

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 28/1998

Arithmetic of Fields

12.07. – 18.07.1998

The organizers of the meeting were Wulf-Dieter Geyer (Erlangen) and Moshe Jarden (Tel Aviv). The 26 talks that were given during the conference fall (roughly) into several categories:

1. Fundamental groups and covers in characteristic p (Bouw, Harbater, Ihara, Lochak, Pop, Saidi, Stevenson, Wewers).
2. Algebraic number theory (Müller, Schmidt, Wiesend, Wingberg).
3. Consequences of the Riemann existence theorem (Dettweiler, Magaard, Völklein).
4. PSC fields, Hilbertian fields, and absolute Galois groups (Geyer, Haran, Jarden, Koenigsmann, Razon).
5. Groups, profinite groups, embedding problems (Black, Brandis, Herfort, Ware, Zalesskii).
6. Arithmetic geometry (Chatzidakis).

Abstracts (in chronological order)

Šafarevič's theorem on solvable groups as Galois groups I

Kay Wingberg (Heidelberg)

In the first of two talks on the famous theorem of Šafarevič "Every finite solvable group occurs as a Galois group over a global field" this result was composed with the related theorems of Schoiz, Reichardt, and Neukirch and the idea of a simplified proof was presented. In particular, a short proof of the so-called "shrinking lemma" was given and its applications to group cohomology were indicated.

Šafarevič's theorem on solvable groups as Galois groups II

Alexander Schmidt (Heidelberg)

We continue the last talk and give a more detailed description of the proof. The essential step is the proof of the following theorem.

Theorem: *Over a global field, every split embedding problem with finite nilpotent kernel has a proper solution.*

Quaternionic Galois Extensions

Roger Ware (Pennsylvania State University)

Let F be a field of characteristic $\neq 2$. A theorem of Witt is used to determine when the quaternion group, Q_8 , of order 8 occurs as a Galois group over F .

Let n be the class of all pro-2 groups with generators $\{y_i, x\}_{i \in I}$ and relations $y_i y_j = y_j y_i$ and $x y_i x^{-1} = y_i^{q+1}$ where $q = 0$ or $q = 2^n$, $n \geq 2$. Let $G_F(2)$, $G_F\pi$ be, respectively, the (pro-2) Galois groups of the quadratic closure and Pythagorean closure of F . Let $\mathcal{F}_0 = \{\mathbb{Z}/2\mathbb{Z}\} \cup n$ and, inductively, let

$$\begin{aligned} \mathcal{F}_{i+1} = \mathcal{F}_i \cup & \{ \mathbb{Z}_2^m \rtimes H \mid H \in \mathcal{F}_i, H \text{ contains an involution} \} \\ & \cup \{ G_1 *_2 G_2 \mid G_1, G_2 \in \mathcal{F}_i, G_1 \text{ is generated by involutions} \}. \end{aligned}$$

Theorem: *If F has a finite number n of orderings, $n \geq 0$, then the following statements are equivalent:*

- (i) Q_8 does not occur over F ;
- (ii) $G_F(2) \in \bigcup_{i \geq 1} \mathcal{F}_i$; and
- (iii) $G_F\pi \in n$.

The ingredients of the proof include techniques from the theory of quadratic forms and Witt rings (a theorem of R. Bos), some constructive valuation theory, and Neukirch's theorem on free pro- p products and cohomology groups.

PSC Galois extensions of Hilbertian fields

Moshe Jarden (Tel Aviv)

Definitions: 1. A field K is **Hilbertian** if for each irreducible polynomial $f \in K[T, X]$ there are infinitely many $a \in K$ such that $f(a, X)$ is irreducible in $K[X]$.

- 2. \tilde{K} (resp., K_s) is the algebraic (resp., separable) closure of K .
- 3. $G(K) = \mathcal{G}(K_s/K)$ is the absolute Galois group of K . For each positive integer e we equip $G(K)^e$ with a Haar measure.
- 4. For each $\sigma \in G(K)^e$ let $K_s(\sigma) = \{x \in K_s \mid \sigma_i x = x \text{ for } i = 1, \dots, e\}$. Then let $K_s[\sigma]$ be the maximal Galois extension of K which is contained in $K_s(\sigma)$. One knows that for almost all $\sigma \in G(K)^e$ (in the sense of the Haar measure) the field $K_s(\sigma)$ is PAC

over K (assuming K is countable and Hilbertian). This means that for each separable dominant rational map $\varphi: V \rightarrow \mathbb{A}^r$ of absolutely irreducible varieties over K there exists $\mathbf{x} \in V(K_s(\sigma))$ such that $\varphi(\mathbf{x}) \in K^r$.

5. A **local prime** of K is an equivalent class \mathfrak{p} of absolute values of K such that the completion $\hat{K}_{\mathfrak{p}}$ of K at \mathfrak{p} is a local field (thus, $\hat{K}_{\mathfrak{p}}$ is locally compact in the \mathfrak{p} -adic topology) and $\hat{K}_{\mathfrak{p}}/K$ is a separable extension.

6. Let \mathcal{S} be a set of local primes of K . For each $\mathfrak{p} \in \mathcal{S}$ let $K_{\mathfrak{p}} = \hat{K}_{\mathfrak{p}} \cap K_s$. Then let

$$K_{\text{tot},\mathcal{S}} = \bigcap_{\mathfrak{p} \in \mathcal{S}} \bigcap_{\sigma \in G(K)} K_{\mathfrak{p}}^{\sigma}.$$

It is the maximal Galois extension of K in which each $\mathfrak{p} \in \mathcal{S}$ splits completely.

7. A subfield N of $K_{\text{tot},\mathcal{S}}$ which contains K is **PSC** if each absolutely irreducible algebraic variety V over N satisfies the following condition:

$$V_{\text{simple}}(K_{\mathfrak{p}}) \neq \emptyset \text{ for all } \mathfrak{p} \in \mathcal{S} \implies V(N) \neq \emptyset.$$

One can deduce the PSC property of a field N between K and $K_{\text{tot},\mathcal{S}}$ from a 'local global principle' for the ring of integers of N . The latter has been proved on two occasions. As a consequence we have the following result:

Theorem A: *Let K be a global field and let \mathcal{S} be a finite set of local primes of K .*

- (a) (Green-Pop-Roquette) $K_{\text{tot},\mathcal{S}}$ is PSC.
- (b) (Jarden-Razon) *If K is a number field, then $\bar{K}(\sigma) \cap K_{\text{tot},\mathcal{S}}$ is PSC for almost all $\sigma \in G(K)^e$.*

It is known that if N is a subfield of $K_{\text{tot},\mathcal{S}}$ which is PSC and if N' is a field extension of N which is contained in $K_{\text{tot},\mathcal{S}}$, then N' is also PSC. So, Theorem A is a consequence of the following result:

Theorem B: *Let K be a countable Hilbertian field. Let \mathcal{S} be a finite set of local primes of K . Let $e \geq 0$ be an integer. Then, for almost all $\sigma \in G(K)^e$, the field $K_s[\sigma] \cap K_{\text{tot},\mathcal{S}}$ is PSC.*

Theorem B is the main result of a joint work with Wulf-Dieter Geyer.

Very ample divisors and stable functions on curves

Wulf-Dieter Geyer (Erlangen)

This talk explains two methods which are used in a joint work with M. Jarden on which he reported above.

1. Let \mathcal{C} be a projective curve over an algebraically closed field K , let S be the set of singularities and \mathcal{O} be the S -local ring of S , i.e. $\mathcal{O} = \bigcap_{\mathfrak{p} \in S} \mathcal{O}_{\mathcal{C},\mathfrak{p}}$. Let f be the conductor of \mathcal{O} and g be the genus of \mathcal{C} . Then the K -vector space, associated to a divisor \mathfrak{a} , prime to S , $\mathcal{L}_{\mathcal{O}}(\mathfrak{a}) = \{f \in \mathcal{O} \mid \text{div}(f) + \mathfrak{a} \geq 0\} = \sum_{i=0}^n K f_i$ gives rise to a rational map $\varphi_{\mathfrak{a}}: \mathcal{C} \rightarrow \mathbb{P}^n$, $\mathbf{x} \mapsto (f_0(\mathbf{x}) : \dots : f_n(\mathbf{x}))$.

Theorem: $\text{dega} > 2g + 2 \cdot \text{degf} \Rightarrow \varphi_a$ is an isomorphism of \mathcal{C} onto its image.

2. This theorem is applied to curves \mathcal{C} over an arbitrary field K , K infinite. Furthermore: A function $t \in K(\mathcal{C})$ is called **stable**, if $K(\mathcal{C})/K(t)$ is a separable extension of degree d such that the geometric monodromy group $\text{Gal}(\overline{K(\mathcal{C})}/\overline{K(t)})$ is maximal, i.e. the full symmetric group S_d . To construct enough stable elements we use a construction of Konrad Neumann: Replace \mathcal{C} by a birationally equivalent curve \mathcal{C}' which is smooth in n -space up to the following singularities: cusps at the primes whose local rings become singular under constant field extension (only in characteristic p) and a special cusp of high prime multiplicity at a chosen separable prime. Further, assume in characteristic p that a plane projection of \mathcal{C}' has only finitely many double tangents, inflection points and no strange point. Then $t = \sum_{i=0}^n a_i x_i / \sum_{i=0}^n b_i x_i$ is stable for almost all (a, b) .

Solvable absolute Galois groups are metabelian

Jochen Koenigsmann (Konstanz)

Let G_F denote the absolute Galois group of the field F . The main result presented in this talk is that if G_F is solvable (i.e. admits a finite subnormal series with abelian quotients) then there is an exact sequence

$$1 \rightarrow A \rightarrow G_F \rightarrow C_1 \times C_2 \times Z \rightarrow 1$$

with A torsion-free abelian, C_1, C_2 finite cyclic and Z torsion-free procyclic. In particular, G_F is then metabelian. Moreover, if $G_{F(\sqrt{-1})}$ is non-projective, but solvable, then F admits a non-trivial Henselian valuation.

A survey of Grothendieck-Teichmüller theory

Pierre Lochak (Paris)

I presented a survey of recent advances in the new-born Grothendieck-Teichmüller theory which aims at an explicit description of the absolute Galois group of \mathbb{Q} in terms of its outer action on the geometric fundamental groups of \mathbb{Q} -varieties, particularly the moduli spaces of algebraic curves. I especially discussed how in collaboration with L. Schneps and A. Hatcher we extended the theory from the case of genus 0 to the general case, i.e. an explicit description of the outer action of the Galois group on the profinite Teichmüller modular groups of any genus (and any number of marked points).

Referring to the report of F. Pop below, one may describe the goal of the theory – suggested by A. Grothendieck – roughly as follows: find a category \mathcal{M} such that $\mathcal{M} \subseteq \text{Var}_{\mathbb{Q}}$ (perhaps though using more general objects than varieties properly speaking) such that $G_{\mathbb{Q}} \cong \text{Aut}(\pi_1(\mathcal{M}))$ and the right hand side can be given a fairly “explicit” description. Grothendieck singled out moduli spaces of curves as important objects to be included in \mathcal{M} . In the past few years, starting with Drinfeld’s definition of \widehat{GT} (the “Grothendieck-Teichmüller group”), a group with indeed an explicit description, one has built successive refinements of the “Teichmüller tower” T , consisting of the fundamental groups of the moduli spaces of curves (the so-called Teichmüller modular groups). These groups

are connected by homomorphisms coming from the geometry of the moduli spaces. So one can view T as $\bar{\pi}_1(\mathcal{T})$ for some \mathcal{T} and $\text{Aut}(\bar{\pi}_1(\mathcal{T})) = \text{Aut}(T)$ can indeed be explicitly described (as a refinement of Drinfeld's original \overline{GT}). The task remains of determining \mathcal{M} – starting from \mathcal{G}_Q and \mathcal{T} – satisfying the above.

Ramification of primes in the field of moduli of covers of curves

Stefan Wewers (Essen)

Let G be a finite group and $f: X \rightarrow \mathbb{P}_\mathbb{C}^1$ a G -Galois-cover of the projective line. The G -cover f is determined (up to isomorphism) by the branch divisor

$$D_f := \{x \in \mathbb{P}_\mathbb{C}^1 \mid |f^{-1}(x)| < \#G\}$$

and the Nielsen class $[g] \in \text{ni}_r^{\text{in}}(G)$ of f induced by a standard representation $\pi_1(\mathbb{P}_\mathbb{C}^1 - D_f) = \langle \gamma_1, \dots, \gamma_r \mid \prod_i \gamma_i = 1 \rangle$. Here

$$\text{ni}_r^{\text{in}}(G) = \{ (g_1, \dots, g_r) \mid G = \langle g_i \rangle, \prod_i g_i = 1, g_i \neq 1 \} / \text{Inn}(G).$$

Assume that D_f is \mathbb{Q} -rational and let $K = K_f$ be the field of moduli of the G -cover f . Then K is a finite extension of \mathbb{Q} .

Theorem: *Let p be a prime not dividing $\#G$ and $p = p_1^{e_1} \dots p_s^{e_s}$ the splitting of p in K . Then there is a finite (unordered) tuple $E_p(D_f, [g])$ which is 'computable' from D_f and $[g]$ such that (e_1, \dots, e_s) is contained in $E_p(D_f, [g])$. Moreover, suppose the component of the Hurwitz space $H_r^{\text{in}}(G)$ corresponding to the braid orbit of $[g]$ is defined over \mathbb{Q} . Then $E_p(D_f, [g]) = (e_1, \dots, e_s)$ for infinitely many G -covers $f: X \rightarrow \mathbb{P}_\mathbb{C}^1$ with associated Nielsen class $[g]$.*

The proof of this theorem uses the Harris-Mumford compactification and good models over $\mathbb{Z}[1/\#G]$ of the Hurwitz space $H_r^{\text{in}}(G)$.

Formal Patching and Embedding Problems with Local Conditions

David Harbater (University of Pennsylvania)

We consider problems of the following type: Let $Y \rightarrow X$ be a connected Galois cover of affine varieties over an arbitrary field k of characteristic p . Assume that $\text{Gal}(Y/X)$ is of the form Γ/N , where N is a p -group. Is there a connected Galois cover $Z \rightarrow X$ with group Γ that dominates $Y \rightarrow X$ and is étale over Y ? Moreover, given $X' \rightarrow X$, if the pullback $Y' \rightarrow X'$ of $Y \rightarrow X$ is dominated by a (possibly disconnected) Γ -Galois cover $Z' \rightarrow X'$, then can $Z \rightarrow X$ above be chosen so as to pull back to Z' over X' ? If X' is taken to be a closed subset of X , we show that the answer is yes if $Y \rightarrow X$ is of degree prime to p or is étale; or if X is a curve and $Y \rightarrow X$ is tamely ramified. It is also true for curves if X' is a union of finitely many spectra of local fields of X . These results are proven cohomologically. Combining them with patching methods, we obtain stronger results in

the special case that X is an affine curve and k is algebraically closed. If N is an arbitrary quasi- p group (i.e. generated by its Sylow p -subgroups) and $Y \rightarrow X$ is tamely ramified, then the above problem can be solved (strengthening a result of Pop). Moreover if N is any finite group, then there is a Γ -Galois cover $Z \rightarrow X$ dominating $Y \rightarrow X$ and having $\text{rank}_{\Gamma/p(N)} N/p(N)$ additional branch points. (Here $p(N) \subset N$ is the subgroup generated by the Sylow p -subgroups of N , and $\text{rank}_D E$ is defined to be the smallest number of elements of $E \subset D$ which, together with any supplement to E in D , generate D .)

Tame covers of curves

Irene Bouw (Padova)

Let X be a nonsingular irreducible projective curve, defined over an algebraically closed field k of characteristic $p > 0$. Let S be a nonempty set of k -points and $U = X - S$. Put $g = g(X)$ and $r = |S|$. We would like to get some information on $\pi_1^+(U)$. The prime to p part and the p part of $\pi_1^+(U)$ are known, therefore we are interested in quotients G which are extensions of prime to p groups H by p groups P . It is no restriction to suppose that P is elementary abelian. The problem translates to finding the $\mathbb{F}_p[H]$ -Galois module structure of $H^1(Y, \mathcal{O}_Y)^F$, for $Y \rightarrow X$ an H -cover unbranched outside S . There is a bound on the p -rank $\sigma(Y)$ of Y , coming from the $k[H]$ -structure of $H^1(Y, \mathcal{O}_Y)$, which is known. It only depends on the monodromy a and g . We denote it by $B(a, g)$.

Theorem: *Suppose $g = 0$ and either $r \leq 4$ or $p \equiv \pm 1 \pmod{\ell}$ or $p \geq \ell(r - 3)$. Then*

$$\sigma(Y) - \sigma(X) = B(a, g)$$

in case the branch points are sufficiently general.

The analogous statement for non-abelian groups does not hold, in general.

Fundamental groups of projective curves

Katherine F. Stevenson (College Park)

Let X be a smooth connected projective curve over an algebraically closed field k of characteristic p . Let $\pi_1(X)$ be the algebraic fundamental group. The maximal prime-to- p quotient and the maximal p -quotient are both known, so we consider finite quotients of $\pi_1(X)$ of the form $P \rtimes H$, where P is a p -group and H is a prime-to- p group. By modding out by the Frattini subgroup $\Phi(P)$ of P , we may assume that P is an elementary abelian p -group.

Theorem (Pacheco, S-): $\pi_1(X) \rightarrow P \rtimes H \Leftrightarrow$ *there exists an H -Galois cover $Z \rightarrow X$ such that $P \hookrightarrow J_Z[p]$ as an $\mathbb{F}_p[H]$ -module.*

For any irreducible H -character χ defined over k , let V_χ be the corresponding irreducible $k[H]$ -module. Then

$$\begin{aligned} J_Z[p] \otimes_{\mathbb{F}_p} k &= \bigoplus_{\chi} V_{\chi}^{r_{\chi}} \\ P \otimes_{\mathbb{F}_p} k &= \bigoplus_{\chi} V_{\chi}^{m_{\chi}}. \end{aligned}$$

(Note: $\gamma_X = r_X n_X$ is the generalized Hasse-Witt invariant; $\gamma_X \leq \begin{cases} (g-1)n_X^2 & \chi \neq 1 \\ g & \chi = 1. \end{cases}$)

Corollary: If $Z \rightarrow X$ is an H -Galois cover of ordinary curves, then

$$\pi_1(X) \rightarrow P \rtimes H \Leftrightarrow \forall \chi \quad m_X \leq \begin{cases} (g-1)n_X & \chi \neq 1 \\ g & \chi = 1. \end{cases}$$

Such ordinary covers exist in many cases, however Raynaud has recently shown that they do not exist for all H 's.

p -rank of semi-stable models of cyclic covers of curves of degree p

Mohamed Saidi (Bonn)

Let R be a discrete complete valuation ring, with fraction field K of characteristic 0, and residue field k algebraically closed of characteristic $p > 0$. Consider a smooth and proper relative curve X over R with geometrically connected generic fibre $X_K = X \times_R K$. Let $f: Y \rightarrow X$ be a finite Galois covering of group $\mathbb{Z}/p\mathbb{Z}$, with Y normal. Assume that there exists a birational morphism $\tilde{Y} \rightarrow Y$ such that \tilde{Y} is semi-stable. If the branch locus $B \subset X_K$ of the morphism f is empty, Raynaud proved that the p -rank $r_{\tilde{Y}_k}$ of the special fibre $\tilde{Y}_k = Y \times_R k$ of \tilde{Y} is equal to the p -rank r_{X_k} of $X_k = X \times_R k$, under the assumption that the morphism $f_k: Y_k \rightarrow X_k$ between the special fibres is purely inseparable. Under this latter condition we generalize this result to the case where $B = \{x_i\}_{i=1}^m$ is not empty. Let $\bar{B} = \{\bar{x}_j\}_{j=1}^n$ be the specialization of B with $n \leq m$. Let $h := \#\{1 \leq j \leq n \mid m_j \text{ is odd}\}$, where m_j is the number of points among $\{x_i\}_{i=1}^m$ which specialize in x_j . We prove that the p -rank of \tilde{Y}_k is bounded by $r_{X_k} + (\frac{m-h}{2})(p-1)$. In particular, \tilde{Y}_k has maximal p -rank if and only if $2g_X - 2 + h \leq 0$ and X_k is ordinary.

A combinatorial description of $G_{\mathbb{Q}}$

Florian Pop (Bonn)

Let $\text{Var}_{\mathbb{Q}}$ be the category of all \mathbb{Q} -varieties and morphisms of such varieties. Taking the fundamental group functor we get

$$\bar{\pi}_1: \text{Var}_{\mathbb{Q}} \rightarrow \mathcal{G}, \quad X \mapsto \pi_1(\bar{X})$$

into the category \mathcal{G} of all profinite groups and outer homomorphisms. Let $\mathcal{G}_{\mathbb{Q}}$ be the image of $\text{Var}_{\mathbb{Q}}$ under $\bar{\pi}_1$. For every X there exists a canonical representation $\rho_X: G_{\mathbb{Q}} \rightarrow \text{Out}(\pi_1(\bar{X})) = \text{Aut}_{\mathcal{G}}(\pi_1(\bar{X}))$ which behaves functorially. Then we get a homomorphism

$$\iota_{\mathbb{Q}}: G_{\mathbb{Q}} \rightarrow \text{Aut}(\mathcal{G}_{\mathbb{Q}}), \quad \sigma \mapsto (\rho_X(\sigma))_X.$$

We gave a positive answer to the question of Oda-Matsumoto, asking whether $\iota_{\mathbb{Q}}$ is an isomorphism. In particular we have a geometric/combinatorial description of the absolute Galois group of the rationals in the tradition of Grothendieck-Teichmüller theory (nevertheless with $\text{Var}_{\mathbb{Q}}$ instead of $\mathcal{M} = (\text{category of all the } \mathcal{M}_{g,n}, \text{ connecting homomorphisms})$). See also the report by P. Lochak above.

Virtually Free Pro- p Groups

W. Herfort and P.A. Zalesskii

We obtain the following results.

Theorem 1 [HRZ2]: A pro- p group G containing a normal open free pro- p subgroup F of index p is of the form $G = (\prod_{x \in X} (C_p \times H_x)) \amalg H$, where C_p is a cyclic group of order p and H_x, H are free pro- p groups.

Theorem 2 [HZ]: A cyclic p -power extension G of a free pro- p group F is a free pro- p product $G = \prod_{x \in X} N_G(C_{p,x}) \amalg H$, where, for $x \in X$, $C_{p,x} \cong C_p$, $N_G(C_{p,x})$ denotes the normalizer of $C_{p,x}$ in G and H is free pro- p .

Theorem 3 [HRZ1]: Let G be a finite extension of a free pro- p group F of rank 2. Then G is the fundamental group of a finite connected graph of finite p groups. An explicit description of such groups is included.

Theorem 4 [HZ]: A cyclic p -power extension G of a free pro- p group F can be presented as the fundamental group of a connected graph of cyclic p -groups of bounded order (see [HZ]).

Theorem 5 [HZ]: Let F be a free pro- p group.

- (i) There exists a dense abstract free subgroup $F^{\text{abs}} \leq F$ such that each conjugacy class of automorphisms of order p^n in $\text{Aut} F$ intersects precisely one conjugacy class of automorphisms of order p^n in $\text{Aut} F^{\text{abs}}$.
- (ii) The conjugacy classes of automorphisms of F having order q coprime to p are in one-to-one correspondence with the conjugacy classes of automorphisms of order q of $F/\Phi(F)$.

Theorem 6 [HZ]: There exists an example showing the following:

- (i) Not every virtually free pro- p group can be described as in 2) from above;
- (ii) A pro- p analogue of the Kurosh subgroup theorem along the lines of Haran [H] and Melnikov [M] does not hold for arbitrary closed subgroups.

References

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Arithmetic aspects of Galois actions on fundamental groups

Yasutaka Ihara (Kyoto)

This is about one particular Lie algebra \mathcal{D} over \mathbb{Z} and its connection with number theory, esp. with the Galois action on $\pi_1^{\text{pro-}p}(\mathbb{P}^1 - \{0, 1, \infty\})$. The algebra \mathcal{D} is the Lie algebra of "special symmetric outer derivations" of \mathfrak{P}_n (which is stable for $n \geq 5$), where \mathfrak{P}_n is the graded Lie algebra associated with the lower central series of the pure braid group on n strings. It may be regarded as a Lie version/ \mathbb{Z} of " GT^m ".

For each prime p , $\mathcal{D} \otimes \mathbb{Z}_p$ contains a Lie subalgebra \mathfrak{g}_p arising from the $G_{\mathbb{Q}}$ -action on $\pi_1^{\text{pro-}p}(\mathbb{P}^1 - \{0, 1, \infty\})$. The three basic notions related to \mathcal{D} are "weight", "depth" and "beta-map", which correspond to "Tate twist", "meta-abelian rank/ $\mathbb{Q}(\mu_{p^\infty})$ " and the "Soulé character". \mathcal{D} contains a certain set of elements D_m ($m \geq 3$, odd) of weight m , and conjecturally, $\mathcal{D} \otimes \mathbb{Q}$ is freely generated by the D_m 's as Lie algebra/ \mathbb{Q} (which would also imply $\mathcal{D} \otimes \mathbb{Q}_p = \mathfrak{g}_p \otimes \mathbb{Q}_p$.) But the structure of \mathcal{D} seems more delicate, and as interesting as modular forms/ \mathbb{Z} . For example, there is a congruence $2[D_3, D_9] - 27[D_5, D_7] \equiv 0(691)$ in weight 12, and for each even $m \geq 12$, there is associate to \mathcal{D} a certain module whose rank is equal to the dimension of modular cusp forms of weight m on $SL_2(\mathbb{Z})$. It seems likely that $\mathcal{D} \otimes \mathbb{Z}_p$ and \mathfrak{g}_p coincide in weight $m < p$, but possibly not so when $m \gg p$. Some results, examples and conjectures are discussed.

Hilbertian Fields under Separable Algebraic Extensions

Dan Haran (Tel Aviv)

We exhibit a quite general sufficient condition for an algebraic separable extension M of a Hilbertian field K to be Hilbertian. The precise criterion, that uses the notion of *twisted wreath product*, is:

Theorem: *Let M be a separable algebraic extension of a separably Hilbertian field K . Suppose that for every $\alpha \in M$ and every $\beta \in M$, there exist:*

- (i) a finite Galois extension L of K that contains β ; let $G = \mathcal{G}(L/K)$;
- (ii) a field K' such that $K \subseteq K' \subseteq M \cap L$ and K' contains α ; let $G' = \mathcal{G}(L/K')$;
- (iii) a Galois extension N of K that contains both M and L ,

such that for every finite nontrivial group A and every action of G' on A there is no realization K, K', L, F, \hat{F} of $\text{Awr}_{G'} G$ with $\hat{F} \subseteq N$.

Then M is separably Hilbertian.

The criterion is general in the sense that it can be used to prove essentially all hitherto known instances. Furthermore, it provides a new large class of extensions that are Hilbertian. The main result is:

Corollary: Let K be a Hilbertian field and let M_1, M_2 be two Galois extensions of K . Let M be an intermediate field of $M_1 M_2 / K$ such that $M \not\subseteq M_1$ and $M \not\subseteq M_2$. Then M is Hilbertian.

Relative embedding problem

Elena Black (University of Oklahoma)

Joint work with John Swallow, Davidson College

Let K be a field. Let G be a finite group, A a normal subgroup and $Q = G/A$. Let B be a subgroup of A which is normal in G . Suppose K_1/K is a Galois extension of K with a group Q . Consider three embedding problems given by the following commutative diagram:

$$\begin{array}{ccccccc}
 & & 1 & & 1 & & \\
 & & \downarrow & & \downarrow & & \\
 & & B & \xlongequal{\quad} & B & & \\
 & & \downarrow & & \downarrow & & \\
 1 & \longrightarrow & A & \longrightarrow & G & \longrightarrow & Q \longrightarrow 1 \quad (*) \\
 & & \downarrow & & \downarrow & & \parallel \\
 1 & \longrightarrow & A/B & \longrightarrow & G/B & \longrightarrow & Q \longrightarrow 1 \quad (**) \\
 & & \downarrow & & \downarrow & & \parallel \\
 & & 1 & & 1 & & \text{Gal}(K_1/K) \\
 & & & & & & (***)
 \end{array}$$

The relative embedding problem data consists of G -Galois extension L/K which is a proper solution of $(*)$ and E_B/K which is a proper solution to the embedding problem $(**)$ such that $L \cap E_B = K_1$. The relative embedding problem asks if there is a (proper) solution to the embedding problem $(***)$, where $\text{Gal}(E_B/K) = G/B$.

Theorem (Reduction): Let $(G, A, B, K_1/K, E_B/K, L/K)$ be as in relative embedding problem above. Assume A is abelian. Let K_0 be a fixed field in L of $Z_G(A)$, the centralizer of A in G (equivalently, K_0 is a fixed field in E_B of $Z_G(A)/B$). Then

(a) E_B/K corresponds uniquely to a solution F_0/K of the embedding problem associated to the split exact sequence

$$1 \rightarrow A/B \rightarrow A/B \rtimes G/Z_G(A) \rightarrow G/Z_G(A) \cong \text{Gal}(K_0/K) \rightarrow 1.$$

Here $G/Z_G(A)$ acts on A/B via the action of G on A ; and

(b) the proper solutions E/K of the relative embedding problem which are linearly disjoint from L over K_1 are in 1-to-1 correspondence with the proper solutions F/K of the embedding problem associated to the split exact sequence

$$1 \rightarrow B \rtimes 1 \rightarrow A \rtimes G/Z_G(A) \rightarrow A/B \rtimes G/Z_G(A) \cong \text{Gal}(F_0/K) \rightarrow 1,$$

which are linearly disjoint from K_1 over K_0 .

Tate-Shafarevich groups of finite Galois modules over function fields over local and global fields.

Götz Wiesend (Erlangen)

Let F be a function field of one variable over the p -adic field K . For a finite G_F -module M define

$$\text{III}^q(F, M) := \text{Ker}(H^q(F, M) \rightarrow \prod_{P \in \mathcal{P}(F|K)} H^q(F_P, M)).$$

Theorem: There is a canonical pairing

$$\text{III}^q(F, M) \times \text{III}^{4-q}(F, M^\vee(2)) \rightarrow \mathbb{Q}/\mathbb{Z}$$

which gives a perfect duality of finite abelian groups ($q = 1, 2, 3$).

Under various conditions the groups $\text{III}^q(F, M)$ can be explicitly determined. This depends on the reduction behaviour of F .

Now let F be a regular function field of one variable over the number field K . For a finite G_F -module M define

$$\begin{aligned} \text{III}_{\text{geo}}^q(F, M) &= \text{Ker}(H^q(F, M) \rightarrow \prod_{P \in \mathcal{P}(F|K)} H^q(F_P, M)) \\ \text{III}_{\text{ar}}^q(F, M) &= \text{Ker}(H^q(F, M) \rightarrow \prod_{Q \in \mathcal{P}(K)} H^q(FK_Q, M)). \end{aligned}$$

These groups need not be finite.

Proposition: $\text{III}_{\text{geo}}^1(F, M) = 0$ (Hilbert Irreducibility Theorem)

$$\text{III}_{\text{ar}}^1(F, M) = \text{III}^1(K, M^{G_{FK}}) \quad (\text{Inflation-Restriction sequence}).$$

Let E/F be the trivializing extension of $M(-2)$. Define M^* by the short exact sequence

$$0 \rightarrow M^* \rightarrow \text{Inf}_{G_E}^{G_F} M \rightarrow M \rightarrow 0.$$

Theorem: For every finite G_F -module M there are exact sequences

$$\begin{aligned} \bigoplus_Q X_Q \rightarrow \text{III}_{\text{ar}}^3(F, M) \rightarrow \bigoplus_{P \in \mathcal{P}(F|K)} \text{III}^3(F_P, M) \rightarrow \text{III}_{\text{ar}}^1(F, M^\vee(2))^\vee \rightarrow 0 \\ \bigoplus_Q X_Q \rightarrow \text{III}_{\text{geo}}^3(F, M) \rightarrow \bigoplus_{Q \in \mathcal{P}(K)} \text{III}^3(FK_Q, M) \rightarrow 0, \end{aligned}$$

where X_Q is the homology in the complex

$$\mathrm{III}^3(FK_Q, M^*) \rightarrow \mathrm{III}^3(EK_Q, M) \rightarrow \mathrm{III}^3(FK_Q, M).$$

This vanishes at places Q , where E has good reduction. Hence $\bigoplus_Q X_Q$ is finite.

Results of Hrushovski on the Manin-Mumford Conjecture over a number field

Zoé Chatzidakis (Paris)

The Manin-Mumford conjecture states: if C is a curve of genus > 2 defined over a field of characteristic 0, embedded in its Jacobian J , then $C \cap \mathrm{Tor}(J)$ is finite. The following result was proved by Raynaud (see also results by Hindry and McQuillan):

Theorem (char. 0): *Let A be an abelian variety, X a subvariety of A . Then there are group subvarieties A_1, \dots, A_m of A , and elements $c_1, \dots, c_m \in A$ such that*

$$\mathrm{Tor}(A) \cap X = \bigcup_{i=1}^m \mathrm{Tor}(A_i) + c_i.$$

In this talk we show how some tools of model theory allowed Hrushovski to prove this theorem for A a commutative algebraic group defined over a number field K . His proof gives an (almost) explicit bound of the form: $m \leq c(\deg(X))^e$, where c and e are constants depending on A and on two primes of good reduction of A .

Some results in the model theory of difference fields reduce the problem to finding: an element $\sigma \in \mathrm{Aut}(\bar{\mathbb{Q}}/K)$, a polynomial $f(T) \in \mathbb{Z}[T]$ having no roots of unity among its roots, such that the endomorphism $f(\sigma)$ vanishes on $\mathrm{Tor}(A)$. The existence of such objects follows from a result of Weil, with bounds on the degree and the absolute sum of the coefficients of $f(T)$. From this data one obtains the desired bound for m .

On the density property of PSC fields

Aharon Razon (Essen)

Let S be a finite set of primes of a field K such that $\hat{K}_{\mathfrak{p}}$ is a local field, separable over K , and $K_{\mathfrak{p}} = K \cap \hat{K}_{\mathfrak{p}}$ is either a Henselian closure or a real closure of K at \mathfrak{p} . Let $G(K)$ be the absolute Galois group of K and $K_{\mathrm{tot}, S} = \bigcap_{\mathfrak{p} \in S} \bigcap_{\sigma \in G(K)} K_{\mathfrak{p}}^{\sigma}$. A field M is PAC over a subset R if for each absolutely irreducible variety V of dimension r and for each dominating separable rational map $\varphi: V \rightarrow \mathbb{A}^r$ over M there exists $\mathbf{a} \in V(M)$ such that $\varphi(\mathbf{a}) \in R^r$. A subextension M of $K_{\mathrm{tot}, S}/K$ is PSC if each absolutely irreducible variety over M which has a simple $K_{\mathfrak{p}}$ -rational point for each $\mathfrak{p} \in S$ also has an M -rational point. M has the S -density property if every absolutely irreducible variety V over K has an M -rational point in each $\bigcap_{\mathfrak{p} \in S} \bigcap_{\sigma \in G(K)} \mathcal{U}_{\mathfrak{p}}^{\sigma}$, where $\mathcal{U}_{\mathfrak{p}}$ is a nonempty \mathfrak{p} -open subset of $V_{\mathrm{simp}}(K_{\mathfrak{p}})$ for each $\mathfrak{p} \in S$. We have two results.

Theorem 1: *Let M be PAC over K . Then M is PAC over each nonempty S -open subset of K . If K is global, then M is PAC over each separable Hilbert subset of K .*

In a forthcoming paper, Geyer and Jarden use Theorem 1 to prove that if M is PAC over K , then the maximal Galois extension of K inside $M \cap K_{\text{tot},S}$ is PSC.

Theorem 2: *Let M be a subextension of $K_{\text{tot},S}/K$. If M is PSC, then M has the S -density property.*

On linearly rigid tuples

Helmut Völklein (Gainesville)

Definition: Let $g_1, \dots, g_r \in \text{GL}_n(K)$ with $g_1 \cdots g_r = 1$. We say they form a **linearly rigid tuple** if the following holds: For any $h_1, \dots, h_r \in \text{GL}_n(K)$ with $h_1 \cdots h_r = 1$ such that h_i is conjugate g_i for each i , there is $g \in \text{GL}_n(K)$ with $g_i = gh_i g^{-1}$ for all i . Furthermore, we say the tuple (g_1, \dots, g_r) is **(absolutely) irreducible** if the group $\langle g_1, \dots, g_r \rangle$ acts (absolutely) irreducibly in V .

In the case $K = \mathbb{C}$, the following theorem is due to N. Katz, *Rigid local systems*, Princeton Univ. Press 1996. The proof given there uses the cohomology of sheaves on the sphere which become locally constant after deleting finitely many points.

Theorem: *Let (g_1, \dots, g_r) be an absolutely irreducible tuple in $\text{GL}_n(K)$ with $g_1 \cdots g_r = 1$. Let δ_i be the codimension of the centralizer of g_i in $M_n(K)$. Then*

$$(1) \quad \delta_1 + \cdots + \delta_r \geq 2(n^2 - 1).$$

If

$$(2) \quad \delta_1 + \cdots + \delta_r = 2(n^2 - 1),$$

then the tuple is linearly rigid. In case K is algebraically closed, the tuple is linearly rigid if and only if (2) holds.

Rigid local systems and realizations of Galois groups

Michael Dettweiler (Erlangen, Gainesville)

This talk is about a joint work of Stefan Reiter (Heidelberg, Gainesville) and myself. We prove the

Theorem: *The linear groups $\text{GL}_{2m+1}(q)$ and the unitary groups $U_{2m+1}(q)$ occur as Galois groups over the rationals if q is an odd prime power and $\Phi(q-1) \leq m$, resp. $\Phi(q+1) \leq m$ in the unitary case (Φ denotes the Euler function).*

This theorem is proved by studying two new classes of rigid generators of the corresponding groups. These satisfy the product relation and have $m+1$ perspectivities and two unipotent elements with a maximal number of two-blocks in the Jordan form in the *type A-case*, resp. have m perspectivities and two unipotent elements having one three-block and a maximal number of two-blocks in the *type B-case*. We then demonstrate how

one can find those rigid generators using Katz' Existence Algorithm. This algorithm can be found in N. Katz: Rigid local systems, Princeton Press, 1996.

The Guralnick/Thompson conjecture for groups of bounded genus

Kay Magaard (Detroit)

In 1990 Guralnick and Thompson published the following Conjecture: *For every non-negative integer g there exists a finite set of finite simple groups $\mathcal{E}(g)$ such that, if $L/\mathbb{C}(t)$ is a finite extension of genus $\leq g$ and \hat{L} the normal closure of L , then the composition factors of $\text{Gal}(\hat{L}/\mathbb{C}(t))$ are either cyclic, alternating, or in $\mathcal{E}(g)$.*

Recent work of Daniel Frohardt and I settles the conjecture in the affirmative.

Relative conditions for the splitting of finite groups over normal subgroups

Albrecht Brandis (Heidelberg)

Notations: Let G be a finite group, A a normal subgroup of G . Then $G//A$ means: G splits over A . $\text{Cpl}_G(A) := \{L \leq G, LA = G, L \cap A = E, |E| = 1\}$. Let $A \leq H \leq G$ and $\text{g.c.d}([G : H], |A|) = 1$.

Question: Under which conditions does

$$(*) \quad H//A \iff G//A$$

hold?

The following theorem (Gaschütz 1951) is well known: *If A is abelian then $(*)$ holds.* This theorem does not remain true even if A is metabelian (s. Huppert I). If A is not abelian the embedding of H in G plays a fundamental role.

Lemma: *If $K \in \text{Cpl}_H(A)$ then there exists $L \in \text{Cpl}_G(A)$ and $K \leq L$ iff*

$$(**) \quad \forall g \in G \exists a \in A : H \cap K^g \leq K^a \text{ holds.}$$

As a consequence one has the following examples:

Theorem: *If A is metabelian then $(*)$ holds iff there exists $K \in \text{Cpl}_H(A)$ such that property $(**)$ holds.*

Theorem: *If $H \trianglelefteq G$ and A is solvable then $(*)$ holds iff there exists $K \in \text{Cpl}_G(A)$ such that property $(**)$ holds.*

If H is a p -Sylow subgroup of G then one can apply the theory of fusion by Alperin. In this case criteria are given such that $N_G(H)//A \iff G//A$ holds.

The rational function analogue of a question of Schur

Peter Müller (Heidelberg)

In 1923 Schur considered the following problem. Let $f \in \mathbf{Z}[X]$ be a polynomial that induces a bijection on the residue fields $\mathbf{Z}/p\mathbf{Z}$ for infinitely many primes p . His conjecture, that such polynomials are compositions of linear and Dickson polynomials, was proved by M. Fried in 1970. Here we investigate the analogue for rational functions, also the base field may be any number field. As a result, there are several classes and some sporadic cases of functions which fulfill the analogous property. The infinite series, besides the polynomial ones, come from rational isogenies of elliptic curves.

The classification of these series proceeds as follows. First we translate the arithmetic hypothesis to a permutation group theoretic property of the pair of arithmetic and geometric monodromy group. The complicated analysis of this property is based on the classification of the finite simple groups. Then we use arithmetic arguments to either rule out many candidates, or to actually prove existence. This part is based on Mazur's results about rational points on the modular curves $X_0(p)$ and $X_1(p)$, results about the Galois images in $GL_2(p)$ coming from action of the absolute Galois group $G_{\mathbf{Q}}$ on p -torsion points of elliptic curves, the theory of complex multiplication, and the techniques used in the inverse regular Galois problem.

The work is joint with Robert M. Guralnick and Jan Saxl.

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