MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Stochastic Geometry and Spatial Statistics

04.01–10.01.1998

This conference, organized by Adrian Baddeley (University of Western Australia), Dietrich Stoyan (TU Bergakademie Freiberg) and Wolfgang Weil (Universität Karlsruhe), continued a sequence of Oberwolfach conferences on stochastic geometry and related topics. While the last conference in 1991 concentrated on applications in stereology, the emphasis was this time on interrelations between stochastic geometry and spatial statistics. The topics treated varied from purely theoretical work in integral geometry to statistical applications of spatial models to geological problems. The conference also showed an increasing interest in geometric models and tools from spatial statistics in physical applications including the large–scale structure of the universe and statistical physics.

The conference had 43 participants. In view of this large number, the program was divided into 11 sessions (workshops) on different areas. Each session started with a main talk followed by a number of short communications. This schedule also left enough time for discussions which were used extensively by the participants. The title of the sessions also reflect the variety of themes presented at the conference:

- Stochastic geometry and convexity
- Random sets
- Integral geometry
- Edge effects
- Tessellations
- (Applied) spatial statistics
- Shape theory
- Physical applications
- Stereology
- Markov models and random fields
- Poisson processes and Boolean models

Dank einer Unterstützung im Rahmen des EU-Programmes TMR (Training and Mobility of Researchers) konnten zusätzlich einige jüngere Mathematiker zu der Tagung eingeladen werden. Dies ist einerseits eine hervorragende Förderung des wissenschaftlichen Nachwuchses und gibt andererseits den etablierten Kollegen die Gelegenheit, besonders begabte junge Mathematiker kennenzulernen.

Vortragsauszüge – Abstracts

R. V. AMBARTZUMIAN

Integral geometry of pregeodesics on 2–manifolds

A family of curves Γ on a two-dimensional manifold is called pregeodesic if the combinatorial behavior of the curves copies the well-known local behavior of the geodesics on a surface. In the lecture a class of combinatorial valuations on Γ is defined. A valuation depends on a function $F(s)$ that maps pregeodesic segments into $(-\infty, \infty)$. The main question is, for which functions $F(s)$ these valuations can be extended to signed measures on Γ? In the case the extension exists and is a nonnegative measure, the function $F(s)$ becomes a metric for which Γ is the family of geodesics. This observation connects the topic with Hilbert's fourth problem posed for the family Γ. Nonnegativity of the extension (if the latter exists) can be guaranteed by the requirement that the directional indicatrix of $F(s)$ is convex at each point of the manifold. A differential equation has been derived yielding a necessary and sufficient condition for signed measure generation. The case of circular directional indicatrix (pointwise isotropic $F(s)$) was studied in some detail. Along with several uniqueness results, description of pregeodesic families in Euclidean planar domains which admit geodesic interpretation with respect to a pointwise isotropic metric has been obtained.

A. BADDELEY

Practical maximum pseudolikelihood for spatial point patterns

We describe a technique for computing approximate maximum pseudolikelihood estimates of the parameters of a spatial point process. The method is an extension of Berman and Turner's (1992) device for maximising the *likelihoods* of inhomogeneous spatial Poisson processes. For a very wide class of spatial point process models, the likelihood is intractable, while the pseudolikelihood (Besag, 1975) is known explicitly, except for the computation of an integral over the sampling region. Approximating this integral by a finite sum yields an approximate pseudolikelihood which is formally equivalent to the likelihood of a loglinear model with Poisson responses. This can be maximised using standard statistical software for generalised linear or additive models, provided the conditional intensity of the process takes an 'exponential family' form. Using this approach we are able to rapidly fit a wide variety of spatial point process models of Gibbs type, incorporating spatial trends, interaction between points, dependence on spatial covariates, and mark information.

Joint work with Rolf Turner.

V. BENEŠ

Simulation of fibre processes

Discrete approximation of a fibre process on a fine grid is studied. Simulation by means of a Markov vector chain is suggested. Existence of a process with prescribed length, orientation and curvature distribution is discussed. Conditions for negligible overlappings are looked for in a given model.

CH. BUCHTA

The convex hull of random points – Recent developments

Let C be a convex body in $\mathbb{R}^d$. Choose n points from C, independently and according to the uniform distribution on C. Their convex hull C_n is a polytope contained in C. A survey on some results of the last ten years concerning the expected volume of C_n, the expected number of vertices of C_n, or, more generally, the expected number of k-faces of C_n $(k = 0, 1, \dots, d-1)$ is given. Special attention is paid to the asymptotic behaviour as n tends to infinity.

S. Chiu

Central limit theory for inhomogeneous birth–growth processes in $\mathbb{R}^d$

A Poisson point process Ψ in d–dimensional Euclidean space and in time is used to generate a birth–growth model: seeds are born randomly at locations x_i in $\mathbb{R}^d$ at times $t_i \in [0, \infty)$. Once a seed is born, it begins to create a cell by growing radially in all directions with speed $v > 0$. Points of Ψ contained in such cells are discarded, i.e. thinned. We study the asymptotic distribution of the number of seeds in a region, as the volume of the region tends to infinity. When $d = 1$, we establish conditions under which the evolution over time of the number of seeds in a region is approximated by a Wiener process. When $d \geq 1$, we give conditions for asymptotic normality. Rates of convergences are given in all cases.

N. Cressie

Statistical modeling and inference in space (and time)

The general spatial model involves a random process $Y(.)$ indexed in a random set $D \subset \mathbb{R}^d$. Three common examples are

(i) Geostatistical Models: D fixed, $\mathrm{vol}(D) > 0$, $\mathrm{Var}(Y(\underset{\sim}{s})) < \infty$ for all $\underset{\sim}{s} \in D$, and $\mathrm{Cov}(Y(\underset{\sim}{s}), Y(\underset{\sim}{u}))$ specified;

(ii) Lattice Models: D fixed, D at most countable and $[y(\underset{\sim}{s}_i) | \{y(\underset{\sim}{s}_j) : j \neq i\}]$ specified for all $i = 1, 2, \ldots$;

(iii) Point Process: D random, D a sample point process observed in a bounded window $A \subset \mathbb{R}^d$, and intensities $\{\lambda_n(\underset{\sim}{s}_1, \ldots, \underset{\sim}{s}_n)\}$ specified.

In practice, data are observed with error. Attribute error occurs, when $Z(.)$ is observed but one is still interested in predicting $Y(.)$; e.g., $Z(\underset{\sim}{s}) = Y(\underset{\sim}{s}) + \varepsilon(\underset{\sim}{s})$, where $\varepsilon(.)$ is additive measurement error. Location error occurs when locations $\{\underset{\sim}{s}_1, \ldots, \underset{\sim}{s}_n\}$ are observed at $\{\underset{\sim}{u}_1, \ldots, \underset{\sim}{u}_n\}$; write $\underset{\sim}{\Delta}_i \equiv \underset{\sim}{u}_i - \underset{\sim}{s}_i$ and assume for example that $\underset{\sim}{\Delta}_1, \ldots, \underset{\sim}{\Delta}_n$ are i.i.d. uniform on a disc centered at $\underset{\sim}{0}$.

Spatio-temporal models should incorporate dynamic structure into the purely spatial model. For example, spatio-temporal geostatistical models can be formulated that are autoregressive in time with a spatially colored error process; then a spatio-temporal Kalman filter can be developed.

I. Dryden

Statistical shape analysis

Shape analysis involves the geometrical study of objects where location, scale and rotation information can be ignored. Typically an object can be represented by a set of points called landmarks and we describe methods for the shape analysis of such points in the presence of randomness. An example of a typical application might be to test for a difference in mean shape in MR scans of Schizophrenia patients and normal subjects.

We describe some shape distances based on Procrustes methods, which turn out to be practically identical for fairly similar shapes. We consider some shape coordinates which can be used for inference using multivariate normal models, providing shape variability is small. Different coordinate choices lead to very similar inferences, for small variability.

Shape models are described which lead to a likelihood based inference procedure. In particular, an isotropic offset normal model is proposed for the Schizophrenia data which leads to a powerful test procedure. It turns out that the Schizophrenia and control groups are significantly different in mean shape.

Methods for describing any shape differences are required, and deformation methods and transformation grids provide a useful tool.

Finally we describe some recent extensions for averaging images and for object recognition.

T. ERHARDSSON

Compound Poisson approximation for the number of components of the uncovered set in the Johnson-Mehl model

The one–dimensional *Johnson-Mehl* crystallization model, also called the *linear birth-growth* model, can be briefly described as follows. Let ξ be a *Poisson point process* on $\mathbb{R} \times [0, \infty)$ with the intensity measure $\ell \times \Lambda$, where ℓ is the Lebesgue measure on $\mathbb{R}$ and Λ is any locally finite Borel measure on $[0, \infty)$ such that $\Lambda([0, \infty)) > 0$. Denote the points of ξ by $\{(x_i, y_i); i \in \mathbb{N}\}$, and let for each $i \in \mathbb{N}$ an interval start to grow in $\mathbb{R}$ from x_i with constant speed in both directions at time y_i. The union of growing random intervals thus generated will cover any finite interval $(0, L] \subset \mathbb{R}$ after an a.s. finite random time.

We consider, for any $z > 0$, the *uncovered set* at time z, by which we mean the set which does not yet at time z belong to the union of growing random intervals described above. Denote by $S(z, L)$ the *number of components intersecting* $(0, L]$ of the uncovered set at time z. The author (1996) gave an approximating Poisson distribution for $\mathcal{L}(S(z, L))$, together with an explicit total variation distance bound for the approximation error; the principal tool used in the proof is the coupling version of the *Stein-Chen method* for Poisson approximation. Under a mild assumption on Λ, the bound converges to 0 as $L \to \infty$ and $z \to \infty$ in a proper fashion. However, the rate of convergence can in some cases be very slow. A reasonable assumption is that this is caused by *clumping* of the components of the uncovered set.

In this talk we will indicate how, instead, an approximating *compound Poisson* distribution for $\mathcal{L}(S(z, L))$ can be found, together with an explicit total variation distance error bound. The proof relies on the fact that $S(z, L)$ can be interpreted as the number of visits by a Markov chain to a "rare" set, and uses a variation on Stein's method for Markov chains developed by the author (in preparation). It will be shown that for many choices of Λ, the rate of convergence as $L \to \infty$ for this upper bound is considerably faster than the rate of convergence for the Poisson approximation error upper bound described above; in the important example when $\Lambda = \lambda\ell$ for some $\lambda > 0$, the rate of convergence is $O(\frac{\log L}{L})$, as compared to $O(\frac{1}{\log L})$ for the Poisson approximation error bound. A further consequence of this is that the rate of convergence for the Poisson approximation error bound is in such cases the *correct* rate of convergence for the error, and cannot be improved upon.

R. GILL

Edge effects and missing data

The talk surveys nonparametric inference in missing data problems. The motivation is that an "edge problem" in spatial statistics can be thought of as a missing data problem, allowing a rich resource of ideas, models and techniques to be brought into play. We start by discussing Kaplan and Meier's (1958) product-limit estimator. We sketch early derivations of its (astonishing) large sample properties. We show how product-integration and martingale theory (via the Duhamel equation) together lead to a more illuminating derivation. We reevaluate the derivation of the estimator as NPMLE showing how an NPMLE is nothing else than an (infinite dimensional) M-estimator, i.e., the solution of a collection of unbiased estimating equations. This gives two important messages for spatial statistics:
(1) an NPMLE can be equally appropriate and equally analysable in the spatial statistics context; and
(2) it also makes sense even when the "likelihood" in question is just a pseudo, quasi, (or whatever) likelihood.

Two examples (of KM) are given - estimation of the empty space function from a windowed point process, and estimation of the line segment length distribution from a windowed line segment process. We return then to the NPMLE in more generality, in the context of nonparametric missing data problem under coarsening at random. We mention the availability of the striking van der Laan identity also when the "likelihood" is not the data-generating probability density but a convenient pseudo, ... construct. This allows another (better) solution to the windowed line-segment process problem discussed earlier.

L. HEINRICH

Random closed sets in $\mathbb{R}^d$

This lecture presents a short overview about classical results of random sets theory combined with reviewing some recent developments in the asymptotic theory of random closed sets (RACS's). RACS's provide a powerful tool for modeling irregularly shaped and randomly scattered clumps in an Euclidean space $\mathbb{R}^d$. Although the origins of the theory of RACS's can be traced back into the first half of the 20^{th} century, the development of the modern theory and statistics of RACS's (in LCS–spaces) began with the appearance of D. G. Kendall's paper "Foundation of a Theory of Random Sets" in 1974 and G. Matheron's monograph "Integral Geometry and Random Sets" in 1975.

First we discuss in some detail the basic notion of the "hitting (Choquet, capacity) functional" $T_Z(.)$ of a RACS Z which is then used to characterize weak convergence, ergodicity and mixing and to express several numerical characteristics of a (stationary) RACS.

Essential classes of RACS's are briefly considered; among them simple point processes, manifold and flat processes, skeletons of random tessellations, germ–grain models, sample paths and excursion (level) sets of Gaussian random fields and diffusion processes. Emphasis is put on some asymptotic results for normalized Minkowski sums and union sets of random compact sets. Furthermore, we deal with germ-grain models with i.i.d.

grains being independent of the germ process. We give conditions ensuring closedness of this model, that is of the union of shifted grains and formulate quite simple conditions ensuring ergodicity, mixing and tail triviality in terms of the underlying germ process. An estimate of the absolute regularity coefficient of the germ–grain model is presented. Some asymptotic results on the behaviour of empirical volume fraction, covariance and contact distribution as well as on minimum–contrast estimates for parameters of RACS's complete this lecture.

H. HERMANN

Formation of random and partially ordered structures in solids during phase transformation

The preparation of materials with well-defined two-phase microstructures from the homogeneous amorphous state is an actual topic in solid state physics and materials science. In most cases the physical properties are very sensitive to the details of the microstructure, e.g. mean size and size distribution, volume fraction and geometrical arrangement of crystallites. For the interpretation of experimental data on the microstructure and the understanding of the physical properties it is necessary to have available models for the time evolution of the structural transformation and the final state of the generated microstructure. Several types of germ–grain models are used to simulate transformation processes, to calculate experimentally accessible structure parameters and to analyze experimental data. It is shown that amorphous precipitates in bulk amorphous metallic alloys are, in the final state of their evolution, distributed according to a hardcore structure. The precipitates are created by nucleation and diffusion where the hardcore interaction is caused by a spherical depletion zone around each precipitate.

D. HUG

The average number of normals of a convex body in Minkowski space

In 1944, Santaló raised the question of determining bounds on the expected (average) number of normals that can be drawn from a random point inside a convex body to its boundary. Meanwhile, this problem has been treated in Euclidean spaces. We now investigate the corresponding problem in the setting of Minkowski (relative) geometry. By a Minkowski space we mean a finite dimensional affine space endowed with a norm which is not Euclidean in general. In other words, we consider $\mathbb{R}^n$ with a strictly convex gauge body B as the basic structuring element.

Let $K \subset \mathbb{R}^n$ be a convex body. A B-normal of K is any line $x + \mathbb{R}b$, where $x \in \partial K$ and b lies in the intersection of B and a support plane H' of B which is parallel to and similarly situated as a support plane H of K with $x \in H$. The number of B-normals of K passing through a point $p \in K$ is denoted by $n_B(K,p)$. We assume that the random point considered is uniformly distributed in K. Then the expected number of B-normals of K passing through such a random point is given by

$$n_B(K) = \frac{1}{\lambda(K)} \int_K n_B(K,p) \mathrm{d}\lambda(p),$$

if λ denotes any translation invariant Haar measure on $\mathbb{R}^n$.

Theorem *Let $K, B \subset \mathbb{R}^n$ be convex bodies with nonempty interiors, $o \in \operatorname{int} B$, and assume that the support function h_B of B is of class C^2. Then*

$$2 \leq n_B(K) \leq \frac{\lambda(K + DK)}{\lambda(K)} - 1,$$

where $DK := K + (-K)$. The estimates are sharp. If $n = 2$, then it is sufficient to assume that B is strictly convex.

The proofs use a combination of arguments from convex geometry and some basic geometric measure theory. In particular, we introduce support measures in Minkowski spaces.

J. Hüsler

Peeling and the smallest parallelepiped

Let n independent random points be given with uniform distribution in the d-dimensional unit cube $[0,1]^d$. The smallest parallelepiped A which includes all the n random points is analysed. We investigate the asymptotic behaviour of the volume of A as n tends to ∞. Using a point process approach, we derive also the asymptotic behaviour of the kth smallest parallelepipeds $A_n^{(k)}, k \geq 1$, which are defined by iteration by deleting all the boundary points X_i of $A_n^{(k-1)}$ and constructing the smallest parallelepiped which includes the remaining inner points of $A_n^{(k-1)}$ where $A_n = A_n^{(1)}$. We derive an explicit asymptotic law for the volumes of $A_n^{(k)}$. In addition we deal with cases where the X_i are not uniformly distributed in the unit cube and derive more general limit results.

M. Kiderlen

Blaschke means of sections of stationary random sets and particle processes

Consider the mean normal measure $S(X,.)$ of a stationary random set X in the extended convex ring in d-space. $S(X,.)$ is a Borel measure on the unit sphere S^{d-1} and can be seen as a local version of the surface area density. The stereological question is discussed, whether $S(X,.)$ is determined by lower dimensional sections of X.

In the design based approach X is intersected with a random isotropic subspace ζ, independent of X. The mean normal measure of $X \cap \zeta$ is defined on the unit sphere $S^{d-1} \cap \zeta$ in ζ. If this measure is considered as a measure on S^{d-1}, then, in general, the average (with respect to ζ) does not determine $S(X,.)$ uniquely. To remedy this situation, a geometrically motivated spherical lifting operator is applied to the mean normal measure of the random section before averaging. Then, the resulting average determines $S(X,.)$, if ζ is at least of dimension two.

The results carry over to stationary processes of particles in the convex ring.

K. Kiêu

Precision of systematic counts

Counting points or particles in space requires sampling, especially when the population of interest is large. Sampling designs based on quadrats, slices or bricks are commonly used in microscopy. Uniform sampling provides unbiased estimators. When the particles are not small compared to the sampling unit size, unbiased rules as described by Gundersen (1977), Howard et al. (1985) and Sterio (1984) should be used.

When sampling units are systematically distributed, one can assess the sampling precision by using the so-called transitive methods first described by Matheron (1965). These methods are quite general and can be applied to many types of systematic measurements.

In Kiêu et al. (1998), a specific method for systematic counts has been developed. In particular, this method predicts sampling precision taking into account both the number of sampling units and the sampling unit size. A key point of this approach is the assumption that the interpoint distribution is smooth.

The method is mainly for one–dimensional sampling (i.e. the location of a sampling unit is determined by a one–dimensional parameter). It can be applied to multi–dimensional sampling when the sampling design is built by nesting one–dimensional sampling designs.

CH. LANTUÉJOUL

Conditional simulation of a Poisson point process

Let $\mathcal{P}$ be a d-dimensional Poisson point process, and let A be a bounded, open and connected subset of $\mathbb{R}^d$, which is the union of p open subsets $A_1, \ldots, A_p$. How can we produce realizations of $\mathcal{P}$ in A, such that each domain A_i contains a prespecified number of points n_i? In spite of its simple formulation, this problem is by no means trivial. The p subsets may give rise to 2^p intersections; their individual volumes are not necessarily explicitly known. Even for small p, standard procedures, such as the rejection technique, may fail in practice.

In this talk, we present a simulation algorithm inspired by genetic optimization techniques (Goldberg, 1978). It consists of building a Markov sequence of point processes, the ergodic distribution of which is precisely the conditional distribution required. The practical setup of this algorithm includes its initialization, the design of its transition kernel, as well as the study of its rate of convergence. Following Propp and Wilson (1996), this algorithm can be made exact, or at least perfect in the sense of Kendall (1996). The compatibility test of the conditions (for any $i = 1, \ldots, p$, A_i contains n_i points exactly), which is crucial in practice, is also investigated in depth.

More generally, Poisson point processes constitute the basic ingredient for various models for random sets and random functions encountered in stochastic geometry (Poisson or Voronoi tessellations, certain types of self similar random functions...). The genetic algorithm considered above can be adapted to carry out conditional simulations of these models. Many examples will be given.

G. LAST

Stationary flows and an inversion formula for Palm probabilities

We consider a surface process Φ tessellating the space $\mathbb{R}^d$ into cells together with a random velocity field $u = \{u(x) : x \in \mathbb{R}^d\}$ that is smooth on each cell but may jump on the boundary Φ. The pair (Φ, u) is (jointly) stationary and u is divergence–free on each cell. We prove and discuss a formula expressing the stationary expectation of a random variable in terms of the Palm probability P_Φ of the surface measure associated with Φ. This formula involves the flow α having the velocity field u and generalizes a well-known classical result for $d = 1$. As an application we use P_Φ to formulate necessary and sufficient conditions for the flow α to be volume–preserving.

T. M. LIEBLING

On power diagrams, ceramics and hour glasses

Power diagrams and their dual triangulations can be tracked back at least as far as to Descartes, who used them to study stellar arrangements. They are generalizations of Voronoi Diagrams and Delaunay triangulations, which have been rediscovered time and again reflecting the fact that they arise quite naturally when modelling all kinds of spatial situations and processes. In this talk we will survey interdisciplinary research carried out at EPFL over the past years, demonstrating the usefulness of two and three dimensional power diagrams in efficient simulation model building. We will illustrate their crucial role in polycrystal growth simulation and distinct element simulation of granular media evolution. In the case of polycrystal growth simulations, their mathematically proven adaptability to (a large proportion of) convex space partitions made large scale succinct and realistic modelling of evolving spatial tessellations possible, thus for the first time realistically reproducing 3d normal grain growth on a large scale. In the distinct element modelling of granular media it accelerated collision detection between grains from $O(n^2)$ to $O(n)$, where n is the number of grains. It was possible to perform very large scale simulations that satisfactorily match experimental observations on convection, segregation, penetration and avalanches to the extent that they are available.

Based on work with A. Mocellin (Nancy) and H. Telley, F. Righetti, X. Xue and D. Müller (Lausanne).

M.–C. VAN LIESHOUT

Markov random sets

In this talk I will survey Markov random sets. These form a class of random sets that are interesting, both in their own right and as prior distributions in a wide variety of applications.

Particular attention will be given to size–biased Markov random sets, generalising Sivakumar & Goutsias's morphologically constrained lattice models. We indicate how granulometries may be useful in analysing binary images, and define a suitable size distribution function as a tool in exploratory data analysis. A new Hanisch-style estimator is suggested to estimate this size distribution function and used to construct Markov random set models which favour certain sizes above others. Applications and examples on real and simulated data sets are included.

S. MASE

Intensities of hard core Gibbs processes and the closed packing density

Intensities of hard core Gibbs processes as functions of the activity z are studied. We show that intensities can attain the closest packing density as $z \to \infty$. Both local and global hard core processes are considered.

Also it is shown that the intensity is an increasing function of z except at those z where the phase transition occurs.

J. Mecke

Typical cell and 0-cell

There are treated variations on the subject, "the 0-cell in a stationary random tessellation is in a sense larger than the typical cell".

It is well-known that the mean volume of the 0-cell in a stationary random tessellation in the d-dimensional Euclidean space ($d = 1, 2, \ldots$) is not smaller than the mean volume of the typical cell. It can be shown that a corresponding result is true for all moments.

A stronger result, a stochastic order relation for the distributions of the volumes of the typical cell and the 0-cell is derived. By a suitable interpretation, it is possible to say that almost surely the volume of the typical cell is not greater than the volume of the 0-cell.

In the case of hyperplane tessellations, it is even true that a random polytope with the distribution of the typical cell can be imbedded as a subset in the 0-cell.

K. Mecke

Morphological extensions of the contact distribution function

The morphological characterization of complex spatial patterns is becoming more and more important in Statistical Physics. We propose a method for the description of spatial patterns formed by a coverage of point sets, where each point is decorated by a sphere of radius r. The method is based on the complete family of Minkowski functionals $M_\nu(r)$ ($\nu = 0, \ldots, d$), which includes the topological Euler characteristic and geometric descriptors to specify the content, shape and connectivity of spatial sets as functions of the radius r. Calculating the Minkowski functionals for a given covering we obtain a quantitative characterization of the scale-dependent morphology of the underlying spatial point process, which generalizes the spherical contact probability $H_s(r) = M_0(r)$. Many applications of these morphological measures are possible in Statistical Physics, for instance, estimating the percolation threshold in porous media, describing the scaling behavior during spinodal decomposition kinetics, and defining order-parameters for dissipative structures such as Turing patterns in chemical reaction-diffusion systems. We illustrate the descriptive power of the functions $M_\nu(r)$, in particular of the Euler-Characteristic $M_d(r)$ by estimating them for three point patterns: the distribution of holes in thin polymer films, of particles in fluid systems, and of galaxies in the universe.

T. Norberg

Particular application and modelling geological structures with Markov random fields

A class of Markov random fields is introduced in which it is possible to write the energy function Ψ in a canonical way as the mean of the negative log likelihood of, say $k = 1, 2, \ldots,$ Markov chains. (Recall that the probability $p(\mathbf{x})$ of a configuration $\mathbf{x} \in S^V$ is proportional to $\exp(-\Psi(\mathbf{x}))$. The actual value of k depends on the site space V, which typically is a rectangular section of Z^2 or a cubic section of Z^3, but need not be; S denotes the state space, which is finite.) Then Ψ has an interpretation Ψ_A, which is defined for a part $\mathbf{x}_A$ of the whole configuration $\mathbf{x} = \mathbf{x}_A\mathbf{x}_B$ ($\mathbf{x}_B$ being the unknown part; $A \cup B = V$, $A \cap B = \emptyset$). This opens up the possibility for finding the energy function $\widehat{\Psi}$ that minimises $\Psi_A(\mathbf{x}_A)$:

$$\widehat{\Psi} = \arg\min_{\Psi} \Psi_A(\mathbf{x}_A).$$

An example of how this program is carried out in a geological setting is given. Also the maximum a posteriori predictor $\widehat{\mathbf{x}}_B$ of $\mathbf{x}_B$ is calculated.

The Ising model with no external field fits well into this canonical energy framework. It suggests the following canonical minimum energy estimator of the inverse temperature β in the case of a complete observation $\mathbf{x}$:

$$\widehat{\beta} = \frac{1}{2k} \log \frac{e(\mathbf{x})}{d(\mathbf{x})}.$$

Here $e(\mathbf{x})$ and $d(\mathbf{x})$ denote the number of neighbour pairs that are equal and unequal, respectively.

V. K. Oganian

Parametric versions of Hilbert's fourth problem

Let H be the class of sufficiently smooth metrics defined in the Euclidean plane for which the geodesics are the usual Euclidean lines. The general problem is to describe all metrics from H which at all points possess the length indicatrix from a prescribed parametric class of convex figures. As a tool, a differential equation is proposed derived from the "triangular deficit principle" formulated in an earlier paper of R. V. Ambartzumian. We demonstrate that for the case where the length indicatrix is segmental this equation leads to a complete solution. There is a partial result stating that in the case of Riemann metrics the orientation of the ellipses should necessarily be a harmonic. We also prove that in the class H there exist Riemannian metrics that are not translation invariant.

J. Rataj

Curvatures of flat sections

Given a set X of positive reach in a Euclidean space, the (generalised) curvature measures can be introduced as integrals over the unit normal bundle of the symmetric functions of principal curvatures of given order, see Zähle (1986). Under a mild technical assumption, translative and kinematic formulae for the curvature measures of flat sections of X can again be expressed by means of integrals over the unit normal bundle of X (Rataj, 1997). As a consequence, we obtain expressions for the symmetric functions of principal curvatures of the flat sections. The formulae mentioned are illustrated on the example of planar sections of a spatial body.

K. RITTER

Adaptive sampling for integration of random functions

We discuss integration of Gaussian random functions $Y(t), t \in [0,1]^d$, with inhomogeneous local smoothness. A single realization may be observed at a finite number of sampling points $t_1, \dots, t_n \in [0,1]^d$, and the correct local smoothness is unknown. Errors are defined in quadratic mean sense.

In the univariate case, $d = 1$, we present adaptive (sequential) sampling designs that lead to asymptotically optimal predictors for $\int_0^1 Y(t)dt$. On the other hand any nonadaptive design will fail.

Joint work with T. Müller-Gronbach, FU Berlin.

M. SCHLATHER

What is a random Voronoi tessellation? A further definition

Let S be a σ-compact metrizable space and $\overline{\mathbb{R}}$ be the extended real axis, i.e., $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$. The following construction principle of a deterministic tessellation in S is considered: Given a set of upper semi-continuous functions $\{f_1, f_2, \dots\}$ with values in $\overline{\mathbb{R}}$, a cell corresponding to $f_j, j = 1, 2, \dots$, consists of all the points $x \in$ S where $f_j(x) = \inf\{f_1(x), f_2(x), \dots\}$. This is a generalization of the construction of the Voronoi tessellation, where $f_j(y) = ||y - x_j||$ for a given set of points $\{x_1, x_2, \dots\}$. The dead leaves model as well as many variations of the Voronoi tessellation (e.g. the Johnson–Mehl tessellation and the tessellation corresponding to the postmen–problem) follow also this construction principle.

In order to define a *random* Voronoi tessellation in a wide sense, the upper semi-continuous functions are identified with the boundary of their hypographs, an element of CL(S $\times \overline{\mathbb{R}}$). Here CL(.) denotes the space of closed sets provided with the Fell topology. Let Y be the closure of the set of upper semi-continuous functions in CL(S $\times \overline{\mathbb{R}}$). Then a Voronoi tessellation v (in a wide sense) can be defined as a mapping from CL(Y) in CL(CL(S)) by $v(\varphi) = \text{clos}\,(\bigcup_{f\in\varphi}\{x \in \mathrm{S} : f \cap \inf(\varphi) \cap (\{x\} \times \overline{\mathbb{R}}) \neq \emptyset\})$. Here $\inf(\varphi) = \partial(\bigcap_{f\in\varphi}\{(x,y) \in \mathrm{S} \times \overline{\mathbb{R}} : \text{there exists } z \geq y \text{ with } (x,z) \in f\})$. If Φ is a random variable with values in CL(Y), then $v(\Phi)$ is a random Voronoi tessellation.

V. SCHMIDT

Series expansion for functionals of spatial point processes

An expansion technique is discussed for the mean value of a class of functionals of a spatial marked point process with respect to its factorial moment measures. This complements previous studies for point processes on the real line, by extending the results to point processes on a general Polish space. The method is applied in order to derive asymptotic expansions for characteristics of the Boolean model. Differentiability properties of Poisson driven functionals are also discussed.

M. SCHMITT

Digitization and connectivity

Consider a closed set X in the n-dimensional space. We digitize it on the square lattice with spacing $1/2^n$. We investigate the links between the connectivity of X and the connectivity of its digitized versions. We show that:

- the good digitization procedure is to approximate X by the union of all the squares of the lattice it hits.
- for the topological connectivity (X cannot be decomposed into two non empty disjoint closed sets) and the geodesic connectivity with bounded diameter (X is connected and bounded for its geodesic distance) the digitization scheme works nicely. X is connected if and only if all its digitized versions are connected. This result is not true for the arc connectivity (two points are linked by a path).
- if X is simply connected (X and X^c connected), all its digitized sets may be not simply connected. However the sequence of simply connected digitized sets obtained by filling their holes also converges to X.
- the set of topologically connected sets is closed in the set of compact non empty sets, hence measurable for the hit-or-miss sigma algebra. In other words, the probability that a random closed compact set is connected makes sense.

R. SCHNEIDER

Flat processes and convexity

With a random process of geometric objects (e.g. flats, surfaces, convex particles) one can sometimes associate a certain auxiliary convex body and then use results from convex geometry to solve uniqueness or extremum problems for the original process. This "method of associated zonoids", invented by Matheron and later extended mainly by J. A. Wieacker, is illustrated with some examples for stationary Poisson processes of r–dimensional flats in Euclidean n–space. The limitations of the method are also demonstrated, by mentioning closely related results that require different methods. The new results concern, for example, the unique determination of a stationary Poisson hyperplane process by the order-k intersection densities of its intersection processes with r–dimensional subspaces, extremal problems for r–flat processes hitting a convex test body of given size, and a duality for Poisson flats interchanging two parameters that measure how close the flats come together in the mean.

M. STEIN

Estimating the large–scale structure of the universe through quasar absorption lines

Absorption lines in quasar spectra allow for the detection of distant galaxies that are on the line of sight between the Earth and a quasar. We apply edge–corrected methods for estimating the second moment structure of a stationary point process to an extensive new catalog of galaxies detected from quasar absorption spectra. We find a sharp transition in the correlation structure of the galaxy locations at around 20 Mpc/h.

D. STOYAN

Tessellations and Gibbs processes

In order to give a basis for a discussion session the following problems were presented.

1) Mean value formulae for Voronoi tessellations with respect to non– homogeneous Poisson processes. In particular, cell volume and cord length were discussed.

2) A new model of a process of forming cracks on surfaces. It is based on the Poisson Voronoi tessellation and produces random subsets of the system of edges of this tessellation. For the edge length density at time t an explicit formula was given.

3) Application of non–homogeneous Gibbs processes for modelling tree stands in forests. The relationships between short and long range variability parameters were particularly considered.

E. VAN ZWET

Statistics of windowed line-segment processes

We investigate the estimation of the distribution of the length of fractures in a rock surface. The censoring due to vegetation, soil and water is very irregular so that the area where we observe the cracks is not convex. This means that we might observe several pieces of a single crack. Because it is difficult at best to assess if two pieces belong to the same crack, we simplify by pretending that all observed pieces are independent. In this simpler model we find the non parametric maximum likelihood estimator of the length distribution. We then show that the estimator retains its consistency if we use the real, dependent, data.

E. B. VEDEL JENSEN

Local stereology

During the last 15 years a new branch of stereology has been developed, called local stereology. The objectives are still the same, namely inference about parameters like volume and surface area, but the sampling designs are of a new type. They are designed for the analysis of a spatial structure which can be regarded as a neighbourhood of a point, called the reference point. Local sampling designs are based on sections through this reference point. The model example is the case, where the spatial structure is a biological cell which can be regarded as a neighbourhood of its nucleus or nucleolus.

In the lecture, a review of this field is given. Two new research projects in local stereology are also discussed. The first one concerns the *variances of local stereological estimators*. It is pointed out that there are close connections between local stereology and geometric tomography. Results from the latter field are useful in the study of when local stereological estimators are exact. The second research project concerns *particle size distributions*. It is discussed how local stereology can increase the efficiency of estimators of particle size.

R. VITALE

Gaussian processes and convexity

For a number of years, connections between Gaussian processes and the theory of convex bodies have been intensively studied. In this vein, I described a link involving the Wills functional used in lattice point enumeration, which leads to stochastic bounds of deviation type. I further described the use of stochastic methods for extending the definition of intrinsic volumes to infinite dimensions and a tightened mean isoperimetric inequality for a class of random planar convex bodies.

R. WAAGEPETERSEN

Simulated tempering and simulation of the hard core point process

Simulation of a highly packed Poisson hard core process is difficult since usual Markov chain Monte Carlo (MCMC) techniques based on single point updates typically get stuck in the first highly packed point pattern reached by the algorithm. It is demonstrated how the MCMC technique of simulated tempering can be applied to obtain a fast mixing simulation algorithm.

W. WEIL

Densities of mixed volumes for Boolean models

It is a well-known fact that the quermass densities $\overline{V}_j(X)$ of a stationary and isotropic Boolean model X with convex grains in $\mathbb{R}^d$ determine the intensity γ of the underlying Poisson particle process Y uniquely. This is no longer true in the non-isotropic case since then the formulae for $\overline{V}_j(X)$ involve mean values of Y for mixed functionals from Translative Integral Geometry. Therefore, densities $\overline{V}(X\,[j], K\,[d-j])$ of mixed volumes of X with arbitrary convex bodies K are considered and expressed in terms of γ and certain mean values of Y. In dimensions $d = 2, 3, 4$, these densities of mixed volumes determine γ, whereas the case $d \geq 5$ remains open.

Extensions to non-stationary Boolean models are also considered.

R. J. WILSON

Estimation methods for random set models

A major problem in fitting random set models to image data is the disparity between the model derivation and the mechanism which gave rise to the image. For example, mineral textures may be modelled using the Boolean model but the formation of the texture is different to and more complex than the construction of the model. Thus, the fitted model is simply a means of representing the observed image and does not have an interpretation beyond that. As a consequence, care needs to be taken in both fitting and interpreting random set models:

(a) A particular image may not have properties which are consistent with a given class of models – the inconsistencies may be quite subtle.

(b) The appropriate fitting criteria for a given model may lead to fitted models whose properties do not match the important properties of the image, although the model, with different parameter estimates, may match these properties well enough.

In this presentation, various estimation methods and the properties of the resulting estimators will be presented. The limitations of these methods when applied to real data will be discussed. Particular reference will be made to estimation methods for models based upon excursion sets of random fields.

S. Zuyev

Variational analysis of functionals of a Poisson process

Let F be a functional of a Poisson process whose distribution is determined by the intensity measure μ. Considering the expectation $\mathbf{E}_\mu F$ as a function on the cone $\mathbb{M}$ of positive finite measures μ, we derive closed form expressions for the Fréchet derivatives of all orders that generalise the perturbation analysis formulae for Poisson processes. Variational methods developed for the space $\mathbb{M}$ allow us to obtain first and second-order sufficient conditions for different types of constrained optimisation problems for $\mathbf{E}_\mu F$. We study in detail optimisation in the class of measures with a fixed total mass a and develop a technique that often allows us to obtain the asymptotic behaviour of the optimal intensity measure in the high intensity settings when a grows to infinity. We give applications of our methods to design of experiments, spline approximation of convex functions, optimal placement of stations in telecommunication studies and others. We scetch possible numerical algorithms of the steepest descend type based on the obtained explicit form of the gradient.

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