

MATHEMATISCHES FORSCHUNGSMINISTITUT OBERWOLFACH

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Homotopy Theory

13.09. - 19.09.1998

The meeting was organized by Mike Hopkins (MIT), Karlheinz Knapp (Wuppertal), and Erich Ossa (Wuppertal). 48 participants from Europe, Japan and the USA attended. There were 20 lectures and three contributed talks about a wide variety of topics in homotopy theory, such as stable homotopy theory, equivariant homotopy theory, elliptic cohomology, Hopf rings, localization, and algebraic K-theory. Several talks presenting details and extensions of unpublished work on chromatic theory aroused particular interest. Some other talks focused on applying homotopy theory to problems in geometry.

Mark Mahowald

### Finitely presented spectra and Brown Comenetz duality

This talk represents joint work with Charles Rezk.

Brown and Comenetz introduced a notion of duality into stable homotopy. Hopkins and Gross showed that this notion, in certain situations, is closely connected with Spanier-Whitehead duality. In this talk I wish to explore this connection and investigate it in connection with the Adams spectral sequence. In particular I will study a class of spectra, called *fp-spectra*. These are connective,  $p$ -complete spectra whose mod  $p$  cohomology is finitely presented over the Steenrod algebra. This class of spectra include  $BP\langle n \rangle$ , connective  $K$ -theories, and some spectra whose  $L_n$  localization is the  $L_n$  localization of some finite spectra. An interesting example is a connective cover of  $L_2 S^0$  at  $p > 3$ .

Andrew Baker

### Isogenies of elliptic curves and operations in elliptic cohomology

If  $k$  is a commutative ring, an *oriented elliptic curve*  $(\mathcal{E}, \omega)$  over  $k$  is a 1-dimensional irreducible abelian variety  $\mathcal{E}$  equipped with a non-vanishing invariant 1-form  $\omega$ . A rule which assigns to each equivalence class of oriented elliptic curves  $(\mathcal{E}, \omega)$  a section  $F(\mathcal{E}, \omega)$  of  $\Omega^1(\mathcal{E})^{\otimes k}$  is called a *modular form of weight  $k$*  over  $k$  if it transforms under a morphism of abelian varieties  $\varphi: \mathcal{E}_1 \rightarrow \mathcal{E}_2$  under which  $\varphi^* \omega_2 = \lambda \omega_1$  according to the rule

$$\varphi^*(F(\mathcal{E}_2, \omega_2)) = F(\mathcal{E}_1, \omega_1).$$

If  $k$  contains  $1/6$ , then oriented elliptic curves are classified by the graded ring of modular forms  $Ell_* = \mathbb{Z}[1/6][Q, R, \Delta^{-1}]$ . Elliptic homology and cohomology are defined using Landweber's Exact Functor Theorem and an associated genus by

$$Ell_*( ) = Ell_* \otimes_{MU_*} MU_*( ), \quad Ell^*( ) = Ell_* \otimes_{MU_*} MU^*( ).$$

An *isogeny*  $\varphi: (\mathcal{E}_1, \omega_1) \rightarrow (\mathcal{E}_2, \omega_2)$  consists of a finite degree morphism of abelian varieties  $\varphi: \mathcal{E}_1 \rightarrow \mathcal{E}_2$ ; on 1-forms it induces  $\varphi^* \omega_2 = \lambda_\varphi \omega_1$  for some  $\lambda_\varphi \in k$ . If  $\lambda_\varphi = 1$  then  $\varphi$  is a *strict isogeny*. An isogeny  $\varphi$  factors uniquely as

$$\mathcal{E}_1 \rightarrow \mathcal{E}_1 / \ker \varphi \xrightarrow{\cong} \mathcal{E}_2,$$

where the first arrow is a strict isogeny and the second is an isomorphism.

The category of oriented elliptic curves over  $\mathbb{C}$  and their isogenies can be used to describe a large part of the stable operation algebra  $Ell^* Ell$ , and the dual object  $Ell_* Ell$  is a certain algebra of functions on this category.

Hecke operations are defined in elliptic (co)homology by symmetrizing over all isogenies of a fixed degree  $n$  in the universal case. One application of these operations is to computing part of the  $E_2$ -term of the Adams spectral sequence for spheres in elliptic cohomology.

Using a theorem of Tate we define a certain completion of the category of separable isogenies between supersingular elliptic curves over  $\mathbb{F}_p$ ,  $\text{SepIsog}_{ss}$ . A certain class of functions on this category can be identified with part of the dual operation algebra of supersingular elliptic cohomology and this is used to identify the supersingular Adams  $E_2$ -term, using a topological splitting of  $\text{SepIsog}_{ss}$ .

Goro Nishida

### Ring spectra maps from $B\mathbb{Z}/p^r$ to the Morava $K(n)$ -spectrum

Let  $\overline{K}(n)$  be the spectrum of Morava  $K(n)$ -theory with  $\pi_* \overline{K}(n) = W_{\mathbb{F}_p}[t, t^{-1}]$  where  $W_{\mathbb{F}_p}$  is the ring of Witt vectors and  $\deg t = 2$ . Let  $K$  be the fraction field of  $W_{\mathbb{F}_p}$ . For a finite extension  $L$  of  $K$  let  $\mathcal{O}_L$  denote the ring of integers in  $L$ . Let  $\overline{K}_{\mathcal{O}_L}$  be the spectrum representing  $\{ \cdot, \mathbb{K} \}^* \otimes_{W_{\mathbb{F}_p}} \mathcal{O}_L$ . Let  $B\mathbb{Z}/p^r$  be the classifying space of  $\mathbb{Z}/p^r$ . Since  $B\mathbb{Z}/p^r$  is an H-space, the spectrum  $B\mathbb{Z}/p^r_+$  is a ring spectrum. Then the set of all ring spectra maps

$$f: B\mathbb{Z}/p^r_+ \rightarrow \overline{K}(n)_{\mathcal{O}_L}$$

turns out to be a group under the cup product.

**Theorem 1** *There exists a finite ramified extension  $L$  of  $K$  such that the group of ring spectra maps is isomorphic to  $(\mathbb{Z}/p^r)^n$ .*

In the Hopf algebra  $\overline{K}(n)^0(B\mathbb{Z}/p^r) \otimes_{W_{\mathbb{F}_p}} \mathcal{O}_L$ , an element  $u$  corresponds to a map of ring spectra if and only if  $u$  is grouplike, i.e.,  $\Delta u = u \otimes u$ . Using the self-duality of a bicommutative finite dimensional Hopf algebra over an algebraically closed field, we can reduce our problem to determining the dual group structure

$$\mathrm{Hom}_{\mathrm{alg}}(\overline{K}(n)^0(B\mathbb{Z}/p^r), \mathcal{O}_L),$$

but this is known by local class field theory.

As an application, we consider the mod  $p$  reduction  $K(n)_*$  with  $K(n)_* = \mathbb{F}_p[t, t^{-1}]$ . Then  $K(n)^0((B\mathbb{Z}/p)^m)$  is regarded as a representation of  $GL_m(\mathbb{F}_p)$  over  $\mathbb{F}_p$ .

**Theorem 2** *As elements in the representation ring  $R_{\mathbb{F}_p}(GL_m(\mathbb{F}_p))$ , we have  $K(n)^0((B\mathbb{Z}/p)^m) = \mathbb{F}_p[M_{n,m}(\mathbb{F}_p)]$ , where the right hand side is the group ring of the additive group of  $(n \times m)$ -matrices with the natural  $GL_m(\mathbb{F}_p)$ -action.*

Dominique Arlettaz

### Homotopical properties of the K-theory space of the ring of integers

(joint work with C. Ausoni, M. Mimura, K. Nakahata, N. Yagita)

The algebraic K-theory of a ring  $R$  is the study of the homotopy type of the infinite loop space  $KR = BGL(R)^+$ . The purpose of this talk is to investigate this space in the case of the ring of integers  $R = \mathbb{Z}$ . The recent calculation of the 2-torsion of the groups  $K_i(\mathbb{Z})$  by J. Rognes and C. Weibel (based on Voevodsky's work) implies the following relationship between  $K\mathbb{Z}$  and Bökstedt's space  $JK(\mathbb{Z}, p)$  after 2-completion.

**Theorem.** *Let  $p$  be any prime  $\equiv 3$  or  $5 \pmod{8}$ .*

(a) *There is a homotopy equivalence  $K\mathbb{Z}_2 \simeq JK(\mathbb{Z}, p)_2$ .*

(b) *There is a commutative diagram*

$$\begin{array}{ccccc} SU_2 & \longrightarrow & K\mathbb{Z}_2 & \longrightarrow & BO_2 \\ \downarrow & & \downarrow f_p & & \downarrow \\ U_2 & \longrightarrow & (K\mathbb{F}_p)_2 & \longrightarrow & BU_2, \end{array} \quad (1)$$

where rows are fibrations and where  $f_p$  is induced by the reduction mod  $p$ .

This enables us to deduce precise results on the homotopy type of  $K\hat{\mathbb{Z}}_2$ . For instance, we can completely determine the Hurewicz homomorphism  $K_i(\mathbb{Z}) \rightarrow H_i(GL(\mathbb{Z}); \mathbb{Z})$  at the prime 2, for all positive integers  $i$ , and compute the 2-part  $(\rho_i)_2$  of the order  $\rho_i$  of the Postnikov  $k$ -invariants  $k^{i+1}(K\mathbb{Z}) \in H^{i+1}(K\mathbb{Z}[i-1]; K_i(\mathbb{Z}))$  (where  $K\mathbb{Z}[i-1]$  denotes the  $(i-1)$ -st Postnikov section of  $K\mathbb{Z}$ ): for all positive integers  $i$ ,

$$(\rho_i)_2 = \begin{cases} ((\frac{i-1}{2})!)_2 & \text{if } i \equiv 1 \pmod{4} \text{ and } i \geq 5, \\ 2 & \text{if } i \equiv 2 \pmod{8} \text{ and } i \geq 10, \text{ or } i = 3 \text{ or } 7, \\ 16 & \text{if } i \equiv 3 \pmod{8} \text{ and } i \geq 11, \text{ or } i = 15, \\ 2(i+1)_2 & \text{if } i \equiv 7 \pmod{8} \text{ and } i \geq 23, \\ 1 & \text{otherwise.} \end{cases}$$

The following calculation of the mod 2 cohomology of the space  $K\mathbb{Z}$  also follows from the above theorem.

**Theorem.** *There is an isomorphism of Hopf algebras and of modules over the Steenrod algebra*

$$H^*(K\mathbb{Z}; \mathbb{Z}/2) \cong H^*(BO; \mathbb{Z}/2) \otimes H^*(SU; \mathbb{Z}/2).$$

Therefore,  $H^*(K\mathbb{Z}; \mathbb{Z}/2) \cong \mathbb{Z}/2[w_1, w_2, \dots] \otimes \Lambda(u_3, u_5, \dots)$ , where  $\deg(w_i) = i$  and  $\deg(u_{2k-1}) = 2k-1$ . For all  $k \geq 2$ , the exterior generators  $u_{2k-1}$  are inductively defined as follows by using the homomorphism  $f_p^*: H^*(K\mathbb{F}_p; \mathbb{Z}/2) \rightarrow H^*(K\mathbb{Z}; \mathbb{Z}/2)$  for any prime  $p \equiv 5 \pmod{8}$ :

$$u_{2k-1} = f_p^*(e_k) + \sum_{j=1}^{k-2} w_j^2 u_{2k-2j-1},$$

where the  $e_k$ 's are the exterior generators (of degree  $2k-1$ ) of  $H^*(K\mathbb{F}_p; \mathbb{Z}/2)$ . If one wants to study the space  $K\mathbb{Z}$  at an odd prime  $l$ , one can again consider Bökstedt's space  $JK(\mathbb{Z}, p)_l$  for suitable primes  $p$  and prove the following statement.

**Theorem.** *If  $l$  is a Vandiver prime, then  $JK(\mathbb{Z}, p)_l$  is a direct factor of  $K\hat{\mathbb{Z}}_l$ .*

This implies in particular that the  $l$ -part  $(\rho_i)_l$  of the order of the Postnikov  $k$ -invariant  $k^{i+1}(K\mathbb{Z})$  satisfies  $(\rho_i)_l \geq ((\frac{i-1}{2})!)_l$  for all integers  $i \equiv 1 \pmod{4}$  (with  $i \geq 5$ ).

Bob Oliver

### Fixed point free actions on $\mathbb{Z}$ -acyclic 2-complexes

The talk centered around the following theorem, proven in joint work with Yoav Segev.

**Theorem** *A finite group  $G$  has an essential action on a 2-dimensional  $\mathbb{Z}$ -acyclic CW complex without fixed points if and only if  $G$  is isomorphic to one of the simple groups  $PSL_2(2^k)$  for  $k \geq 2$ ,  $PSL_2(q)$  for  $q \equiv \pm 3 \pmod{8}$  and  $q \geq 5$ , or  $Sz(2^k)$  for odd  $k \geq 3$ .*

Here, a  $G$ -complex is called *essential* if there is no normal subgroup  $1 \neq N \triangleleft G$  with the property that for each  $H \subseteq G$ , the inclusion  $X^{HN} \rightarrow X^H$  induces an isomorphism on integral homology. (Otherwise,  $X^H$  is a  $G/H$ -complex with "essentially" the same homological properties.) One can show that a 2-dimensional  $G$ -complex is essential if and only if there is no  $1 \neq N \triangleleft G$  for which  $X^N \neq \emptyset$ .

There is a classical example of an action of  $A_5 = PSL_2(4)$  on an acyclic 2-complex without fixed points: the only such example already known. It is easy to see that this is the smallest dimension where such an action is possible, since any 1-dimensional  $\mathbb{Z}$ -acyclic complex is a tree. In an earlier paper, Aschbacher and Segev showed that the only finite simple groups which could possibly have fixed point free actions on 2-dimensional  $\mathbb{Z}$ -acyclic complexes are groups of Lie type and Lie rank one, and the sporadic group  $J_1$ . What Segev and I did was to determine which of

these remaining groups do have such actions, and show that no non-simple groups have essential actions.

Paul Goerss

### Hopf rings, Dieudonné modules, and $E_*\Omega^2 S^3$

Let  $E$  be a commutative ring spectrum and  $E_n = \Omega^\infty \Sigma^{-n} E$ . Then the cup product on  $E_*$  gives the graded collection of Hopf algebras  $H_* E = \{H_* E_n\}_{n \in \mathbb{Z}}$  the structure of a Hopf ring; that is, a commutative ring object in the category of colagebras over  $\mathbb{F}_p$ . The study of such goes back to Milgram and plays a role in the study of unstable cohomology operations for  $E^*$ .

There is an equivalence of categories  $D_* : \mathcal{HA} \rightarrow \mathcal{D}$  due to Schoeller from graded bicommutative Hopf algebras over  $\mathbb{F}_p$  to the category of graded Dieudonné modules. The functor  $D_*$  turns the Hopf ring  $H_* E$  into a graded  $E^*$  Dieudonné algebra  $D_* H_* E$ . The functor on spectra  $X \mapsto D_* H_* \Omega^\infty X$  is part of a homology theory represented by a spectrum  $B(n)$ , the Brown-Gitler spectrum. The spectra  $B(n)$  are stable retracts, by the Snaith splitting, of  $\Omega^2 S^3_+$  and one obtains a surjective homomorphism of  $E_* \Omega^2 S^3_+ \rightarrow D_* H_* E$

$$E_* \Omega^2 S^3_+ \longrightarrow D_* H_* E$$

which is an isomorphism in favorable cases, such as  $E$  Landweber exact and concentrated in even degrees. Since  $BP_* \Omega^2 S^3_+$  has been calculated by Ravenel, one can recover the calculation of  $D_* H_* E$  in the Landweber exact case due to Hunton, Hopkins, Turner, and others.

John Hunton

### Applications of homotopy theory to quasicrystallography

joint work with Alan Forrest and Johannes Kellendonk

A *pattern* in  $\mathbb{R}^d$  is taken to be a locally finite arrangement of compact subsets. We say that it is *quasiperiodic* if any finite portion of it (i.e., the part of the pattern visible in some spherical window) repeats infinitely often under translation, with only bounded gaps between occurrences, but the whole pattern has itself no global translational symmetries. A well known example is that of the Penrose tiling.

From a quasiperiodic pattern (q.p.p.)  $P$  we can construct a topological space - we take the set of all translations of the pattern and metrize it essentially by saying that two patterns are "close" if they almost agree on a "large" ball about the origin; this can be made rigorous in a variety of equivalent ways. Done correctly this gives a precompact space which we complete to the space  $MP$  associated to the q.p.p.  $P$ .

We consider in detail the q.p.p.'s which arise by the "strip and projection" method. This takes as data an irrationally sloped subspace  $\mathbb{R}^d$  in a larger space  $\mathbb{R}^N$ ,  $0 < d < N$ , together with a chosen neighbourhood  $K \times \mathbb{R}^d$  of the subspace. The projection of the integer lattice points  $\mathbb{Z}^N \cap K \times \mathbb{R}^d$  lying within the strip onto  $\mathbb{R}^d$  gives a q.p.p. of points on  $\mathbb{R}^d$ .

We can then ask about the topological invariants (cohomology,  $K$ -theory etc.) of the associated space  $MP$ . These can be shown to provide important geometric information about the underlying pattern. For example, work of Bellissard shows that its  $K$ -theory contains information about the spectrum of the Schrödinger operator on a "quasicrystal" formed by atoms sitting on the points of the original q.p.p. (such structures seem to occur in nature). The rational cohomology of  $MP$  provides an invariant for discussing self similarity properties of the pattern.

We give techniques that interpret the topological invariants in terms of group homology and this is used to show that for large ranges of initial data projection method patterns do not display

self-similarity (arising from a substitution system). The same conclusion is also shown to hold for generic projection patterns. The group homological description is sufficiently practical to allow for complete computation of the homology groups for specific patterns and we illustrate this by describing the computation for the Penrose tiling.

Takuji Kashiwabara

## Homological algebra for coalgebraic modules and Morava $K$ -theory of infinite loop spaces

Given a generalized homology theory  $h$  and a spectrum  $E$ , what can we say about  $h_*(E_*)$ ? There have been many case by case computations, but very few systematic phenomenon is known so far. Ravenel and Wilson computed  $h_*(BP_*)$  and showed that it has nice properties. For  $BP$ -module spectrum  $M$ , using the knowledge of  $h_*(BP_*)$  and the  $BP^*$ -module structure of  $M^*$ , one can construct a natural algebraic model  $h_*(BP_*) \otimes_{h_*(BP^*)} h_*[M^*]$  that comes with a natural map to  $h_*(M_*)$ . Now it is natural to ask when this map is isomorphism. Hunton and Hopkins showed that it is isomorphism when  $M$  is Landweber-exact (although the notion of  $\otimes$  didn't exist at the time).

However, the theory of coalgebraic modules (module objects in the category of coalgebras) developed by Hunton and Turner suggests that this could be proved using the derived functor of  $\otimes$ . As a matter of fact we have the following:

**Theorem 1** Let  $h = K(n)$ ,  $H\mathbb{Z}/p$ , or  $HQ$ , and  $M$  a reasonable  $BP$ -module spectrum. If  $CTor_i^{h_*(BP^*)}(h_*(BP_*), h_*[M^*])$  vanishes for  $i > 0$ , then the natural map  $h_*(BP_*) \otimes_{h_*(BP^*)} h_*[M^*] \rightarrow h_*(M_*)$  is isomorphism.

From this point of view, the result of Hunton and Hopkins is a consequence of the following:

**Theorem 2** Let  $h = H\mathbb{Z}/p$ . If  $(p, v_1, \dots, v_n, \dots)$  is a regular sequence on  $M$ , then  $CTor_i^{h_*(BP^*)}(h_*(BP_*), h_*[M])$  vanishes for  $i > 0$ .

Recently Wilson and the author computed  $K(n)_*(BP < q >_*)$ , and showed that the map in question is isomorphism for  $q \geq n - 1$ . This, together with the above motivates the following conjecture:

**Conjecture 3** Let  $h = K(n)$ . If  $(p, v_1, \dots, v_{n-1})$  is regular on  $M$ , then

$$CTor_i^{h_*(BP^*)}(h_*(BP_*), h_*[M])$$

vanishes for  $i > 0$ .

As a matter of fact, we have

**Theorem 4** The conjecture holds if  $n = 1$  or  $M$  is  $I_n$ -complete.

This result and a detailed analysis on the functor

$$CTor_i^{K(1)_*(BP^*)}(K(1)_*(BP_*), K(1)_*[M])$$

for  $i = 0, 1$  (as a matter of fact it vanishes if  $i > 1$ ) lead to the determination of  $K(1)_*(M_*)$  when  $M$  is not necessarily  $p$ -torsion free, notably one can recover the Hopkins-Ravenel-Wilson's theorem for  $K(1)$ -case.

Sarah Whitehouse

## Operads and Gamma Homology of Commutative Rings

Joint work with Alan Robinson on *gamma homology* is described. This is the natural homology theory for  $E_\infty$ -algebras. It specialises to give a new homology theory for discrete commutative rings. The motivation for this theory is that the obstructions to an  $E_\infty$  multiplicative structure on a spectrum lie (under mild hypotheses) in the  $\Gamma$ -cohomology of the corresponding dual Steenrod algebra, just as the obstructions to an  $A_\infty$ -structure lie in the Hochschild cohomology of that algebra.

Cyclic operads and algebras over them are introduced, in order to describe a 'realization' of an algebra over a suitably cofibrant cyclic operad. For an  $E_\infty$ -operad and an algebra over it, this realization is called the gamma cotangent complex of the algebra and its homology is gamma homology. There is also a cyclic version of realization, which gives rise to cyclic gamma homology.

A useful check on the constructions is given by calculating the  $A_\infty$ -analogues; we get Hochschild and cyclic homology. A natural filtration of the cotangent complex gives a spectral sequence for the gamma homology of a commutative algebra  $A$  with coefficients in an  $A$ -module  $M$ :

$$E_{p-1,q}^1 \cong H_q(\Sigma_p; V_p \otimes A^{\otimes p} \otimes M) \Rightarrow H\Gamma_{p+q-1}(A; M),$$

where  $V_p$  is the tree representation of the symmetric group  $\Sigma_p$ . This is used to show that rationally gamma homology agrees with André/Quillen homology. In general the theories are different. Other properties of the theory include flat base change and transitivity theorems.

Finally, we discuss two spectral sequences which relate to calculating the gamma homology of the Eilenberg-Mac Lane spectrum for  $\mathbb{F}_2$  over the sphere.

Martin D. Crossley

## Conjugation Invariants in the dual Steenrod Algebra

We study the canonical conjugation or anti-automorphism,  $\chi$ , in the dual Steenrod algebra,  $\mathcal{A}_*$ , with a view to calculating the subspace of invariant elements,  $\mathcal{A}_*^\chi$ . This problem arises in Whitehouse's work on Gamma homology.

It is well-known that the dual Steenrod algebra is polynomial on generators  $\xi_1, \xi_2, \xi_3, \dots$  in degrees 1, 3, 7,  $\dots$  and that  $\chi$  is a multiplicative map defined using the product and coproduct in  $\mathcal{A}_*$ .

It is trivial that  $\mathcal{A}_*^\chi$  is a subalgebra of  $\mathcal{A}_*$  and that in each degree its dimension is at least half that of  $\mathcal{A}_*$ . Up to degree 42 it is minimal subject to this constraint. In general, however:

**Theorem 1**  $(\dim \mathcal{A}_{d-1})/2 \leq \dim(\chi - 1)(\mathcal{A}_d) \leq (\dim \mathcal{A}_d)/2$  and hence

$$(\dim \mathcal{A}_d)/2 \leq \dim(\mathcal{A}_*^\chi)_d \leq \dim \mathcal{A}_d - (\dim \mathcal{A}_{d-1})/2.$$

For example, in degree 42, where  $\dim \mathcal{A}_d = 92$ ,  $\dim \mathcal{A}_{d-1} = 86$ , so the theorem says  $46 \leq \dim(\mathcal{A}_*^\chi)_d \leq 49$ . In fact  $\dim(\mathcal{A}_*^\chi)_d = 47$ . So the bounds are not perfect, but they're not bad.

The lower bound on  $\dim(\mathcal{A}_*^\chi)_d$  comes from the above lemma. The upper bound hinges on the following curious result.

**Lemma 2** *The monomials ending in 1, i.e. things of the form  $\xi_1^{a_1} \xi_2^{a_2} \dots \xi_{k-1}^{a_{k-1}} \xi_k^1$  have linearly independent images under  $\chi - 1$ .*

To get from the lemma to the theorem one simply counts the monomials ending in 1.

**Question 3** What's the first degree containing an invariant which involves  $\xi_n$ ?

Well, it is at least  $2^n + 1$ , by:

**Lemma 4**  $\xi_n$  is not a summand in any invariant, nor is  $\xi_1 \xi_n$ .

But the proof of this lemma fails for  $\xi_1^2 \xi_n$  and in fact, for  $3 \leq n \leq 7$ , we know that this is the leading term of an invariant, e.g. for  $n = 3$ ,  $(\chi - 1)(\xi_2 \xi_3)$  is divisible by  $\xi_1$  and the quotient has  $\xi_1^2 \xi_n$  as its leading term.

**Conjecture 5** For each  $n \geq 3$  there exists an invariant  $d_n \in A_{2^n+1}^X$  with  $\xi_1^2 \xi_n$  as its leading term.

These elements are important because they are easily seen to be indecomposable, i.e. necessary algebra generators of  $A^X$ .

Assuming this conjecture to be true, we make the following generating conjecture:

**Conjecture 6**  $A^X$  is generated as an algebra by  $\xi_1$ ,  $a_n = \xi_n \chi \xi_n$  for  $n \geq 2$ ,  $b_{m_1, \dots, m_n} = (\chi - 1)(\xi_{m_1} \cdots \xi_{m_n})$  where  $2 \leq m_1 < \cdots < m_n$ ,  $n \geq 2$  and sequences  $(2, n)$  are excluded, and  $d_n$  for  $n \geq 3$ , taking the place of  $b_{2,n}$ .

Again we know that these elements are all indecomposable so  $A^X$  is not polynomial - it has far too many generators. So the following result came as a great surprise to us:

**Theorem 7**  $A[\xi_1^{-1}]^X = A^X[\xi_1^{-1}] = k[\epsilon_2, \epsilon_3, \dots]$  where  $k = \mathbb{F}_2[\xi_1, \xi_1^{-1}]$ ,  $\epsilon_2 = \xi_2 \chi \xi_2$  and, for  $n \geq 3$ ,  $\epsilon_n = (\chi - 1)(\xi_2 \xi_n)$ .

The proof relies on classifying which monomials occur as leading terms of invariants. One easy result shows that all such monomials must have a certain property (even  $\xi_2$  exponent) and then one observes that all monomials with this property do actually occur as leading terms of monomials in the  $\epsilon_n$ 's which are visibly invariant and algebraically independent. One then proceeds by a transfinite induction.

(Joint work with Sarah Whitehouse)

Franklin P. Peterson

## The Global Structure of the Dickson Algebra

The following gives the global structure as an unstable  $A$ -algebra for the Dickson algebra on  $k$  variables. Let  $W_k$  be the free unstable  $A$ -module on one generator  $u$  of dimension  $2^{(k-1)}$  modulo the left ideal generated by  $Sq^{2^j}(u)$  for  $j = 0, \dots, k-3$ . Then  $D_k$  is isomorphic to  $U(W_k)$  with one more relation, namely  $Sq^{2^{(k-2)}}(u) \cdot u = Sq^{2^{(k-1)}} Sq^{2^{(k-2)}}(u)$ .

To calculate  $\text{Hom}(D_k, -)$  in the category of unstable  $A$ -algebras is now an easy corollary.

The main step in the proof is to find an  $A$ -submodule of  $D_k$  which is isomorphic to  $W_k$ . This is the cyclic  $A$ -module generated by the bottom Dickson invariant. An additive basis for  $W_k$  is given by the elements  $\omega(n) = \sum x_1^{i_1} \cdots x_k^{i_k}$ , where  $\sum i_j = n$  and each  $i_j$  is either 0 or a power of 2. This symmetric sum is invariant under  $GL(k, \mathbb{Z}/2)$  if and only if  $2^{k-\alpha(n)} | n$  and it is those elements which are in  $W_k$ . The proof is somewhat complicated.



Tibor Beke

## Locally presentable categories and topos theory with applications to abstract homotopy theory

Add the following hypotheses to Quillen's axioms for a (closed) model category: that the underlying category be locally presentable, and that the full subcategory of its category of morphisms with objects the weak equivalences be accessible. This extra set-theoretic handle makes it possible to prove theorems on the existence and localization of model categories more easily and more functorially. An added advantage is that the theory of accessible functors and categories applies to sheaves of structures (ie. algebraic objects in a topos) equally well. (The current theory of accessible categories is mainly due to Makkai, Adamek and Rosicky, but it is rooted in work of Grothendieck, Gabriel and Ulmer, conceived precisely for such purposes.)

Our main theorem gives a sufficient condition for a category and subcategory as above to be part of a model category; a very similar result has recently been announced by J. Smith. Either makes it possible to give a proof of the existence of  $f$ -localizations (in the sense of Dror-Farjoun and Bousfield) in simplicial sets that generalizes almost verbatim to the analogous construction for simplicial objects in a topos; this is the technical ingredient needed for the "homotopy theory of site with an interval", due to Voevodsky and Morel. As another application, there exists a model structure on simplicial algebraic objects in a topos, created by the forgetful functor into simplicial "sheaves". (Here "algebraic" means the single-sorted, finitary equational theories of universal algebra.) This generalizes the "sA" model structure developed by Quillen, and answers a question he left open at the end of "Homotopical Algebra".

Carles Casacuberta

## Implications of large-cardinal principles in homotopical localization

A functor  $E$  in the category of simplicial sets is called *homotopy idempotent* if it preserves weak equivalences and comes equipped with a natural transformation  $\text{Id} \rightarrow E$  inducing weak equivalences  $EX \simeq EEX$  for all  $X$ . For any map  $f: A \rightarrow B$  there is a homotopy idempotent functor  $L_f$ , described by Bousfield and Farjoun, with the property that, for each  $X$ , the map  $X \rightarrow L_f X$  is homotopy universal among maps  $X \rightarrow Y$  where  $Y$  is fibrant and satisfies  $f^*: \text{map}(B, Y) \simeq \text{map}(A, Y)$ . Farjoun has asked if every homotopy idempotent functor is weakly equivalent to  $L_f$  for some map  $f$ .

In a joint article with Dirk Scevenels and Jeff Smith, we prove that the answer to this question is affirmative if Vopěnka's Principle holds, yet it is impossible to prove that the answer is affirmative using the ordinary ZFC axioms of set theory (Zermelo-Fraenkel axioms with the axiom of choice). Vopěnka's Principle (VP) states that no locally presentable category contains a large discrete subcategory, that is, given a proper class  $A$  of objects in a locally presentable category, there is a nonidentity arrow  $A \rightarrow B$  for some  $A$  and  $B$  in  $A$ . This statement cannot be proved using ZFC, since its truth implies the consistency of ZFC.

If VP holds, then every full subcategory  $S$  closed under filtered colimits in a locally presentable category has a set  $X$  of presentable objects such that every object of  $S$  is a filtered colimit of objects from  $X$ . From this fact we infer that the local complement of any (possibly proper) class of fibrant simplicial sets is the class of  $L_f$ -equivalences for some map  $f$ . This has two important consequences:

**Theorem 1** *If VP holds, then for every homotopy idempotent functor  $E$  there is a map  $f$  such that  $L_f \simeq E$ .*

**Theorem 2** *If VP holds, then  $h^*$ -localization exists for every cohomology theory  $h^*$ .*

It is not known if the statement of Theorem 2 can be proved without using VP. On the other hand, we show that it is impossible to prove the statement of Theorem 1 using ZFC, without VP. Indeed, assuming that measurable cardinals do not exist — this assumption is consistent with ZFC — we exhibit a homotopy idempotent functor in the category of reduced simplicial sets which is not equivalent to  $L_f$  for any map  $f$ . To this aim, let  $\mathcal{A}$  be the class of groups  $\mathbb{Z}^*/\mathbb{Z}^{<\kappa}$  for all regular cardinals  $\kappa$ , where  $\mathbb{Z}^*$  denotes the abelian group of all functions  $\kappa \rightarrow \mathbb{Z}$ , and  $\mathbb{Z}^{<\kappa}$  is the subgroup of functions whose support has cardinality lower than  $\kappa$ . Let  $P_{\mathcal{A}}$  be the idempotent functor in the category of groups which sends every group  $G$  onto its largest quotient admitting no nontrivial homomorphisms from groups in  $\mathcal{A}$ . Let  $EX = \overline{W}(P_{\mathcal{A}} \pi_1(X))$ , where  $\overline{W}$  denotes the classifying space functor. Then  $EX$  is homotopy idempotent, yet, if it is equivalent to  $L_f$  for some map  $f$ , then we infer that  $\text{Hom}(\mathbb{Z}^*/\mathbb{Z}^{<\kappa}, \mathbb{Z}) = 0$  for some cardinal  $\kappa$ . This implies that  $\kappa$  is measurable, contradicting our assumption.

Stefan Schwede

### Formal groups and stable homotopy of commutative rings

We discuss properties of a certain  $(A_{\infty})$ -ring spectrum  $DR$ , functorially associated to any commutative ring  $R$ .  $DR$  is characterized by the property that its modules have the same homotopy theory as spectra of commutative simplicial  $R$ -algebras.

We present an explicit construction which associates to every 1-dimensional commutative formal group law over  $R$  a map of ring spectra from  $H\mathbb{Z}$  to  $DR$ . This way the homotopy classes of ring spectrum maps  $H\mathbb{Z} \rightarrow DR$  can be identified with strict isomorphism classes of formal group laws. We also express the space of ring spectrum maps in terms of formal group data and the homotopy units of  $DR$ .

Hal Sadofsky

### The homotopy type of the $K(n)$ localization of the Brown-Comenetz dual of $L_n S^0$

In joint work with Mike Hopkins we calculate  $E_{n,*}(IL_n S^0 \wedge X)$  as a module over the Galois extended stabilizer group. Here  $I$  is the Brown-Comenetz duality functor,  $X$  is finite type  $n$ , and  $E_{n,*}$  is the homology theory based on Johnson-Wilson theory such that

$$E_{n,*} = W_{F_p,*}[[u_1, \dots, u_{n-1}]]\langle u, u^{-1} \rangle, |u| = -2.$$

This calculation is motivated by Gross and Hopkins's result identifying the dualizing module for  $S_n - E_{n,*}$  modules. We calculate that  $E_{n,*}(IL_n S^0 \wedge X)$  is  $E_{n,*+n^2}(X)$  tensored with that dualizing module.

This formula has the corollary that  $L_{K(n)} IL_n S^0 \in \text{Pic}_n$ , and allows one to deduce the result announced by Hopkins and Gross in the Bulletin of the AMS relating  $IL_n X$  to a suspension of  $L_n DX$  in case  $X$  is finite type  $n$  and annihilated by  $p$  ( $DX$  is the Spanier-Whitehead dual of  $X$ ).

Our calculation uses standard results about the cohomology of profinite groups, together with results of Hopkins and Miller that provide an action of the Morava stabilizer group on the spectrum  $E_n$ , and results of Devinatz and Hopkins that provide a construction of homotopy fixed points with respect to this action.

Stefan Stolz

# Positive scalar curvature metrics and the Baum-Connes Conjecture

According to the Gromov-Lawson-Rosenberg Conjecture a smooth closed spin manifold  $M$  of dimension  $n \geq 5$  with fundamental group  $\pi$  admits a metric of positive scalar curvature if and only if an index obstruction  $\alpha(M) \in KO_n(C^*\pi)$  in the  $K$ -theory of the real group  $C^*$ -algebra  $C^*\pi$  vanishes. This conjecture has been verified for groups  $\pi$  with periodic cohomology in joint work with Botvinnik and Gilkey; but last year the conjecture turned out to be too optimistic when Schick produced a counterexample for  $\pi = \mathbb{Z}^4 \times \mathbb{Z}/3$  and  $n = 3$ . However, there is a "stable" version of the conjecture according to which the vanishing of  $\alpha(M)$  should imply the existence of a positive scalar curvature metric on the product  $M \times B \times \dots \times B$  of  $M$  with enough copies of the "Bott manifold"  $B$ , which is any simply connected 8-dimensional spin manifold with  $\hat{A}(M) = 1$ .

The stable conjecture can be reformulated by saying that the kernel of the "assembly map"  $A: KO_n(B\pi) \rightarrow KO_n(C^*\pi)$  is equal to the subgroup  $KO_n^+(B\pi)$  consisting of elements of the form  $f_*[N]$ , where  $N$  is a spin manifold admitting a positive scalar curvature metric,  $[N]$  is its  $KO$ -fundamental class, and  $f$  is a map from  $N$  to  $B\pi$ . Joint work with Rosenberg led to a proof of the stable conjecture for finite groups  $\pi$ ; it is based on an "Artin" induction argument which shows that elements in the kernel of the assembly map come from elements in the  $K$ -theory of cyclic subgroups of  $\pi$ .

For general groups, it is useful to factor the assembly map in the form

$$KO_n(B\pi) = KO_n^*(E\pi) \longrightarrow KO_n^*(E\pi) \xrightarrow{\beta} KO_n^*(pt) = KO_n(C^*\pi),$$

where  $E\pi$  denotes the universal  $\pi$ -space with finite isotropy groups, and  $KO^*$  is a suitably defined equivariant  $KO$ -homology which for finite groups  $\pi$  agrees with the usual equivariant theory. The Baum-Connes Conjecture claims that  $\beta$  is an isomorphism for **all groups**. The author showed that injectivity of  $\beta$  (which has been proved for quite a few groups, unlike surjectivity) implies the stable conjecture for  $\pi$ . The proof again involves reducing down to the cyclic subgroups of  $\pi$ .

Katsumi Shimomura

## The Adams-Novikov differentials on the mod 2 Moore spectrum

In the same way as the case  $p > 2$ , we study the Adams-Novikov differentials in the Adams-Novikov  $E_2$ -term  $E_2^*(L_2M_2)$  for the homotopy groups  $\pi_*(L_2M_2)$  of the mod 2 Moore spectrum  $M_2$  through the Bockstein spectral sequence  $E_2^*(L_2M(1,4)) \Rightarrow E_2^*(L_2M_2)$ . Here  $BP_*(M(1,4)) = BP_*/(2, v_1^4)$ . The Bockstein  $E_1$ -term  $E_1^*(L_2M(1,4))$  is the direct sum of modules  $M$  and  $N$ , where  $M$  is "generated" by  $h_{10}$  and  $h_{11}$ , and  $N$  by  $\zeta_2$ . The Adams-Novikov differentials on  $M$  can be read off from Hopkins and Mahowald's calculation of  $\pi_*(EO_2)$ . In order to consider differentials on  $N$ , we consider a spectrum  $X_j^i$  such that  $BP_*(X_j^i) = (BP_*/(2, v_1^j))[t_1]/(t_1^{i+1})$ . Note that  $X_0^i = M(1,4)$ . Then we know the  $E_\infty$ -term  $E_\infty^*(L_2X_j^\infty)$ , in fact,  $E_\infty^*(L_2X_j^\infty) = E_2^*(L_2X_j^\infty)$ , which implies immediately that  $E_\infty^*(L_2X_j^{15}) = E_2^*(L_2X_j^{15})$  for  $j \leq 4$ . By the cofibrations  $X_j^7 \rightarrow X_j^{15} \rightarrow \Sigma^{18}X_j^7$  and  $X_j^3 \rightarrow X_j^7 \rightarrow \Sigma^8X_j^3$ , we obtain  $E_\infty^*(L_2X_j^i)$  for  $i = 3, 7$  and  $j \leq 4$ . In particular,

**Theorem.** The  $E_\infty$ -term for  $\pi_*(L_2X_3^1)$  is the tensor product of  $\Lambda(h_{30}, \rho_2)$  and a  $K(2)$ -module

$$\zeta_2 K(2) \cdot [h_{20}]/(h_{20}^2) \oplus K(2) \cdot [h_{20}]/(h_{20}^{2^r})$$

for some integer  $r \geq 9$ .

Christian Nassau

### An improved algorithm for the computation of a minimal resolution of the mod 2 Steenrod algebra $A$

So far minimal resolutions of  $A$  have only been computed with the straightforward 'brute force' method which does not take any nontrivial structural results about  $A$  into account. In this talk I describe how some easy vanishing lines for  $\text{Ext}_B(\mathbb{F}_2)$  - where  $B \subset A$  is a finite sub Hopf algebra of  $A$  - can be used to speed up the computation considerably.

Nobuaki Yagita

### Highly homotopy non-commutativity of Lie groups

Lie groups are highly non-commutative. This fact follows, for example, from the structure of Lie algebras. Even when we consider in the homotopy category, highly noncommutativities also holds. However the Pontrjagin ring structures of the mod  $p$  ordinary homology are not sufficiently highly noncommutative. For example the Pontrjagin ring of  $H_*(G; \mathbb{Z}/2)$  for the exceptional Lie group  $G = G_2$  or  $F_4$ , is commutative. The Pontrjagin ring of ordinary mod  $p$  cohomology of each finite Lie group is nilpotent, because it is finite dimensional.

V. Rao first noticed that the Morava K-theory  $K(n)_*(-)$  is a powerful theory to show this homotopy non-commutativity. He showed that for an adequate  $n$ ,  $K(n)_*(SO(2m+1))$  is as non-trivial as possible. Moreover, he proved that if a Lie group  $G$  has  $p$ -torsion in homology, then for some  $n$ ,  $K(n)_*(G)$  is not nilpotent. In this paper we study the Pontrjagin product structure of  $K(n)_*(G)$  for smallest  $n$  such that  $K(n)_*(G) \cong K(n)_* \otimes H_*(G; \mathbb{Z}/p)$ . Note that if  $n$  is large enough, then the product is induced from that of  $H_*(G; \mathbb{Z}/p)$ . Of course  $K(n)_*(G)/(v_n = 1)$  is additively isomorphic to  $H_*(G; \mathbb{Z}/p)$ . However its Pontrjagin ring structure is quite different; indeed, the Pontrjagin ring  $K(2)_*(G_2)/(v_2 = 1)$  is not nilpotent but  $H_*(G_2; \mathbb{Z}/2)$  is commutative.

The Pontrjagin product of  $K(4)_*(E_8)$  for  $p = 2$  has a very interesting structure, namely the Pontrjagin ring is generated by one odd degree element and six even degree elements so that odd degree primitive elements are expressed as vertices of a cube and the adjoint actions of even degree generators are expressed as oriented edges on the cube.

Wojciech Chachólski

### An $A$ -complication and an $A$ -Blanc-Stover resolution

Let  $A$  be a finite complex. For an  $A$ -cellular space  $X$  we define its  $A$ -complication  $l(X)$  by induction. We say that  $l(X) = 0$  if  $X$  is a retract of  $\bigvee \Sigma^i A$ . We say that  $l(X) \leq n$  if  $X$  is a retract of a pointed homotopy colimit  $\text{hocolim}_i F$ , where  $F: I \rightarrow \text{Top}_*$  is a pointed diagram such that, for all  $i$ ,  $F(i) \leq (n-1)$ . Thus spaces of  $A$ -complication 1 are those  $A$ -cellular spaces that can be built using only primary information about  $A$ , i.e., only maps between wedges of suspensions of  $A$ .

**Example 1.** Let  $A = S^n$ . Stover proved that, for any  $S^n$ -cellular space  $X$ ,  $l(X) \leq 1$ .

**Example 2.** Let  $A = M(\mathbb{Z}/p, n)$ . Then  $\lim_{r \rightarrow \infty} l(M(\mathbb{Z}/p^r, n+1)) = \infty$ .

**Example 3.** Let  $A = \bigvee_{r \geq 0} M(\mathbb{Z}^r/p, n)$ . For any  $A$ -cellular space  $X$ ,  $l(X) \leq 1$ .

**Proposition.** Let  $A$  be a finite,  $p$ -torsion complex. Assume that  $A \simeq \Sigma B$ . If there exists  $N$ , such that, for an  $A$ -cellular space  $X$ ,  $l(X) < N$ , then the Bousfield class of  $A$  is equivalent to that of a  $\mathbb{Z}/p$ -Moore space.

In view of the above Proposition and Example 3, the way to obtain spaces of arbitrary  $A$ -complication is either to play a "game" with torsion or higher type.

The main theorem is as follows:

**Theorem.** *Let  $B$  be finite, and*

$$d = (\text{dimension of top cell in } B) - (\text{dimension of bottom cell in } B).$$

*Let  $A = \Sigma^d B$ . For any  $A$ -cellular space  $X$ , there is a natural map  $X' \rightarrow X$  such that:*

- $l(X') \leq 1$ ,
- *If  $F = \text{Fib}(X' \rightarrow X)$ , then  $\text{map}_*(A, F)$  is a  $d$ -PolyGEM.*

(Joint work with W.G.Dwyer, M.Intermont)

John Greenlees

### Equivariant bordism and equivariant formal groups

This is a report on joint work with M. Cole and I. Kriz.

Let  $A$  be a finite abelian group. One may consider tom Dieck's equivariant homotopical bordism,  $MU_A^*$ , and more generally, equivariant complex oriented cohomology theories.

The idea of an equivariant formal group law is to model  $E_A^*(\mathbb{C}P(U))$  where  $\mathbb{C}P(U)$  is a classifying space for equivariant line bundles. Thus (i)  $\mathbb{C}P(U)$  is a group object in the homotopy category (ii) the components of the fixed point set  $\mathbb{C}P(U)^A$  has components corresponding to the dual group  $A^*$  and (iii) we need an orientation.

**Definition** An  $A$ -equivariant formal group law (Afgl) is a topological  $k$ -algebra  $R$  such that

- (i)  $R$  is a Hopf  $k$ -algebra
- (ii) there is a map  $\theta : R \rightarrow k^{A^*}$  of Hopf algebras, and the topology of  $R$  is defined by the kernel of  $\theta$  and it is complete for the topology and
- (iii) there is a regular element  $y(\epsilon)$  in  $R$  which generates the kernel of  $R \rightarrow k^{A^*} \rightarrow k$  (projection onto the  $\epsilon$ 'th factor).

The examples at and away from the Euler class ideal were discussed, as were multiplicative Afgl's and the representing rings identified. In fact one may find an additive basis of  $R$ , and hence deduce there is a representing ring  $L_A$  for Afgl's.

As for  $MU_A^*$ , one may show (i) that  $MU$  is topologically universal in the sense that if  $E$  is complex oriented there is an equivariant ring map  $MU \rightarrow E$  and (ii) the map  $L_A \rightarrow MU_A^*$  is surjective and the kernel is divisible, torsion and nilpotent in suitable senses. This shows (iii) that  $MU_A^*$  represents Afgl's over any Noetherian ring. It is conjectured that the map is injective. The strategy of proof was described.

Ethan Devinatz

### On the non-existence of the Toda $V(n)$ 's

This is a report on work of Lee Nave of the University of Washington. Recall that a  $p$ -local finite spectrum is said to be a Toda  $V(n)$  if its Brown-Peterson homology is isomorphic to  $BP_*/(p, v_1, \dots, v_n)$  as a  $BP_*BP$ -comodule. Nave proves that, for any prime  $p \geq 7$ , the spectrum  $V(\frac{p-3}{2})$  does not exist. (One can of course do better for  $p < 7$ .) The proof makes use of the Hopkins-Miller spectrum  $E_{p-1}^{hG}$ , where  $G \cong \mathbb{Z}/(p) \rtimes \mathbb{Z}/(p-1)^2$  is a maximal finite subgroup of the Morava stabilizer group  $S_n$ . The main step is to prove that if  $V(r-1)$  exists,  $r = \frac{p-3}{2}$ , then the element  $v_r^2$  in  $H^0(G, E_{p-1}^{hG}(r-1))$  does not survive to  $\pi_*(E_{p-1}^{hG} \wedge V(r-1))$ . This is achieved by proving that if it does survive, it must be in the image of  $\pi_*(E_{p-1}^{hG} \wedge V(2)) \rightarrow \pi_*(E_{p-1}^{hG} \wedge V(r-1))$ . However, one can prove that  $v_r^2$  is not in the image of  $H^0(G, E_{p-1}^{hG}(2)) \rightarrow H^0(G, E_{p-1}^{hG}(r-1))$ , a contradiction.

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