Tagungsbericht 42/1998

Modeltheory

October 25 - October 31, 1998

This conference was organized by J. Ershov (Novosibirsk), A. Prestel (Konstanz) and M. Ziegler (Freiburg). There were 19 plenary sessions, most of the talks belonging to one of the following topics:

- Local Global Principles for Fields and Rings
- Fields of finite Morley Rank and Bad Fields
- Valuation Theory and p-adic Fields

A. Baudisch :

A free pseudospace

(by Andreas Baudisch and Anand Pillay)

We construct a non CM-trivial stable theory in which no infinite field is interpretable. In fact our theory will also be trivial and ω-stable, but of infinite Morley rank. A long term aim would be to find a non CM-trivial theory which has finite Morley rank and does not interpret a field. The construction is direct, and is a "3-dimensional" version of the free pseudoplane.

Building the "free pseudospace" turns out to be a little less of a trivial matter than we had first thought, although it is quite naive. Hrushovski has pointed out to us that our example is related to an unpublished example due to himself of a stable non-equational theory.

L. Darnière :

Local-Global Principle for rings

Let us say that a ring A with fraction field K satisfies the *Local-Global principle with respect to its henselisations (LGPH)* if and only if:

i. For every non zero a in A, the set of all maximal ideals of A wich do not contain A is quasi-compact (with respect to Zariski's topology).

ii. Every variety defined over K having a smooth point in the henselisation of each valuation ring centered at a maximal ideal of A, has a smooth point in A.

It appears that this property is first order and provides a general and very flexible framework for the model-theoretic study of rings. Specializing this general study to some particular cases leads to natural notions of PAC, PRC or PpC rings, which applies to various "concrete" examples of rings with fraction fields algebraic over $\mathbb{Q}$ or $F_p[T]$. Let us give two exemples of applications, among others:

a. The ring of all absolutely p-adic integers – that is those algebraic integers which belong to $\mathbb{Q}_p$ and such that all their conjugates over $\mathbb{Q}$ still belong to $\mathbb{Q}_p$ – has a decidable complete theory.

b. A PAC domain A of positive characteristic, that is a domain satisfying $(LGPH)$ with a PAC fraction field of positive characteristic, has a decidable theory if and only if its fraction field has so. If moreover the theories of the fraction field and the residual lattice of A are model-complete (the residual lattice associated to any ring is an interpretable structure which has been introduced by Prestel and Schmid, it plays a key role for the model theory of rings satisfying $(LGPH)$) then so is the theory of the r-domain $(A, \leq_A)$, and this last result remains true in zero characteristic.

J. Denef :

Motivic analogons of the Poincaré series associated to the p-adic points on a variety

(Joint work of Denef and Loeser)

In this talk we introduce, for any algebraic variety X over $\mathbb{C}$, two power series P_{geom} and P_{arithm} , whose coefficients are in the Grothendieck ring of motives. They are analogons of the Poincaré series P_p , associated to the p-adic points on a variety over $\mathbb{Q}_p$.

For this we study the scheme of formal arcs on an algebraic variety and its images under truncations. These were first considered by J.F.Nash. The Poincaré series of these images yields P_{geom}. For p big enough, replacing the coefficients A_i of P_{geom} by $\#A_i(F_p)$, does not generally yield P_p. However, we construct P_{arithm} so that this is nevertheless true. We prove that P_{geom} and P_{arithm} are rational power series. The main tools which are used are quantifier elimination for valued fields and pseudo-finite fields, and motivic integration (a notion first introduced by Kontsevich). Geometry and motivic integration on the scheme of formal arcs is used to obtain several new geometric invariants of singularities (e.g. the stringy Hodge numbers of Batyrev). These methods also yield a new approach to the study of the Hodge spectrum of a singularity.

J. Ershov :

Nice local-global fields

A valuation ring of a field of a characteristic 0 is called *quasiclassical* if its henselisation is elementary equivalent to a ring of p-adic integers for some prime p or to an ultraproduct of such rings. A field is called *nice local-global* if the family of all quasiclassical valuation rings of the field forms a boolean family satisfying (LGP) (local-global principl) and $(CLEP)$ (continuity of the local elementary properties).

For three classes – $mNLGF$, $fNLGF$ and $uNLGF$ – of nice local-global fields (defined by the properties of the enriched absolute Galois groups: minimal, free or universal) a decidability of the elementary theory was proved and natural necessary and sufficient conditions for elementary equivalence or embeddings was found.

B. Green :

On the geometry of order p automorphisms

Let R be a complete discrete valuation ring dominating the Witt ring $W(k)$ of an algebraically closed field k of characteristic $p > 0$. The aim of this talk is to discuss new results (obtained jointly with M. Matignon) on order p automorphisms σ of the formal power series ring $R[[Z]]$ and the geometry of their fixed points.

Let F_σ denote the set of fixed points and $\mathcal{D}^o$ be the minimal model of the p-adic disc in which the points in F_σ specialize to distinct smooth points. Then by studying the differential data associated to the irreducible components of the special fibre of $\mathcal{D}^o$ we show that if $|F_\sigma| \leq p$ the fixed points are all equidistant in the p-adic norm. By studying the Hurwitz data associated with the fixed points in the above situation, we obtain a characterization of order p automorphisms of the disc, provided there is no inertia at π.

The study of order p automorphisms of the p-adic disc is motivated by the lifting problem for galois covers of smooth curves from characteristic p to characteristic 0.

[G–M 1] B. Green, M. Matignon: *Liftings of Galois Covers of Smooth Curves*, Compositio Mathematica, Vol 113 no. 3 (1998), 239–274.

[G–M 2] B. Green, M. Matignon: *Order p automorphisms of the open disc over a p-adic field*, to appear in the Journal of the American Mathematical Society.

W. Henson :

Model Theory of L_1-spaces

We discuss the model theory of Banach lattices of the form $L_1(\mu)$ where μ is an atomless measure. This is done with respect to the logic of positive bounded formulas and approximate satisfaction. Let T be the theory of this class of structures in this logic. T is complete and its models are exactly the class of $L_1(\mu)$-spaces described above. T admits quantifier elimination and is categorical for separable models. T is stable (in the sense of Iovino); indeed, T is ω-stable with respect to the topology on its type spaces that is given by the d-metric. (The d-distance between n-types measures how close their sets of realizations are with respect to norm- distance in models.) QE plus separable categoricity yields a strong form of near-isometric homogeneity for L_1. The key mathematical idea behind these results are those in the abstract structure theory of measure algebras, including especially Maharam's theorem in its relative form. The stability result strengthens results of Krivine and Maurey (by including lattice operations) and of Haydon (by showing separability of the type spaces in a finer topology than the strong topology). A different set of ideas leads to the same results for the class of atomless $L_1(\mu)$-spaces *as Banach spaces*.

M. Jarden :

Local global principles for large algebraic fields

We consider here a field K with a finite set S of *local primes*. Each $\mathfrak{P} \in S$ is an equivalent class of absolute values such that the completion $\hat{K}_{\mathfrak{P}}$ is a local field, i.e., locally compact. Let $K_{\mathfrak{P}} = K_s \cap \hat{K}_{\mathfrak{P}}$ and let $K_{\mathrm{tot},S} = \bigcap_{\mathfrak{P}\in S} \bigcap_{\sigma \in G(K)} K_{\mathfrak{P}}^{\sigma}$. Here $G(K)$ is the absolute Galois group of K. We equip it with a Haar measure. For each $\sigma = (\sigma_1, \ldots, \sigma_e) \in G(K)^e$ let $K_s(\sigma)$ be the fixed field of $\sigma_1, \ldots, \sigma_e$ in K_s and let $K_s[\sigma]$ be the maximal Galois extension of K which is contained in $K_s(\sigma)$.

Theorem (Local global principle for fields, Geyer-Jarden): *Suppose that K is a countable Hilbertian field. Then, $K_s[\sigma] \cap K_{\mathrm{tot},S}$ is PSC for almost all $\sigma \in G(K)^e$.*

Let O be the ring of integers of K. For each algebraic extension L of K let O_L be the integral closure of O in L.

Theorem (Local global principle for rings, Jarden-Razon): *Suppose that K is a global field. Then, for almost all $\sigma \in G(K)^e$ the following holds: Let $F_0 = K_s(\sigma) \cap K_{\mathrm{tot},S}$. Denote the maximal purely inseparable extension of F by F_{ins}. Consider an affine absolutely irreducible variety V over F_{ins}. Suppose that $V_{\mathrm{simp}}(O_{F_{\mathfrak{P}}}) \neq \emptyset$ for each $\mathfrak{P} \in S_F$ and $V(O_{F_{\mathfrak{P}}}) \neq \emptyset$ for each $\mathfrak{P} \in P_F \setminus S_F$. Then $V(O_F) \neq \emptyset$.*

J. Koenigsmann :

Elementary characterization of fields by their absolute Galois group

A field K is said to be *elementarily characterized by its absolute Galois group* $G_K := Gal(K^{sep}/K)$ if the fields with isomorphic absolute Galois group are exactly those which are elementarily equivalent to K. Examples of such fields are real closed fields, certain p-adically closed fields (e.g. all finite abelian extensions of $\mathbb{Q}_p$) and fields elementarily equivalent to power series fields over those p-adically closed fields with exponents from $\mathbb{Z}_{(q)}$. There is very strong evidence that these are all fields elementarily characterized by their absolute Galois group.

F.-V. Kuhlmann :

Rational place = existentially closed?

Take $F|K$ to be a function field and P a place of F which is trivial on K. If $FP = K$ then P is called a *rational place*. We say that $(F|K, P)$ is *weakly uniformizable* if there is a model of $F|K$ on which P is centered at a smooth point. The following is well-known: *If K is existentially closed in F, then $F|K$ admits a rational place which is weakly uniformizable.* For the converse, one has:

Theorem 1: *Assume that K admits a henselian valuation w (or, more generally, that K is a "large field"). Assume further that $F|K$ admits a rational place P.*

a) If K is perfect, then K is existentially closed in F.

b) If P is weakly uniformizable, then K is existentially closed in F.

Part b) is well-known. Which places are weakly uniformizable? Local uniformization is not known in positive characteristic, but we can give a partial answer:

Theorem 2: (K. 1997) *Rational Abhyankar places of rank 1 and rational discrete places are weakly uniformizable.*

We call a place an *Abhyankar place* if it satisfies equality in the Abhyankar inequality, i.e., if $\mathrm{trdeg}\, F|K = \mathrm{rr}\, v_P F + \mathrm{trdeg}\, FP|K$, where $\mathrm{rr}\, v_P F$ is the rational rank of the value group of P.

Theorem 3: (K. 1997) *If K is perfect and P is a rational place of $F|K$, then there is a finite extension $\mathcal{F}|F$ and an extension of P to $\mathcal{F}$ such that P is still a rational place of $\mathcal{F}|K$ and $(\mathcal{F}|K, P)$ is weakly uniformizable.*

Since K is existentially closed in F if it is existentially closed in $\mathcal{F}$, this proves part a) of Theorem 1 via part b).

For v_P denoting the valuation associated with P, we demonstrate that a certain kind of existential sentences which hold in $(F, v_P \circ w)$ will also hold in (K, w) (although in general it cannot be expected that (K, w) be existentially closed in $(F, v_P \circ w)$). To prove this by an embedding lemma, we use a result about the density of certain "nice" places in the space of all rational places of $F|K$, with respect to an "existentially constructible topology". Part b) of Theorem 1 can be proved in a similar way, also in the case of large fields.

S. Kuhlmann :

The Exponential-Logarithmic Power Series Fields

In this talk we describe the non-archimedean models of the elementary theory of an o-minimal expansion of the reals. We focus on models of the expansion of the ordered field of the real numbers by the real exponential function. The notion of o-minimality was introduced by van den Dries, who observed that sets which are parametrically definable in an o-minimal expansion of the reals share many of the geometric properties of semi-algebraic sets.

Inspired by the theory of real places and real closed fields, and seeking the analogy to the semi-algebraic case, we systematically develop an exponential analogue for all important notions and methods. We use this abstract machinery to describe explicitly the algebraic structure of the models, and to give concrete constructions. These constructions use power series fields. If a field has a place onto an ordered residue field, then the order can be lifted up to the field through the place. It is not surprising that exponentials cannot be lifted through arbitrary places. Indeed, we showed (cf. [K–K–S]) that power series fields *never* admit exponentials, but that every power series field $\mathbb{R}((G))$ carries a *prelogarithm*. We give an explicit formula for the basic pre-logarithm $\log_0$ on power series fields with any given value group of the form $\mathbb{R}^{\Gamma_0}$, where Γ_0 is a totally ordered set. Going to the union over an increasing chain of power series fields, we make this logarithm $\log_0$ surjective. We call the so-obtained field the *exponential-logarithmic power series field* and denote it by $R((\Gamma_0))^{EL}$. The logarithm $\log_0$ does not satisfy the growth axioms, and we develop a method modify $\log_0$. We show how to use any increasing automorphism σ of Γ_0 to *twist* $\log_0$ as to obtain a logarithm $\log_\sigma$ having the right growth rate. Its inverse $\exp_\sigma$ yields a model of real exponentiation (and restricted analytic functions). This method enables us also to construct exponential fields with arbitrary principal exponential ranks (cf. [K–K]). In this way, our construction exhibits the relation between order automorphisms of the value sets and the growth rates of the constructed exponentials. We also show that $\mathbb{R}((\Gamma_0))^{EL}$ admits countably infinitely many exponentials of distinct exponential ranks, for *any* Γ_0. This contrasts the impression of rigidity which is given by the notation $\mathbb{R}((\Gamma_0))^{EL}$ (cf. [KS] chapter 5).

[KS] S. Kuhlmann: *Ordered Exponential Fields*, Habilitationsschrift, submitted.

[K–K] F.-V. Kuhlmann, S. Kuhlmann: *The exponential rank*, to appear in: Proc. of the Special Semester on Real Algebraic Geometry and Ordered Structures, Amer. Math. Soc. Contemporary Mathematics Series.

[K–K–S] F.-V. Kuhlmann, S. Kuhlmann, S. Shelah: *Exponentiation in Power Series Fields*, Proc. Amer. Math. Soc. 125 (1997), 3177–3183.

A. Macintyre :

Elementary Aspects of Weil Cohomology Theories

At first glance, the notion of Weil Cohomology Theory is not first-order, involving as it does notions of functor, finite-dimensionality, Poincare Duality and intersection theory. However, it turns out to be first-order. To see this, one first has to develop intersection theory in a constructible way. This is not straightforward, as the basic equivalence relations involved in defining the intersection ring are not known to be first-order (and, indeed, there is evidence that some , e.g rational equivalence, are not first-order). One has to proceed in a much more "logical" way, unwinding (in Kreisel's sense) the constructions done in Fulton's book. This done, one gains certain uniformities, and then, exploiting the axiomatic development of the Lefschetz Trace Formula from Weil's axioms, and using the Weak Lefschetz axiom one derives uniform bounds (within constructible families) for the dimensions of the cohomology groups. Thereafter it is fairly routine to give a first-order rendition of Weil's axioms.

Two consequences were mentioned in the talk. Firstly, one can form ultraproducts of Weil theories, to create new ones. Secondly, if one interprets the Standard Conjectures as being about arbitrary Weil theories, one can show that they imply very strong uniformities. In particular, they imply that numerical equivalence is constructible.

D. Marker :

Intersecting Varieties with Tori

We prove the following theorem of Boris Zilber and indicate how it is related to the construction of bad fields. By a *torus* I mean a coset of an algebraic subgroup of $(K^*)^n$.

Theorem: *Let K be algebraically closed, let $W \subset K^n$ be an irreducible variety and let T be the smallest torus containing W. There is a finite set of families of proper subtori of T such that if S is a proper subtorus of T,*

$dim(W \cap S) > 0$ and $dim(W \cap S) > dimW + dimS - n$, then

S is contained in one of the tori in one of the families.

F. Point :

On p-minimal groups

In collaboration with F.O. Wagner, we have obtained the following results (see [PW]). Let L be a language containing the language of totally ordered groups, let G be an L-structure which is a (not necessarily abelian) totally ordered group. The L-structure G is p-minimal if every L-definable (possibly with parameters) subset of G is a finite union of cosets of subgroups of G intersected with intervals with end points in $G \cup \{\pm\infty\}$.

We characterize the pure group structure of a p-minimal group G. Namely G is abelian, has finitely many definable convex subgroups (say d) ($G_0 = \{0\}, ..., G_d = G$). In case d=1, if the order is discrete, $(G, +, 0, \leq)$ is elementarily equivalent to $(\mathbb{Z}, +, 0, \leq)$ and if the order is dense, G is divisible and its theory is o-minimal (for a definition of o-minimality, see [KPS]). So a p-minimal group belongs to a class of groups described by V.Weispfeinning [W], namely the classes $\sum_{a,d}$ and $\sum_{b,d}$, where G_{i+1}/G_i is elementarily equivalent to $(\mathbb{Z}, +, 0, \leq)$ for all $i = 1, \ldots, d-1$ and G_1 is elementarily equivalent to $(\mathbb{Z}, +, 0, \leq)$ or $(\mathbb{Q}, +, 0, \leq)$. Those are decidable classes in the language of ordered groups augmented by the congruences and finitely many constants interpreted by the smallest positive elements in each discrete quotient.

In the case the order is discrete, we show that there are proper p-minimal expansions of p-minimal groups, e.g. $\langle \mathbb{Z} \times \mathbb{Z}, +, \leq, 1_1, 1_2, f \rangle$, where $f : (x, y) \mapsto (0, x)$ is the shift function (see [BPoW]). This phenomenon does not happen in the case d=1, by a result of Michaux and Villemaire [MV]. In the dense case, the question is still open whether there exists an o-minimal expansion of $(\mathbb{Q}, +, <, 0)$.

We don't know whether p-minimality is preserved by elementary equivalence, but essential periodicity is preserved; it is defined as follows: for every definable non-trivial convex subgroup H and every definable subset X of G, there is some interval $I = [h, h']$ with $h \in H$ and $h' > H$, such that $X \cap I$ is equal to the intersection with I of a finite union of cosets of subgroups of G. We show that a definable function in an essentially periodic group G is piecewise linear and that the L-definable subsets of G^n are locally periodic. Moreover, if the theory of G is p-minimal, we obtain that the definable functions are piecewise linear.

Quasi-o-minimality has been introduced in [BST] and studied in [BPW]. One can show that a group G is essentially quasi-o-minimal iff its theory is p-minimal (here essentially means that it becomes quasi-o-minimal after adding some new constants).

[BPW] O. V. Belegradek, Y. Peterzil, F. O. Wagner: *Quasi-o-minimal structures*, to appear in J. Symb. Logic.

[BPoW] O.V. Belegradek, F. Point, F.O. Wagner: *A quasi-o-minimal group without the exchange property*, MSRI preprint series 1998-051.

[BST] O. V. Belegradek, A. P. Stolboushkin, M. A. Taitslin: *Generic queries over quasi-o-minimal domains*, in: Logical Foundations of Computer Science (Proc. 4th International Symposium LFCS'97, Yaroslavl, Russia, July 1997) S. Adian and A. Nerode, eds., Lecture Notes in Computer Science 1234, Springer-Verlag (1997), 21–32.

[KPS] J. F. Knight, A. Pillay, C. Steinhorn: *Definable sets in ordered structures II*, Trans. Amer. Math. Soc. **295** (1986), 565–592.

[MV] C. Michaux, R. Villemaire: *Presburger Arithmetic and recognizability of sets of natural numbers by automata*, Ann. Pure Appl. Logic **77** (1996), 251–271.

[Mu] A. Muchnik: *Definable criterion for definability in Presburger arithmetic and its applications*, Institute of new technologies (1991), preprint (in russian).

[PW] F. Point, F.O. Wagner: *Essentially periodic ordered groups*, preprint

[W] V. Weispfenning: *Elimination of quantifiers for certain ordered and lattice-ordered groups*, Bull. Soc. Math. Belg., Ser. B, **33** (1981), 131-155.

B. Poizat :

Corps de rang deux

Un corps K algébriquement clos tout nu, à fait une structure monodimensionnelle de rang de Morley un. Si on ajoute un prédicat désignant un ensemble infini algébriquement indépendant, à devient bidimensionnel de rang oméga ; il en est de même si on en nomme un sous corps algébriquement clos. Un corps différentiellement clos de caractéristique nulle, c'est encore de rang oméga, mais à a énormément de dimensions.

Tous ces exemples ont pour générique un type régulier, ayant pour rang une puissance de oméga, auquel la structure ajoutée est orthogonale : Berline et Lascar, après avoir montré que le rang U d'un corps était un monô me, avaient conjecturé qu'il devait en être toujours ainsi. Notre but est de les contrarier et de construire des corps oméga-stables avec une structure ajoutée mettant en cause le générique, cela grâce au procédé industriel de fabrication d'objets paranormaux mis au point par Hrushovski (lequel est intervenu à plusieurs reprises dans les présentes constructions).

Premier exemple : on amalgame des corps algébriquement clos k de degré de transcendence fini, avec un prédicat unaire - les points noirs - en utilisant pour formule de prédimension d(k) = deux fois le degré de transcendence moins le nombre de points noirs ; on obtient dans un premier temps un corps de rang w.2 avec un ensemble noir de rang w , et après collapsage un corps de rang deux avec un ensemble de points noirs de rang un.

Deuxième exemple : on amalgame des corps algébriquement clos k de caractéristique p et de degré de transcendence fini, avec un prédicat unaire - les points rouges - formant un groupe additif, en utilisant pour formule de prédimension d(k) = deux fois le degré de transcendence moins la dimension linéaire sur le corps premier des points rouges ; on obtient dans un premier temps un corps de rang w.2 avec un groupe additif rouge de rang w , et après collapsage un corps de rang deux avec un groupe additif rouge de rang un.

Troisième exemple : on amalgame des paires de corps algébriquement clos k/c de caractéristique 0 et de degré de transcendence fini sur c , avec un prédicat unaire - les points rouges - formant un c-espace vectoriel, en utilisant pour formule de prédimension d(k) = deux fois le degré de transcendence sur c moins la dimension linéaire sur c des points rouges ; on obtient dans un premier temps un corps de rang w2.2 avec un groupe additif rouge de rang w2 , et après collapsage un corps de rang w.2 avec un groupe additif rouge de rang w .

Quatrième exemple : on amalgame des corps algébriquement clos k de degré de transcendence fini, avec un prédicat unaire - les points verts - formant un groupe multiplicatif divisible, en utilisant pour formule de prédimension d(k) = deux fois le degré de transcendence moins la dimension linéaire sur le corps des rationnels des points verts ; on obtient dans un premier temps un corps de rang w.2 avec un groupe multiplicatif de rang w , et après collapsage un corps de rang deux avec un groupe multiplicatif vert de rang un.

Dans tous ces exemples, le trait essentiel qui rend l'amalgamation possible est le côté modulaire (ou trivial) de la dimension retranchée. Les deux premiers cas sont faciles, la situation se décrivant finiment ; le troisième ne l'est guère moins, à ceci près que la clôture autosuffisante varie avec le petit corps c ; pour traiter le dernier, on a besoin de montrer que la dimension multiplicative d'une variété est définissable, ainsi que d'un récent théorème de Zilber impliquant que, dans certaines situations, l'indépendence multiplicative s'exprime finiment (ces deux résultats appellent à la rescousse l'Algèbre Différentielle).

A. Razon :

Model Theory of Pseudo Algebraically Closed domains

Let $\mathcal{O}$ be a Dedekind domain with trivial Jacobson radical and with global quotient field K. Then $\tilde{\mathcal{O}}(\sigma) = \tilde{\mathcal{O}} \cap \tilde{K}(\sigma)$ satisfies Rumely's local-global principle for almost all $\sigma \in G(K)^e$ (see the report of M. Jarden, this meeting). The following theorems are model-theoretic applications of this result.

Theorem 1: *The theory* Almost$(\mathcal{O}, e)$ *of all sentences in the language of rings which are true in* $\tilde{\mathcal{O}}(\sigma)$ *for almost all* $\sigma \in G(K)^e$ *is decidable. Moreover, in case* $\mathcal{O}$ *is* $\mathbb{Z}$ *or* $\mathbb{F}_p[t]$, Almost$(\mathcal{O}, e)$ *is even primitive recursively decidable.*

Definition: Let R be an integral domain with quotient field E.

(a) R is a *PAC domain* if it satisfies: (i) *E is PAC* over *R*; (ii) *R is Bezout*; (iii) R satisfies Rumely's local-global principle with radical relations; and (iv) *every nonzero nonunit in R is a product of two relatively prime nonunits.*

(b) R is of *type* (o) if $R \neq E$ and $\text{Rad}(R) = (0)$.

Theorem 2: *An integral domain A is a model of* $\mathrm{Almost}(\mathcal{O}, e)$ *iff A is a PAC domain of type (o) s.t. $\tilde{K} \cap A$ is integral over $\mathcal{O}$ and* $\mathrm{Quot}(A)$ *is perfect and e-free.*

Theorem 3: *If R and S are PAC domains of type (o) s.t. $\tilde{K} \cap R$ and $\tilde{K} \cap S$ are integral over $\mathcal{O}$ and with perfect and e-free quotient fields E and F, respectively, then $R \equiv_{\mathcal{O}} S$ iff $\tilde{K} \cap E \cong_K \tilde{K} \cap F$.*

Theorem 4: *The PAC domains of type (o) with their Jacobson radical relation are exactly the r_0-domains which are existentially closed in every regular r_0-domain extension.* (See the report of L. Darniere, this meeting, for the definition of an r_0-domain.)

Theorem 5: (a) *Let T_a (resp., T_r) be the theory of r_0-domains augmented in such a way that an r_0-domain $(R, \leq)$ is a substructure of an r_0-domain $(R', \leq')$ iff $(R, \leq) \subseteq (R', \leq')$ and* $\mathrm{Quot}(R)$ *is algebraically closed in* $\mathrm{Quot}(R')$ *(resp.,* $\mathrm{Quot}(R')$ *is a regular extension of* $\mathrm{Quot}(R)$*). Then T_a (resp., T_r) has a model companion whose models are the PAC domains of type (o) with their Jacobson radical relation s.t. their quotient fields are* 1*-imperfect (resp., ω-imperfect) and ω-free.*

(b) *Let T_e be the theory of r_0-domains augmented in such a way that an r_0-domain $(R, \leq)$ is a model of T_e if there exists a Galois extension E_e of Quot(R) with Galois group $\hat{F}_e$ and, moreover, $(R, \leq)$ is a substructure of an r_0-domain $(R', \leq')$ iff $(R, \leq) \subseteq (R', \leq')$ and* $\mathrm{Quot}(R') \cap E_e = \mathrm{Quot}(R)$. *Then T_e has a model companion whose models are the PAC domains of type (o) with their Jacobson radical relation s.t. their quotient fields are perfect and e-free.*

H. Schoutens :

Bounds in cohomology

Many properties in algebraic geometry or commutative algebra are valid in zero characteristic but not in prime characteristic or vice versa. This dichotomy between zero and prime characteristic is also present in the methods of proof, or even in the lack of a proof. From a model theoretic point of view, this is at first surprising, as by the Lefschetz Principle, there is a transfer of first order properties between algebraically closed fields of different characteristic. One might therefore be tempted to conclude that most algebraic-geometric properties are not first order. However, it turns out that quite the opposite is true but that the reason why transfer fails in full generality is because the real dichotomy is between "all primes" and "almost all primes".

We will illustrate the methods of translating algebraic-geometric statements into first order statements in the base field using the example of the Bass Conjecture. The latter states that any local affine ring over an algebraically closed field is Cohen-Macaulay, if and only if, it admits a finitely generated module of finite injective dimension. Firstly, with an affine local ring we mean a localisation at a prime ideal of a quotient $K[X_1, \dots, X_n]/(f_1, \dots, f_s)$

of a polynomial ring. We say that its complexity is at most d, if the f_i and the generators of the prime ideal all have degree at most d. The tuple of coefficients is a code for the affine local ring in the field K. Next, one observes that every finitely generated module M can be realised as a cokernel of a matrix Γ. Therefore we say that M has complexity at most d, if all entries of Γ have degree at most d and also its dimensions are at most d. Whence the tuple of coefficients of the entries of Γ is a code for M. Finally, in order to translate the Bass Conjecture into a first order statement in these codes, we need to encode the concepts of Cohen-Macaulayness and injective dimension. This can be done most easily by means of cohomological characterisations. Our main result on bounds in cohomology is then the following:

Let F be a functor on the category of finitely generated R-modules. Assume that F is additive, covariant and right exact. If, moreover, multiplication with an element from R is preserved under this functor then we can find, for each pair of natural numbers d and i some natural number d_i, with the property that for every module M of complexity at most d, the image of M under the i-th derived functor L_iF is a module of complexity at most d_i. This theorem applies in particular to the Tor and Ext functors.

Returning to our example, Peskine and Szpiro proved the Bass Conjecture in positive characteristic using properties of the Frobenius. They then found an ad hoc procedure to conclude its validity also in zero characteristic. However, our approach shows that such a lifting to zero characteristic is an immediate consequence of the definable nature of the problem.

P. Scowcroft :

p-adically closed fields without cross-sections

A p-adically closed field K is a field elementarily equivalent to the field of p-adic numbers in the language L of rings. One may view such a K as a valued field (K, v) with value group a $\mathbb{Z}$-group – an ordered Abelian group elementarily equivalent to $(\mathbb{Z}, +, <, 0, 1)$ – because "$v(x) < v(y)$" is L-definable in the field of p-adic numbers. The core field of a p-adically closed K is the maximal p-adically closed subfield of the field of p-adic numbers that may be embedded in K. A cross-section for (K, v) as above is a homomorphism, from the value group into K's multiplicative group, that is a right inverse to v.

After reviewing results concerning the existence of p-adically closed K with or without cross-sections – see "Cross-sections for p-adically closed fields," J. Algebra 183 (1996), 913-928 – I discussed the following

Theorem: Let K be a p-adically closed field whose core field is not the field of p-adic numbers, and D be a proper elementary extension of K's value group. There is a p-adically closed extension L of K, with value group D and core field the same as K's, that lacks a cross-section.

K. Tent :

Defining fields in groups and geometries

The Cherlin-Zil'ber Conjecture, viz. that a simple group of finite Morley rank is an algebraic group over an algebraically closed field, has been the starting point for much research in model theory. While this conjecture is still open, we have made progress by proving the conjecture for several important classes of groups.

Theorem 1: [KRT] *k be an infinite field, and let $G(k)$ denote the group of k-rational points of a k-isotropic absolutely simple k-algebraic group G. Then k is definable in $G(k)$. Thus $G(k)$ has finite Morley rank if and only if the field k is algebraically closed. If under the same assumptions G is k-isotropic and only k-almost simple, then k is either real closed or algebraically closed.*

Dropping the assumption that G is the group of k-rational points of some algebraic group, we have the following result.

Theorem 2: [KTVM] *If G is an infinite simple group of finite Morley rank with a spherical BN-pair of Tits rank ≥ 3, then G is (interpretably) isomorphic to a simple algebraic group over an algebraically closed field.*

Spherical irreducible buildings of Tits rank ≥ 3 are uniquely determined by their rank 2 residues (i.e. polygons). So the crucial step for the proof is the following

Theorem 3: [KTVM] *If $\mathcal{P}$ is an infinite Moufang polygon of finite Morley rank, then $\mathcal{P}$ is either the projective plane, the symplectic quadrangle, or the split Cayley hexagon over some algebraically closed field.*

It follows that any infinite simple group of finite Morley rank with a spherical Moufang BN-pair of Tits rank 2 is either $\mathbf{PSL}_3(K), \mathbf{PSp}_4(K)$ or $\mathbf{G}_2(K)$ for some algebraically closed field K. This shows that polygons play an important role in the study of groups of finite Morley rank. However polygons of finite Morley rank are not *tame*:

Theorem 4: [Te] *There are many almost strongly minimal n-gons for all n not interpreting an infinite group whose automorphism group acts transitively on the set of ordered $n+1$-gons.*

The automorphism group of such a strongly homogeneous polygon thus has a BN-pair, which, of course, cannot be definable. In contrast to these examples we have the following results:

Theorem 5: [Te1] *A projective plane has strongly minimal point rows and definable projectivity groups if and only if it is a Pappian plane over an algebraically closed field, i.e. corresponds to $\mathbf{PSL}_3(K)$. An infinite generalized quadrangle of finite Morley Rank is the symplectic quadrangle over an algebraically closed field, i.e. corresponds to $\mathbf{PSp}_4(K)$, if and only if all points are regular and the line pencils are strongly minimal with definable projectivity groups.*

[KTVM] L. Kramer, K. Tent, H. Van Maldeghem: *Simple groups of finite Morley Rank and Tits buildings*, to appear in Israel Journal of Mathematics.

[KRT] L. Kramer, G. Röhrle, K. Tent: *Defining k in $G(k)$*, to appear in J. Algebra

[Te1] K. Tent: *A note on the model theory of generalized polygons*, to appear in J. Symbolic Logic.

[Te2] K. Tent: *Very homogeneous generalized n-gons of finite Morley rank*, submitted.

F. O. Wagner :

Minimal Fields

An infinte structure $\mathcal{M}$ is minimal if every definable subset (using parameters in $\mathcal{M}$) is finite or cofinite. Minimal groups have been classified long ago by Reineke [Rei]: they are either abelian divisible with only finitely many elements of any given order, or elementary abelian of prime exponent p for some prime p. However, a corresponding classification of minimal fields has not yet been obtained; Podewski [Po] conjectured them to be algebraically closed. (It is easy to see that every algebraically closed field is minimal, since it allows elimination of quantifiers.) Schmücker [Sch] has shown that a minimal field has no extension of degree five. I shall use dimension-theoretic methods to prove the conjecture in characteristic 2, and show a partial result in other characteristics, namely that no algebraic extension of a minimal field has an Artin-Schreier extension.

[Po] K.-P. Podewski: *Minimale Ringe*, Math., Phys., Semesterberichte,22,1973,pages193–197

[Rei] J. Reineke: *Minimale Gruppen*, Zeitschr., math., Logik Grundl., Math.,21,1975, pages357–359

[Sch] R. Schmücker: *Almost algebraically closed fields*, preprint 1995

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