MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Komplexe Analysis

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Die diesjährige Tagung Komplexe Analysis fand unter der Leitung von J.-P. Demailly (Grenoble), K. Hulek (Hannover) und T. Peternell (Bayreuth) statt. Insgesamt nahmen 41 Mathematiker aus 8 Ländern an der Tagung teil und es wurden 19 Vorträge gehalten. Einige der Schwerpunkte der Themen waren: Klassifikationstheorie, Mirrorsymmetrie und Calabi-Yau-Varietäten, Gromov-Witten Invarianten, Abbildungen von Mannigfaltigkeiten, Verschwindungs- und Dualitätssätze sowie homogene Räume und Gruppenoperationen. Das Tagungsprogramm ließ viel Zeit für Diskussionen und für die Zusammenarbeit der Tagungsteilnehmer, was auch ausgiebig genutzt wurde.

Dank einer Unterstützung im Rahmen des EU-Programmes TMR (Training and Mobility of Researchers) konnten zusätzlich einige jüngere Mathematiker zu der Tagung eingeladen werden. Dies ist einerseits eine hervorragende Förderung des wissenschaftlichen Nachwuchses und gibt andererseits den etablierten Kollegen die Gelegenheit, besonders begabte junge Mathematiker kennenzulernen.

Abstracts

K. Amerik: Morphisms between smooth projective threefolds

(Partly joint work with A. Van de Ven (Leiden))

Theorem: If X, Y are smooth projective 3-folds such that $b_2(X) = b_2(Y) = 1$ and $f : X \to Y$ is a morphism, then $\deg f$ is bounded in terms of the discrete invariants of X and Y. (Conjecturally this should be true for any dimension when $Y \neq \mathbb{P}^n$ or an elliptic curve.)

The case where Y is of general type is trivial and the case where Y is Fano is proved using the classification of Fano 3-folds. In the case when Y has trivial canonical class, there is the following general result:

If X, Y are smooth projective of dimension n, if $\mathrm{NS}(X) = \mathrm{NS}(Y) \cong \mathbb{Z}$ and if $c_1(Y) = 0$ in $H^2(Y, \mathbb{R})$, then the degree of a morphism $f : X \to Y$ is bounded unless Y is flat, i.e. is a quotient A/G where A is abelian and G is a finite freely acting group.

In dimension 3, we prove that such a quotient $Y = A/G$ cannot have $b_2(Y) = 1$. This result is elementary for dimension 2 and 4, and we hope that, according to the conjecture, it should be true in general.

D. Barlet: Transferring ampleness of the normal bundle to the incidence divisor

We present our recent joint work with Jon Magnusson (Iceland) on this subject. Our motivations for this problem came from

1) M. Kaddar: local construction of the relative fundamental class in Deligne cohomology for an analytic family of n-cycles in a complex manifold Z, which allows to associate a line bundle on $C_n(Z)$ (cycle space of Z) to any codimension $(n+1)$-cycle in Z,

2) the problem to try to pass from positivity of the normal bundle in high codimension to a codimension 1 analogue.

I) Incidence divisor: We explain how to build up a functorial Cartier divisor structure on the incidence set in the following situation: $Y \subset Z$ is a local complete intersection of codimension $n+1$ in the manifold Z, $(X_s)_{s\in S}$ is an analytic family of n-cycles in Z such that $p^{-1}(Y) \xrightarrow{\pi} S$ is finite and with an empty interior image (here $\tilde{Z} \subset S \times Z$ is the graph of the family and $p : \tilde{Z} \to Z$ and $\pi : \tilde{Z} \to S$ are the projections).

II) Integration of meromorphic cohomology classes. The main result is to build up a functorial map $\int : H^{n+1}_{[Y]}(\Omega^n_Z) \to H^1_{[\Sigma_Y]}(O_S)$ in the previous setting, which is a filtered map (here $H^{\bullet}_{[A]}$ denotes the algebraic part of the local cohomology, the filtration is given by the image on the $\mathrm{Ext}^{\bullet}_{O_Z}(O_Z/I_A^{\nu}, -)$ where I_A is the defining ideal of A and where $\nu \in \mathbb{N}^*$). We denote by Σ_Y the (Cartier) incidence divisor constructed in I.

III) Transfer of ampleness. We explain that for S and Y compact and $N_{Y/Z}$ ample, we are able to build up a lot of sections of $N_{\Sigma_Y|S}$, so with suitable hypotheses on the way the cycles $(X_s)_{s\in S}$ meet Y, we get that the Kodaira dimension of $[\Sigma_Y]$ (as a line bundle in S) is maximal.

IV) Transfer of metric. We explain how in our previous situation (but now completely local again) it is possible to define, in a natural way, a singular continuous metric on $N^*_{\Sigma_Y|S}$ from a C^2 metric on $N^*_{Y/Z}$ and to get a rather precise control about the curvature.

A. Beauville: Complex manifolds with split tangent bundle

Let X be a compact Kähler manifold. We expect that any direct sum decomposition $T_X = \oplus_{i\in I}E_i$ of its tangent bundle comes from a splitting of the universal covering space of X as a product $\prod_{i\in I}U_i$, in such a way that the given decomposition $T_X = \oplus E_i$ lifts to the canonical decomposition $T_{\prod U_i} = \oplus T_{U_i}$. We prove this assertion when X is a Kähler-Einstein manifold or a Kähler surface. Simple examples show that the Kähler hypothesis is necessary.

M. de Cataldo: The Douady space of a complex analytic smooth surface

(Joint work with L. Migliorini (Florence))

Let X be a quasi-projective nonsingular complex surface. Göttsche computed a generating function for the Betti numbers of the Hilbert scheme $X^{[n]}$ of n-points on X. Nakajima used this result to endow the vector space $\bigoplus_{n\geq 0} H^*(X^{[n]}, \mathbb{Q})$ with a structure of irreducible highest weight representation of a certain Heisenberg / Clifford algebra associated with X.

Göttsche proved his formula using the Weil conjectures. Later, Göttsche-Sorgel proved the same fomula using the Decomposition Theorem of Beilinson-Bernstein-Deligne(-Gabber) and Saito.

We prove such a decomposition theorem for every complex analytic smooth surface (i.e. not necessarily quasi-projective, e.g. non-Kähler) directly, i.e. using the geometry of the symmetric products of X and avoiding the methods of perverse sheaves.

In this more general context, the Hilbert schemes are replaced by the Douady spaces. Moreover, choosing as incarnation for the constant complex on the Douady spaces the complex of currents, the Decomposition Theorem is proved, not as a statement in the derived category, but directly via a concrete morphism of complexes.

As consequences, we can prove the analogue of Göttsche's formulas and of Nakajima's theorem for X an arbitrary complex analytic smooth surface.

In addition, it has been noted that the dimensions of the topological K-theory of $X^{[n]}$ ($\mathbb{Q}$-coefficients) coincide with the corresponding equivariant K-theory $K_{S_n}(X^n)$. Bezrukavnikov-Ginzburg prove that this is realized by a natural isomorphism. We propose a similar natural isomorphism by a different method which stems from the Decomposition Theorem.

F. Catanese: Singular bidouble covers and the construction of interesting algebraic surfaces

I describe work in progress motivated by the following problem raised by Enriques:

Problem: Construct surfaces with $p_g = 4$, birational canonical map ϕ_1, and K^2 high.

Franchetta constructed such surfaces with $K^2 = 6, \dots, 10$ ($K^2 = 5$ being obvious). Burniat obtained the values $K^2 = 11, \dots, 16$. Enriques had conjectured that $K^2 = 24$ should be the maximum possible.

Theorem: There exists such a surface with $p_g = 4$, ϕ_1 birational, $K^2 = 28$.

The result is based on a systematic study of bidouble covers, that is, finite flat Galois covers with group $(\mathbb{Z}/2)^2$. A basic model is as follows: consider the equations

$$\operatorname{rk}\begin{pmatrix} x_1 & w_1 & w_2 \\ w_1 & x_2 & w_3 \\ w_2 & w_3 & x_3 \end{pmatrix} = 1 \qquad (*)$$

describing the cone over the Veronese surface V. $V \cong \mathbb{P}^2$, and if we take coordinates (y_i) in $\mathbb{P}^2$ such that $x_i = y_i^2$, $w_k = y_i y_j$, then the projection $\pi : V \to \mathbb{P}^2$ given by the diagonal entries is the standard model of a bidouble cover $\mathbb{P}^2 \to \mathbb{P}^2$, $(y_i) \mapsto (x_i = y_i^2)$. There is indeed a structure theorem for bidouble covers, which, in case where the base is smooth, states:

Structure Theorem (Catanese, Hahn-Miranda): Let $Y \xrightarrow{\pi} X$ be a finite flat $(\mathbb{Z}/2)^2$ cover, with char $k \neq 2$. Then there exist line bundles L_i on X, with coordinate w_i, and divisors D_j on X, with equation (x_j) such that Y is defined, in the total space of the vector bundle $L_1 \oplus L_2 \oplus L_3$, by equations $(*)$.

A basic formula is that, since Y is C-M, $\pi_* \omega_Y = \omega_X \oplus \bigoplus_i \omega_X(L_i)$.

One observes that Y is smooth if the divisors D_j are smooth and transversal.

But one can relax on this assumption, and then take a minimal resolution S of Y. One studies, in the case of surfaces, how a triple (μ_1, μ_2, μ_3) of multiplicities at a point p (of the respective divisors D_i) affect K^2 and χ. One has a small table

(1,1,1)	$K^2 \downarrow 1$	$\chi =$ same,
(3,1,0)	$K^2 \downarrow 1$	$\chi \downarrow 1$,
(2,2,0)	$K^2 \downarrow 4$	$\chi \downarrow 1$,
(2,1,0)	$K^2 \downarrow 1$	χ equal.

Therefore, if we want to wive well in surface geography, the first two singularities are very good. For instance, P. Burniat used m (1,1,1) points and 3 (3,1) points to construct surfaces with $p_g = 0$ and $K^2 = 6 - m$ ($0 \le m \le 4$). He chose for D_j 3 lines through a point in $\mathbb{P}^2$.

A. Gathmann: *Gromov-Witten- and degeneration invariants*

The goal of degeneration invariants is to determine numbers of curves in a smooth complex projective variety X satisfying incidence conditions with given generic subvarieties of X, and in addition satisfying certain multiplicity conditions to a fixed hypersurface $Y \subset X$. We will restrict our attention to curves of genus zero.

To state the idea of the degeneration techniques, let $Y \subset X$ be given and consider the example that one wants to count rational curves in X (with given homology class) intersecting generic points $P_1, \dots, P_n \in X$. One then moves one P_i after the other away from its generic position such that it now lies in Y. In each such "degeneration step", some of the curves satisfying the incidence conditions will become reducible, splitting off an irreducible component contained in Y. The other components will in general have higher multiplicities to Y at the point where they meet the component contained in Y. The number of these reducible curves can then be calculated recursively by the same methods, similarly to the Gromov-Witten case.

R. Vakil has developed these techniques for the case where $X = \mathbb{P}^r$ and Y is a hyperplane. If one wants to generalize these methods to the case of arbitrary X and Y, one needs a theory of virtual fundamental classes for the moduli spaces under consideration. This has been established so far in the case where $X = \mathbb{P}^r$ and Y is an arbitrary (possibly non-convex) smooth hypersurface. In this case, one can use the Veronese embedding of degree $\deg Y$, such that Y becomes a hyperplane section in X. This embedding can be used to define virtual fundamental classes on the moduli spaces considered, and to carry over the results of Vakil to the hypersurface case. As a result, one gets relations between the Gromov-Witten invariants of X and Y and the degeneration invariants of $Y \subset X$.

M. Gross: Mirror symmetry and special Lagrangian fibrations

We gave a refined version of the Strominger-Yau-Zaslow mirror symmetry conjecture. Roughly put, this says that mirror pairs of Calabi-Yau manifolds X and $\check{X}$ possess special Lagrangian T^n-fibrations $f : X \to B$ and $\check{f} : \check{X} \to B$ which are "dual".

The refined version of this conjecture gives a hint as to how to construct symplectic and complex structures on the mirror. The complex structure on X is determined by a holomorphic n-form Ω, and the symplectic structure by the Kähler form ω of a Ricci-flat metric. The form ω along with a B-field $\underline{B} \in H^1(B, R^1 f_* \mathbb{R}/\mathbb{Z})$ should determine the complex structure on $\check{X}$, while Ω should determine the symplectic form $\check{\omega}$ on $\check{X}$. We gave a detailed explanation of how the latter is accomplished, along with some suggestion of how the former might be accomplished.

S. Helmke: Global generation of adjoint linear systems

In 1985 T. Fujita raised the following

Conjecture A: Let X be a smooth projective variety over $\mathbb{C}$ and H an ample divisor on X. Then $|K_X + mH|$ is base point free if $m > n = \dim X$ or if $m = n$ and $H^n > 1$.

The case $X = \mathbb{P}^n$ with a hyperplane section H shows that the bound on m is optimal. Later this was generalized to the

Conjecture B: Let (X, Δ) be KLT, projective and L a nef divisor on X such that $L^n > n^n$ and $K_X + \Delta + L$ is Cartier. If $x \in \mathrm{Bs}(K_X + \Delta + L)$, then there is a subvariety $Z \ni x$ such that $L^d Z < n^d$, where $d = \dim Z$.

Clearly conjecture B implies conjecture A. In the talk I explained some technique which leads to a proof of conjecture B at least if X is smooth of dimension ≤ 5.

Y. Kawamata: Deformations of canonical singularities and invariance of plurigenera

I reported some recent progress on the extension problem of pluricanonical forms from a divisor to the ambient space. The main theorems are as follows:

Theorem 1 (Siu, K., Nakayama): Let $f : X \to S$ be a proper flat morphism of algebraic varieties. Assume that the fibers $f^{-1}(s) = X_s$ have only canonical singularities and that the generic fiber X_η satisfies the abundance condition: $\kappa(X_\eta) = \nu(X_\eta)$. Then the plurigenera $P_m(X_s)$ are constant on s for all $m \geq 0$.

Theorem 2 (K., Nakayama): Let $f : X \to S$ be a flat deformation of singularities. Assume that a fiber X_s has only canonical (resp. terminal) singularities. Then so have all the nearby fibers.

The proof uses a tricky induction on multiplier ideals developed by Siu.

S. Kebekus: Quasihomogeneous projective threefolds

Let X be a smooth projective threefold and G a connected algebraic group acting algebraically on X. In this context all steps of the minimal model program are equivariant. If one assumes additionally that G acts almost transitively, i.e. that the G-action has an open orbit, then the minimal model program always leads to a contraction of fiber type over a base Y, i.e. to a Mori fiber space. In this talk we classify these varieties in the case where G is linear algebraic and not solvable.

The assumption "linear algebraic" is made to rule out varieties which are bundles over their Albanese torus.

The following are the main examples which occur in the classification. Here a "linear bundle" is a variety of the form $\mathbb{P}(E)$, where E is a vector bundle. Call $\mathbb{P}(E)$ "splitting" if E is.

- The special Fano varieties V_5 and V_{22}^S.
- The weighted projective spaces $\mathbb{P}_{(1,1,2,3)}$ and $\mathbb{P}_{(1,1,1,2)}$.
- Varieties over $Y \cong \mathbb{P}_1$ which are locally isomorphic to a deformation of a quadric surface, and certain quotients of these varieties.
- Singular varieties arising as very special quotients of splitting linear $\mathbb{P}_1$-bundles over $Y \cong \mathbb{P}_2$.
- Linear $\mathbb{P}_1$-bundles over Hirzebruch surfaces Y which are constructed from a trivial $\mathbb{P}_1$-bundle by repeatedly performing certain well-described elementary transformations.

C. Laurent: Some new separation theorems for the Dolbeault cohomology

(Joint work with Jürgen Leiterer, Berlin)

We use Serre duality for cohomology with support to prove the following theorem:

Theorem: Let X be an n-dimensional complex manifold which is q-concave-q^*-convex, $1 \leq q \leq n-1$ and $1 \leq q^* \leq n$, and E a holomorphic vector bundle over X. We denote by Φ the family of closed subsets of X which are compact at the concave end of X, then the dual family Φ^* of Φ consists of all closed subsets of X which are compact at the convex end of X. The following three statements hold:

(i) If $\max(q+1, q^*) \leq r \leq n$, then $\dim H^{0,r}_{\Phi}(X,E) < +\infty$.

(ii) If $0 \leq r \leq \min(n-q, n-q^*+1)$, then $H^{0,r}_{\Phi^*}(X,E^*)$ is separated.

(iii) If $0 \leq r \leq \min(n-q-1, n-q^*)$, then $\dim H^{0,r}_{\Phi^*}(X,E^*) < +\infty$.

From this theorem we deduce the following results:

Corollary 1: Let X be an n-dimensional complex manifold, $n \geq 3$, which is q-concave-$(n-q)$-convex, $1 \leq q \leq n-1$. If $q < \frac{n}{2}$, then for any vector bundle E over X, $H^{0,n-q}(X,E)$ is separated.

Corollary 2: Let Y be a compact complex space of dimension n, whose singular set S consists of a finite number of points. Set $X = Y \backslash S$, let some subset S_0 of S be fixed and denote by Φ the family of all closed subsets C of X such that $Y \backslash C$ is a neighborhood of S_0. Then for any holomorphic vector bundle E over X, $H^{0,n-1}_{\Phi}(X,E)$ is separated.

R. Lazarsfeld: Vanishing theorems and the effective Nullstellensatz

In recent years, there has been a great deal of interest in the effective Nullstellensatz for homogeneous ideals in $\mathbb{C}[T_0, \ldots, T_n]$. In joint work with L. Ein, we prove a statement which strengthens in several respects and clarifies the geometric underpinnings of results of Brownawell and Kollár and others. The result is the following:

Theorem: Let X be a smooth complex projective variety of dimension n, let L be an ample line bundle on X, and let $D_1, \ldots, D_m \in |L|$ be divisors in the corresponding linear series, where for simplicity we suppose that the D_j have no common components. Set $B = D_1 \cap \cdots \cap D_m$ (scheme-theoretic intersection), $Z = B_{red}$, and $J = \sum \mathcal{O}_X(-D_i) \subset \mathcal{O}_X$ the ideal sheaf of B. Then there exists a canonical decomposition $Z = \bigcup_{i=1}^{t} Z_i$ of Z as a union of t irreducible and reduced subvarieties, plus integers $r_i > 0$, such that one has:

(i) $\sum r_i \deg_L(Z_i) \leq \deg_L(X) = \int_X c_1(L)^n$,

(ii) $J_{Z_1}^{[r_1(n+1)]} \cap \cdots \cap J_{Z_t}^{[r_t(n+1)]} \subset J$, where $J_{Z_i}^{[r]} = r$-th symbolic power of J_{Z_i} (so $J_{Z_i}^{[r]}$ is the sheaf of germs of functions vanishing to order $\geq r$ at the general point of Z_i).

(iii) if A is a line bundle on X such that $A \otimes L^{-(n+1)}$ is ample, and if $s \in \Gamma(X, \mathcal{O}_X(K_X + A))$ is a section vanishing to order $\geq r_i(n+1)$ on Z_i, then we can write $s = \sum s_j h_j$, where $s_j \in \Gamma(X, \mathcal{O}_X(D_j))$ is the section defining D_j, and $h_j \in \Gamma(X, \mathcal{O}_X(K_X + A - L))$.

Previous work corresponds to the case $X = \mathbb{P}^n$.

M. Mella: Applications of CLC minimal centers to projective geometry

A morphism $f : Y \longrightarrow X$ with connected fibers between normal varieties is called Fano-Mori if $-K_Y$ is f-ample. To such a morphism is naturally associated a line bundle L and a rational number r, and the holomorphic sections of L help to understand the geometry of $f : Y \longrightarrow X$.

Using the Kawamata Base Point Free technique and the notion and properties of CLC minimal centers we answer some problems concerning sections of L for special values of r.

In particular we prove the Mukai conjecture, concluding in such a way the Mukai classification, and the Andreatta-Wisniewśki conjecture about relative base point freeness of L when $\dim F(f) \leq r+1$.

Y. Miyaoka: On the canonical degree of curves on a surface of general type

Let X be a smooth projective (or compact Kähler) manifold over $\mathbb{C}$ and $C \subset X$ an irreducible, possibly singular, curve of geometric genus g. Then we can show

Theorem: If K_X is nef, then the "canonical degree" KC is bounded from above by a function in $g(C)$, $c_1^2(X)$ and $c_2(X)$.

In particular, if X is a surface of general type, we have

Corollary: The number of rational / elliptic curves on a surface of general type is bounded by a function depending only on the diffeomorphism type of the surface.

The proof of the theorem is obtained by closely looking at the symmetric tensors of Ω_X^1 and their restriction to the curve C, which are highly unstable whenever C has large canonical degree compared to the genus.

W. Oxbury: SO(8)-bundles and trigonal curves

Let (C, g_3^1) be a trigonal curve and $G(C) = \{(p,q) \in \mathbb{C} \times \mathbb{C} \mid g_3^1 - p - q \text{ effective}\}$ the associated Galois S_3-curve over $\mathbb{P}^1$. A Galois Spin_8-bundle (on C) means a Spin_8-bundle $F \longrightarrow G(C)$ such that for all $g \in S_3$ one has $g^*F = F^g$, where F^g denotes the triality action on the structure group.

The talk described recent work with S. Ramanan (TIFR, Bombay) in which we construct a moduli space M_C of such (semistable) bundles with properties:

(i) $\mathrm{SU}_C(2) \hookrightarrow M_C$ as semistable boundary ($\mathrm{SU}_C(2)$ = moduli space of semistable rank 2 vector bundles $E \longrightarrow C$ with $\det E = 0$),

(ii) M_C is smooth of dimension $7g(C) - 14$ away from $\mathrm{SU}_C(2)$.

(iii) for all $\eta \in J_C[2]\backslash\{0\}$ non-trivial 2-torsion point there exists a commutative diagram

$$\begin{tikzcd} \mathrm{Prym}(C,\eta) \arrow[r, "\simeq"', "\text{Recillas}"'] \arrow[d, dash, "\text{direct image}"'] & \mathrm{Jac}(R) \arrow[d, dash] \\ & \mathrm{SU}_R(2) \arrow[d, dash] \\ \mathrm{SU}_C(2) \arrow[r, hook] & M_C. \end{tikzcd}$$

I explained the motivation for these constructions from the case $g(C) = 4$, where we expect $M_C \xrightarrow{\sim}$ unique Heisenberg-invariant quartic in $|2\Theta| \cong \mathbb{P}^{15}$ singular along the image of $\mathrm{SU}_C(2)$.

S. Usui: Partial compactification of arithmetic quotients of classifying spaces of Hodge structures

(Joint work with Kazuya Kato)

This work is an attempt to add "points at infinity" to the classifying space of polarized Hodge structures (PH for short) of arbitrary weight. It bases on the following two ideas:

(1) We introduce the notion of "polarized log Hodge structures" (PLH for short). PLH is defined by using the theory of logarithmic structures of Fontaine-Illusie and it works well in the analysis of degenerations of PH. Our principle is that we can enlarge the classifying space of PH as $D :=$ (classifying space of PH) $\subset$ (classifying space of PLH) $=$ (space of nilpotent orbits).

(2) We introduce the notions of "generalized analytic spaces" and "generalized fs log analytic spaces". In fact, our extended space $\Gamma\backslash D_\Sigma \supset \Gamma\backslash D$ is not in general an analytic space. Sometimes, it is like the space $S := \{(x,y) \in \mathbb{C}^2 \mid x \neq 0\} \cup \{(0,0)\}$. The space $\Gamma\backslash D_\Sigma$ and the above space S are "generalized analytic spaces" or even "generalized fs log analytic spaces". A crucial point is that, to endow a structure of "generalized analytic space" on such above set S, we endow it with the so-called "strong topology" which is strictly stronger than the topology as a subspace of $\mathbb{C}^2$.

Our main result says that the "generalized fs log analytic space" $\Gamma\backslash D_\Sigma$ is a moduli space of polarized log Hodge structures. This realizes one of the dreams of Griffiths in Bull. AMS 76 (1970).

This work is a continuation of the study (Usui, Tôhoku J. Math. 47 (1995)) on a part of $\Gamma\backslash D_\Sigma$, and is also a continuation of the study (K. Kato and C. Nakayama, Prepubl. Univ. Tokyo UTMS95-16) on logarithmic complex geometry. We use Schmid's theory in the proofs.

Remark: In the case D being a Hermitian symmetric domain, our $\Gamma\backslash D_\Sigma$ coincides with a toroidal compactification of Mumford et al. So ours are generalizations of theirs.

C. Voisin: The Griffiths group of a general Calabi-Yau threefold is not finitely generated

If X is a smooth projective complex variety, $\dim X = n$, one defines the intermediate Jacobians of X as follows: $J^{2k-1}(X) = H^{2k-1}(X,\mathbb{C})/(F^kH^{2k-1}(X) \oplus H^{2k-1}(X,\mathbb{Z}))$, where $F^kH^{2k-1}(X) = \bigoplus_{p\geq k} H^{p,2k-1-p}(X)$.

The Abel-Jacobi map Φ_X^k is defined on the group $B_{hom}^k(X)$ of cycles homologous to zero, with values in $J^{2k-1}(X) \simeq F^{n-k+1}H^{2n-2k+1}(X)/H_{2n-2k+1}(X,\mathbb{Z})$. It associates to $Z = \partial\Gamma$ the element $\int_\Gamma \in F^{n-k+1}H^{2n-2k+1}(X)$, which is well-defined up to the periods.

One knows that if the complex torus $J^{2k-1}(X)$ does not contain any non-trivial complex subtorus with tangent space contained in $H^{k-1,k}(X) \subset TJ^{2k-1}(X)$, the Abel-Jacobi map factors through the countable group $\mathrm{Griff}^k(X) = B_{hom}^k(X)/B_{alg}^k(X)$. This is the case if X is a general Calabi-Yau threefold and $k = 2$. We prove the following, which generalizes theorems of Clemens and Griffiths.

Theorem: Let X be a non-rigid Calabi-Yau threefold. Then $(\mathrm{Im}\,\Phi_{X_t}) \otimes \mathbb{Q}$ is infinite-dimensional over $\mathbb{Q}$ for a general deformation X_t of X. In particular, $\mathrm{Griff}(X_t) \otimes \mathbb{Q}$ is infinite-dimensional over $\mathbb{Q}$.

J. Winkelmann: Compact manifolds with large automorphism groups

For a real Riemannian differentiable manifold M of (real) dimension n the dimension of the group of isometries is bounded by $\frac{n(n+1)}{2}$. If this bound is reached, then M must be isomorphic to one of the following: H^n, $\mathbb{R}^n$, $\mathbb{P}_n(\mathbb{R})$, S^n. By the theorem of Bochner and Montgomery for every compact complex manifold X the automorphism group $\mathrm{Aut}(X)$ has the structure of a finite dimensional complex Lie group. In view of the above mentioned fact on Riemannian manifolds it is natural to ask whether there exists a bound on the dimension of the automorphism group which depends only on the dimension of X. For arbitrary compact complex manifolds this is wrong: Easy counterexamples are provided by the Hirzebruch surfaces. On the other hand, results of Borel and Remmert imply that $\dim \mathrm{Aut}(X) \leq n^2 + 2n$ for every Kähler compact homogeneous complex manifold X and that this bound is achieved only for $X \simeq \mathbb{P}_n(\mathbb{C})$. Remmert raised the question whether the Kähler assumption in this statement is really necessary. Akhiezer proved that there is some function $f : \mathbb{N} \to \mathbb{N}$ such that $\dim \mathrm{Aut}(X) \leq f(\dim X)$ for every compact complex homogeneous manifold X. In an article which is accepted for publication in the Inventiones mathematicae, we (D. Snow and J. Winkelmann) show that there is a sequence of compact complex homogeneous manifolds X_k with $\dim(X_k) = 3k+1$ and $\dim \mathrm{Aut}(X_k) \geq 3k + 3^k$. Thus the bound $f(n)$ for the dimension of $\mathrm{Aut}(X)$ cannot be better than exponential in $\dim X$.

J. Wolf: Complex flag manifolds and double fibration transforms

Let G be a complex semisimple Lie group, Q a parabolic subgroup, $Z = G/Q$ the corresponding complex flag manifold, and G_0 a real form of G. There are only finitely many G_0-orbits on Z. Certain of these orbits give a geometric construction of, and lead to analytic information on, the tempered representations of G_0. Those are the irreducible unitary representations that occur in the Plancherel formula for G_0, in other words that are involved in the left regular representation of G_0 on $L^2(G_0)$. However there are many other irreducible unitary represen-

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tations. Those others occur in interesting situations but are not yet well understood. Here I indicate an approach that may be useful.

Fix an open orbit $D = G_0(z)$ in Z. If K_0 is an appropriate maximal compact subgroup of G_0 then $Y = K_0(z)$ is a complex submanifold of Z. That done, the "linear cycle space" $M_D =$ component of Y in $\{gY \mid g \in G$ and gY contained in $D\}$ is a Stein manifold. Also, D is $(s+1)$-complete where Y has complex dimension s. So one expects cohomology only in degree s for negative vector bundles $E \to D$. Now consider the double fibration $\nu : W_D = \{(z', Y') \in D \times M_D \mid z' \in Y'\} \to M_D$, $\mu : W_D \to D$, and a sufficiently negative homogeneous holomorphic vector bundle $E \to D$. For various reasons one expects to capture certain representations of G_0 in the form of Fréchet space representations as its action on $H^s(D; \mathcal{O}(E))$. But estimates on the subquotient space $H^s(D; \mathcal{O}(E))$ are not so easy, so one tries to carry that analytic problem over to a problem of estimating functions on the Stein manaifold M_D. For this one wants

(a) a natural injection $j : H^s(D; \mathcal{O}(E)) \to H^s(W_D; \mathcal{O}(\mu^* E))$ by pullback and tensor over the structure sheaf of W_D,

(b) a natural injection $H^s(W_D; \mathcal{O}(\mu^* E)) \to H^0(M_D; R^s(\mathcal{O}(\mu^* E)))$ by the Leray spectral sequence of the proper holomorphic map ν, and

(c) explicit description of the image of the composite map $P : H^s(D; \mathcal{O}(E)) \to H^0(M_D; R^s(\mathcal{O}(\mu^* E)))$.

That composite map is the double fibration transform. The most famous case is the Penrose transform, where $G_0 = SU(2,2)$. In any case the description should be in terms of a system of PDE from which one can make good a priori estimates.

Here I describe what I know about these double fibration transforms in the case where G_0/K_0 is a bounded symmetric domain. This is joint work with Roger Zierau (Oklahoma).

E-mail addresses of the participants

L. Alessandrini	lucale@mate.polimi.it
K. Amerik	amerik@mozart.ujf-grenoble.fr
M. Andreatta	andreatt@science.unitn.it
D. Barlet	barlet@iecn.u-nancy.fr
I. Bauer	ibauer@cfgauss.uni-math.gwdg.de
T. Bauer	bauerth@mi.uni-erlangen.de
A. Beauville	beauville@dmi.ens.fr
F. Campana	campana@iecn.u-nancy.fr
M. de Cataldo	mde@math.wustl.edu
F. Catanese	catanese@uni-math.gwdg.de
O. Debarre	debarre@math.u-strasbg.fr
J.-P. Demailly	demailly@ujf-grenoble.fr
W. Ebeling	ebeling@math.uni-hannover.de
P. Eyssidieux	eyssi@picard.ups-tlse.fr
A. Gathmann	gathmann@math.uni-hannover.de
M. Gross	mgross@maths.warwick.ac.uk
P. Heinzner	heinzner@cplx.ruhr-uni-bochum.de
S. Helmke	helmke@kurims.kyoto-u.ac.jp
A. Huckleberry	ahuck@cplx.ruhr-uni-bochum.de
K. Hulek	hulek@math.uni-hannover.de
Y. Kawamata	kawamata@ms.u-tokyo.ac.jp
S. Kebekus	stefan.kebekus@uni-bayreuth.de
B. Kreussler	kreusler@mathematik.uni-kl.de
J. Landsberg	jml@picard.ups-tlse.fr
C. Laurent	christine.laurent@ujf-grenoble.fr
R. Lazarsfeld	rlaz@math.lsa.umich.edu
M. Lehn	lehn@uni-math.gwdg.de
M. Mella	mm@dpmms.cam.ac.uk
Y. Miyaoka	miyaoka@kurims.kyoto-u.ac.jp
W. Oxbury	w.m.oxbury@durham.ac.uk
M. Paun	paun@ujf-grenoble.fr
T. Peternell	thomas.peternell@uni-bayreuth.de
C. Peters	chris.peters@ujf-grenoble.fr
G. Sankaran	gks@maths.bath.ac.uk
G. Schumacher	schumac@mathematik.uni-marburg.de
B. Siebert	bernd.siebert@ruhr-uni-bochum.de
S. Usui	usui@math.wani.osaka-u.ac.jp
C. Voisin	voisin@math.jussieu.fr
P. Wilson	pmhw@dpmms.cam.ac.uk
J. Winkelmann	jwinkel@member.ams.org
J. Wolf	jawolf@math.berkeley.edu

Tagungsteilnehmer

Prof.Dr. Lucia Alessandrini
Dipartimento di Matematica
Politecnico di Milano
Via Bonardi 9

I-20133 Milano

Dr. Thomas Bauer
Mathematisches Institut
Universität Bonn
Beringstr. 1

53115 Bonn

Dr. Katja Amerik
Institut Fourier
UMR 5582 du CNRS
BP 74
100 rue de Maths

F-38402 Saint-Martin d'Heres

Prof.Dr. Arnaud Beauville
Departement de Mathematiques et d'Informatique
Ecole Normale Superieure
45, rue d'Ulm

F-75005 Paris Cedex

Prof.Dr. Marco Andreatta
Dipartimento di Matematica
Universita di Trento
Via Sommarive 14

I-38050 Povo (Trento)

Prof.Dr. Frederic Campana
Dept. de Mathematiques
Universite de Nancy I
B.P. 239

F-54506 Vandoeuvre-les-Nancy Cedex

Prof.Dr. Daniel Barlet
Departement de Mathematiques
Universite de Nancy I
Boite Postale 239

F-54506 Vandoeuvre les Nancy Cedex

Mark de Cataldo
Max-Planck-Institut für Mathematik
Vivatsgasse 7

53111 Bonn

Dr. Ingrid Bauer
Fachbereich Mathematik
Universität Göttingen
Bunsenstr. 3/5

37073 Göttingen

Prof.Dr. Fabrizio Catanese
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

37073 Göttingen

Prof.Dr. Olivier Debarre
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. Mark Gross
Dept. of Mathematics
Cornell University
White Hall

Ithaca , NY 14853-7901
USA

Prof.Dr. Jean-Pierre Demailly
Institut Fourier
UMR 5582 du CNRS
BP 74
100 rue de Maths

F-38402 Saint-Martin d'Heres

Dr. Peter Heinzner
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Wolfgang Ebeling
Institut für Mathematik
Universität Hannover
Postfach 6009

30060 Hannover

Stefan Helmke
Research Institute for
Mathematical Sciences
Kyoto University, Kitashirakawa
Sakyo-Ku

Kyoto 606-01
JAPAN

Dr. Philippe Eyssidieux
Laboratoire Emile Picard
CNRS UMR 5580
UFR MIG Universite Paul Sabatier
118 route de Narbonne

F-31000 Toulouse Cedex 4

Prof.Dr.Dr.h.c. Alan T. Huckleberry
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Andreas Gathmann
Institut für Mathematik
Universität Hannover
Welfengarten 1

30167 Hannover

Prof.Dr. Klaus Hulek
Institut für Mathematik (C)
Universität Hannover
Postfach 6009

30060 Hannover

Prof.Dr. Yujiro Kawamata
Dept. of Mathematics
University of Tokyo
Komaba, Meguro

Tokyo 153-8914
JAPAN

Prof.Dr. Robert Lazarsfeld
Dept. of Mathematics
University of California
405 Hilgard Avenue

Los Angeles , CA 90095-1555
USA

Dr. Stefan Kebekus
Mathematisches Institut
Universität Bayreuth

95440 Bayreuth

Dr. Manfred Lehn
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

37073 Göttingen

Dr. Bernd Kreussler
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße

67663 Kaiserslautern

Prof.Dr. Massimiliano Mella
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB-Cambridge , CB2 1SB

Prof.Dr. Joseph M. Landsberg
Universite Paul Sabatier
U.F.R. M.I.G.
118, route de Narbonne

F-31062 Toulouse Cedex

Prof.Dr. Yoichi Miyaoka
Dept. of Mathematics
Faculty of Science, RIMS
Kyoto University
Kitashirakawa, Sakyo-ku

Kyoto 606-8224
JAPAN

Prof.Dr. Christine Laurent
Laboratoire de Mathematiques
Universite de Grenoble I
Institut Fourier
B.P. 74

F-38402 Saint-Martin-d'Heres Cedex

Dr. William M. Oxbury
Dept. of Mathematical Sciences
The University of Durham
Science Laboratories
South Road

GB-Durham , DH1 3LE

Dr. Mihai Paun
Institut de Recherche
Mathematique Avancee
ULP et CNRS
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Dr. Bernd Siebert
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA

44780 Bochum

Prof.Dr. Thomas Peternell
Fakultät für Mathematik und Physik
Universität Bayreuth

95440 Bayreuth

Prof.Dr. Sampei Usui
Dept. of Mathematics
Osaka University
Toyonaka

Osaka 560
JAPAN

Prof.Dr. Chris A.M. Peters
Laboratoire de Mathematiques
Universite de Grenoble I
Institut Fourier
B.P. 74

F-38402 Saint-Martin-d'Heres Cedex

Prof.Dr. Antonius Van de Ven
Mathematisch Instituut
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Gregory Sankaran
School of Mathematical Sciences
University of Bath
Claverton Down

GB-Bath Somerset BA2 7AY

Prof.Dr. Claire Voisin
Institut de Mathematiques
Analyse Algebrique
Universite Pierre et Marie Curie
4, place Jussieu, Case 247

F-75252 Paris Cedex 5

Prof.Dr. Georg Schumacher
Fachbereich Mathematik
Universität Marburg

35032 Marburg

Prof.Dr. Pelham M.H. Wilson
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB-Cambridge , CB2 1SB

Dr. Jörg Winkelmann
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Prof.Dr. Joseph Wolf
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA
Universitätsstr. 150

44801 Bochum