürgen Hausen.)

Width of lattice–free simplices
Jean–Michel Kantor

Among integral polytopes (vertices with integral coordinates) , lattice-free polytopes - intersecting the lattice ONLY at their vertices- are of particular interest in combinatorics and geometry of numbers. A natural question is to measure their " width" (with respect to the integral lattice).There were no known examples of lattice-free polytopes with width bigger than 2 .We prove the following Theorem : Given any positive number α strictly inferior to $\frac{1}{e}$, for d large enough there exists a lattice-free simplex of dimension d and width superior to αd.

Abstracts of informal talks:

Two remarks about coherent triangulations
Günter Ewald

Remark 1 extends a theorem of Gelfand, Kapranov, and Zelevinski about characterizing coherent triangulations of a polytope by Gale transforms to arbitrary cell decompositions. Remark 2 applies the theorem to present a class of non–coherent cell decompositions.

Problems on the Minkowski sums of convex lattice polytopes
Tadao Oda

Let P and P' be convex lattice polytopes in $M_{\mathbf{R}} := \mathbf{R}^r \supset \mathbf{Z}^r =: M$. We have

$$(*) \qquad (M \cap P) + (M \cap P') \subset M \cap (P + P'),$$

where $P + P'$ is the Minkowski sum of P and P', while the left hand side means

$$\{m + m' | m \in M \cap P, m' \in M \cap P'\}.$$

We need not have the equality in $(*)$ even if both P and P' are nice (e.g., absolutely simple).
Hopefully, we have the equality in $(*)$ at least when P is absolutely simple and P' is obtained from P by independent parallel translations of facets.
The problem is closely related to the *diagonal ideal* of the product of a smooth projective toric variety with itself, which in itself is an important object of study in algebraic geometry.

The intersection cohomology vanishing and the g–theorem
Tadao Oda

In his proof of the necessity part of the g–theorem, Stanley (1980) used a consequence $(**)$ of the strong Lefschetz theorem for toric projective varieties with at

worst orbifold singularities. McMullen (1993) proved the consequence entirely in the framework of polytopes in terms of the polytope algebras.
On the other hand, Ishida (1994) succeeded in describing the intersection cohomology (with respect to various perversities) of toric varieties entirely in terms of fans, and proved directly the vanishing theorems which in the general case are important consequences of the decomposition theorem for the intersection cohomology.
We can derive (**) from Ishida's vanishing theorems, hence get another proof of the necessity part of the g–theorem entirely in the framework of polytopes and fans.

Topology of the combinatorial Grassmannian
Laura Anderson

I will discuss a combinatorial model, due to MacPherson, for the Grassmannian of k–planes in $\mathbf{R}^n$. By viewing an oriented matroid as a "combinatorial analog to a vector space", one can define the combinatorial Grassmannian as the order complex of a certain poset of oriented matroids. Recent results suggest that this combinatorial model captures much of the topology of the real Grassmannian, offering new combinatorial approaches to subjects such as characteristic classes.

A combinatorial formula for the Bando–Calabi–Futaki characters on toric orbifolds
Yasuhiro Nakagawa

Let X be a toric orbifold, and L a holomorphic line bundle on X with $c_1(L) > 0$. When L is the anti–canonical line bundle K_X^{-1} of X, we consider the existence problem for an Einstein-Kähler metric whose Kähler form represent the de Rham cohomology class $2\pi c_1(L) = 2\pi c_1(X)$. As to such existence of Einstein–Kähler metrics, an obstruction, which is called the Futaki character, is known. For the general L, Bando, Calabi, and Futaki generalized this Futaki character to an obstruction to the existence of constant scalar curvature Kähler metrics whose Kähler form represent the de Rham cohomology class $2\pi c_1(L)$, which we call the Bando–Calabi–Futaki character. In this talk, we shall give a combinatorial formula for the Bando–Calabi–Futaki character on toric orbifolds, which allows us to calculate this character on toric orbifolds easily. Moreover, we also discuss about Calabi's structure theorem for the automorphism group, which is an obstruction to the existence of an extremal Kähler metric.

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