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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Arbeitsgemeinschaft mit aktuellem Thema: The Nilpotence Theorem in Stable Homotopy Theory

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Organizers: Hans-Werner Henn (Strasbourg)
Wilhelm Singhof (Düsseldorf)

The aim of this Arbeitsgemeinschaft was to understand the proof of the "Nilpotence Theorem" in stable homotopy theory which had been conjectured by Ravenel around 1976 and was proved by Devinatz, Hopkins and Smith in 1988. It describes one of the rare occasions where an important geometric property is faithfully reflected in algebra by means of a homology theory.

A generalized homology theory is a functor h_* from a topological category (for example the category $\mathcal{CW}$ of CW-spaces and continuous maps between them) to an algebraic category (for example the category $\mathcal{AB}_*$ of graded abelian groups). Such a functor is required to have a number of properties familiar from ordinary homology: in particular, if a map $f : X \to Y$ is homotopic to a constant map, then the induced map $h_*f : h_*X \to h_*Y$ is trivial, and if ΣX denotes the suspension of X, then f induces a map $\Sigma f : \Sigma X \to \Sigma Y$, and $h_*(\Sigma f)$ coincides with h_*f up to a dimension shift.

Complex bordism theory MU_* is a generalized homology theory which is important for a number of reasons: it is conceptually interesting because of its close relation with the theory of formal groups, and concrete calculations with this theory are often possible.

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Consider a map $f : \Sigma^d X \to X$ from the d-fold suspension of X to X; any such map is called a self-map of X. We can "iterate" f; then we write $f^r := f \circ \cdots \circ \Sigma^{(r-2)d} f \circ \Sigma^{(r-1)d} f : \Sigma^{rd} X \to X$ and call this the r-th power of f. We say that f is nilpotent if a sufficiently high suspension of a sufficiently high power of f is null homotopic.

The following striking theorem is what is commonly known as the "Nilpotence Theorem" (or at least one form of it):

Theorem 1. *Let $f : \Sigma^d X \to X$ be a self-map. Then f is nilpotent if and only if $MU_*(f)$ is nilpotent.*

Only one special case of this theorem was known to be true before the proof of the general result by Devinatz, Hopkins and Smith, namely Nishida's Nilpotence Theorem (1973):

Theorem 2. *Let $f : \Sigma^d S^n = S^{d+n} \to S^n$ be a self-map of the n-sphere with $d \geq 1$. Then f is nilpotent.*

An important theoretical consequence of the general Nilpotence Theorem, Theorem 1, is the Thick Subcategory Theorem.

Definition. *A full subcategory C of CW is called thick if it satisfies the following conditions:*

1. *If X is in C and Y is homotopy equivalent to X then Y is in C.*
2. *If $X \vee Y$ is in C, then X and Y are in C.*
3. *If $X \to Y \to Z$ is a cofibre sequence and if two of the spaces X ,Y and Z are in C then so is the third.*

The Thick Subcategory Theorem gives a complete classification of the thick subcategories of the stable homotopy category of all p-local finite complexes, where p is a prime number. (A CW-space is called p-local finite if it is the p-localization of a finite complex.) These subcategories admit a simple description in terms of what MU_* does to the spaces belonging to them.

The Thick Subcategory Theorem is one of the tools needed to prove the Periodicity Theorem, which was the second of the two main theorems to which the Arbeitsgemeinschaft was dedicated. This theorem asserts the existence of periodic self maps (which are very far from being nilpotent) and is best stated in terms of the Morava K-theories $K(n)$. For a fixed prime number p there exists a sequence of periodic homology theories $K(n)_*$, $n = 1, 2, \ldots$ which in some sense generalize complex K-theory; in fact $K(1)_*$ is very closely related to complex K-theory. It is convenient to define $K(0)_*$ to be equal to rational homology for every prime p. The Periodicity Theorem reads as follows:

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Theorem 3. *If X is a p-local finite CW-complex and n is minimal among those non-negative integers such that $\overline{K(n)}_*X \neq 0$, then for some suitable d there exists a stable self map $f : \Sigma^d X \to X$ with the following properties:*

1. *f induces an isomorphism $K(n)_*f : K(n)_*\Sigma^d X \to K(n)_*X$.*

2. *For any $m > n$ the map $K(m)_*f : K(m)_*\Sigma^d X \to K(m)_*X$ is trivial.*

Moreover, up to taking sufficiently high powers this map is stably unique.

Before this theorem was proven, only very few periodic self maps were known. They had been discovered by Adams, Toda and Larry Smith. Each of them had led to an infinite family of non-trivial elements in the stable homotopy groups of spheres. Although the periodicity theorem has not produced so far further families, it provides some new systematic insight into the stable homotopy groups of spheres and the stable homotopy category of finite CW–complexes.

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ABSTRACTS OF THE TALKS PRESENTED

WOLFGANG K. SEILER, MANNHEIM:

Introduction

First some classical results on homotopy groups of spheres were summarized: Hurewicz's theorem, the Hopf map, Serre's finiteness theorem, Freudenthal's suspension theorem. Stable homotopy groups were defined, Nishida's theorem mentioned, Whitehead's J–homomorphism from the stable homotopy of SO_n to the stable homotopy of S^n constructed and its image described.

In the second part of the talk, nilpotent and periodic self-maps were defined, and the Nilpotence Theorem and the Periodicity Theorem were stated. Finally, the classical examples α, β and γ of periodic maps were described and applied to the construction of infinite families $\{\alpha_t\}_{t\in\mathbf{N}}$, $\{\beta_t\}_{t\in\mathbf{N}}$ and $\{\gamma_t\}_{t\in\mathbf{N}}$ of nontrivial elements in the stable homotopy of spheres.

HENNING KRAUSE, BIELEFELD:

Spectra

In my lecture I introduced the stable homotopy category of spectra. First I explained the Spanier–Whitehead category as stabilization of the usual homotopy category of CW-spaces and listed a number of basic properties, e.g. the triangulated structure and the smash product. To introduce the category $\mathcal{S}$ of all spectra, I followed Margolis' exposition and listed five properties which determine $\mathcal{S}$ up to phantom maps. I also gave a sketch of the construction of $\mathcal{S}$, following Adams' classical exposition. Finally, I discussed some examples (suspension spectra, Moore spectra, Eilenberg–Mac Lane spectra) and presented the representability theorems of Brown and Adams for (co)homological functors from $\mathcal{S}$ into the category of abelian groups.

DAGMAR M. MEYER, PARIS:

Localization of spectra

First we discussed localization of spectra at a single prime p, which is given simply by the smash product with the Moore spectrum $M(\mathbb{Z}_{(p)})$, where $\mathbb{Z}_{(p)}$ denotes the integers localized at p.

Then the concept of localization in an arbitrary category $\mathcal{C}$ with respect to a class of morphisms $\mathcal{W}$ in $\mathcal{C}$ was introduced. We specialized to the case where $\mathcal{C} = \mathcal{S}$ and $\mathcal{W} = \{f \in \operatorname{mor}\mathcal{S} \mid E_*f \text{ is an isomorphism }\}$ for a fixed spectrum E. Following the approach presented in Margolis' book "Spectra

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and the Steenrod algebra" I sketched a proof of the existence of a functorial localization with respect to $\mathcal{W}$ in this specific case.

Next we looked at some of the more important properties of this "E_*-localization". I also introduced the class $\mathcal{A}(\mathcal{S})$ of all Bousfield classes of spectra and described the partial ordering on $\mathcal{A}(\mathcal{S})$.

Finally, E_*-localization in the special case $E = M(G)$ was discussed. Here $M(G)$ is the Moore spectrum corresponding to the abelian group G. I introduced types of acyclicity of abelian groups and presented Bousfield's explicit description of the localization $X_{M(G)}$ for an arbitrary spectrum X.

BERNHARD HANKE, MÜNCHEN:

The Adams spectral sequence

In this talk I explained the construction and some of the properties of the Adams spectral sequence, which is an important tool in the calculation of homotopy groups. Starting with two spectra X and Y and a commutative ring spectrum E one constructs a "fibre tower" over Y which, after applying the functor $[X, -]$, gives rise to an exact triangle. The associated spectral sequence has an E_2-term which in many cases can be described in purely algebraic terms involving E_*E, E_*X and E_*Y and using derived functors in the category of comodules over an Hopf algebroid with certain flatness properties. Convergence of the Adams spectral sequence was discussed relying on the results in Bousfield's paper on localization of spectra with respect to homology, where fairly general convergence statements are given. Under favourable circumstances one can expect the Adams spectral sequence to converge to $[X, Y_E]$, where Y_E denotes the Bousfield E_*-localization of Y. Multiplicative structures of the Adams spectral sequence were briefly mentioned.

MARKUS SZYMIK, GÖTTINGEN:

The Steenrod algebra

The Steenrod algebra $\mathcal{A}^*$ was defined as the endomorphism algebra of classical cohomology with coefficients in $\mathbb{F}_2$. To show non-triviality of $\mathcal{A}^*$, the Steenrod squares Sq^i were constructed via the description of cohomology classes as maps into Eilenberg-MacLane spaces. The Sq^i are generators of $\mathcal{A}^*$ as an algebra. The so-called Adem relations were used as a motivation for the description of a basis of $\mathcal{A}^*$ given by Cartan and Serre. The Cartan formula is related to the fact that $\mathcal{A}^*$ is a Hopf algebra. Then Milnor's results on the multiplication and comultiplication in the dual Hopf algebra $\mathcal{A}_*$ in terms of the elements ξ_i were described. Although the situation at the prime

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two is emphasized, the results for odd primes were also mentioned in the end, so that the Margolis elements could be defined in the general setting.

MANFRED LEHN, GÖTTINGEN:

BU and MU – definitions and basic properties

The classifying spaces $BU(n)$ were introduced and identified as colimits of complex Grassmannians. BU was defined as colimit of $BU(n)$'s and its classifying property for K-theory was stated. We sketched a proof of how to reduce the K-theoretic version of Bott periodicity to the statement that an explicitly (in terms of matrices) given map $S^1 \times BU \to SU$ is a homotopy equivalence. The Thom spaces $MU(n)$ and the classical Thom spectrum MU were introduced. Finally, we proved the Pontryagin-Thom isomorphism $\Omega_*^U \cong \pi_* MU$.

THOMAS LEHNKUHL, GÖTTINGEN:

The (co-)homology of BU and MU

Theorem *(a) $H^*(BU;\mathbb{Z})$ is the polynomial ring $\mathbb{Z}[c_1, \ldots]$. Moreover it has a canonical Hopf algebra structure.*
(b) $H_*(BU;\mathbb{Z})$ is just the dual of $H^*(BU;\mathbb{Z})$.
(c) $H_*(MU;\mathbb{Z})$ is isomorphic to $H_*(BU;\mathbb{Z})$ as an algebra and $H^*(MU;\mathbb{Z})$ is isomorphic to $H^*(BU;\mathbb{Z})$ as a coalgebra.

We also give an explicit formula for the cooperation of the dual Steenrod algebra on $H_*(MU;\mathbb{Z}/p)$.

HOLGER REICH, GÖTTINGEN

Computation of $\pi_* MU$

Using results of the previous lecture we determined the structure of $H_*(MU)$ as a comodule over the dual Steenrod- algebra. We computed the E_2-term $\mathrm{Ext}_{A_*}^{s,t}(\mathbb{Z}/p, H_*(MU))$ of the corresponding Adams spectral sequence and finally proved that $\pi_*(MU)$ is a polynomial ring with one generator in each even degree.

VICTOR V. BATYREV, TÜBINGEN:

MU and formal group laws

Let E be a ring spectrum. Using the canonical classifying mapping $\mathbb{C}P^\infty \times \mathbb{C}P^\infty \to \mathbb{C}P^\infty$, one defines a formal group law $F(x,y) = \sum a_{i,j} x^i y^j \in$

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$\pi_*(E)[[x,y]]$ over $\pi_*(E)$, if an orinetation $x^E \in \tilde{E}^\bullet(\mathbb{C}P^\infty)$ has been chosen. If $E = MU$ and the orientation $x^{MU} \in \widetilde{MU}^2(\mathbb{C}P^\infty)$ is defined by $\omega : \mathbb{C}P^\infty = BU(1) \to MU(1)$, then one obtains a formal group law $F_{MU}(x,y)$ over $\pi_*(MU)$. There is a universal group law over the lazard ring L so that one obtains the homomorphism $\theta : L \to \pi_*(MU)$. The theorem of Quillen says that θ is an isomorphism. Moreover, one gets an embedding $L \hookrightarrow \mathbb{Z}[b_1, \dots, b_n, \dots]$ which can be identified with the homology ring $H_*(MU)$. The variables $b_1, b_2, \dots, b_n$ may be considered as coefficients of a formal power series $\sum_{i \geq 0} b_i x^{i+1} = \exp x$. One obtains the logarithm of the formal group F_{MU} as $\log y = \sum_{i \geq 0} m_i y^{i+1}$, where the m_i can be expressed using the coefficients b_i. The ring $MU_*MU = \pi_*(MU \wedge MU)$ can be considered (together with the structure map $\pi_*MU \to MU_*MU$) as a Hopf algebroid, whose structure mappings $\epsilon, c, \eta_R, \eta_L$, and Δ can be explicitly determined in terms of their values on $\{m_i\}$ and $\{b_i\}$.

Florence Lecomte, Strasbourg:

Brown–Peterson theory BP; properties of MU_*MU and BU_*BU comodules (part I)

The talk deals with the localization at the prime p of the Lazard ring L and MU-theory introduced in previous lectures. We define p-typical formal group laws and show that they are classified by a direct summand V of $L_{(p)}$. We introduce the Brown-Peterson spectrum BP as a retract of $MU_{(p)}$ verifying $\pi_*(BP) = V$ and describe the Hopf algebroid (BP_*, BP_*BP). We finally state two theorems by Landweber; one which lists all invariant ideals of BP_* and the filtration theorem which describes the structure of BP_*BP-comodules which are finitely presented as BP_*-modules.

Michael Joachim, Münster:

Brown–Peterson theory BP; properties of MU_*MU and BU_*BU comodules (part II)

We presented applications of the Landweber filtration theorem. In particular we proved the Landweber exact functor theorem on $\mathcal{C}$, the category of comodules over BP_*BP that are finitely presented as BP_*-modules. The latter was used to define a family of full subcategories $\mathcal{C}_n$ in $\mathcal{C}$. These subcategories are thick, i.e. if $0 \to M' \to M \to M'' \to 0$ is a short exact sequence in $\mathcal{C}$ then M is in the subcategory if and only if M' and M'' are. We then showed the Thick Subcategory Theorem for $\mathcal{C}$ which says that the $\mathcal{C}_n$ and the trivial subcategory consisting just of the zero module are actually all non-trivial

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thick subcategories in C. After that we turned to applications for spaces and spectra. We used the Landweber exact functor theorem to define homology theories $E(n)_*$, and we discussed the counterpart of the Thick Subcategory Theorem for the category FH of p-local finite CW-complexes. The latter led us to a tower of maps

$$X \longrightarrow \quad \cdots \to L_n X \to L_{n-1} X \to \cdots \to L_0 X$$

where $X \in FH$ and L_n denotes the Bousfield localization of X with respect to $E(n)_*$. We then presented the Chromatic Convergence Theorem saying that $X \simeq \operatorname{holim} L_n X$ for all $X \in FH$ and concluded the talk with the definition of the algebraic chromatic filtration on $\pi_*(X)$.

NORA GANTER, BONN:

Morava K-theories, the Nilpotence Theorem in terms of Morava K-theories, and the Thick Subcategory Theorem in stable homotopy theory (part I)

Our first aim is to construct a tower of BP-quotient spectra, i.e. BP-module spectra that arise via a tower of cofibre sequences

$$\begin{tikzcd}
BP =: P(0) \arrow[r] & P(1) \arrow[r] \arrow[dl] & P(2) \arrow[r] \arrow[dl] & P(3) \arrow[r] & \cdots \\
P(0) \arrow[u, "v_0=p"] & P(1) \arrow[u, "v_1"] & P(2) \arrow[u] & &
\end{tikzcd}$$

where the vertical arrows are given by multiplication with generators of BP_*. Since these form a regular sequence, the corresponding coefficient rings have the form

$$P(n)_* = BP_* / < v_0, \ldots, v_{n-1} > .$$

The construction uses Baas-Sullivan theory of bordism with singularities, which gives rise to a similar tower of MU-module spectra. Localizing at p gives the above tower.

The example of dividing out by one bordism class of $MU_*(-)$ is discussed in detail.

Spanier-Whitehead duality and its basic properties are briefly introduced.

BIRGIT RICHTER, BONN:

Morava K-theories, the Nilpotence Theorem in terms of Morava K-theories, and the Thick Subcategory Theorem in stable homotopy

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theory (part II)

After we have seen how to construct quotient spectra of BP via the Baas-Sullivan method we prove some properties of $P(n)$ and of the connective spectra for Morava k-theory, $k(n)$. We prove that the mod p cohomology of a quotient spectrum of BP is a quotient of the Steenrod algebra by left ideals generated by elements corresponding to the remaining generators in the homotopy of the quotient spectrum. We sketch a proof that the spectra $P(n)$ are ring spectra: Shimada and Yagita constructed a pairing for quotient spectra of MU which yields a pairing on $P(n)$. We give arguments why the phantoms that could be an obstruction to a ring structure vanish in this case. Similar arguments show that the $P(n)$ and the $k(n)$ are BP-module spectra.

The construction of the spectrum $K(n)$ is done by an exact functor theorem for the $P(n)$ due to Yagita.

The $K(n)$ with coefficients $K(n)_* = \mathbf{F}_p[v_n, v_n^{-1}]$ are BP-module spectra, and they have a ring structure. The Bousfield class of $K(n)$ is the same as the one for $B(n)$. This is used to prove that the Bousfield class of $P(n)$ is the same as that of $K(n) \vee P(n+1)$.

TILMAN BAUER, BONN:

Morava K-theories, the Nilpotence Theorem in terms of Morava K-theories, and the Thick Subcategory Theorem in stable homotopy theory (part III)

The third part concentrates on the derivation of the Morava K-theoretic formulation of the Nilpotence Theorem from the MU version and the proof of the Thick Subcategory Theorem. First, we introduce the notion of fields in the category of spectra with Morava K-theories as examples. We show that the Künneth theorem holds for fields. Then, using the universal coefficient spectral sequence for the module $K(n+1)$ over E, where $E_*(Y) := E(n+1)_* \otimes_{BP_*} P(n)_*Y$, we prove that for finite spectra X, the vanishing of the $(n+1)$st Morava K-theory implies the vanishing of the nth. The Nilpotence Theorem states that if f is a map from the sphere spectrum to any spectrum which induces the zero morphism in all Morava $K(n)$-theories then it is smash nilpotent. We reduce this problem to the MU-theoretic version which is proved in later talks.

Then we show that any thick subcategory of the category of p-local finite spectra is one of a countable set $C_0 \supseteq C_1 \supseteq \dots$ of subcategories determined by the vanishing of $K(n)$-theory.

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Anton Deitmar, Heidelberg:

The Periodicity Theorem (part I)

The Periodicity Theorem answers the question for which spectra the periodicity of the nth Morava K-homology $K(n)_*$ given by the element v_n can be realized by self-maps.

A self-map v of a spectrum is called a v_n-map if some power v^i of v gives some power of v_n on $K(n)_*X$ and v^i gives zero on $K(m)_*X$ for all $m \neq n$. At first it is shown that such maps are essentially unique, i.e. given two v_n-maps v, w on X there exist $i, j \in \mathbb{N}$ such that $v^i = w^j$. Next the Periodicity Theorem is stated. This theorem says that for a p-local finite spectrum X there is a v_n-map if and only if X is annihilated by $K(m)_*$ for $m < n$. The "only if" part is shown and the fact that the category of all X admitting a v_n-map is thick. By the Thick Subcategory Theorem this reduces the proof of the Periodicity Theorem to the construction of one particular v_n-map. This construction is performed in the next two talks. Finally a brief discussion of the Telescope Conjecture is given. It is shown that the Telescope Conjecture implies the partial coincidence of the geometric and the algebraic chromatic filtration.

Alexander Schmidt, Heidelberg:

The Periodicity Theorem (part II)

In the first part we introduced the notion of an N-endomorphism of a subcategory $\mathcal{C} \subseteq \mathcal{C}_0$, where $\mathcal{C}_0$ is the category of p-local finite spectra and $\mathcal{C}$ is assumed to be closed under suspensions. We presented a classification theorem for these N-endomorphisms and explained shortly how to derive this from the Thick Subcategory Theorem.

In the second part of the talk we explained how to construct a so-called v_n-self-map on a spectrum X which is "strongly type n" after Ravenel. We gave a series of technical lemmas around the Adams spectral sequence which are needed in the construction process and we tried at least to indicate why the constructed self-map has the required properties.

Raphael Albrecht, Bonn:

The Periodicity Theorem (part III)

The theorem is concluded by exhibiting a p-local finite spectrum $X \in \mathcal{C}_n$, i.e. of type n. There are three steps (in the exposition I've restricted myself to the case $p = 2$):

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(i) A partially type n-spectrum X is constructed, i.e. such that the Margolis elements P^0_t, for all $0 < t \le n$, act nontrivially on $H^*(X;\mathbb{F}_2)$, P^0_{n+1} does act trivially on $H^*(X;\mathbb{F}_2)$, and $\mathrm{rk}_{\mathbb{F}_2} H^*(X;\mathbb{F}_2) = \mathrm{rk}_{K(n)_*} K(n)^*(X)$.

(ii) For any m we find integers $0 < k_m \le k_{m-1} \le \ldots \le k_1$ such that for $k = \sum k_i$ there exists a nontrivial idempotent e_m in the ring $\mathbb{Z}_{(2)}[\Sigma_k]$. Moreover, we show that for a vector space V over $\mathbb{F}_2$ the subspace $e_m V^{\otimes k}$ is nontrivial if and only if $\dim_{\mathbb{F}_2} V \ge m$.

(iii) Let X be partially type n, then for all $l > 0$, $H^*(X^{\wedge l})$ contains a nontrivial direct summand which is a free module over the algebra generated by the Margolis elements P^s_t for $0 \le s < t \le n$. We use (ii) and an appropriate choice of l, m, k such that $H^*(e_m X^{\wedge lk})$ is a nontrivial free module over this algebra. (Here $e_m X^{\wedge lk}$ is the homotopy colimit of the infinite iteration of the map induced by e_m.) Finally we conclude by noting that this module occurs as a direct summand. $Y := e_m X^{\wedge lk}$ is thus strongly of type n and by the results of part II of type n.

DON STANLEY, BERLIN:

The proof of the Nilpotence Theorem (part I)

We introduced the James construction JX which is a model for $\Omega\Sigma X$. The James-Hopf maps $h_k : JS^{2n} \to JS^{2nk}$ were constructed. The fibres of $h_{p'}$ were shown to be $J_{p'-1}S^{2n}$. We introduced the configuration spaces $C(\mathbb{R}^n, X)$ which are models for $\Omega^n\Sigma^n X$. $C(\mathbb{R}^n, X) = \mathrm{colim}_j F_j C(\mathbb{R}^n, X)$, where $F_j C(\mathbb{R}^n, X)$ denotes the union of the configurations of at most j points. Let $D_j X = F_j C(\mathbb{R}^n, X)/F_{j-1}C(\mathbb{R}^n, X)$. We showed that stably $\Omega^n\Sigma^n X \simeq \bigvee_{j\ge 0} D_j X$ (Snaith splitting). We described $H_*(\Omega^2 S^{2n+1};\mathbb{F}_p)$ and the image $H_*(D_j S^{2n-1};\mathbb{F}_p) \subset H_*(\Omega^2 S^{2n+1};\mathbb{F}_p)$ under the inverse of the Snaith splitting map. We constructed a map $D_j \to H\mathbb{Z}/p$ and showed that this map has connectivity which increases with j.

VOLKER EISERMANN, BONN:

The proof of the Nilpotence Theorem (part II)

The composition, the smash-product and the ring spectrum version of the Nilpotence Theorem have been introduced. It has been shown how the other two versions follow from the ring spectrum version. Two sequences of spectra have been described which were used in the proof. The sequence $X(n)$ has the

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properties $X(0) = S$, $X(\infty) := \mathrm{hocolim}_n X(n) = MU$. The sequence $G_i(n)$ has the properties $G_0(n) = X(n)$, $G_\infty(n) := \mathrm{hocolim}_i G_i(n) = X(n+1)$. The two main steps in the proof of the Nilpotence Theorem (ring spectrum version) have been outlined. The first (minor) step has been carried out. For this the Adams spectral sequence for $G_i(n)$ based on $X(n+1)$ has been discussed. The crucial step was the existence of a vanishing line for this spectral sequence.

DAVID J. GREEN:

The proof of the Nilpotence Theorem (part III)

In my talk I completed the proof of the Nilpotence Theorem by showing that the Thom spectra $X_{p^k-1}(n)$ and $X_{p^{k+1}-1}(n)$ have the same Bousfield class, following the lecture notes of Hopkins and Ossa. This involves a sequence of Thom spectra Y_r^ξ going from $X_{p^k-1}(n)$ $(r=0)$ to $X_{p^{k+1}-1}(n)$ $(r = p-1)$. The space Y_r is defined to be a covering of the rth space J_r in the James model for ΩS^{2np^k+1}. A self-map b of Y_0^ξ is constructed in a way analogous to the construction of the Wang sequence of a fibration over a sphere. The desired result is equivalent to the telescope of b being contractible.

To show this, an action of $\Omega^2 S_+^{2np^k+1}$ on Y_0^ξ is constructed, and b is shown to be multiplication by a homotopy class α of $\Omega^2 S_+^{2np^k+1}$. This double loop space has a multiplicative Snaith splitting, α factors through the weight p piece, and the N-fold smash product of α factors through the weight Np piece D_{Np}. Hence b^{N+1} factors through $D_{Np_*}(b)$. But the D_{Np} are asymptotically Eilenberg-MacLane spectra, and b is zero in homology, proving the contractability of the telescope.

Report written by Dagmar M. Meyer

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