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Spectral Theory and Stochastic Analysis

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organized by M. van den Berg (Bristol)
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The link between spectral theory and stochastic analysis was landmarked by the papers of Mark Kac "Can you hear the shape of a drum", Frank Spitzer "Heat flow, Brownian motion and electrostatic capacity" and Edward Nelson "Dynamical theories of Brownian motion" in the 60's. The subject was diversified considerably over the last three decades and influenced the development in quantum mechanics and quantum field theory: Schrödinger operators with random potentials are used to describe models of amorphic substances, evolution equations can be solved by running an appropriate stochastic process, the spectral theory for boundary value problems can be treated quantitatively via strong Markov and martingale properties.

The conference brought together experts of both fields. The main subject covered by the talks include

- random Schrödinger operators
- spectral properties for generators of diffusions
- differential geometry and scattering theory
- Dirichlet and Neumann problems
- Feynman-Kac and Feyman-Kac-Ito formulae
- Feller properties for Markov semigroups

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Abstracts:

Some pseudodifferential generators: Spectral problems, infinite dimensional processes and quantum fields

S. Albeverio

We discuss examples of pseudodifferential operators in finite and infinite dimensions, related spectral problems and stochastic analytic questions. We mention that the study of the negative spectrum of the Hamiltonian describing N particles with 2-body point interactions (heuristically also arising from the non-relativistic limit of a ϕ^4 quantum field model) is related to the one of the bound state spectrum of N-body Schrödinger operators with nice potentials, in the situation of "zero energy resonances" (accumulation of eigenvalues $E_n \to 0$, Efimov effect). We discuss a conjecture (by S. Albeverio, R. Høegh-Krohn, T. T. Wu) concerning the asymptotics $\frac{E_n}{E_{n+1}} \to$ universal constant, showing how it is related to an extension of the theory of Alchieser-Kac-Szegő for the asymptotics of determinants of certain integral operators in the case of meromorphic symbols (recent work by Albeverio and K. Makarov). We also show how this study is related to the one of the self-adjoint realizations and spectra of relativistic Hamiltonians of the form $\sqrt{-\Delta + m^2} - \frac{\lambda}{|x|}$ in $L^2(\mathbb{R}^3)$, especially in the case $\lambda > 2/\pi$. We briefly mention work on pseudodifferential operators (by Albeverio and P. Kurasov) and probabilistic aspects of processes associated with pseudodifferential operators. We then pass to the discussion of infinite dimensional differential and pseudodifferential operators. We show how certain partial pseudodifferential equations arise in connection with the construction of homogenous and Markovian random fields. In particular we discuss constructions starting for Gaussian and non-Gaussian (Lévy-type) white noises. The latter lead to interesting examples, even in space-time dimension 4, of relativistic local quantum fields of gauge-type, satisfying all Marchio-Strocchi-Wightman axioms and leading to a non-trivial S-matrix (the first known non-trivial models having these properties, work by S. Albeverio, H. Gottschalk and J.L. Wu).

Applications of spectral theory to global regularity and decay

M. Ben-Artzi

Consider the self-adjoint extension H of $-\Delta$ and note that, if $\{E(\lambda)\}$ is the associated spectral family,

$$(E(\lambda)f, g) = \int_{|\xi|^2 \le \lambda} \hat{f}(\xi)\overline{\hat{g}(\xi)}d\xi,$$

(where $\hat{f}$ is the Fourier transform of f), hence,

$$\frac{d}{d\lambda}(E(\lambda)f,g) = \frac{1}{2\sqrt{\lambda}}\int_{|\xi|^2=\lambda} \hat{f}(\xi)\overline{\hat{g}(\xi)}d\omega_\xi, \quad \lambda > 0,$$

which defines a continuous bilinear form $< A(\lambda)f, g >$ on $H^s(\mathbb{R}^n) \times H^s(\mathbb{R}^n)$, for any $s > \frac{1}{2}$. Here $H^s(\mathbb{R}^n)$ is the Sobolev space of order s. In particular, $A(\lambda) \in B(L^{2,s}, L^{2,-s}),\ \ \lambda > 0$, where

$$L^{2,s} = \{f| \int_{\mathbb{R}^n}(1+|x|^2)^s|f(x)|^2dx < \infty\}.$$

Using suitable trace estimates, one can prove that $\|A(\lambda)\|_{B(L^{2,s},L^{2,-s})} \leq C_{s,n}\lambda^{-\frac{1}{2}}$, $\lambda > 0$, $s > \frac{1}{2}$, for $n \geq 3$, and $\|A(\lambda)\|_{B(L^{2,1},L^{2,-1})} \leq C$. Using the representation $e^{-itH} = \int_0^\infty e^{it\lambda}A(\lambda)d\lambda$ and the above estimates, a duality argument leads to the estimate

$$\int_{\mathbb{R}^n}\int_{\mathbb{R}^n}(1+|x|^2)^{-1}|u(t,x)|^2dtdx \leq C\|u_0\|^2_{L^2}, \tag{1}$$

where $u(t,x) = \exp(itH)u_0$. In fact, using the $\lambda^{-\frac{1}{2}}$ decay one can throw derivatives into the estimate (1), obtaining, with a suitable gauge invariance,

$$\int_{\mathbb{R}^n}\int_{\mathbb{R}^n}|x|^{2\alpha-2}||D|^\alpha u|^2dxdt \leq C\|u_0\|^2_{L^2}, \quad \alpha < \frac{1}{2},$$

with a slight modification (due to Ben-Artzi–Klainerman) for $\alpha = \frac{1}{2}$, and, for $\alpha = 0$,

$$\sup_{a,v\in\mathbb{R}^n}\int_{\mathbb{R}^n}\int_{\mathbb{R}^n}\frac{|u(t,x)|^2}{|x-a-vt|^2}dxdt \leq C\|u_0\|^2_{L^2}$$

(with "best constant" due to Simon).

In the above considerations, the operator H can be replaced by appropriate functions $f(H)$, for f, say, smooth and increasing. Specifically, the case $f(-\nabla) = (-\Delta+1)^{\frac{1}{2}}$ is the "relativistic Schrödinger operator" (equivalently, the Klein–Gordon Eq.) and the estimate (1) is still valid in this case.

The Hölder continuity of $A(\lambda)$ leads, in view of $R(z) = (H-z)^{-1} = \int \frac{A(\lambda)}{\lambda - z}d\lambda$ and the Privaloff theorem, to the "Limiting Absorption Principle", namely,

$$R^\pm(\mu) = \lim_{z\to\mu\pm i\cdot 0} R(z) \text{ exists in } B(L^{2,s}, L^{2,-s}), \quad s > \frac{1}{2}. \tag{2}$$

Using more refined trace theorems for noncompact manifolds one can show that (2) holds for general simply characteristic operators, thus yielding apriori estimates for solutions of the wave equation or the Klein–Gordon equation in space-time.

Also, $L^1 \to L^\infty$ heat kernel estimates can be obtained, using the representation $e^{-tH} =$

$\int e^{-t\lambda}A(\lambda)d\lambda$. As in the smoothing (Schrödinger) case, one proceeds by duality, proving first an $L^1 \to L^2$ estimate.

A clumping process connected with Ruelle's probability cascades

E: Bolthausen

We describe a continuous time Markov process on the set of partitions of the natural numbers. The only transitions allowed are clumpings of classes. This process appears naturally in connection with Derrida's generalized random energy model in the formulation of Ruelle. Several key features of the cavity method in spin glass theory have natural interpretations in an abstract framework.

Remarks on the Neumann heat kernel

K. Burdzy

The "hot spots" conjecture of J. Rauch (1974) says that the second Neumann eigenfunction in a Euclidean domain attains its maximum on the boundary of the domain.
Theorem. (K. B. and W. Werner) *The "hot spots" conjecture is false.*

Recent progress in quantum KAM methods

P. Duclos

We extend, improve and simplify previous works, [1], [2], on global pure pointness of Floquet Hamiltonians, with a non necessarily smooth time dependent potential. In particular we have bypassed the use of the adiabatic regularisation which greatly reduces the required regularity of the potential and we give explicit quantitative estimates in terms of the relevant (physical) parameters for the method to work.

[1] J. Bellissard, *Stability and instability in quantum mechanics*, in "Trends and Developments in the Eighties", Albeverio and Blanchard eds., World Scientific, Singapore 1985, pp. 1-106

[2] P. Duclos and P. Šťovíček: *Floquet Hamiltonians with pure point spectrum*, Comm. Math. Phys. 177 (1996), 327-347

Existence and completeness of scattering systems by using the theory of operator ideals

S. Eder

There is a very deep interplay between operator ideals and scattering theory. The basics for showing this are Pearson's theorem and the theory of p-summing operators.

First Pearson's theorem is improved. Then by passing to L_2-operators we show how the assumptions in Pearson's theorem can be verified by using the theory of operator ideals.

One of the main achievements is that Pearson's estimate can be formulated in such a way that the $L_\infty \to L_1$ norm of a sandwiched difference determines the behaviour of the difference between the wave and identification operator. As a byproduct we obtain existence and completeness of scattering systems (H_2, J, H_1) for wide classes of operators H_2, J, H_1.

Spectrum and scattering for complete Riemannian manifolds

K. D. Elworthy

One of the nicest class of examples of the relationships between the behaviour of diffusion processes and the spectral properties of Schrödinger operators comes from Nelson's stochastic mechanics. The starting point was a 1980 paper by Shucker showing that stochastic mechanical diffusions in Euclidean space corresponding to states of a free quantum mechanical evolution have an asymptotic velocity which is given by the inverse Fourier transfom of the initial state. This was extended in the mid 80's by Carlen to more general systems and Carlen and I considered versions for Riemannian manifolds using his approach. The special case of quantum mechanics on hyperbolic spaces appeared in 1993: We showed that for hyperbolic 3- space Shucker's result holds with the Fourier transform replaced by the Fourier Radon transform. In unpublished work we showed that for "quasi-free" systems on simply connected negatively curved complete manifolds, under minor technical conditions stochastic mechanical particles still have an asymptotic velocity (which is non-zero). With some other technical conditions this velocity was described by a scattering transform which partially diagonalizes the Hamiltonian. This transform is described analytically and exists in more general situations. Again with technical assumptions which do not have an obvious geometric significance, except that the related classical mechanics should have trajectories emanating out to infinity from some subdomain (e.g. as the geodesics leaving a point in hyperbolic space in the free case). This was taken up more recently and refined with Feng-Yu-Wang. The range of the scattering transform described above should be all scattering states. With Wang the related essential spectrum was described in terms of the behaviour near infinity of the Laplacian of certain exhaustion functions on M e.g.

the distance from a point, submanifold, or suitable domain. This led to results for the Laplace–Beltrami operator improving existing ones by Kumura (which in turn generalised earlier results of Donnelly, for negative curvature, and by Li for non-negative curvature).

[1] E. Carlen and K.D. Elworthy, *Stochastic and quantum mechanical scattering on hyperbolic spaces*, in "Asymptotic Problems in Probability Theory: Stochastic Models and Diffusions on Fractals", Proc. Taniguchi Symp 1990, K.D. Elworthy and N. Ikeda, eds., Pitman Research Notes in Maths. 283, pp. 3-14

[2] Elworthy, K.D. and Wang F-Y.: *On the esssential spectrum of the Laplacian on Riemannian manifolds.* Warwick Preprint 30/1997 Maths Dept. Warwick University, Coventry CV47AL

Dia- and paramagnetism for nonhomogeneous magnetic fields

L. Erdős

Diamagnetism of the magnetic Schrödinger operator and paramagnetism of the Pauli operator are rigorously proven for nonhomogeneous magnetic fields in the large field, in the large temperature and in the semiclassical asymptotic regimes. New counterexamples are presented which show that neither dia- nor paramagnetism are true in a robust sense (without asymptotics). In particular, we demonstrate that the recent diamagnetic comparison result by Loss and Thaller [Comm. Math. Phys. **186** (1997), 95–107] is essentially the best one can hope for.

On the essential spectra of Neumann Laplacians on domains and associated trees

D. Evans

Let Ω be a domain in $\mathbb{R}^2$, Γ an ordered tree of finite degree which is mapped into Ω by a locally Lipschitz function u, and τ a locally Lipschitz function which maps Ω onto Γ. A typical candidate for $u(\Gamma)$ is the skeleton of Ω, that is, the set of points in Ω which have more than one near point on the boundary $\partial\Omega$: Fremling has proved that if Ω does not contain a halfplane, its skeleton is connected and expressible as the countable union of paths of finite length. Examples satisfying our requirements include horns, spirals, rooms and passages, snowflakes, etc. In the lecture, recent joint work with Y. Saito was presented, the objective in this being the determination of the spectral properties of the Neumann Laplacian $-\Delta_{\Omega,N}$ on Ω in terms of the geometric and metric properties of Ω. Specifically, the relationship between essential spectra of $-\Delta_{\Omega,N}$ and a natural differential operator of Sturm-Liouville type on Γ was investigated.

Wegner estimates for random operators

P.D. Hislop

Localization of electron wave functions and classical waves is an expected result of random perturbations of background media. If the background is described by an operator H_0 exhibiting a spectral gap G, the localized states are expected at energies near the band edges of G. A key step in the proof of localization is the proof of a Wegner estimate for various models. Let V_Λ be the restriction of V_ω, the perturbation, to a region $\Lambda \subset \mathbb{R}^d$. Local Hamiltonians have the form $H_0 + V_\Lambda$ for electrons and $H_\Lambda = (1 + V_\Lambda)^{-1/2} H_0 (1 + V_\Lambda)^{-1/2}$ for classical waves. A Wegner estimate states that the probability that H_Λ has eigenvalues near $E_0 \in G$, say in $[E_0 - \eta, E_0 + \eta]$, is bounded above by $C_\omega \eta |\Lambda|^\sigma$, $\sigma \geq 1$. We discuss a proof of this estimate for $\sigma = 2$ following the work of Kirsch, Stollmann, Stolz. This result is used to establish band-edge localization for the breather model. The proof uses estimates on the spectral shift function and estimates on the localization of eigenfunctions of H_Λ (joint work with J.M. Combes).

Pseudodifferential operators generating Markov processes

W. Hoh

We consider generators of Markov processes with jumps. It is well-known that the generator of such a process is a Lévy-type operator. We use another equivalent representation as a pseudo differential operator $-p(x, D)$. The symbols $p(x,\xi)$ of such operators are characterized by the property that $\xi \mapsto p(x,\xi)$ is a continuous negative definite function for all fixed x. We consider the problem which growth behaviour of the symbol with respect to x is allowed such that a corresponding process remains conservative. i.e no explosion occurs. It turns out that the maximal growth is determined by the behaviour of the symbol for small values of ξ. In particular, if the ξ-dependence of the symbol is controlled in terms of the negative definite function $|\xi|^\alpha$ of a symmetic α-stable process, then an estimate $p(x,\xi) \leq c(1 + |x|^\alpha) \cdot |\xi|^\alpha$ for the symbol implies conservativeness.

Correlation structure of intermittency in the parabolic Anderson model

F. den Hollander

Consider the Cauchy problem

$$\frac{\partial u}{\partial t}(x,t) = \Delta u(x,t) + \xi(x)u(x,t) \qquad (x \in Z^d, t \geq 0)$$

with initial condition $u(x,0) \equiv 1$, where Δ is the discrete Laplacian and $\{\xi(x) : x \in Z^d\}$ is an i.i.d. field of R–valued random variables. Let $\langle\cdot\rangle$ denote expectation w.r.t. the ξ–field and let $H(t) = \log\langle e^{t\xi(0)}\rangle < \infty$ for all $t \geq 0$. Under the condition

$$\lim_{t\to\infty} tH''(t) = \rho \in (0,\infty),$$

we prove that

$$\lim_{t\to\infty} k_t(x,y) = \frac{1}{\|w_\rho\|_2^2} \sum_{z\in Z^d} w_\rho(x+z)w_\rho(y+z),$$

where $k_t(x,y) = \langle u(x,t)u(y,t)\rangle / \langle u^2(0,t)\rangle$ is the correlation coefficient of the u–field, and $w_\rho = v_\rho^{\otimes d}$ with v_ρ the ground state of the 1-dimensional nonlinear difference equation

$$\Delta v(x) + 2\rho v(x)\log v(x) \qquad (x \in Z),$$

provided this ground state is unique modulo translations. For large ρ we can prove uniqueness. A numerical analysis suggests that uniqueness holds for all ρ, but the proof is open (joint work with J. Gärtner).

Error bound estimates for Kac's transfer operator and the Lie–Trotter product formula

T. Ichinose

Kac's transfer operator is an operator of the kind

$$K(t) = e^{-tV/2}e^{-tH_0}e^{-tV/2},$$

where $V(x)$ is a real-valued continuous function in $\mathbf{R}^d$ bounded from below and H_0 is the nonrelativistic / relativistic Schrödinger operator $-\frac{1}{2}\Delta$ / $\sqrt{-\Delta+1}-1$ with mass 1. When $t > 0$ is small, $K(t)$ can be regarded as a transfer operator for some lattice models in statistical mechanics studied by M. Kac [1966 Brandeis Lecture] to discuss a mathematical mechanism for a phase transition. It is important to see whether the first eigenvalue of $K(t)$ is asymptotically degenerate as $t \downarrow 0$. Such a spectral information may be obtained from that of the Schrödinger operator $H = H_0 + V$, if we can show a norm estimate like $\|K(t) - e^{-tH}\| = O(t^{1+a})$ for small $t > 0$ with $a > 0$. We have given such a kind of estimates in L^p operator norm, $1 \leq p \leq \infty$, and in trace norm. As another application the result yields the Lie–Trotter product formula in L^p operator norm and in trace norm with error bounds $O(n^{-a})$ as $n \to \infty$. This extends the result obtained by B. Helffer in L^2 operator norm for the nonrelativistic Schrödinger operator $H = -\frac{1}{2}\Delta + V$ with potential $V(x)$ satisfying $|\partial^\alpha V(x)| \leq C_\alpha \langle x \rangle^{(2-|\alpha|)+}$ to the case for the relativistic as well as nonrelativistic Schrödinger operator H with more general potentials $V(x)$.

The present lecture is based on recent joint works with Satoshi Takanobu and Hideo Tamura.

Fractional derivatives, non-symmetric Dirichlet forms and the drift form

N. Jacob

Using fractional derivatives we show that the drift form $\int_{-\infty}^{\infty} u(x) \frac{dv(x)}{dx} dx$ can be approximated by non-symmetric Dirichlet forms. A similar result holds for the drift form in $\mathbb{R}^n$ with variable coefficients if the coefficient functions satisfy certain regularity and commutator conditions. An abstract result on fractional powers of Markov generators allows to extend this observation to generalized Dirichlet forms. Another consequence is that the bilinear form induced by an arbitrary Lévy process is the limit of non-symmetric Dirichlet forms (joint work with R.L. Schilling).

A "dominated-type" convergence theorem for the Feynman integral defined via the Trotter product formula

G.W. Johnson

It is known that under conditions sufficiently general to include the standard potentials of nonrelativistic quantum mechanics, the FI (Feynman integral) via the Trotter product formula, the modified FI (defined via a product formula involving imaginary resolvents), and the analytic in time operator-valued FI all exist and agree with one another and with the unitary group arising from the usual Hamiltonian approach to quantum mechanics. Further, rather satisfactory stability theorems are known for the 2nd and 3rd approaches to the Feynman integral mentioned above. A recent result of G.W. Johnson and J.G. Kim completes this picture by giving a stability theorem as described briefly in the title above. We now state this result taking the space dimension equal to 3 for convenience (all of the results mentioned above are included in the forthcoming book, The Feynman Integral and Feynman's Operational Calculus by G.W. Johnson and M.L. Lapidus, Oxford U. Press).

Theorem. *Let V, V_m, $m = 1, 2, \dots$ be Lebesgue measurable, $\mathbb{R}$-valued functions on $\mathbb{R}^3$. Assume that V_m converges to V "dominatedly" in the following sense:*

(a) $V_m \to V$ Lebesgue-a.e. in $\mathbb{R}^3$,

(b) $V_{m,+} \leq U$ for some $U \in L^2_{loc}(\mathbb{R}^3)$,

(v) $V_{m,-} \leq W$ for some $W \in L^2_{loc}(\mathbb{R}^3)_u$.

Then $\mathcal{F}^t_{TP}(V_m) \to \mathcal{F}^t_{TP}(V)$ in the strong operator topology, uniformly in t on compact subsets of $\mathbb{R}$ ($\mathcal{F}^t_{TP}(V)$ denotes the FI via the Trotter product formula associated with the potential V at time t).

Added Note: The proof given by G.W. J. and J.G. Kim is rather easy and depends on the dominated-type convergence theorem of Lapidus and the theorem in the book of G.W. J. and Lapidus showing agreement (under quite general conditions) of the modified FI, $\mathcal{F}^t_M(V)$ of Lapidus and $\mathcal{F}^t_{TP}(V)$. As discovered during the conference, an even easier proof can be given using a result of Jürgen Voigt [J. Operator Theory 20 (1988), 117-131]. See also the paper of Vitali Liskevich [J. Funct. Anal. 151 (1997), 281-305].

Time Asymptotics for the Burgers' equation

W. Kirsch

We consider the initial value problem for the (viscous) Burgers' equation with a periodic force term, i.e.:

$$u_t + uu_x = \frac{1}{2}\Delta + V_x$$

for a periodic potential V and

$$u(0, x) = u_0(x)$$

Ya.G. Sinai [J. Statist. Phys. 64 (1991), 1-12] proved that for "typical" u_0 the solution $u(t, x)$ converges to a periodic function v_0 as $t \to \infty$.

We give a new proof of Sinai's result which uses methods from spectral theory to investigate the Schrödinger semigroup e^{-tH} for the operator $H = -\frac{1}{2}\Delta + V$. In fact the limit function v_0 turns out to be $\frac{\phi'}{\phi}$ for the (periodic) ground state of the operator H (joint work with Almut Kutzelnigg).

Cohomology of loop spaces and Hochschild cohomology

R. Léandre

We do a calculus on an algebraic model (Hochschild space), a calculus on the Brownian bridge and define a stochastic cohomology of the based loop space. We show that the two cohomology groups are equal, by using Driver's flow.

Bounded and L^2 Harmonic Forms on Universal Covers

Xue-Mei Li

This is based on joint work with K.D. Elworthy and S. Rosenberg [1]. We relate certain curvature conditions on a complete Riemannian manifold to the existence of bounded and L^2 harmonic forms, using a probabilistic representation of heat semigroups on forms. In the case where the manifold is the universal cover of a compact manifold, we obtain topological and geometric information about the compact manifold.

Let h be a smooth function, δ^h the adjoint of the exterior differential d on $L^2(M, e^{2h}dx)$ and $\Delta^{h,q} = (d+\delta^h)^2$ the Witten Laplacian on q-forms. The Weitzenbock term $\mathcal{R}^{h,q}$ on q forms is given by $\mathcal{R}^{h,q} = \Delta^{h,q} + \text{trace}\nabla^2$ with the convention $\mathcal{R}^{h,-1} = \mathcal{R}^{h,n+1} = 0$ and we identify $\mathcal{R}^{h,q}$ with its adjoint operator on q-vector. Let $\underline{\mathcal{R}}^{h,q}(x)$ be the infimum of $\mathcal{R}^{h,q}(v)$ over all q-vectors of norm 1 and set

$$\underline{R}_q(x_0) = \int_0^\infty E \exp^{-\frac{1}{2}\int_0^t \mathcal{R}^{h,q}(x_s)ds}\, dt.$$

Here $\{x_s\}$ is a Brownian motion starting from x_0 with drift ∇h.

By a C^1 *h-harmonic* form (or harmonic form if $h=0$) we mean a form ϕ such that $\delta^h\phi = 0$ and $d\phi = 0$. Let $H^k(M)$, $BH^k(M)$, and $L^2H^k(M)$ be respectively the space of h-harmonic, bounded h-harmonic and L^2 h-harmonic k-forms.

Proposition. *Let $0 \le k \ne 1 \le n$. Assume the Witten heat semigroup is conservative. Suppose $\mathcal{R}^{k-1}$ and $\mathcal{R}^{k+1}$ are bounded from below with both $\sup_{x\in M} \underline{\mathbb{R}}_{k+1}(x)$ and $\sup_{x\in M} \underline{\mathbb{R}}_{k-1}(x)$ finite. Then $BH^k(M) = \{0\}$ if $L^2H^k(M) = \{0\}$.*

Recall that a function f is *strongly stochastic positive* if

$$\limsup_{t\to\infty} \frac{1}{t} \sup_{x_0\in K} \log E\left(\exp^{-\frac{1}{2}\int_0^t f(x_s)ds}\right) < 0$$

for each compact set K [3]. For compact manifolds strongly stochastic positivity of f is equivalent to the spectral positivity of the operator $\Delta + f$. If $\tilde{M}$ is an infinite universal cover of some compact manifold M with a non-zero harmonic form and such that $\mathbf{R}^{k\pm1}$

are strongly stochastic positive ($k \neq 0, 1$), then by the above proposition there is a non-zero L^2 harmonic form on $\tilde{M}$. The corresponding result for $k = 0$ given in [2] yields a finite fundamental group result since 1 is a bounded harmonic function and L^2 harmonic functions only exists if M has finite volume. There is a corresponding result for $k = 1$.

[1] K. D. Elworthy, Xue-Mei Li, and S. Rosenberg. *Bounded and L^2 harmonic forms on universal covers*, Geom. and Funct. Anal. Vol 8 (1998), 283-303

[2] Xue-Mei Li, *On extension of Myers' theorem,* Bull. London Math. Soc. 27. (1995), 392-396

[3] K. D. Elworthy, Xue-Mei Li, and S. Rosenberg, *Curvature and topology: spectral positivity,* in: "Methods and Applications of Global Analysis", Voronezh series on new developments in global analysis), Voronezh University Press, Voronezh 1993

On L^p-theory of Markov semigroups

V. Liskevich

We prove some general inequalities for the generators of Markov semigroups; as a consequence the sharp sector of analyticity is obtained. Perturbation theory by form bounded Markov generators is developed. The uniqueness problem is discussed.

The second part of the talk was devoted to the case of non-symmetric operators of the form $-\nabla \cdot a \cdot \nabla + b \cdot \nabla$ with measurable coefficients, on $L^p(\Omega)$, $\Omega \subset \mathbb{R}^d$ an open set. It is shown that, under the assumption that $b \cdot a^{-1} \cdot b$ is form bounded w.r.t. $-\nabla \cdot a \cdot \nabla$ with bound β, the extension generating a semigroup of quasi-contractions exists on L^p, $p \in [\frac{2}{2-\sqrt{\beta}}, \infty]$. In all cases when the semigroup is constructed it is proved that the spectra of generators are p-independent. The examples showing the sharpness of the results are discussed.

Spectral bifurcations and multiscattering phenomena

S.A. Molchanov

Discussions around Anderson's conjecture: There is a transition from Σ_{pp} to Σ_{ac} for $d \geq 3$ in the spectral problem $H\psi = \Delta\psi + \sigma\xi(x, \omega)\psi = \lambda\psi$, ($\xi(\cdot)$ i.i.d. r.v.), as the coupling constant σ tends to 0. This transition is proven for several inhomogeneous models:

(a) 1D Schrödinger operators with sparse random potentials (soliton gas potentials)

(b) Multidimensional sparse potential (a.c. spectrum is typical)

(c) Surface waves (random potentials on the boundary of half-space)

The spectral analysis of self-adjoint Jacobi matrices

S. Naboko

Unbounded Jacobi matrices J with power-like weights are considered. The main question here concerns when the absolutely continuous component of the spectrum of J covers the whole real axis. Some sufficient conditions for this are given. Also we present explicit formulae for the asymptotics of the generalized eigenvectors of J. To obtain these results we used the so-called "grouping in block" method for the large product of the transfer matrices associated to J. Beside this the detailed analysis of this product is based on the Gilbert-Pearson subordinacy theory, generalized Behncke-Stolz lemma, Kiselev's ideas for the discrete Schrödinger operators with decaying potentials, Harris-Lutz-like transforms and some facts from harmonic analysis. Also some constructive examples of embedded eigenvalues have been presented (joint work with J. Janas).

Capacitary estimates for the bottom eigenvalue of self-adjoint operators in abstract Hilbert spaces

A. Noll

The notion of capacity in abstract Hilbert spaces is used to prove new capacitary upper and lower bounds for the spectral shift of a self-adjoint operator which is subjected to a domain perturbation. This leads, among other results, to a generalization of Thirring's inequality. Moreover, it is possible to obtain the following estimate for the spectral shift $\lambda' - \lambda$:

$$c_1 r^{d-2m} \leq \lambda' - \lambda \leq c_2 r^{d-2m},$$

valid for certain differential operators of order $2m$ which are perturbed by imposing additional Dirichlet boundary conditions on some ball of radius r.

Heat kernel bounds for multplicative perturbations of elliptic operators

El-M. Ouhabaz

Let (X, d, μ) be a space of homogenous type and consider on $L^2(X)$ an operator A which has a heat kernel $p_t(x, y)$. Let $b \in L^\infty(X)$ bounded below by a positive constant. Denote by $k_t(x, y)$ the heat kernel of the multiplicative perturbation bA. Our aim is to show that Gaussian upper bounds (i.e. $|p_t(x, y)| \leq Cv(x, \sqrt{t})^{-1} exp\frac{-cd^2(x,y)}{t}$ where $v(x, r)$ denote the volume of the ball of center x and raduis r), Gaussian lower bounds and Hölder continuity carry over from $p_t(x, y)$ to $k_t(x, y)$. In order to achieve this, a key idea is to show that Hölder continuiy of heat kernels can be characterized in terms of a version of Gagliardo-Nirenberg inequality.

Pointwise convergence of generalized Fourier-Bessel series

M.A. Pinsky

$$L := \frac{d^2}{dr^2} + \frac{\Delta'(r)}{\Delta(r)} \frac{d}{dr} \qquad \text{where} \qquad \frac{\Delta'(r)}{\Delta(r)} \sim \frac{2\alpha + 1}{r} (r \downarrow 0), \qquad \alpha > -\frac{1}{2}$$

Eigenfunctions $\phi_n(r)$ are defined by the singular Sturm-Liouville problem:

$$\begin{aligned} L\phi_n(r) + \lambda_n \phi_n(r) &= 0 \quad (0 < r < a), \\ \cos\beta\, \phi_n(a) + a \sin\beta\, \phi_n'(a) &= 0 \quad \text{for some } \beta \in [0, \pi). \end{aligned}$$

Without loss of generality, normalize so that $\phi_n(0) = 1$. The formal Fourier expansion of a piecewise smooth function is written

$$f(r) \sim \sum_{n=1}^{\infty} A_n \phi_n(r) \quad \text{where} \quad A_n = \frac{\int_0^a f(r)\phi_n(r)\Delta(r)\, dr}{\int_0^a \phi_n(r)^2 \Delta(r)\, dr} \qquad n = 1, 2, \ldots$$

We decompose f as follows: $f = f_I + f_{II}$, where

f_I is identically zero for $0 < r < a - \delta$, for some $\delta > 0$ (boundary trouble)

f_{II} is identically zero for $a - \delta \leq r < a$, for some $\delta > 0$ (internal trouble)

Theorem 0. *The expansions for $f_I(r), f_{II}(r)$ converge for all $r \in (0, a]$.*

Theorem I. *The expansion for $f_I(0)$ converges as follows:*
(Dirichlet BC): If $\beta = 0, \alpha < 1/2$, then the series converges.
If $\beta = 0, 2k + \frac{1}{2} \leq \alpha < 2k + \frac{5}{2}$ for some $k = 0, 1, \ldots$, then the series converges iff $f(a) = 0, Lf(a) = 0, \ldots, L^k f(a) = 0$. (Neumann-Robin BC): If $0 < \beta < \pi, \alpha < 3/2$ then

the series converges. If $\beta > 0, 2k + \frac{3}{2} \leq \alpha < 2k + \frac{7}{2}$ for some $k = 0, 1, \ldots$, then the series converges iff $\cos \beta f_j(a) + a \sin \beta f_j(a) = 0$ for $j = 0, 1, \ldots k$ where $f_0 = f$ and $f_j = L^j f$ for $j = 1, \ldots, k$.

Theorem II. *The expansion for $f_{II}(0)$ converges as follows:*
If $\alpha < 1/2$, then the series converges. If $2k + \frac{1}{2} \leq \alpha < 2k + \frac{3}{2}$ for some $k = 0, 1, \ldots$, then the series converges iff $r \to f(r)$ is smooth of class C^k.

Remarks: $\Delta(r) \equiv r^{2\alpha+1}$ corresponds to classical Fourier-Bessel series. The dimension parameter is $d = 2\alpha + 2$.

Limiting absorption principle for singularly perturbed feller operators

W. Renger

We establish the stability of a limiting absorption principle for Feller operators under singular perturbations. Here a Feller operator is an operator which is defined as the generator of a strong Markov processes with the Feller property, the process being defined by its transition density.

On an abstract Hilbert space level the result is the following:

Suppose a limiting absorption principle (LAP) holds for a self-adjoint, semibounded operator H_1 in a Hilbert space $\mathcal{H}_1$. That is, suppose there is a dense subspace $X \subset \mathcal{H}_1$ and an open set $\Delta \subset \mathbb{R}$ such that $R_1^{\pm}(\lambda) := \lim_{\varepsilon \downarrow 0} R(\lambda \pm i\varepsilon)$ exists in the norm topology of $\mathcal{B}(X, X^*)$ (the space of bounded operators from X to X^*) for all $\lambda \in \Delta$.

Let H_2 be a densily defined, self-adjoint and semibounded operator in the Hilbert space $\mathcal{H}_2$ and J a bounded operator from $\mathcal{H}_1$ to $\mathcal{H}_2$ with JJ^* equal to the identity on $\mathcal{H}_2$. Suppose that H_2 is a small perturbation of H_1 in the sense that the difference between some powers of the resolvents $R_j(a)$ is compact: $J^* R_2(a)^m J - R_1(a)^m \in \mathcal{B}_\infty(X^*, X)$ for some $m \in \mathbb{N}$. On this abstract level we show that (except for possibly a discrete set of eigenvalues) a LAP holds for H_2.

This theory is then applied to study potential and especially domain perturbations of Feller operators. Assume that a LAP holds for the unperturbed operator H_1 on $L^2(M)$ (M some appropriate measure space) and consider a perturbed operator $H_2 = (H_1 + V)_\Sigma$ that is produced by restricting $H_1 + V$ to a set $\Sigma \subset M$ via Dirichlet boundary conditions. We give sufficient conditions in terms of V and the equilibrium potential of the set $\Gamma = M \setminus \Sigma$ to ensure that a limiting absorption principle holds for H_2. If, for instance, H_1 is the Laplacian this theory allows to treat the usual short range potentials, but it also allows domain perturbations by sets Γ which may be unbounded – the condition imposed on Γ is slightly stronger than requiring that its capacity is finite.

L^p-analysis of finite and infinite dimensional diffusion operators

M. Röckner

Oriented towards applications to problems in Stochstic Analysis, more precisely stochastic differential equations in infinite dimensions, we discuss the analysis of (finite and) infinite dimensional singular diffusion operators in L^p-spaces w.r.t. suitably chosen measures. As an important case study we consider the differential operator coming from the stochastic quantization of Euclidian field theory in finite volume. In particular, we present our recent uniqueness result (obtained in a joint work with Vitali Liskevich) in this case. More precisely, this result states that the closure of the corresponding singular diffusion operator in $L^p(\mu)$, where μ is the Euclidian finite volume quantum field, generates a strongly continuous contraction semigroup for all $p \geq 1$. This immediately implies the uniqueness of all solution Markov processes to the underlying stochastic differential equation whose transition semigroup consists of contractions on $L^p(\mu)$.

Bochner's subordination for processes, semigroups, and infinitesimal generators.

R.L. Schilling

Subordination (in the sense of S. Bochner) is a technique that allows to construct new processes and semigroups from given ones. Probabilistically it is a time-change w.r.t. an independent one-sided Lévy process, from an analytic point of view it is given by a Bochner integral $T_t \rightsquigarrow \int_0^\infty T_s \, \mu_t(ds)$ where $\{\mu_t\}$ is a convolution semigroup of measures carried on the half-axis. We prove a new representation of the generator of a subordinate semigroup as a limit of bounded operators. Our construction yields, in particular, a characterization of the domain of the generator. The generator of a subordinate semigroup can be viewed as a function of the generator of the original semigroup. For a large class of these functions (the operator monotone or complete Bernstein functions) we show that operations at the level of functions have their counterparts at the level of operators.

Non commutative structures and quantum dynamical semigroups

K.B. Sinha

The theory of classical Markov processes begins with a given transition probability satisfying Chapman-Kolmogorov equation or equivalently with a given Markov semigroup and then one constructs a Markov process so that its expectation gives back the semigroup one had started with. In quantum (or non-commutative) theory, one starts with a semigroup of completely positive maps on a suitable unital von Neumann algebra $\mathcal{A}$ (the algebra of observables) and tries to construct a (non-commutative) stochastic process so that its (vacuum) expectation gives back the CP-semigroup.

First, starting with the unbounded version of the formal Lindblad generator (a sort of non-commutative Laplacian) a "minimal" CP-semigroup on $\mathcal{A}$ is constructed and a few equivalent criteria of its conservativity (i.e. preservation of the identity of $\mathcal{A}$ are established. One also studies a large class of conservative extensions of the minimal semigroup. This part is analogous in spirit to the self-adjointness problem in a Hilbert space. Next, having constructed the semigroup, one chooses and fixes a $\star$-representation π of $\mathcal{A}$ in a suitable Hilbert space. Then the basic (non-commutative) stochastic processes are constructed on a Fock space, closely related to the representation space, and using these processes as (stochastic operator-valued) integrators, the stochastic flow on $\mathcal{A}$ is constructed such that the expectation semigroup w.r.t. the fock vacuum is precisely the CP-semigroup one had started with.

Eigenvalue gaps and conditioned processes

R.G. Smits

If one considers the ergodic process which solves the stochastic differential equation

$$dX_t = dW_t + \frac{\nabla\phi_1(X_t)}{\phi_1(X_t)}dt,$$

where $\phi_1(x)$ is the ground state for the Dirichlet Laplacian on a domain Ω, the mixing rate for such process is exponentially fast with exponent $\lambda_2 - \lambda_1$, the spectral gap. We examine the geometric influence the domain has on the gap, paying particular attention to the special class of convex domains where extremal domains are given by thin rectangles and sectors of circles. We also consider analogous questions for the Neumann and Robin boundary value problem whose eigenfunctions/values are solutions to

$$-\Delta u = \lambda u \text{ in } \Omega, \quad \frac{\partial u}{\partial n} + \alpha u = 0 \text{ on boundary of } \Omega$$

which serves as an intermediary between reflecting and killed Brownian motion.

Recent developments in the spectral theory of random operators

P. Stollmann

In this talk we start with a survey on recent results about localization for typical random operators, including models from solid state physics and models for acoustic waves. We then discuss in some detail a specific feature entering in these proofs, known under the name of Wegner estimates. Roughly speaking, Wegner estimates establish at least some spreading of eigenvalues for random operators. After presenting a simple proof for a very general Wegner estimate, we point out the difficulties arising in one of the most prominent examples, the Bernoulli–Anderson model in higher dimension. For this model there is no proof of Wegner estimates so far, and therefore the occurence of localization has not been established.

Understand Non-Monotonicity in Random Operators!

G. Stolz

At the current time the amount of rigorous knowledge about localization phenomena for random operators is heavily model dependent. Much more is known for Anderson-type models than for other physically important models as for example the random displacement or Poisson models. Mathematically this is caused by a lack of monotonicity in the latter models, i.e. the operators do not depend monotonously on the basic random parameters. Monotonicity, however, crucially enters most known methods to establish localization by being used in the proofs of Wegner estimates as well as in spectral averaging.

In the talk some recent ideas to overcome the problem of non-monotonicity are discussed. They include the use of results from inverse spectral theory and the new method of two-parameter spectral averaging. Applications of these ideas include the proof of exponential localization at all energies for the one-dimensional random displacement model and Poisson model. With the exception of a result by Klopp for a quasiclassical version of the random displacement model the corresponding problems are open in higher dimension.

Dirichlet forms and geometric analysis

T. Sturm

In this talk, a probabilistic approach to generalized harmonic maps $f : M \to N$ with values in the metric space N is presented. Classical harmonic maps between Riemannian manifolds are critical values of the energy functional $\mathcal{E}(f) = \int_M \|df\|^2 dm$.

In probabilistic terms, this can be expressed as $\mathbb{E}_m \langle f(X) \rangle_1$, the mean of the quadratic variation of the N-valued process $f(X)$ where X denotes Brownian motion on M.

Nowadays for several reasons one is also interested in harmonic maps with values in singular target spaces N. For metric spaces (N, d), an appropriate way to define $\mathcal{E}(f)$ is as the limit (for $t \to 0$) of $\mathcal{E}^t(f) = \frac{1}{t}\mathbb{E}_m d^2(f(X_t), f(X_0))$. However, as illustrated by an example, this limit will not always exist. We discuss conditions on M *or* on N which imply (monotone) convergence. We also present continuity results for maps with finite energy as well as Hölder continuity results for harmonic maps.

Quantum symmetries and stochastic analysis

J.C. Zambrini

We prove the theorem of Noether in quantum mechanics, which provides richer information about quantum symmetries than the usual method in Hilbert space (i.e. essentially, the theorem of Stone). Generically, the associated quantum constant of motion are time dependent.

The result has been found via a probabilistic analogy with quantum theory using two heat equations (adjoint with respect to the time parameter) instead of the Schrödinger equation. The stochastic version of Noether's theorem provides martingales of the class of underlying stochastic processes. The whole framework can be formulated on a Riemannian manifold.

The probabilistic counterpart of quantum mechanics underlying the method "Euclidian Quantum Mechanics" can be regarded as a probabilistic reinterpretation of Feynman's strategy.

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