MATHEMATISCHES FORSCHUNGSINSTITUT
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Mathematical Methods in Tomography

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The conference was organized by F.A. Grünbaum (Berkeley), A.K. Louis (Saarbrücken) and F. Natterer (Münster).

Mathematical problems arising from medical imaging or of possible future impact in this field were in the center of interest. The different kinds of tomography the speakers dealt with are

- X-ray Computerized Tomography (2D, 3D)
- Vector Tomography
- Ultrasound Tomography
- Optical Tomography
- Impedance Tomography
- Positron Emission Tomography

The conference was attended by 44 participants and 30 talks were given. The pleasant atmosphere in Oberwolfach inspired many valuable discussions that will give impulses for future research.

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VORTRAGSAUSZÜGE - ABSTRACTS

S. Arridge

Optimisation methods in optical tomography

We consider the problem of Optical Tomography in the diffusion domain. Given $y = M[\Phi]$ and q on the boundary of a domain Ω where M is a measurement operator and q a boundary source with Φ the solution to

$$-\nabla \cdot \kappa(x)\nabla\Phi + \left(\eta(x) + \frac{\partial}{\partial t}\right)\Phi = q.$$

Problem: Recover $\kappa(x)$, $\eta(x)$ in Ω.

Conjecture: Recovering two coefficients requires two measurement maps.

We consider temporal filtered transforms of $\Phi(x,t)|_{\partial\Omega}$ and propose iterative non-linear algorithms for the inverse problem.

Y. Berest

Huygens' principle and lacunas for hyperbolic operators with variable coefficients

Lacuna of a linear hyperbolic differential operator L is a domain inside the propagation cone where the principal fundamental solution of L vanishes identically. The study of lacunas for hyperbolic operators of arbitrary order was initiated by I.G. Petrovsky (1945). Atiyah, Bott and Gårding (1970-73) extended Petrovsky's results to the case of hyperbolic operators with constant coefficients and multiple characteristics. In fact, they developed a profound and complete theory of lacunas for such operators. By contrast, much less is known about lacunas for variable coefficients. In our work we study the question of lacunas for some remarkable class of differential operators with singular coefficients related to finite root systems (Coxeter groups). In fact, we give a generalization of the Petrovsky-Atiyah-Bott-Gårding theory in this case. We obtain, in particular, the analog of the classical Herglotz-Petrovsky-Leray formulas that express the fundamental distribution (locally, outside its singular locus) as an Abelian integral over a certain analytic cycle in the complex projective space $\mathbb{C}P^{n-1}$. This allows us to employ the Petrovsky topological condidtion to study its lacunas. We discuss a number of illustrative examples as well as some relation of the theory of lacunas to the problem of classification of commutative rings of partial differential operators.

J. Boman

Injectivity for generalized X-ray transforms

We consider an X-ray transform

$$R_\rho f(L) = \int_L f(x)\rho_L(x)\,ds, \quad L \in Y,$$

where $f \in C_c(\mathbb{R}^3)$, Y is a 3-dimensional manifold of lines in $\mathbb{R}^3$, and $(x, L) \mapsto \rho_L(x)$ is positive and real analytic. The manifold Y is assumed to be admissible in the sense of Gelfand; for example, Y can be the set of all lines intersecting a real analytic curve γ. It was proved by Todd Quinto and myself (Trans. A.M.S. **335** (1993), 877-890) that if γ is not a plane curve, Ω is an open subset of $\mathbb{R}^3$ not intersecting γ, and Y_0 is an open connected subset of Y containing at least one line not intersecting Ω, then $f \in C_c(\Omega)$ is uniqueley determined by $R_\rho f(L)$ for $L \in Y_0$. A new, simplified proof of this result is presented together with an analogous theorem for 4-dimensional families of lines in $\mathbb{R}^4$.

C. Börgers

Non-uniqueness of optimal radiation treatment plans

We study the nullspace, and more generally the small singular values, of the mapping from radiation intensity distributions to dose distributions in radiation therapy planning. We discuss semi-discrete (finitely many beam directions, all else continuous) and fully discrete model problems with constant attenuation. We also discuss the stability of the nullspace with respect to small perturbations of the dose operator, as introduced for instance by slight spatial variations in the attenuation coefficient or a small amount of scattering.

A.V. Bronnikov

Numerical solution of the identification problem for the attenuated Radon transform

The identification problem for the attenuated Radon transform is to find the attenuation coefficient, which is a parameter of the transform, from the values of the transform alone. Previous attempts to solve this problem used range theorems for the continuous attenuated/exponential Radon transform. We consider a discrete version of transform and make the use of results for the range of finite-dimensional operators. The numerical identification algorithm proposed is based on variable projection minimization using a regularizing Gauss-Newton-type method. The singular value decomposition is applied to compute the orthogonal projector and its derivative. Numerical examples are considered.

Y. Censor

Dykstra's algorithm with Bregman projections for projecting a point onto an intersection of convex sets

We show that Dykstra's algorithm with Bregman projections, which finds the Bregman projection of a point onto the nonempty intersection of finitely many closed convex sets, is actually the nonlinear extension of Bregman's primal-dual, dual coordinate ascent, row-action minimization algorithm. Based on this observation we give an alternative convergence analysis and a new geometric interpretation of Dykstra's algorithm with Bregman projections which complements recent work of Censor and Reich, Bauschke and Lewis, and Tseng.

E. Clarkson

The effects of positivity on regularized and unregularized tomographic reconstruction methods

When the positivity constraint is used with maximum likelihood, or similar unregularized, tomographic reconstruction methods, the result is often an image with many bright spots on a dark background. We find that these 'night-sky' reconstructions always exist for any given data vector, and often unavoidable. When the likelihood maximizing reconstruction is not unique, which is often the case, the data vector nevertheless determines a unique maximal support set for all reconstructions compatible with it. This creates an interplay between smooth reconstructions and local consistency conditions for the system. With maximum *a priori*, or other regularized reconstruction methods, we show how the regularizer, together with the positivity constraint, determines the form of the reconstructed function. To be specific, the reconstructed function is the result of a nonlinear operator, which is determined by the regularizer, applied to a natural pixel function. The number of free parameters is therefore equal to the number of detectors. We find that the statistical properties of the reconstruction may be determined by using this result.

M. Defrise

New developments in cone-beam tomography

We summarize new solutions for the inversion of the 3D X-ray transform with truncated cone-beam (CB) projections. These new solutions are based on a generalization of Grangeat's formula, which allows to calculate the derivative of the 3-D Radon transform for a plan π as a sum of partial contributions provided by the X-ray sources lying in π. Using this result, exact image reconstruction is possible from helical CB data when the data are known in the region B that is bounded in the detector by the CB projections of the upper and lower turns of the helix (Tam, Edholm et al.). An alternative exact algorithm is proposed, which reformulates reconstruction as a 3D filtered-backprojection. We show how the shift-variant filter in this algorithm can be reduced to a 1-D ramp filter in the case of the helix. Furthermore, using an elegant property of the helix (Edholm), we produce modified CB data which vanish on ∂B. This all leads to an efficient algorithm for truncated helical CB data.

A.S. Denisjuk

On reconstruction of a function by spherical means

Let $f(x)$, supp $f \subset \{x_n > 0\} \subset \mathbb{R}^n$ be an unknown function. We consider the problem of reconstruction of f by spherical mean, known for all spheres whith the centers at $\{x_n = 0\}$. This problem appears in seismic tomography, in the synthetic aperture radar image processing, in the inverse problem of determination of velocity from observed back-scattered data.

We will show that this problem is equivalent to Radon transform inverse problem. This equivalence permits to apply to spherical means inverse problem most of results obtained to Radon transform: inversion formulas, Plancherel equations, range conditions. Incomplete data problem is considered as well.

A. Faridani

High-resolution algorithms in local and global parallel-beam tomography

This research is motivated by the high demands on resolution and the use of local tomography in Micro-CT. The problem of efficient data sampling and high-resolution reconstruction is investigated using Shannon sampling theory. A new method for estimating the aliasing error in multidimensional sampling is presented. A characterization of the practically suitable sampling lattices confirms the standard and interlaced lattices as being most advantageous for X-ray CT. Error estimates for the filtered backprojection algorithms are extended to include local tomography. Local reconstructions from real-life data sampled on the interlaced lattice are presented and the relative advantages of standard and interlaced sampling are explained.

D. Finch

Microlocal analysis of cone beam tomography

In joint work with my student Ih-Ren Lav we have obtained simpler proofs of some of the results of Greenleaf and Uhlmann on the restricted x-ray term in the specific setting at the cone beam transform with sources on a curve. In particular we obtain explicit expressions for the principal symbols of both the pseudo-differential and the Fourier integral operator parts of $P_c^t \circ P_c$, where P_c is the restricted line integral transform.

P. Grangeat

Highlights on fully three dimensional image reconstruction in radiology and nuclear medicine

Fully three-dimensional (3D) image reconstruction is an inverse problem that solves for 3D image information from integral measurements. It provides both localization and contrast enhancement between neighboring structures that are lacking in the acquired projection data. In this talk, we will present highlights on fully 3D image reconstruction algorithms used in 3D tomographic imaging techniques such as 3D X-ray Computed Tomography (3D-CT), 3D Totational Radiography (3D-RR) for angiography or radiotherapy, Single Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET). Three-dimensional quantitative reconstruction algorithms that compensate for image degrading effects for accurate 3D imaging will not be discussed.

We will review the general class of reconstruction algorithms associated with transform methods also called filtered backprojection algorithms (FBP). For a specific fully 3D image reconstruction problem, each acquisition line crosses several tomographic slices and dedicated fully three-dimensional image reconstruction algorithms are often required to solve this cross-over effect. The main challenge is to define direct inversion formulae or indirect inversion schemes via intermediate transforms that take into account the new data acqusition geometry. We will give an overview about the state of the art FBP algorithms for the different 3D tomographic techniques using either direct inversion, indirect inversion via line rebinning or indirect inversion via plane rebinning.

Finally, we will present the special case of 3D Rotational Angiography with digital subtraction. A major issue is the restricted number of views that can be acquired during the steady state infusion of the contrast material. Here, the transform methods described previously are not applicable. This results in a highly ill-posed inverse problem in the sense that even if the acquired data were perfect, there are too many unknown image data. As one can see at least partially the structures on the projections, pattern recognition reconstruction algorithms can be derived. An alternative approach is the use of algebraic reconstruction technique (ART). We will present results from a prototype MORPHOMETER device. In this particular example, a priori information related to the sparse structure of objects such as vascular trees can be introducted by entropic or half-quadratic constraints. These can be used to derive, from the Bregman algorithm for constrained optimization, a generalization of the ART algorithm which is more robust for the limited number of projections than the standard ART algorithm.

P. Gritzmann

On the reconstruction of finite lattice sets from their line sums in few directions

A new technique based on high resolution transmission electron microscopy can effectively measure the number of atoms in a crystalline structure lying on each line parallel to a given set of directions. The goal is to reconstruct the cristal from this data. Mathematically this leads to the discrete problem of (approximately) reconstructing a finite set of points in the integer lattics $\mathbb{Z}^3$ from measurements of the number of its points lying on each line parallel to one of a small number of directions specified by nonzero vectors in

$\mathbb{Z}^3$. The talk surveys recent results on this and related problems in discrete tomography with a special emphasis on uniqueness results, on the computational complexity of the problems and on algorithms that have recently be developed and analysed for such tasks.

P. Hähner

An inverse imhomogeneous medium problem in an exterior domain

We consider the inverse problem for the Helmholtz equation

$$\Delta(u^i + u^s) + \kappa^2 n(u^i + u^s) = 0 \text{ in an exterior domain } D_e \subset I\!R^2$$

together with the Dirichlet boundary condition $u^i + u^s = 0$ on ∂D_e and the Sommerfeld radiation condition for u^s, where $\kappa > 0$ denotes the wave number, n is the refractive index, and u^i is an incident wave.

Assuming D_e to be known and that $1 - n$ vanishes in the exterior of a large disk B, we prove that the Cauchy data $(u^i + u^s)_{|\partial B}$, $\frac{\partial}{\partial \nu}(u^i + u^s)$ on ∂B for sufficiently many incoming waves u^i and an interval of wave numbers κ uniquely determine the refractive index n.

To this end we use the limit $\kappa \to 0$ and a conformal mapping to reduce the uniqueness proof to the question whether

$$\text{span } \{u \cdot v : \Delta u = 0,\ \Delta v = 0 \text{ in } A,\ u_{|\Gamma} = 0,\ v_{|\Gamma} = 0\}$$

is dense in $L^2(A)$. Here, A denotes the annular domain $A := \{x \in I\!R^2, 1 < |x| < \rho\}$ and Γ is the unit circle. A positive answer for this latter question can be established with the help of separation of variables.

M. Hanke

Reconstruction of an unknown inclusion using electrical impedance tomography

In electrical impedance tomography currents are applied to a two-dimensional body and the resulting voltages are measured on the boundary. The goal is to use these (overdetermined) boundary data to reconstruct information about the distributed conductivity coefficient σ within the body.

It is known that these boundary data (i.e., the Neumann-to-Dirichlet operator) uniquely define the conductivity coefficient provided that σ is, for example, piecewise analytic. This may correspond to the practical situation that the body consists of a number of inhomogeneities (organs) in a homogeneous background medium. We present a theoretical characterization of the domain of these inclusions (yet, under some restrictions in σ) which is easily translated into a very cheap numerical algorithm for the reconstruction of their domain. Preliminary numerical results will be presented.

This is ongoing work with M. Brühl, A. Kirsch, and M. Pidcock.

I. Kazantsev

Tomographic reconstruction from arbitrary directions using ridge functions: applications and numerical experiments

In this work tomographic reconstruction based on the concept of ridge functions (Logan and Shepp) is considered. We derive a formula (based on some results of Davison and Louis) to calculate the ridge functions from the set of arbitrary projections.

In the case of equally spaced projections formulae for the analytical inversion of matrices encountered in the calculation of ridge functions are obtained.

Some applications of suggested approach are discussed. Results of numerical experiments are presented.

P. Kuchment

A general exponential X-ray transform

A new transform of the X-ray type is introduced. This transform provides a unifying approach to all exponential transforms arising in emission tomograhy (SPECT). It has much more invariance built in than the individual exponential transforms, which makes its study more transparent. In particular, a range description in terms of F. John equation is obtained. This range description is related to the S. Bernstein's separate analyticity theorem. This study also leads to new relations of the exponential transforms with hypergeometric functions.

P. Maaß

Bilinear integral equations and applications to emission tomography

Bilinear equations arise frequently in tomography, e.g. in ultrasound and emission tomography. However so far no convergence analysis - taking into account date errors - exists for any numerical algorithm.

Based on the theory of regularizing non-linear inverse problems with Tikhonov-functionals, we show that the attenuated Radon-transform can be solved with optimal convergence rates.

This result requires some smoothness assumptions on the solution which has to be reflected in the numerical implementation by an additional smoothing step. Results from clinical data show the efficiency of this method.

C. Mennessier

Doppler imaging invertibility

Doppler imaging is an imaging technique in astrophysics to reconstruct the flux on the surface of a rotating star. This inverse problem can be expresflux on the surface of a rotating star. This inverse problem can be expressed as a rotation invariant generalized Radon transform. Thus, using the same approach as the one developed by Quinto, we deduced the existence of some radial functions in the Doppler imaging operator kernel. Then we focused on the sampling geometry in Doppler imaging. Again we extended some well-known results of efficient sampling in classical tomography to the rotation-invariant generalized Radon transform with polynomial weight functions. This result can be applied on two extremely cases in Doppler imaging.

R. Model

Numerical methods in optical tomography

Generally, the recognition of structures in random media such as human tissue by means of optical tomographic methods is of growing interest. Potential applications are given as a tool for medical diagnostics. The resolution is been expected to be lower than for other tomographic methods because of the highly light scattering. Consequently, there is a pressing demand for effective reconstruction methods.

As mathematical model of the description for the light propagation (forward problem) the diffusion approximation of the Boltzmann equation with boundary conditions of the third kind is used, in time domain a parabolic differential and in frequency domain an elliptic equation. The inverse imaging problem then appears to be an identifaction problem with distributed parameters, the absorption and the transport scattering. Some image reconstruction methods are known. In the talk they are outlined and an iterative reconstruction algorithm based on a FEM forward solution and a least squares minimization strategy with regularization is introduced in more detail. Generally, the calculations are time consuming. One possibility of saving the computational effort consists in exploiting a priori information such as the search for single inhomogeneities (absorbers or/and scatterers) within a relative homogeneous object, a typical situation for breast cancer detection. For this case pertubation approaches for the optical parameters and the light propagation may be introduced. The calculations are performed by a 2D FEM algorithm, however, as a time-dependent correction factor is applied, the 3D situation is well approximated in cases of interest.

V. Palamodov

Reconstruction of a function from limited data of arc means

We consider the problem of recovering a function f with compact support in the unit half disc H from the knowledge of integrals of f over halfcircles that are orthogonal to the diameter of H. An explicit method will be given as well as a microlocal analysis of stability of the econstruction.

S.K. Patch

Extrapolation of cone beam CT data

To improve image quality and decrease reconstruction time for volumetric CT reconstructions we explore data extrapolation. CT data collection maps the linear attenuation coefficient of an imaging object to its set of line integrals

$$X : \rho(x_1, x_2, x_3) \mapsto X_\rho(\xi_1, \xi_2; \eta_1, \eta_2).$$

The inverse map is overdetermined. In 1938 F. John showed that

$$\left(\frac{\partial^2}{\partial\xi_1\,\partial\eta_2} - \frac{\partial^2}{\partial\xi_2\,\partial\eta_1}\right) X_\rho(\xi_1, \xi_2; \eta_1\eta_2) = 0.$$

We explore the possibility of solving a boundary problem for John's equation to extrapolate unmeasured line integrals from a minimal set of measured data.

E.T. Quinto

Spheres are fun, and industrial tomography

This talk encompases two of the author's research areas, both of which were motivated by research with Allan Cormack, one of the pioneers in tomography. The author first presents some of his theorems about the spherer transform (including one theorem with Mark Agranovsky). This includes a precise microlocal regularity theorem and a new support theorem for the transform, both of which have theoretical relevance to local SONAR and geophysical testing over both planar and nonplanar source surfaces. The microlocal regularity theorem is related to a neat result on visible singularities presented at this conference by Dr. Palamodov. Strengths and limitations of the author's ideas for reconstructions are discussed.

The second topic involves the author's exterior reconstruction algorithm for nondestructive evaluation. The algorithm is outlined and inverse bounds are given. Reconstructions are provided from Perceptics, Inc., industrial exterior and limited angle exterior data. Limited angle exterior reconstruction is done for a 135° angular range and a 30° angular range. A reconstruction is shown from 1/32 of the original data set. These results were first presented in [Inverse Problems, 1998].

A.G. Ramm

Partial differential operators and local tomography

A necessary and sufficient condition is given for a PDO to be a local tomography operator. Let

$$Bf := F^{-1}[b(x,\xi)\tilde{f}], \quad \tilde{f} := \int e^{i\xi\cdot x} f(x)\, dx\,,$$

$f(x)$ is a compactly supported piecewise-smooth function. Let

$$\hat{f}(\alpha, p) = \int_{l_{\alpha p}} f(x)\, ds, \quad l_{\alpha p} = \{x \in \mathbb{R}^n \ : \ \alpha \cdot x = p\}\,.$$

It is proved that

$$\begin{aligned} Bf &= A\hat{f} := R^*(a_e * \hat{f})\,, \\ \text{where} \quad R^* g(\alpha, p) &= \int_{S^{n-1}} g(\alpha, \alpha - x)\, d\alpha\,, \end{aligned}$$

$$a_e := \frac{a(x,\alpha,p)+a(x,-\alpha,-p)}{2},$$

$$a(x,\alpha,p) := \int_0^\infty \frac{dt\, t^{n-1}}{(2\pi)^n}\, b(x,t,\alpha)\, e^{-itp}, \quad t=|\xi|, \quad \alpha := \frac{\xi}{|\xi|}.$$

$\hat{f} \mapsto Bf$ is called a local tomography operator if

a) sing supp $A\hat{f} =$ sing supp f.

b) supp $a_e \subset [-\rho,\rho]$ where $\rho > 0$ is a small number, ρ does not depend on x and α, and the support is taken with respect to the p-variable. Define $t_+ = \max(t,0)$, $t_- = (-t)_+$.

Theorem: If $b(x,\xi)$ is a hypoelliptic symbol and $t_+^{n-1}b(x,t,\alpha) + t_-^{n-1}b(x,t,\alpha)$ is an entire function of exponential type $\leq \rho$, then A is a local tomography operator.

Remark: Usually $b(x,t,\alpha) = b(x,t,-\alpha)$. In this case

$$t_+^{n-1}b(x,t,\alpha) + t_-^{n-1}b(x,-t,-\alpha) = |t|^{n-1}b(x,t,\alpha).$$

Example: $n=2$, $b=|t|$, $|t|^{n-1}b = |t|^2 = t^2$. This is an entire function of type 0. The corresponding $a_e =$ const $\delta''(p)$ generates the standard local tomography function $A\hat{f} = -\frac{1}{4\pi}\int\limits_{S^1} \hat{f}_{pp}(\alpha,\alpha\cdot x)\,d\alpha$.

A. Rieder

The approximate inverse in computerized tomography: a mathematical foundation

The approximate inverse is a general scheme to obtain stable numerical inversion formulas for linear operator equations of the first kind. Especially, it applies to semi-discrete (under-determined) systems.

Yet, in some concrete applications a crucial ingredient, the so-called reconstruction kernel, can be computed neither numerically nor analytically. To cure this dilemma we propose and analyze a technique which is based on a cure this dilemma we propose and analyze a technique which is based on a singular value decomposition of the underlying (infinite dimensional) operator.

Finally, the abstract results are applied to the reconstruction problem in 2D-computerized tomography.

E.L Ritman

Use of X-Ray micro-CT to develop and/or evaluate a number of novel CT approaches

The primary goal is to provide 3D images of anatomic structure and associated function, throughout intact rat or mouse organs (or comparably sized biopsies from larger animals' organs, at a resulution sufficient for the quanitative description of the characteristics (e.g., size, location, packing, perfusion and drainage) of organs' basic functional units (approximately 0.1 mm^3).

A major issue in this CT imaging approach is the mechanism of contrast of structures of interest. The traditional use of X-ray attenuation as the signal results in limited contrast between various tissue components. To overcome this limitation, high-contrast agents are added to selected anatomic spaces such as the vascular tree. Other workers in this field use micro-CT, especially those who use a synchroton X-ray source, to measure other aspects of X-ray which result in greater contrast. These methods include phase delay of X-ray and fluorescence of selected atomic species. These methods still involve the need to know local X-ray exposure within the object so that the scattered or emitted X-ray from that location can be quantitated appropriately. In the interest of scan speed, concurrent exposure of multiple locations is desirable but does require inverse solutions. With these methods, therefore, a variety of reconstruction algorithms need to be used to generate the tomographic images of interest.

T. Schuster

An efficient solver for 3D vector tomography

The idea of vector tomography is to use the Doppler effect for reconstructing velocity fields of moving fluids. For this we send continuous ultrasound waves along lines to the object which is assumed to be a bounded domain. If the signal runs into an obstacle the frequency of the reflected part of the signal is increased or decreased by the Doppler shift. By measuring the reflected part we get by some signal processing the mean value of the velocity component parallel to the test beam. The geometry is the well known parallel geometry of 2D-CT, but with source and detector at the same position. We describe the obtained data by the Doppler transform of the field and apply the 2D case to the 3D case for getting the 3D Doppler transform. Thus, the vector tomography is mathematically described by an operator equation of first kind with the 3D discrete Doppler transform as operator. To solve this inverse problem we apply a modified form of the approximate inverse. We compute the reconstruction kernels for a special mollifier and reconstruct both, the curl of the velocity field and the solenoidal part of the field itself by only changing the reconstruction kernels. By using translation invariants of the Doppler transform we get a very efficient algorithm for the vector tomography, which can be parallely implemented. Numerical results for a straight flow through a cylinder are presented.

V.A. Sharafutdinov

Inverse problem of determining a connection on a vector bundle

Dealing with the inverse forward scattering problem, G. Uhlmann arrived at the following tomographic problem.

Let γ be a vector bundle over a convex bounded domain $D \subset I\!R^n$, and ∇ be a connection on γ. Let $T_{x,y} : \gamma_x \to \gamma_y$ be the parallel transport in the sense of the connection ∇ along straight line segment with endpoints $x, y \in D$. To what extent is a connection ∇ determined by te parallel transport $T_{x,y}$ that is known for all boundary points $x, y \in \partial D$?

The main result is the following local version of the conjecture.

Theorem: Let γ be an Hermitian vector bundle over a convex bounded domain $D \subset I\!R^n$, and ∇ be a connection on γ compatible with the metric. Assume the curvature form Ω of ∇ to satisfy the inequality

$$\int_0^{|y-x|} t\,\|\Omega(tx+(1-t)y)\|\,dt < \frac{1}{6}\sqrt{n-3/2}$$

for any points $x, y \in D$. There exists a C-neighbourhood U of the connection ∇ such that for every two connections $\nabla', \nabla'' \in U$ the following statement is valid: if the parallel transport $T'_{x,y}$ and $T''_{x,y}$ of the connections ∇' and ∇'' coincide for all boundary points $x, y \in \partial D$, then there exists a gauge transform $A \in \mathrm{Aut}(\gamma)$ such that $A\nabla' = \nabla''$ and $A|_{\partial D} = Id$.

H. Sielschott

New applications of waveform inversion: acoustic pyrometry and breast cancer detection

Acoustic pyrometry is a measurement technique for gas temperature via speed of sound $c(x)$ and gas velocity $v(x)$ in large scale furnaces. Acoustic transducers are fixed at the walls in one horizontal layer. For each measurement one transducer 'fires' and the others receive.

Looking only at the time of flight we get a scalar tomography problem for $1/c(x)$ and a vector tomography problem for $v(x)$. Reconstruction from simulations and measured data are shown.

The whole signal measured by the microphones can be used via waveform inversion which means the solution of an inverse problem for the wave equation. Here $c(x)$ and the source term in the wave equation are to be reconstructed. A propagation-backpropagation algorithm similar to Kaczmarz's method is proposed and numerical results are presented.

Natterer has considered acoustic inverse scattering for breast cancer detection. This is translated into time domain and treated with an algorithm similar to the one for waveform inversion in acoustic pyrometry. Numerical results indicate that tumors can be found via time resolved acoustic measurements when $c(x)$ and the attenuation coefficient are reconstructed.

G. Uhlmann

On the inverse backscattering problem

In this talk we discussed the inverse backscattering problem which consists, roughly speaking, in determining a potential by measuring the reflection produced by the potential on incident plane waves.

It is an open problem in dimensions $n \geq 2$ whether one can uniquely determine the potential from backscattering information. For the case of a potential having jump type singularities across a submanifold in dimension $n \geq 3$. A. Greenleaf and the speaker proved that we can determine from backscattering the location of the singularity and the jump of the potential across the submanifold. We outlined in the talk a proof of this result the Lax-Phillips theory of scattering and microlocalanalysis. We also discussed briefly the inverse backscattering problem for the acoustic equation.

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