

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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**Inverse Wave Scattering Problems and Applications**

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The conference on Inverse Wave Scattering Problems and Applications was organized by Ralph Kleinman (Newark) and Rainer Kress (Göttingen). After Ralph Kleinman's death in February 98 his position as organizer was kept open in order to emphasize on his impact on the theme and the selection of the participants of the conference. The spirit in which the conference was held paid tribute to Ralph Kleinman's scientific achievements.

41 participants from Austria, Belgium, Finland, France, Germany, Great Britain, Italy, New Zealand, Russia, Spain, Sweden, and the USA the invitation. In 24 lectures of 40 minutes duration, followed by lively and stimulating discussions, both theoretical and applied aspects of inverse wave scattering problems were presented. In particular, new insights in uniqueness and stability for inverse problems and new numerical reconstruction procedures were provided. The time before, between and after the talks was used for intensive scientific discussions in small groups and, thanks to the beautiful sunshine, for hiking in the surrounding of the institute. The conference clearly exhibited that inverse wave scattering is a lively and challenging area in applied mathematics.

A special note of thanks is given to the people at the Oberwolfach institute for the friendly and pleasant atmosphere during the whole week.

## Abstracts

**Mikhail I. Belishev**

The BC-method as a triangular factorization

The talk gives some a view point on the BC-method which is an approach to the Inverse Problems based upon their relations to the Boundary Control Theory. We clarify that to reconstruct a Riemannian manifold via dynamical Dirichlet-to-Neuman map (response operator) is to solve a problem of continual triangular factorization. On this way a continual operator analog of a matrix diagonal is introduced. A factorization problem is solved with the help of a special operator construction (operator integral) which generalizes the classical M. Krein - M. Livshits - M. Brodskii integral.

**Martin Brühl**

Reconstruction of an Unknown Inclusion Using Electrical Impedance Tomography

In electrical impedance tomography currents are applied to a two-dimensional body and the resulting voltages are measured on the boundary. The goal is to use these (overdetermined) boundary data to reconstruct information about the distributed conductivity coefficient  $\sigma$  within the body. It is known that these boundary data (i.e., the Neumann-Dirichlet operator) uniquely define the conductivity coefficient provided that  $\sigma$  is, for example, piecewise analytic. This may correspond to the practical situation that the body consists of a number of inhomogeneities (organs) in a homogeneous background medium. We present a theoretical characterization of the domain of these inclusions (yet, under some restrictions on  $\sigma$ ) which is easily translated into a very cheap numerical algorithm for the reconstruction of their domain. Preliminary numerical results will be presented. (Ongoing work with M. Hanke and A. Kirsch.)

**Simon N. Chandler-Wilde**

Inverse scattering by rough surfaces : uniqueness results

We consider the problem of scattering of an acoustic wave by a sound soft, unbounded, one-dimensional rough surface. We review recent results on the unique solvability of the direct problem, for both plane and cylindrical wave incidence. We show that, in the cylindrical wave case (but not the plane wave case) the solution has a well-defined far-field pattern

and we prove a mixed reciprocity result. We then establish two uniqueness results for the inverse problem: that knowledge of the scattered field for a single incident wave is sufficient to identify the rough surface if the rough surface is a priori known to lie in a strip of width less than half a wavelength; and that, without a priori information, the rough surface can be identified by the response to sufficiently many cylindrical waves.

### Margaret Cheney

#### An Asymptotic Wave Interpretation of Sonar Images

This talk considers the inverse problem of determining the shape and medium parameters of the seafloor from sonar data. First we review beamforming techniques and the Kirchhoff (high-frequency) approximation. Next we carry out a stationary phase calculation for various measurement configurations associated with side-scan sonar and pencil-beam sonar. These results provide an interpretation of the information content of sonar images. The pencil-beam sonar, in particular, allows reconstruction of the shape and acoustic impedance of the seafloor.

### Klaus Giebermann

#### On the Numerical Solution of Three-Dimensional Inverse Scattering Problems in Homogeneous and Inhomogeneous Media

We consider the numerical aspects of the inverse obstacle scattering in  $\mathbb{R}^3$ . The goal is to recover the shape of one or more obstacles by the measured farfield  $u^\infty$ , where the corresponding wavelength is of the same order of magnitude as the diameter of the obstacles. Using the point source or the linear sampling method leads to an efficient and fast algorithm for this task. We present an adaptive algorithm based on the linear sampling method and show numerical examples for both homogeneous and inhomogeneous media.

### Natalia Grinberg

#### Inverse Scattering Algorithm for the Elastic Layered Medium

The ISP (inverse scattering problem) for the normally incident time-harmonic plane wave (longitudinal or transverse) is reduced to the ISP for the 1D wave equation (=1D-Helmholtz equation). This, in its turn, is decided by the method similar to the Zacharov-Shabat method for their system (or Gelfand-Levitan-Marchenko method for the Schrödinger equation). The main singular integral operator is, in case of wave equation, not only Fredholm, but can be estimated in the Hilbert space: its  $L_2$ -norm does not exceed the value  $\tanh(\frac{1}{2}Var \log c|_{-\infty}^{\infty})$  (hyperbolic tangence of the total variation of the logarithm of the wave velocity). Since this value is strictly less then 1, we can guarantee, that the series

which gives the solution to the GLM equation, converges fast in  $L_2$ . Hence, the ISP can be solved explicitly (for P- and S-waves). It gives two independent relations for three unknown elastic characteristics:  $\rho$ ,  $\lambda$  and  $\mu$ .

The scattering operator, which maps (definition) the function  $\sqrt{c} - \frac{1}{\sqrt{c}}$  to the scattering data  $\frac{r(k)}{ik}$  is the nonlinear generalization of the Fourier transform. The functions  $\sqrt{c} - \frac{1}{\sqrt{c}}$  and  $\frac{r(k)}{ik}$  satisfy a sort of uncertainty relation. As variation of the logarithm of velocity tends to zero, this relation tends to the Heisenberg uncertainty principle.

**Peter Hähner**

Uniqueness of the Shape of an Anisotropic Medium

We consider an inverse problem related to the following transmission problem for a penetrable, inhomogeneous, anisotropic obstacle  $D \subset \mathbb{R}^3$ : given the domain  $D$ , a positive constant  $\gamma$ , a positive definite matrix-valued function  $A$ , and an incident field  $v$  (i.e., a solution to  $\Delta v - v = 0$  near  $\overline{D}$ ), find fields  $u$  in  $D$  and  $w \in L^2(D_e)$  in the exterior  $D_e$  of  $D$  satisfying

$$\begin{aligned} \operatorname{div}(A\nabla u) - \gamma u &= 0 \quad \text{in } D, & \Delta w - w &= 0 \quad \text{in } D_e, \\ u|_{\partial D} - w|_{\partial D} &= v|_{\partial D}, & \partial u / \partial \nu_A - \partial w / \partial \nu &= \partial v / \partial \nu \quad \text{on } \partial D \end{aligned}$$

( $\partial u / \partial \nu_A := \nu \cdot A\nabla u$ ,  $\nu$  is the outward normal vector).

We prove for the corresponding inverse problem, namely to recover the domain  $D$  from Dirichlet data of the reflected fields  $w|_{\partial B}$  on a large sphere  $\partial B$ , that the Dirichlet data  $w|_{\partial B}$  for all possible incident fields uniquely determine  $D$  provided  $\gamma > 1$  and  $\xi \cdot A(x)\xi \geq \gamma|\xi|^2$  for all  $\xi \in \mathbb{R}^3$ ,  $x \in \overline{D}$ .

The main ingredients for the proof are the well-posedness of the direct transmission problem, a regularity theorem for solutions of the direct problem, and the well-posedness of the following interior transmission problem: given data  $f$  and  $g$  on  $\partial D$ , find  $u$  and  $v$  in  $D$  satisfying

$$\begin{aligned} \operatorname{div}(A\nabla u) - \gamma u &= 0 & \Delta v - v &= 0 \quad \text{in } D, \\ u|_{\partial D} - v|_{\partial D} &= f, & \partial u / \partial \nu_A - \partial v / \partial \nu &= g \quad \text{on } \partial D. \end{aligned}$$

Assuming  $\gamma > 1$  and  $\xi \cdot A(x)\xi \geq \gamma|\xi|^2$  for all  $\xi \in \mathbb{R}^3$ ,  $x \in \overline{D}$ , this interior transmission problem can be transformed into an equivalent fixed point equation which can be solved by the Banach fixed point theorem.

## Frank Hettlich

### A Second Degree Method for Inverse Obstacle Scattering Problems

The nonlinear inverse problem of recovering a scattering obstacle from measurements of the far field pattern is considered. As a possible improvement of known iterative schemes a new method is discussed based on the second degree Taylor expansion of this operator. The existence and a representation of the second domain derivative of the operator is shown for a certain scattering problem. The performance of the method is illustrated by several numerical examples.

## Thorsten Hohage

### Convergence of iterative regularization methods in inverse scattering

We considered the problem of reconstructing the shape of a scatterer from far field measurements of the scattered field corresponding to one incident plane wave and looked at the application of iterative methods to solve this problem. First we presented a simplification of an approach by Potthast to compute the boundary values of the derivative of the scattered field. Then we gave an interpretation of a logarithmic source condition as a closeness condition in Sobolev spaces. Such conditions are sufficient and almost necessary to obtain convergence rates that are logarithmic in the data noise level. Finally we presented and compared theoretical and numerical results for various methods such as Landweber iteration, Levenberg-Marquardt algorithm, Newton-CG method and the iteratively regularized Gauss-Newton method.

## Victor Isakov

### Inverse Problems in Acoustic Noise Detection

The problem of detection of acoustic noise in aircrafts can be posed as the linear integral equation

$$\int_{\Gamma} K(x, y) f(y) d\Gamma(y) = F(x), \quad x \in \gamma$$

where  $\Gamma$  is a boundary of a domain and  $\gamma$  of its subdomain where one can actually implement acoustic measurement.  $K(x, y)$  is a fundamental solution to the Helmholtz equation  $\Delta u + k^2 u = 0$  in this domain. We will discuss uniqueness, stability of  $f$  given  $F$  and numerical solution of the above integral equation.

## Andreas Kirsch

Characterization of the shape of the scattering obstacle by the spectral data of the far field operator

In this talk we study the inverse obstacle scattering problem for time harmonic plane waves under Dirichlet boundary conditions. We derive a factorization of the self-adjoint part of the far field operator  $F$  in the form  $-\frac{1}{2}(F + F^*) = GSG^*$ . Here,  $G$  is the solution operator of the exterior Dirichlet problem and  $S$  the real part of the single layer potential operator. We prove that the ranges of  $\sqrt{-\frac{1}{2}(F + F^*)}$  and  $G$  coincide and use this result to give an explicit characterization of the scattering obstacle which uses only the spectral data of  $\frac{1}{2}(F + F^*)$ .

## Gerhard Kristensson

Direct and Inverse Scattering for Transient Electromagnetic Waves in Nonlinear Media

This paper presents the solution to an inverse scattering problem in nonlinear media for electromagnetic waves. Specifically, the constitutive relations that model the nonlinear response to the electromagnetic field is assumed to be

$$D(x, t) = \epsilon_0 F(E(x, t)), \quad B(x, t) = \mu_0 H(x, t)$$

Here, the function  $F : R \rightarrow R$ . The inverse problem is to recover the function  $F$  from scattering data. The reconstruction algorithm that is presented is model independent. Moreover, two standard models for the nonlinearity are discussed—the Kerr and the saturated Kerr models. In this paper we prove that the use of incident and transmitted fields suffices to reconstruct the function  $F$ . A uniqueness theorem for a symmetric 2D nonlinear hyperbolic system is presented.

## Yaroslav V. Kurylev

Moments' methods for the inverse boundary problem for the acoustic and heat equation

This is a joint work with A. Starkov, K. Peat, M. Kawashita, and H. Soga. We consider an inverse spectral boundary problem for the acoustic operator  $A_\rho u = \rho^{-1} \Delta u$ ,  $u|_{\partial M} = 0$ ,  $M \in \mathbb{R}^m$ ,  $m \geq 2$  (in the talk  $m = 2$ ) and inverse boundary problem for the heat equation  $\rho u_t - \Delta u = 0$ ,  $u|_{t=0} = 0$ ,  $u_{\partial M \times (0, T)} = f$ . The inverse data used is a finite set of first eigenfunctions on the boundary,  $\frac{1}{\lambda_k} \partial_n \phi_k|_{\partial M}$ ,  $k = 1, \dots, N$  for the acoustic case and the values of  $\partial_n u^f|_{\partial M \times [0, T]}$  for a finite number of sources  $f$  for the heat equation.

We develop algorithms of an approximate reconstruction of  $\rho$  from these data, analyze their stability (assuming some measurement error) and discuss some numerical implementation of the scheme for the acoustic case.

## Christophe Labreuche

### On the stability of several ill-posed inverse scattering problems

The most challenging aspect in inverse problems comes from the fact that these latter are generally severely ill-posed. This implies the use of regularization techniques in order to numerically recover satisfactory reconstructions. Roughly speaking, regularization methods consist in approximating an ill-posed problem by a well-posed one (for instance by adding a term in the defect when the inverse problem can be turned into an optimization problem).

Yet, there is another way to remove the ill-posedness. By adding some a-priori information about the inverse problem, it can become stable. In this talk, we focus on the following two inverse problems: the recovery of the shape of a sound soft obstacle, and the recovery of the surface impedance on a known obstacle, from a knowledge of the far field pattern for the scattering of incident time-harmonic acoustic plane waves. In each case, an accurate and detailed study of stability must be carried out to determine this a-priori information that is sufficient to get stability. To this end, one has to examine how the error in the data propagates up to what we wish to recover. In these estimates, a particular care must be given on what is the dependency of the constants (involved in these estimates) upon what we wish to recover. We finally obtain some sharp estimates by using Carleman inequalities.

## Paul A. Martin

### Waves in wood: an inverse problem for telegraph poles

The title problem refers to the use of stress waves to inspect wooden poles for decay. The main part of the lecture was concerned with the modelling of the problem. Thus, it is usual to model wood as an orthotropic elastic solid. This gives a good *local* description: for example, at any point in a tree, three principal directions can be identified, namely longitudinal (along the grain), radial and tangential. Much is known about wave propagation in such solids.

However, the most obvious feature of the cross-section of a tree is the presence of annual rings. So, this structure should be taken into account if wave propagation through a tree or wooden pole is to be modelled properly. For this reason, the tree is modelled as a cylindrically orthotropic elastic solid, giving a *global* description of the wood.

The governing equations were given and simple explicit solutions for motions in a cross-sectional plane were presented. No explicit solutions for non-axisymmetric in-plane motions have been found to date, but a scheme using a generalized Frobenius method (using expansions as series of Bessel functions rather than powers) was outlined.

Finally, some remarks on the inverse problem were made. Thus, a cross-section of the pole can be thought of as annular, with the rotten region in the centre. So, the basic question is: what is the radius of the core?

This work was motivated by discussions with a small company in Manchester which has a stress-wave device for assessing the strength of decayed poles. However, it does not have any underlying theory, so it would like to understand why the device works!

**Christine De Mol**

Wavelets for inverse problems

We analyze under which circumstances and to what extent it is possible, in imaging experiments, to get "super-resolution", i.e. to supersede the classical Rayleigh resolution limit of half a wavelength of the probing radiation. To discuss this problem, we use the simple example of the so-called inverse diffraction problem from plane to plane. In the case of near-field imaging, we show that the use of the information conveyed by evanescent waves allows to increase considerably the classical far-field limit and we assess the achievable resolution as a function of the signal-to-noise ratio and of the distance between the source and measurement planes. In the case of far-field imaging, we show that super-resolution can be achieved through the use of prior knowledge about the solution and in particular about its support. We demonstrate, however, that a significant resolution improvement can only be obtained when the space-bandwidth product is very small, i.e. for subwavelength sources. We recall some regularized inversion algorithms which can be used to compute the corresponding solutions and we propose a wavelet-based generalization of the Gerchberg-Papoulis algorithm, which is expected to have enhanced super-resolution capabilities for spatially inhomogeneous objects.

**Adrian I. Nachman**

Numerical inversion using eigenfunctions of the scattering operator

We describe a numerical procedure for reconstructing the variable speed of sound from scattering measurements at fixed frequency. A regularized version of the problem leads, by a Lagrange multiplier calculation, to an expansion in terms of products of retransmitted fields of eigenfunctions of the far-field operator. When linearized around a constant background this expansion yields a fast inversion procedure analytically equivalent to the filtered backpropagation formula of Devaney and Beylkin. We show quantitative imaging of inhomogeneous media, nonlinear inversion when the Born approximation fails, reconstruction from data corrupted by noise, as well as from experimental data.  
(Joint work with T. D. Mast, R. C. Waag, C. Drager, F. Lin.)

## Petri Ola

### Recovering Singularities from Backscattering in $\mathbb{R}^2$

Consider the Schrödinger operator

$$H_q = (-\Delta + q(x) - k^2) \quad \text{in } \mathbb{R}^2.$$

Let  $u(x, \theta, k) = e^{ik(\theta, x)} + C(2)|x|^{-1/2}A(\hat{x}, \theta, k) + o(|x|^{-1/2})$  be an outgoing solution of  $H_q u = 0$  corresponding to plane wave  $e^{ik(\theta, \cdot)}$ . We prove the following theorem:

Theorem: If

$$q_1, q_2 \in H_\delta^{s_0}, \quad \delta > \frac{2s_0^2 + 9s_0 + 3}{2s_0 - 1}$$

and they have the same backscattering data  $\{A_j(\theta, -\theta, k); k \in \mathbb{R}, \theta \in S^1\}$ , then

$$q_1 - q_2 \in H_{loc}^{\min\{1, \frac{1+5s_0}{4}\}} + BC(\mathbb{R}^2).$$

The proof relies on an recent characterization of  $W^{1,p}(\mathbb{R}^2)$ -functions due to Hajlasz (96) ( $1 < p \leq \infty$ ) and Triebels's maximal inequality.

(Joint work with L. Päivarinta, and V. Serov.)

## Michele Piana

### Regularized Sampling Method for Solving Inverse Scattering Problems in the Resonance Region

The regularized sampling method is a fast and simple algorithm for the solution of non linear inverse scattering problems. The application of the method requires no low- or high-frequency approximation and no apriori information about the number or the nature of the scatterers. The method provides an estimate of the support of the scatterer and not the point values of the refraction index. The implementation of the algorithm is based on the solution of a linear Fredholm integral equation of the first kind. Since this is a severely ill-posed problem Tikhonov regularization method with generalized discrepancy principle is adopted to reduce the numerical instability. An interesting feature is that the regularization parameter can be used as reliable indicator of the support of the scattering object.

## **Roland Potthast**

On mixed reciprocity relations in inverse scattering.

Reciprocity relations are of great importance in the treatment of direct and inverse scattering problems. Mixed reciprocity relations relate the far field pattern of incident point-sources with the scattered field for incident plane waves. Here we derive mixed reciprocity relations for some acoustic and electromagnetic scattering problems and show their applicability to solve inverse obstacle scattering problems. Questions of uniqueness, stability and reconstruction are investigated.

## **Wolfgang Rieger**

Inverse electromagnetic medium scattering: reconstruction of two-dimensional inhomogeneous anisotropic objects

We consider the inverse scattering problem of reconstructing the permittivity and conductivity tensors of inhomogeneous anisotropic or biaxial cylindrical objects. The material properties are reconstructed using scattering data from time-harmonic electromagnetic plane waves with the electric field vector perpendicular to the cylinder axis (TE-polarization). The inverse scattering problem formulated as nonlinear optimization problem is numerically solved using a variable metric method. This method involves exact first-order gradients. Since the inverse scattering problem is severely ill-posed, a regularization term is used to ensure stability and uniqueness of the solution. We apply either the total variation penalty method as regularization functional or Tikhonov regularization. Numerical examples were presented for both the biaxial and anisotropic case. (Joint work with Günther Lehner and Wolfgang M. Rucker.)

## **Erkki Somersalo**

Dynamical Inverse Problems

The talk is a review of joint works with D. Baroudi, J. Kaipio and M. Vauhkonen. The problem considered here is to solve inverse problems where the parameters to be determined change in time during the measurements. Two examples are discussed in more detail: In electrical impedance tomography (EIT), one seeks to estimate the internal impedance distribution of a body by current/voltage measurements on the surface of the body. The typical procedure is to apply a full set of linearly independent current patterns on the body and measure the corresponding voltages (this set of measurements is a full frame). If the number of electrodes is large, the body may change during this measurement procedure. As an example, one can think of heart beats in medical imaging applications or fluid flow in process tomography.

The second example deals with temperature distribution monitoring by impedance measurements: A set of electrically conducting wires are spanned across the space whose temperature is being monitored. The temperature changes of the wires cause changes in the resistivity and so the impedance measurements can be used indirectly to temperature monitoring. The dynamical nature of this inverse problem comes from the possibly rapid temperature changes in the room as well as from the heat diffusion along the wires.

In the proposed approach, the time evolution of the target is modelled as a stochastic differential equation that leads in the time discretization to a Markov model. The indirect measurement is modelled as a noisy linear observation model. Put together, we have a Kalman-Bucy model where we can apply classical Kalman filtering techniques to estimate the state of the system. A novel feature in our application is a spatial regularization method of the Kalman filtering.

### David Wall

#### Signal Restoration after transmission through an advective and diffusive medium

The human bodies response to metabolic stress is by the release of glucocorticoids. One of the measurements carried out in an attempt to understand more about the hormonal pathways is to measure adrenocorticotrophic hormone (ACTH) release from the pituitary gland.

Some interesting inverse problems involving advection diffusion equations arise from this measurement. We discuss one of these problems and the methods we are using to solve it. We consider in particular an inverse problem associated with mass transport of material concentration down a tube when the flowing medium has a two-dimensional velocity profile. The concentration is measured downstream and from this the temporal variation of the concentration upstream is to be estimated.

### Frank Wübbeling

#### Numerical schemes for Time-Harmonic acoustical inverse scattering in three dimensions

We consider the time-harmonic wave equation in two or three dimensions. Our goal is to reconstruct the scattering potential  $q(x)$  in the Helmholtz-equation

$$\Delta u + k^2(1 + q(x))u = 0$$

from measurements of the wave field  $u = u^i + u^s$  that is produced by different incoming plane waves  $u^i$ .  $u$  is measured on the boundary of some subset  $\Omega$  which is known to contain the support of  $q$ .

In the application of breast cancer detection, the objects are of size  $80\lambda$  where  $\lambda$  is the wavelength, assuming standard ultrasound devices. Because of this, we need a solution to

the nonlinear problem. Also, since tumors and cysts can only be identified by comparing their attenuation, the imaginary part of  $q$  also has to be recovered.

We propose an algorithm for this problem that is numerically manageable in 3D and delivers the expected resolution of  $\lambda/2$ . It is based on a nonlinear version of Kaczmarz' algorithm and employs a technique for solving the initial value Helmholtz problem. Numerical experiments for two and three dimensions are shown.

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