MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 04/1998

Geometric Questions in Partial Differential Equations

25.01. – 31.01.1998

Die Tagung fand unter Leitung von H. Berestycki (Paris), B. Kawohl (Köln) und G. Talenti (Florenz) statt und hatte geometrische Fragen in den partiellen Differentialgleichungen zum Gegenstand. Hierzu gehörten beispielsweise Symmetriefragen für Differentialgleichungen auf Mannigfaltigkeiten, die Gestalt von Körpern minimalen Strömungswiderstandes oder von Lösungen zu Gleichungen vierter Ordnung, sowie Gebietsvariationsprobleme für Eigenwerte. Da die Vortragszeit mehrheitlich auf 20 Minuten begrenzt wurde, ergab sich hinreichend Zeit zum intensiven gemeinsamen Arbeiten. Die hervorragende Atmosphäre des Instituts und eine Abendsitzung mit 8 offenen Problemen trug zu einem regen Austausch von Ideen bei.

Vortragsauszüge

A. Aftalion

A review of overdetermined boundary value problems

A classical result of Serrin asserts that if a smooth bounded domain Ω is such that there exists a positive solution of the overdetermined problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in} \quad \Omega, \\ u = 0, \quad \partial u/\partial \nu = const & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

then Ω is a ball and u is radial. With J.Busca, we are interested in some extensions of this result to the case of exterior domains, and infinite cylinders.

The first study deals with exterior domains $\Omega = \mathbb{R}^N \setminus \overline{\Omega}_1$, where Ω_1 is a bounded domain. We show under certain hypotheses on f that if there exists a solution of an equation of type (1) then the domain Ω_1 is a ball and u is radial. Our proof covers the case $f(u) = u^p$ for $1 \leq p \leq (n+2)/(n-2)$. It uses Kelvin transforms, and consists in writing a symmetry with respect to a hyperplane, as the limit of a family of inversions, in order to apply a variant of the moving plane device.

In order to treat the case of infinite cylinders, we extend the moving plane device of Berestycki, Caffarelli and Nirenberg to solve problem (1) when $\Omega = \omega \times \mathbb{R}^{n-j}$, where ω is a bounded domain of $\mathbb{R}^j$. We prove that necessarily ω is a ball without hypotheses on the behaviour of u at infinity. Our aim is to extend this result when f is non Lipschitz at 0.

In a work with J.Busca and W.Reichel, we study the stability of Serrin's result for bounded domains. We impose in (1) that $\partial u/\partial\nu$ is close to a constant, instead of $\partial u/\partial\nu = const$ and prove that the domain is close to a ball and the solution is nearly radial.

A. Baernstein II

Rearrangement theorems for multiple integrals

Let $M_{k,n}$ be the set of all affine k-planes in $\mathbb{R}^n$, $1 \le k \le n-1$. For $f : \mathbb{R}^n \to [0,\infty)$ and $\pi \in M_{k,n}$, define $T_{k,n}f(\pi) = \int_\pi f\,dx$. For a natural measure $\mu_{k,n}$ of $M_{k,n}$, it is known that

$$(*) \qquad \left(\int_{M_{k,n}} (T_{k,n}f)^q \, d\mu_{m,n}\right)^{1/q} \le C(p,k,n)\left(\int_{\mathbb{R}^n} f^p \, dx\right)^{1/p}$$

where $1 \le p \le \frac{n+1}{k+1}$, $q = \frac{p(n-k)}{n-pk}$. Here are some conjectures about extremal functions for $(*)$.

Conjecture 1 $f(x) = (a + b|x|^2)^{\frac{-1}{2}\frac{n-k}{p-1}}$ is an extremal for $(*)$. Here $a, b > 0$ are constants. Conjecture 1 would be a consequence of Conjecture 2 and 3.

Conjecture 2 For the symmetric decreasing rearrangement f^* of f holds $\int_{M_{k,n}} (T_{k,n}f)^q \, d\mu_{k,n} \le \int_{M_{k,n}} (T_{k,n}f^*)^q \, d\mu_{k,n}$, for $1 \le q \le n+1$.

Conjecture 3 For $g : \mathbb{R}^+ \to \mathbb{R}^+$, $0 < d < \beta$, $q = \frac{p(\beta-\alpha)}{\beta - p\alpha}$, set $u_\alpha f(x) = \int_x^\infty (y-x)^{\alpha-1} f(y)\,dy$. Then

$$\left(\int_0^\infty (u_\alpha f)^q(x) x^{\beta-\alpha-1}\,dx\right)^{1/q} \Big/ \left(\int_0^\infty f^q(x) x^{\beta-1}\,dx\right)^{1/p}$$

is maximized by $f(x) = (a+bx)^{-\frac{\beta-\alpha}{p-1}}$.

So far we can prove Conjecture 2 and 3 in enough cases to show that Conjecture 1 is true when $k = 2$ and $q = 1, 2, \ldots, n+1$, or when $q = 2$.

C. Bandle

Best Sobolev constants and Emden equations for the critical exponent in S^3

It is shown that for geodesic balls in S^3 the best Sobolev constant for the critical exponent is attained if the balls are larger than the hemisphere. On the other hand for balls smaller or equal to the hemisphere the best Sobolev constant is not attained. The discussion is based upon a study of the corresponding Euler equations. By means of symmetrization and harmonic transplantation criteria in arbitrary domains are derived when the best Sobolev constant is attained . It depends on the size of D. For domains with area smaller than the one of the hemisphere the best constant is not attained. The question has been raised and partly solved in a paper by C. Bandle, A. Brillard and M. Flucher (to appear in TRAMS). The discussion of the Emden equation has been done in collaboration with. L. A. Peletier.

F. Brock

Symmetry via continuous rearrangement

Theorem: Let B be a ball in $\mathbb{R}^n$, $(n \geq 2)$, with center 0, $p > 1$, and assume that $f \in C(\mathbb{R}_0^+)$ and satisfies the following conditions:
(i) if $f(u_0) = 0$ for some $u_0 > 0$, then there is some $c > 0$, such that $f(u) \leq c(u_0 - u)^{p-1}$ $\forall u \in [0, u_0]$
(ii) if $f(0) = 0$, then there is some $d > 0$, such that $f(u) \geq -du^{p-1}$ $\forall u \geq 0$.
Then, if $u \in W_0^{1,p}(B) \cap C^1(\overline{B})$ is a nonnegative, nontrivial solution of the equation $-\nabla(|\nabla u|^{p-2}\nabla u) = f(u)$ we have $u = u(|x|)$ and $\frac{\partial u}{\partial r}(x) < 0$ in $B \setminus \{0\}$, $(r = |x|)$.

The proof of this theorem is based on a "local symmetry" result which was recently obtained via continuous Steiner symmetrization and on the strong maximum principle for the p-Laplacian operator. Details can be found in F. Brock: *Continuous rearrangement and symmetry of solutions of elliptic problems.* preprint, Cologne 1997, 124 p. and F. Brock: *Radial symmetry for nonnegative solutions of semilinear elliptic equations involving the p-Laplacian.* to appear in: Proc. Pont-a-Mousson 1997.

A. Burchard

Continuity of Steiner symmetrizsation in $W^{1,p}(\mathbb{R}^n)$

Steiner symmetrization is a simple rearrangement (of sets or functions) that creates a reflection symmetry. It is often used to approximate the spherically symmetric rearrangement. In the talk I discuss why Steiner symmetrization is continuous in the Sobolev spaces $W^{1,p}$ in several variables – in contrast to the spherically decreasing rearrangement, which was proven to be discontinuous in all dimensions above 1 by Almgren/Lieb (1989). Consequently, the spherically decreasing rearrangement cannot be approximated by Steiner symmetrizations in these spaces. A key step in the proof of continuity is to show that Steiner symmetrization of $W^{1,p}$-functions preserves the measure of the set of critical points.

G. Buttazzo

Optimization problems for eigenvalues

We are interested in the following problem: given a bounded open subset Ω of $\mathbb{R}^n$, an elliptic operator of the form $Lu = -\mathrm{div}\,(a(x)Du)$ with $a(x)$ symmetric and uniformly elliptic on Ω, an integer N, a real number $m > 0$, and a nonnegative continuous function $\varphi : \mathbb{R}^N \to \mathbb{R}$, consider the minimization problem $\min\left\{\varphi(\lambda_1(A), \dots, \lambda_N(A)) \;:\; A \subset \Omega,\ |A| \leq m\right\}$ where $\lambda_j(A)$ are the eigenvalues of L defined on $H_0^1(A)$. When φ is monotone increasing in each variable, the existence of an optimal domain can be proved (Buttazzo-Dal Maso, A.R.M.A. 1993). Here we consider the particular case: i) $L = -\Delta$ and ii) $N = 2$ and we show (Bucur-Buttazzo-Figueiredo, preprint 1997) that for every continuous nonnegative $\varphi : \mathbb{R}^2 \to \mathbb{R}$ the minimization problem above admits an optimal domain. The proof is obtained by showing that the set $E = \left\{(\lambda_1(A), \lambda_2(A)) \;:\; A \subset \Omega,\ |A| \leq m\right\}$ is closed in $\mathbb{R}^2$. Open problems are to remove assumptions i) or ii), and to prove that the set E is actually convex.

X. Cabré

Some estimates related to the Alexandroff-Bakelman-Pucci method

The Alexandroff-Bakelman-Pucci estimate and the Krylov-Safanov Harnack inequality are the two basic estimates in the theory of second order elliptic PDEs in nondivergence form with bounded measurable coefficients in a domain of $\mathbb{R}^N$. We prove new versions of these estimates for (nondivergent) elliptic equations in domains of Riemannian manifolds. Assuming the sectional curvature to be nonnegative, we obtain a global Harnack inequality of Krylov-Safanov type. As a consequence we deduce a Liouville theorem for bounded solutions of the homogeneous equation in all the manifold.

We also present a new proof of the isoperimetric problem in $\mathbb{R}^n$. The proof is quite elementary and uses the Alexandroff-Bakelman-Pucci method as main ingredient. Using similar ideas, we give a new and short proof of some Faber-Krahn type inequalities.

A. Chang

Regularity of some 4-th order non-linear PDE

In conformal geometry, one is interested to study conformal covariant operators. One particularly interesting one is a 4-th order differential operator P_4 discovered by Paneitz in 1983.

$$P_4(\phi) = \Delta^2\phi + \delta[\frac{2}{3}Rg - 2\mathrm{Ric}] \cdot d\phi$$

defined for $\phi \in C^\infty(M^4)$, where M^4 is a compact 4-manifold with metric g. P_4 is a natural analogue of $-\Delta$ on M^2 in many ways. In this talk, I reported on some differential equations related to the study of P_4. More specifically, I reported on two problems:

(a) Uniqueness problem (joint work with P. Yang): Suppose $P_4 w + be^{4w} = b$ on (S^4, g_c), where g_c is the canonical metric on S^4. Then $e^{2w}g_c = \phi^*(g_c)$ for some conformal transformation ϕ on S^4. This result can be proved via the method of moving planes, and can be generalized to S^n.

(b) Regularity problem (joint work with P. Yang & M. Gursky): It turns out that the $\langle P_4 w, w\rangle$ is one term in the log-determinant formula $F[w] = \log \frac{\det L_w}{\det L}$, where L =conformal Laplacian, $L_w = L$ w.r.t $g_w = e^{2w}g$. On (M^4, g), we proved that extremal metrics (for $w \in W^{2,2}$) are regular.

E.N. Dancer

Some nonlinear elliptic problems on domains with small holes

We discuss the problem

$$\begin{aligned} -\Delta u &= u^p \quad \text{and} \quad u > 0 \text{ in } \Omega \setminus \cup_{i=1}^k U_{i,\epsilon}, \\ u &= 0 \quad \text{on} \quad \partial(\Omega \setminus \cup_{i=1}^k U_{i,\epsilon}). \end{aligned}$$

Here Ω is a bounded domain in $\mathbb{R}^N$, $U_{i,\epsilon}$ are small holes tending to zero with ϵ and $1 < p < \frac{n+2}{n-2}$. We are interested in the existence of solutions whose sup-norm tends to ∞ as $\epsilon \to 0$. We show whether this occurs depends on the shape of the $U_{i,\epsilon}$. In particular, if $U_{i,\epsilon}$ has the shape of W_i for small ϵ where $0 \in W_i$, then this problem is closely related to the existence of solutions of the exterior problem

$$\begin{aligned} -\Delta u &= u^p \quad \text{and} \quad u > 0 \text{ in } \mathbb{R}^n \setminus W_i, \\ u &= 0 \quad \text{on} \quad \partial W_i. \end{aligned}$$

The existence for this problem is complicated but we show that there is no solutions if $p \leq \frac{n}{n-2}$ or if W_i is nearly star-shaped while there is a solution if W_i has non-trivial homology and p is close to the critical exponent.

J. Denzler

Existence of windows with minimal heat leakage

For a given bounded Lipschitz domain $\Omega \subset \mathbb{R}^n$ and some measurable set $D \subset \partial\Omega$ let $\lambda_1(D)$ be the first Laplace eigenvalue subject to Dirichlet BCs on D and Neumann BCs on $\partial\Omega \setminus D$. For the problem $\min\{\lambda_1(D) | \text{area}(D) = A\}$ an existence proof for minimizers is given. Spherical symmetrization arguments show that for Ω a ball, the minimizer is a spherical cap, and it is unique in this case. I conclude with simple heuristic evidence that the optimal D in a convex domains Ω should be connected.

V. Ferone

Nonsymmetry for Newton's problem of minimal resistance

Let $\Omega \subset \mathbb{R}^2$ be the maximal cross-section of a body. One can describe the front end of the body by a function $u : \Omega \to \mathbb{R}$. Using Newton's model, the resistance of the body, which moves with constant velocity (orthogonal to the cross section) through a fluid, is proportional to the following integral: $F(u) = \int_\Omega (1+|Du|^2)^{-1}\, dx$. The problem of minimizing $F(u)$ has been studied by many authors when Ω is a disc and u is radially symmetric. If one considers the minimization of $F(u)$ without any symmetry assumption on Ω and on u, the existence of a minimizer has been proven, for example, in the class of admissible functions $C_M = \{u \in W^{1,\infty}_{loc}(\Omega) : 0 \leq u \leq M \text{ in}\, \Omega,\, u \text{ concave}\}$ (see [Buttazzo–Ferone–Kawohl, Math. Nachr., 173 (1995)]). A natural question is the following one: if Ω is a disc, do the minimizers of $F(u)$ in C_M need to be radially symmetric? A negative answer to the above question has been given in [Brock–Ferone–Kawohl, Calc. Var., 4 (1996)]. Indeed, denoting by v_r the function which minimizes $F(u)$ in the class $C_M \cap \{$radially symmetric functions$\}$, it is possible to construct a function $\bar{u} \in C_M$, which is not radially symmetric, such that $F(\bar{u}) < F(v_r)$. We finally observe that the question about the symmetry of the minimizers of $F(u)$ can have a different answer if one changes the class of admissible functions. For example, the existence of a minimizer of $F(u)$ can be proven in the class of bodies with prescribed surface area and, when Ω is a disc, a radially symmetric minimizer in such a class exists (see [Ferone–Kawohl, preprint, 1997]).

M. Flucher* and M. Rumpf

Tool design in electrochemical machining with threshold current, an ill-posed free-boundary problem

Tool design in BCM leads to the following Cauchy problem for the electric potential. Given the shape of the workpiece Γ_0 (anode) design a cathode Γ_1 such that

$$\begin{aligned} -\Delta u &= 0 \quad \text{between } \Gamma_0 \text{ and } \Gamma_1 \\ u &= 0, \quad \partial u/\partial\nu = E_0 \quad \text{on } \Gamma_0 \\ u &= 1 \quad \text{on } \Gamma_1 \end{aligned}$$

where E_0 denotes the threshold current. After reformulating it as an evolutionary problem for the level sets of the potential a simple von Neumann stability analysis confirms ill-posedness and instability of the Cauchy problem. Therefore solving it numerically is only possible after regularization – for instance by adding artificial viscosity. The stability analysis suggests the appropriate form of these terms leading to satisfactory numerical results. (http://www.math.unibas.ch/~flucher/)

W. Gangbo* and R. McCann

Wasserstein distance and computer vision

We study the geometry of the support of the optimal measure γ_0 minimizing the functional $I[\gamma]$ where

$$(M) \qquad I[\gamma] := \int_{\mathbb{R}^d\times\mathbb{R}^d} \|x-y\|^2\, d\gamma[x,y],$$

the infimum being performed over the set $\Gamma(\mu^+,\mu^-)$ of all measures that have μ^+ and μ^- as their marginals. Here μ^+ and μ^- are two probability measures on $\mathbb{R}^d\times\mathbb{R}^d$. We prove that (M) admits a unique minimizer γ_0 provided that the supports of μ^+ and μ^- are two strictly convex, uniformly convex hypersurfaces and μ^+,μ^- do not have atoms. Under the above assumptions we prove that the support of γ_0 lies in the union of two smooth maps.

H.-C. Grunau

Positive boundary data in clamped plate equations

This talk concerns joint work with Guido Sweers (Delft, The Netherlands). Positivity phenomena in higher order elliptic Dirichlet problems as e.g. in the clamped plate equation are in general rather subtle. Until now most people concentrated on positivity with respect to the right-hand side, i.e. on positivity of the Green function itself. Here it depends on the domain and on the particular form of the operator whether there are comparison principles or not. In this talk we focus on the role of the boundary data, i.e., on positivity of certain Poisson kernels. While it is expected that the Poisson kernel of highest order behaves similarly as the Green function, it may be surprising that for some Dirichlet problems of arbitrary order and in any dimension there is also a positivity result with respect to a second Poisson kernel. Furthermore we report on a perturbation theory for this result.

G. Huisken* and Tom Ilmanen
Inverse mean curvature flow in asymptotically flat manifolds
In this work we study solutions $F : M^n \times [0,T] \to (N^{n+1}, g)$ of the inverse mean curvature flow

$$(*) \qquad \frac{d}{dt}F = \frac{1}{H}\nu, \quad F(\cdot, 0) = F_0,$$

where ν is the outer unit normal to a closed hypersurface, H the mean curvature and F_0 some initial data in a smooth Riemannian manifold (N^{n+1}, g). An equivalent level set formulation uses a scalar function $u : N^{n+1} \to \mathbb{R}$ satisfying

$$(**) \qquad D_i\left(\frac{D_i u}{|Du|}\right) = H = |Du| \text{ in } N^{n+1}, \quad u = 0 \text{ on } F_0(M^n),$$

such that the level sets $\{u(x) = t\}$ represent the evolving surfaces $M_t^n = F(\cdot, t)(M^n)$. The lecture describes weak solutions of $(**)$ and indicates how they can be used to prove the Penrose inequality for asymptotically flat three-manifolds. Moreover, it is shown how an upper bound for the speed $(\frac{1}{H})$ can be derived for star-shaped surfaces, implying that the weak solution of $(**)$ is actually smooth outside some compact set in an asymptotically flat manifold.

V. Kondratiev
Blow-up results for nonlinear parabolic problems
We consider the parabolic equation

$$u_t = \sum_{i,j=1}^{n} (a_{ij}(x,t)u_{x_j})_{x_i} + \sum_{i=1}^{n} a_i(x,t)u_{x_i} + a_0(x,t)|u|^{\sigma_1 - 1}u = 0$$

with the boundary condition

$$\frac{\partial u}{\partial \nu} - b_0(x,t)|u|^{\sigma_2 - 1}u = 0$$

where $x \in \Omega$, Ω is a bounded Lipschitz domain, the coefficients are measurable bounded functions and $1 < \sigma_1 < 2\sigma_2 - 1$. We prove that there is no positive solution of this problem in $\Omega \times [0, \infty)$ if

$$\lim_{t\to\infty} h \int_{\partial\Omega\times[0,t]} b_0(x,\tau)\, ds_\gamma\, d\tau - \int_{\Omega\times[0,t]} a_0(x,t)\, dx\, dt = +\infty$$

where $h = \text{const.}$ is small enough. We study also the asymptotic behaviour of solutions tending to zero as $t \to \infty$. The case of the Dirichlet problem is considered also.

R.S. Laugesen

Vibrating strings and cylinders with variable mass density

Suppose we have a vibrating, homogeneous string that is fixed at one endpoint and free at the other. If we perturb the mass density of the string by moving mass towards the free end, then intuition suggests the vibration will slow down. This is actually true only for the fundamental mode, and the higher frequencies of vibration may either increase or decrease. Nonetheless, we will show that certain *sums* of eigenvalues do change monotonically under perturbation of the mass towards the free end. The spectral zeta function is one such spectral sum.

These extremal results generalize to cylindrical membranes. Also, corresponding results hold for strings *fixed* at both ends: one can move mass towards the middle of the string in order to slow it down. But the results fail to generalize to cylinders, under the purely fixed boundary conditions.

Finally, under a stronger hypothesis on the mass density we do succeed in extending the method to the "generalized cylinder" $\{\text{interval}\} \times M$, where M is a compact homogeneous Riemannian manifold.

H.A. Levine

A system of reaction diffusion equations arising in the theory of reinforced random walks

Cells may interact in a variety of ways. For example, there may be long range(hormonal) interaction, intermediate range interaction via the production and release of diffusible substances or short range interactions due to local modifications of the environment such as the production and release of substances which modify the extra-cellular matrix. There may even be contact interactions via surface recognition molecules or cell-to-cell exchange of low molecular weight substances via gap functions. Examples which combine several of these interactive processes occur in the study of fruiting bodies such as myxococcus fulvus or the dictyostelium discoideum amoeba. The fruiting body cycle begins with the development of spores which germinate and develop in vegetative growth until starved of nutrients. In this latter case the vegetative growth aggregates to form a new fruiting body to start the cycle once more. This process is far from being completely understood. Dispersal often involves mechanisms that may include correlations in movement. For example, the movement of an organism in response to external stimuli may include a 'taxes' dependence on flux densities, avoidance phenomena or orientation of cells. It is well accepted that dispersal in general is one of correlated or reinforced random walks [1]. Consequently it is important to address the following questions:

1) How are the microscopic details of detection of cells to stimuli and their response reflected in the macroscopic parameters of a continuous description?
2) Is aggregation possible without long range signaling via a diffusible attractant?

In their attempt to address these questions Othmer and Stevens [3] have developed a number of mathematical models of chemotaxis to illustrate aggregation leading (numerically) to non constant steady states (which appear to be stable, at least numerically), blow up resulting in the formation of singularities (in finite time) and collapse or the formation of a spatially uniform steady state. In [3], they recorded the results of their numerical experiments. We present some theorems and plausible arguments that explain their numerical observations.

[1] Davis, B., *Reinforced Random Walks* Probability Theory Related Fields **84** (1990) 203-229.
[2] Levine, H. A. and Sleeman, B. D., *A system of reaction-diffusion equations arising in the theory of reinforced random walks* SIAM J. Appl. Math. (in press)
[3] Othmer, H. G. and Stevens, A., *Aggregation, blow and up and collapse: the ABCs of taxis. and reinforced random walks* SIAM J. Appl. Math. **57**(1997).

Congming Li

Prescribing scalar curvature on S^n

We consider the prescribing scalar curvature equation

$$-\Delta u + \frac{n(n-2)}{4}u = \frac{n-2}{4(n-1)}R(x)u^{\frac{n+2}{n-2}} \tag{1}$$

on S^n for $n \geq 3$. In the case R is rotationally symmetric, the well-known Kazdan-Warner condition implies that a necessary condition for (1) to have a solution is: $R > 0$ somewhere and $R'(r)$ changes signs.

(a) Is this a sufficient condition?

(b) If not, what are the necessary and sufficient conditions?

These have been open problems for decades. We answered question (a) negatively in an earlier paper. We showed that a necessary condition for (1) to have a solution is that $R'(r)$ changes signs in the region where R is positive. Is this condition also sufficient? We proved that if $R(r)$ satisfies the 'flatness condition', then the necessary and sufficient condition for (1) to have a solution is that $R'(r)$ changes signs in the region where $R > 0$. This essentially answers question (b). We also generalized this result to non-symmetric functions R. Here the additional 'flatness condition' is a standard assumption which has been used by many authors to guarantee the existence of a solution. Based on a theorem in a recent paper, we also show that for some rotationally symmetric R, (1) is solvable while none of the solutions is rotationally symmetric. This is an interesting result in the study of symmetry breaking.

M. Loss

On the Laplace operator penalized by mean curvature

This work, which is jointly with Evans Harrell, is concerned with linear differential operators of the form $H = -\Delta - q$ defined on curves, surfaces, and hypersurfaces. Here $-\Delta$ is the corresponding Laplace–Beltrami operator and q is a quadratic expression in the principal curvatures κ_j. In the particular case where $q = \sum_j \kappa_j^2$, the operator H plays a role in the evolution of phase interfaces in materials. A stability analysis of these interfaces led Alikakos and Fusco (in: *The spectrum of the Cahn-Hilliard operator for generic interface in higher space dimensions*, Indiana U. Math. J. 4, 1993, pp. 637–674) to formulate the following spectral geometric conjecture:

Conjecture (Alikakos and Fusco) *a) Suppose that Ω is a simply connected, smooth, compact surface in $\mathbb{R}^3$. The second eigenvalue of H with $q = \sum_j \kappa_j^2$*

is maximized at 0 precisely when Ω is a sphere.
b) Suppose that Ω is a simple, closed, smooth curve in the plane. The second eigenvalue of H with $q = \kappa^2$ is maximized at 0 precisely when Ω is a circle.

Let $h = \sum_{j=1}^{d} \kappa_j$ where the κ_j are the principal curvatures of a d-dimensional hypersurface immersed in $\mathbb{R}^{d+1}$. Our main result is the following:

Theorem *Let Ω be a smooth compact oriented hypersurface of dimension d immersed in $\mathbb{R}^{d+1}$; in particular self-intersections are allowed. The metric on that surface is the standard Euclidean metric inherited from $\mathbb{R}^{d+1}$. Then the second eigenvalue λ_2 of the operator $H = -\Delta - \frac{1}{d}h^2$ is strictly negative unless Ω is a sphere, in which case λ_2 equals to zero.*

In particular this proves the conjecture of Alikakos and Fusco.

K. Mikula

Numerical methods for PDEs arising in geometry driven image analysis

We discuss several degenerate parabolic PDEs, and their numerical solutions, related to multiscale analysis of 2D and 3D images as well as image sequences. We consider that $u(x,t)$, the general image intensity function, satisfies the regularized anisotropic diffusion equations accompanied with slow and fast diffusion effect

$$u_t = \nabla \cdot (g(|\nabla G_\Gamma * u|)\nabla\beta(x,u))$$

for $t \in [0,T]$, $x \in \Omega \subset \mathbb{R}^N$, with $\beta(x,u)$ a nondecreasing Lipschitz continuous in u, G_Γ a smoothing kernel and g Lipschitz continuous, $g(0) = 1$, $0 < g(s) \to 0$ for $s \to \infty$. The zero Neumann boundary conditions are considered and the processed image $u_0(x)$ gives the initial condition. The numerical method is based on finite element discretization in space and a kind of Jäger-Kačur algorithm in scale t.

In order to smooth the image silhouettes we apply the level set equation to the initial image. In case of denoising of time image sequences, we combine the ideas of Guichard's multiscale analysis with anisotropic diffusion equations of Perona&Malik. The computational results on 3D echocordiographic images are presented.

L. Caffarelli and V. Oliker*

The Minkowski method in the problem of mirror design

We consider the problem of recovering a closed convex reflecting surface such that for a given point source of light (inside the convex body bounded by the surface) the reflected directions cover a unit sphere with prescribed in advance density. In analytic formulation the problem leads to an equation of Monge-Ampère type on the unit sphere. In this talk we describe the problem in terms of certain associated measures and establish existence of weak solutions.

M. Peletier

Towards a characterization of the body of least resistance

We study the minima of the functional $\int_\Omega f(\nabla u)$. The function f is not convex, the set Ω is a domain in $\mathbb{R}^2$ and the minimum is sought over all concave functions on Ω with values in a given bounded interval. The most well-known example of problems of this type is the problem of the body of least resistance as posed by Newton, where $f(p) = 1/(1+|p|^2)$.

Brock, Ferone and Kawohl (Calc. Var. PDE, 4 (1996)) have proved that a minimizer u satisfies $\det D^2u = 0$ in any open set in which it is twice continuously differentiable. In particular u can not be strictly concave in such a set. We generalize this statement to be independent of the regularity: in any open subset the minimizer is non-strictly concave.

The proof of this result relies on techniques that are completely different from those used by Brock, Ferone and Kawohl. The main tool is the usage of perturbations of the form $(u-\theta)_+$ where θ is an affine function. An important step consists in showing, under the hypothesis of (local) strict concaveness of u, that in the direction of one of these perturbations the functional has a strictly negative first variation.

J. Prajapat

The moving plane method on manifolds

1) We adapt the method of "moving planes" to prove symmetry results of solutions of differential equations and domains – in particular Serrin's result, and the Gidas-Ni-Nirenberg result for hyperbolic space $\mathbb{H}^N$ and sphere S^n. We define "reflections" for these manifolds which generate the isometry groups, which correspond to the usual Euclidean reflection. This work is done jointly with Prof. S. Kumaresan.

2) Here we define H-reflections for the Heisenberg group. We consider the equation $\Delta_H + u^{\frac{Q+2}{Q-2}} = 0$ on the Heisenberg group, u positive, $\Delta_H =$ Heisenberg Laplacian, $Q = 2n+2$ which is the critical exponent equation in this setup. We describe a group of transformations which leave Δ_H invariant and prove using "moving plane techniques" that every solution of the above

equation is of the form $u(z,t) = C|t + i|z|^2 + z \cdot \bar{\mu} + \lambda|^{-n}$ with $C > 0$, $\lambda \in \mathbb{C}$, $\mathrm{Im}\lambda > 0$, $\mu \in \mathbb{C}^N$. This work is in collaboration with Prof. I. Birindelli.

J.M. Rakotoson

Relative rearrangement: A review of results

After giving the definition of the relative rearrangement for functions defined on a measure space (X, μ) with μ a nonnegative, complete and nonatomic measure, we give various examples of applications like Euler equations for multiconstraint problems appearing in ideal fluids, pointwise estimates for solutions of quasilinear equations in weighted Sobolev spaces, models in plasma physics for Stellerator geometry and Tokamak geometry, regularity of the first derivative of the weighted monotone rearrangement.

W. Reichel

Overdetermined boundary value problems and characterizations of balls in electrostatics, capillarity and heat flow

Consider a body $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ with a metal surface $\partial\Omega$. A charge-distribution $\rho : \partial\Omega \to [0, \infty)$ creates a single-layer potential with potential energy $E(\rho)$. In a suitable class of charge-distributions it can be shown that the variational problem $\min E(\rho)$ subject to $\int_{\partial\Omega} \rho(x)\, d\sigma_x = 1$ has a minimizer ρ^*, the *equilibrium distribution*. The corresponding single-layer potential satisfies an overdetermined boundary value problem $\mathbb{R}^N \setminus \overline{\Omega}$ with constant Neumann and Dirichlet data on $\partial\Omega$. The following conjecture was formulated by P. Gruber: *Ω is a ball $\Leftrightarrow \rho^* \equiv const.$*

In the class of bounded C^2-domains we have proved this conjecture by making the moving plane method of Alexandroff-Serrin available for exterior domains. In the class of (possibly non-smooth) convex domains, the characterization also holds and is proved in collaboration with O. Mendez (Univ. of Missouri, Columbia) with the help of maximum principles, convex analysis and the isoperimetric inequality.

Similar overdetermined problems arise in capillarity and steady-state heat-flow. In collaboration with A. Aftalion and J. Busca (ENS, Paris) we consider the *stability* of these characterizations: *if the Neumann and Dirichlet data are almost constant than the underlying domain is almost a ball.*

G. Sweers

Critical constants for positivity of noncooperative elliptic systems

Consider the noncooperative model system

$$\begin{cases} -\Delta u &= f - \varepsilon v & \text{in } \Omega, \\ -\Delta v &= \varepsilon u & \text{in } \Omega, \\ u = v &= 0 & \text{on } \partial\Omega. \end{cases}$$

<u>Theorem</u> *If Ω is a bounded Lipschitz domain in $\mathbb{R}^n$, then there exists $\varepsilon_\Omega > 0$ such that for all $\varepsilon \in (0, \varepsilon_\Omega)$ we have $f \geq 0 \Rightarrow u \geq 0$.*

The result, which is uniform in f, cannot be proven by a direct application of the classical maximum principle. However, if $|\varepsilon| < \lambda_1$, it is sufficient to prove that $(I - \varepsilon G)\, G$ is a positive operator for $\varepsilon < \varepsilon_\Omega$ where G is the Green function for $-\Delta$ with zero Dirichlet boundary condition. Using Cranston-Fabes-Zhao [TAMS 307, 1988], who showed a result called the 3G-Theorem which states that

$$M = \sup_{x,y\in\Omega} \frac{\int_\Omega G(x,z)\, G(z,y)\, dz}{G(x,y)} \text{ is bounded,}$$

the theorem follows for $\varepsilon_\Omega = M^{-1}$. See Mitidieri-Sweers [Math.Nach. 173, 1995]. For $n = 1$ or $n \geq 2$ and restricting to radially symmetric functions on $\Omega = B$ (a ball) one finds the surprising identity $M = \sum_{i=1}^\infty \frac{1}{\lambda_i}$, where $\{\lambda_i\}_{i=1}^\infty$ is the set of eigenvalues of the Dirichlet Laplacian. See Sweers [SIAM J.M.A. 20, 1989] and Caristi-Mitidieri [Delft Pr.R. 14, 1990].

N.S. Trudinger

Hessian measures

In joint work with X.J. Wang, the notion of k-Hessian measure, $k = 1, \ldots, n$ in Euclidean n-space $\mathbb{R}^n$, has been introduced through extension of the k-Hessian operator F_k, defined for $u \in C^2(\Omega)$ by $F_k[u] = S_k(\lambda[D^2u])$, where $\lambda = (\lambda_1, \ldots, \lambda_n)$ denotes the eigenvalues of the Hessian matrix D^2u and S_k is the k-th elementary symmetric function, $S_k(\lambda) = \sum_{i_1 < \ldots < i_k} \lambda_{i_1} \ldots \lambda_{i_k}$. Our basic result asserts the weak continuity of the k-Hessian measure with respect to convergence in measure. In this talk, I describe the extension to mixed k-Hessian measures for the $k-$vector functions $u_1, \ldots, u_k \in \Phi^k(\Omega)$, the set of proper k-convex functions in Ω; the mixed k-Hessian operator F_k is defined for $u_1, \ldots, u_k \in C^2(\Omega)$ by

$$F_k[u_1, \ldots, u_k] = S_k(\lambda^1, \ldots, \lambda^k) = \frac{1}{k!} \sum_{\mathrm{card}\{i_1,\ldots,i_k\}=k} \lambda^1_{i_1} \ldots \lambda^k_{i_k}.$$

where $\lambda^s = (\lambda_1^s, \ldots, \lambda_k^s)$ denotes the eigenvalues of D^2u^s. The proof in the more general setting facilitates some simplifications of the special case $u_1 = \ldots = u_k$, where $F_k[u_1, \ldots, u_k] = F_k[u]$.

L. Veron

Generalized boundary value problems for semilinear elliptic equations

Consider a positive solution u of (1) $\Delta u = u^q$ $(q > 1)$ in a bounded open smooth subset $\Omega \subset \mathbb{R}^N$; then what can we say about the "generalized value" of u on $\partial\Omega$? In a common work with M. Marcus, we prove that there exists a unique outer-regular positive Borel measure ν such that $u(x) \to \nu$ as $\delta(x) = \mathrm{dist}(x, \partial\Omega) \to 0$ in a sense which is appropriate to such Borel measures. Conversely, given a positive outer-regular Borel measure ν on $\partial\Omega$ we study the existence of solutions u of (1) such that $u(x) \to \nu$ as $\delta(x) \to 0$ in the above sense (we say that $\nu = \mathrm{Tr}|_{\partial\Omega}(u)$).

When $1 < q < q_c = (N+1)/(N-1)$ the correspondence $u \leftrightarrow \mathrm{Tr}|_{\partial\Omega}(u)$ between the set of positive solutions of (1) in Ω and the set of outer-regular positive Borel measures on $\partial\Omega$ is $1-1$.

When $q \geq q_c$ we found necessary and sufficient conditions on a outer-regular positive Borel measure ν on $\partial\Omega$ such that there exists a positive solution u of (1) with $\mathrm{Tr}|_{\partial\Omega}(u) = \nu$. Usually such a u is not unique.

G. Buttazzo and A. Wagner*

On the optimal shape of a rigid body supported by an elastic membrane

We investigate the optimal shape of a three dimensional body supported by a membrane. Optimal means, that the potential energy of the configuration (body and membrane) is minimal among an admissible class of bodies. We assume the displacement of the membrane u to be parameterizable over a given set Ω while the shape of the body is given as a graph over an unknown set $\omega \subset \Omega$ with prescribed volume:

$$\text{Body} := \{(x,y) \in \mathbb{R}^n \times \mathbb{R} : x \in \omega, \phi(x) \leq y \leq c\}$$

We consider the following minimization problem:

$$\min_{u \in H_0^{1,2}(\Omega)} \min_{\{(\phi,c):\phi \text{ meas.}, c \in \mathbb{R}\}} \int_\Omega |\nabla u|^2 \, dx + k \int_\omega (c^2 - \phi^2(x)) \, dx$$

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under the constraints: (i) the maximal height and (ii) the volume of the body is prescribed.

Finding necessary conditions on the minimizer allows us to simplify this functional and to give the suitable class of admissible (ϕ, c). We conclude with the proof of existence of a minimizer.

M. Wiegner

Blow-up for a degenerate diffusion equation

We consider degenerate diffusion problems of the type

$$\begin{aligned} u_t &= u^p \Delta u + u^q && \text{on } \Omega \times (0,T), \quad \Omega \subset \mathbb{R}^N, \\ u &= 0 && \text{on } \partial\Omega \times (0,T), \\ u(x,0) &= u_0(x) > 0 && \text{in } \Omega. \end{aligned}$$

Depending on the relation between p and q and the size of Ω, there is a wide variety of behaviour. Especially interesting is the case $q = p + 1$ and large domains, where always finite time blowup occurs. Problems are e.g.

the blowup rate $\max_{x \in \Omega} u(x,t)^p (T - t)$ for $t \to T$

the blowup profile $L(x) = \lim_{t \to T} u(x,t) / \max_x u(x,t)$

the blowup set $S = \{x | u(x,t) \to \infty \text{ for } t \to T\}$

We give answers to these questions in the case $N = 1$ and symmetric data, while in the general case only partial results are obtained by now.

E. Zuazua

Geometric aspects of the decay of thermo-elastic waves

In this lecture I present some results on the decay of thermo-elastic waves in bounded domains as $t \to \infty$. We recall the pioneering results of C. Dafermos, showing that every solution tends to zero if and only if the domain is such that the Lamé system has no divergence-free eigenfunctions. This is known to hold generically with respect to the domain. It is also known that this property fails when the domain is a ball. In 2-d this problem is equivalent to Pompeiu's problem that has not been solved completely by now. We also present a joint work with J.L. Lions on the control of the Stokes flow in a 3-d cylinder by uni-axial forces. We show that this problem has a positive answer if and only if the 2-d cross section satisfies the property above. We then analyze the decay of magneto-elastic waves. It turns out that, generically with respect to the domain, all solutions decay. In 2-d, singular domains exist in which solutions do not decay. They are polygons with sides of an appropriate slope. The existence of singular domains in 3-d is an open problem. Finally, we discuss the uniform decay of thermo-elastic waves. We report on a recent joint work with G. Lebeau showing that for a large class of domains (in particular convex ones) the decay may not be uniform. We use the Gaussian beams of J. Ralston to exhibit solutions of the Lamé system with arbitrarily small energy concentrated on the longitudinal component. This result has also been proved recently by H. Koch by a different method. Finally, in 2-d, we prove that, for most domains, smooth solutions decay at a polynomial rate.

APPENDIX: Open problems

Problem 1: Let Ω_1, Ω_2 be two disjoint bounded domains in $\mathbb{R}^N$ such that $\mathbb{R}^N \setminus (\Omega_1 \cup \Omega_2)$ is connected. Consider the following overdetermined problem:

$$\begin{aligned} \Delta u &= 0 && \text{in } \mathbb{R}^N \setminus (\partial\Omega_1 \cup \partial\Omega_2), \\ u &= c_i = \text{const.} > 0 && \text{on } \partial\Omega_i, i = 1,2, \\ \tfrac{\partial u}{\partial \nu} &= d_i = \text{const.} < 0 && \text{on } \partial\Omega_i, i = 1,2 \end{aligned}$$

and let $u = 0$ at ∞ for $N \geq 3$ and let u have log-decay at ∞ for $N = 2$. Show that this is impossible. The conjecture extends to k pairwise disjoint domains $\Omega_1, \ldots, \Omega_k$ for $k \geq 2$. (Suggested by W. Reichel)

Problem 2: Consider the boundary value problem

$$\Delta u + f(u) = 0 \text{ in } \Omega,$$
$$0 < u \in L_\infty, u = 0 \text{ and } \tfrac{\partial u}{\partial \nu} = \text{const. on } \partial\Omega,$$

where Ω is unbounded. Prove that the boundary of Ω is a hyperplane, a cylinder or a sphere. (Suggested by H. Berestycki)

Problem 3: In a cylinder $\omega \times \mathbb{R}$, where $\omega \subset \mathbb{R}^{N-1}$ is the bounded cross-section, consider $-\Delta u = f(u)$ with zero-Dirichlet or zero-Neumann boundary condition. Find conditions on f and u such that $u(x', \cdot)$ $(x' \in \omega)$ is either (i) symmetrically de- or increasing, (ii) monotonically in- or decreasing or (iii) periodic. (Suggested by F. Brock)

As Congming Li has noted, boundedness of u is not sufficient to conclude (i) or (ii) or (iii). The function $u(x,y) = \cos(\sqrt{5}y) + \cos(2x)\cos(y)$ satisfies $\Delta u + 5u = 0$ in $[0,\pi] \times \mathbb{R}$ and $\partial u/\partial \nu = 0$, but none of (i)–(iii).

Problem 4: Morrey (1952) conjectured that *rank-one convexity* does not imply *quasiconvexity* of 2×2 matrix valued functions. A special case of this conjecture was proved by Sverak (1992). A conjecture of Ivaniec states that $\int_{\mathbb{C}} |\bar{\partial} f|^p \leq (p^* - 1)^p \int_{\mathbb{C}} |\partial f|^p$, where $p^* = \max(p, \frac{p}{p-1})$ and $1 < p < \infty$. Ivaniec's conjecture is true for $p = 2$ and in the general case an estimate with 4-times the constant is also known.

Here is a further conjecture: Define $L(z,w) = 2|z| - 1$ if $|z| + |w| \geq 1$ and $L(z,w) = |z|^2 - |w|^2$ if $|z| + |w| \leq 1$. Then, if $f \in Lip(\mathbb{C}, \mathbb{C})$, the inequality $\int_{\mathbb{C}} L(\partial f, \bar{\partial} f) \geq 0$ holds, where extremal functions are of the form $f(z) = g(r)e^{i\theta}$ with $g > 0$, g Lipschitz and $|g'(r)| \leq \frac{1}{r} g(r)$ for all $r > 0$.

If this conjecture is true, then Ivaniec's conjecture is true. If this conjecture is false, then Morrey's conjecture is true. (Suggested by A. Baernstein)

Problem 5: Consider the Steklov-eigenvalue problem:

$$\Delta u = 0 \text{ in } \Omega \subset \mathbb{R}^N, \quad |\nabla u| = 1 \text{ and } \frac{\partial u}{\partial \nu} = pu \text{ on } \partial\Omega.$$

Payne and Philippin conjectured that Ω has to be a ball. It was proved by Alessandrini and Magnanini that this is false in $N = 2$. Moreover for $N \geq 3$ it can be shown that Ω is a ball if additionally u is known to be the function $u = x_k$ for some $k \in \{1, \dots, N\}$.
On a ball in $N = 2$ the functions $r^N \cos(N\theta)$ satisfy the overdetermined Steklov-problem. More generally for $N \geq 3$ one has solutions for the form $r^N U$, where U satisfies

$$\Delta_{S^{N-1}} U + \lambda U = 0, \quad |\nabla_{S^{N-1}} U|^2 = 1 - p^2 U^2.$$

For $N = 2k$ they are of the form $x_1^2 - x_2^2 + x_3^2 - x_4^2 + \dots$.

In a recent paper of Wang, it was shown that for an equation of the form $|\nabla_M U|^2 = g(U)$ the level sets of U are submanifolds. The problem is now to prove or disprove that the set Ω in the Steklov problem is a ball. (Suggested by R. Magnanini)

Problem 6: For the operator $-\frac{d^2}{dx^2} - V$, $V > 0$ we know that the negative eigenvalues λ_i have the property $\sum |\lambda_i|^2 \leq c_\gamma \int V^{\gamma+1/2}$, for $\gamma > 1/2$. The existence of such a constant was proved by Weidl ($\gamma = 1/2$) and by Thirring ($\gamma > 1/2$). The problem is to determine the best constant c_γ. For $\gamma \geq 3/2$ the best constant is known. We conjecture that c_γ is determined by assuming that there is exactly one negative eigenvalue and by considering

$$\max \frac{-\int (\frac{df}{dx})^2 + \int v f^2}{\int v^{\gamma+1/2} \int f^2}.$$

In particular if $\gamma = 1/2$ then the optimal V is a multiple of the delta-function. (Suggested by E. Lieb)

Problem 7: Consider the hyperbolic equation

$$u_{tt} + u_{xx} + u^q - lu = 0 \text{ for } (t,x) \in \mathbb{R} \times S^1$$

for $u > 0$, $l > 0$ and $q > 1$. The equation has two conserved quantities $\frac{d}{dt} \int_{S^1} u_t u_x = 0$ and $\frac{d}{dt} E = 0$ for an energy E. The ansatz $u(x,t) = w(\alpha t + \beta x)$ leads to $(\alpha^2 + \beta^2) w'' + w^q - lw = 0$ on S^1. The problem is to prove that any solution is of this type. If $(q-1)l \leq \lambda_1 = 1$, then only $w = \text{const.}$ are solutions. The question arises, whether one can conclude the existence of ϕ of $\phi'' + \phi^q - l\phi = 0$ on $\mathbb{R}$ such that $u \approx \phi$ as $t \to \infty$. We know that if $|u|_{L^\infty(\mathbb{R} \times S^1)} \leq (\frac{l+\lambda_1}{q})^{\frac{1}{q-1}}$, then $|u(x,t) - \bar{u}(t)|_{L^\infty(S^1)} \to 0$ as $t \to \infty$ and hence the result is true. Moreover one knows the existence of a constant c_q such that $|u|_{L^\infty} \leq C_q l^{\frac{1}{q-1}}$. This problem has an interesting link to the equation $\Delta u + u^{\frac{n+2}{n-2}} = 0$, since the ansatz $u(r,\sigma) = r^{-\frac{n-2}{2}} w(r,\sigma)$ with $t = \ln \frac{1}{r}$ leads to the equation $u_{tt} + \Delta_{S^{N-1}} u - lu + u^{\frac{n+2}{n-2}} = 0$. (Suggested by L. Veron)

Problem 8: Consider the problem

$$\begin{aligned} u_t &= \sqrt{1 + |Du|^2} S_k(u) \text{ in } \Omega \times (0,T), \\ u(x,0) &= u_0(x) \text{ for } x \in \overline{\Omega}, \\ u(x,t) &= \phi(x) \text{ for } (x,t) \in \partial\Omega \times (0,T), \end{aligned}$$

where $S_k(u)$ is an elementary symmetric function of the principle curvatures of the graph of u and u is such that the problem is parabolic. Are there any solutions for $1 < k \leq n$ without any assumptions on Ω other than Ω bounded and $\partial\Omega \in C^\infty$? For $k = 1$, $S_1 = H$, the flow converges to the solution of a variational problem. (Suggested by V. Oliker)

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