MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 45/1998

APPLIED PROBABILITY

29.11.-05.12.1998

The meeting has been organized by Arie Hordijk (University of Leiden, The Netherlands) and Volker Schmidt (University of Ulm, Germany).

There were talks and discussions dealing with the following subjects:

- Large deviations and heavy tails
- Long range dependence
- Queueing networks
- Heavy traffic
- Performance analysis
- Coupling and renewal theory
- Subexponential distributions
- Point processes
- Markov processes
- Duality and admission control
- Processor sharing and priorities
- Simulation and change analysis
- Tandem networks
- Conservation laws and fluid models

Apart from the regular talks there was a poster session on "Stochastic Models in Telecommunications".

Richard Serfozo, Editor-in-Chief of Queueing Systems: Theory and Applications (QUESTA) donated to the MFO library a complimentary subscription to the journal. QUESTA is a leading international journal in applied probability.

ABSTRACTS

SCHEDULING STRATEGIES AND LONG-RANGE DEPENDENCE

ANANTHARAM, V.
UNIVERSITY OF CALIFORNIA, BERKELEY, USA

Let $T_n, n \in \mathbb{Z}$ be the points of a Poisson process and let $\tau_n, n \in Z$ be i.i.d. non-negative random variables with $P(\tau_n > t) = t^{-\alpha}L(t)$ where $1 < \alpha < 2$ and $L(t)$ is slowly varying. Let $A(t) = \sum_{n \in \mathbb{Z}} 1(T_n \le t \le T_n + \tau_n)$. Then $(A(t), t \in I\!R)$ is a long-range dependent stationary stochastic process, i.e. $\int_{-\infty}^{\infty} |h(u)|du = \infty$, where $h(u) = \mathrm{E}[A(t)A(t+u)]$.
Processes of this type are believed to be important models for traffic in communication networks. Motivated by practical considerations, we studied the following problems:

1. Does long-range dependence persist when a process of this type is regulated by a leaky bucket flow control scheme? We show that it does.
2. Can one construct tractable queueing networks whose arrival processes are of this type? We show that such networks can be built by using the S-queues of Walrand with traffic flowing between the queues by Bernoulli routing. Further, the inter-queue traffic processes are all also long-range dependent.
3. Can control be used to mitigate the effects of long-range dependence on queues? We show that it can. Indeed, while the stationary sojourn time of a customer at a first come first served single server queue fed by an arrival process of this type is regularly varying with infinite mean, one can find causal stationary scheduling policies for which the stationary sojourn time of a customer is regularly varying with finite mean.

ON A CONTROL PROBLEM FOR RISK PROCESSES AND QUEUES

ASMUSSEN, S.
LUND UNIVERSITY, SWEDEN

Let A_t be compound Poisson with drift $\mu = \mathrm{E}A_t/t$ and $S_t' = x + \mu(1+\eta)t - A_t$ the generated risk process, $\psi'(x)$ its ruin probability. We compare S_t' with the controlled risk process $S_t = x + (1+\eta)\int_0^t A_s/s ds - A_t$ with ruin probability $\psi(x)$. With heavy tails, $\liminf_{x\to\infty} \psi(x)/\psi'(x) \ge 1$. With light tails, it can be deduced from a recent result of Nyrhinen that $\liminf_{x\to\infty} \psi(x)/\psi'(x) = \infty$. We supplement by studying the claims history leading to ruin: The claims have been few and small from the outset so that the premium has been set too low. This is done both in the compound Poisson setting and in a diffusion limit, where it turns out that $\psi(x) = \psi'(x) = e^{-2x}$.

QUEUEING NETWORKS WITH CROSS TRAFFIC

BACCELLI, F. [1]
ENS, PARIS, FRANCE

The TCP protocol is the basic transmission control mechanism of the Internet. In this joint work with Thomas Bonald, we propose a mathematical model for this protocol. This model is based on the perturbation of a (max,+)-linear system (representing the window flow control) by cross-traffic (representing the offer of the other users on the controlled connection). This leads to a random dynamical system, which boils down to a stochastic (max,+)-linear system in the absence of cross traffic. We use ergodic theory to prove the existence of a maximal throughput for the controlled flow and to build a minimal stationary regime for such a system. We then show that this maximal throughput is extremely sensitive to the variability of the cross-traffic, and also to small parametric variations. An example where the boundary of the stability region (i.e. the maximal throughput) of the controlled connection is a self-similar curve illustrates this sensitivity.

QUEUEING NETWORKS WITH INDEPENDENT RESOURCES

BAMBOS, N. [2]
STANFORD UNIVERSITY, USA

Consider a set of Q parallel queues and a service mechanism (server) that can be in any one of M possible states at any point in time. When the server is in the m-th state, the q-th queue receives service at rate r_m^q. There is also a differential cost that the system incurs when the server is in state m. We study the problem of dynamically choosing the server's state so as to maximize the throughput of the system, that is, the maximum load that the system can tolerate without excessive build-up of backlog in the queues. We also discuss the problem of dynamically selecting the server states so as to minimize the long-run aggregate cost to run the system, given a feasible load and quality of service constraints (say, average backlogs less than certain thresholds) that each queue has to satisfy. We then extend the results to acyclic networks of nodes of the type described above. The motivation for considering this class of models and the related problems comes from power control in wireless communication networks and also fast scheduling in high-speed switching systems.

[1] The paper is available on the author's web page.
[2] Joint work with M. Azmony.

RISK AND DUALITY IN MULTIDIMENSIONS

BLASZCZYSZYN, B.[3]
UNIVERSITY OF WROCLAW, POLAND

We present, in discrete time, general-state-space dualities between content and insurance risk processes that generalize the stationary recursive duality of Asmussen and Sigman and the Markovian duality of Siegmund (both of which are onedimensional). The main idea is to allow a risk process to be set-valued, and to define ruin as the first time that the risk process becomes the whole space. The risk process can also become infinitely rich which means that it eventually takes on the empty set as its value. In the Markovian case, our results connect deeply with stochastic geometry. As a motivating example, in the multidimensional Euclidean space our approach yields a dual risk process for the Kiefer and Wolfowitz vector in the classic $G/G/c$ queue, and we include a simulation study of this dual to obtain estimates for the ruin probabilities.

LARGE DEVIATIONS FOR REAL-VALUED MARKOV CHAINS

BOROVKOV, A.
INSTITUTE OF MATHEMATICS, NOVOSIBIRSK, RUSSIA

Let $X(n)$, $n = 0, 1, \ldots$, be a real-valued Markov chain with initial position $X(0)$. We consider spatially asymptotically homogeneous Markov chains for which increments $\xi(x) = X(n+1) - X(n)$, given $X(n) = x$, converge weakly in distribution to ξ. For such Markov chains we found under broad conditions the logarithmic and fine asymptotics of $P(X(n) > x)$ as $n \to \infty, x \to \infty$. The case of stationary Markov chains ($n = \infty$) is not excluded. Assumptions include existence of a stationary distribution (ergodicity of the Markov chain) and conditions concerning distributions of $X(0)$, ξ and closeness of $\xi(x)$ and ξ as $x \to \infty$.

GENERALIZED PROCESSOR SHARING WITH LONG-TAILED TRAFFIC SOURCES

BORST, S.
CWI, AMSTERDAM, THE NETHERLANDS

In this talk we analyze the queueing behaviour of long-tailed traffic sources under the Generalized Processor Sharing (GPS) discipline. Under mild stability conditions, we show that the tail behaviour of the buffer content of an individual source with long-tailed traffic characteristics is equivalent to the tail behaviour when that source is served in isolation at a *constant* rate, which is equal to the link rate minus the aggregate average rate of all other sources. Thus, asymptotically, the buffer content of a source is only affected by the traffic characteristics of the other sources through their aggregate average rate. In particular, the source is

[3] Joint work with K. Sigman.

essentially immune from excessive activity of sources with "heavier"-tailed traffic characteristics.

QUEUES WITH HEAVY TAILS

BOXMA, O. J.[4]
EINDHOVEN UNIVERSITY OF TECHNOLOGY, THE NETHERLANDS

Recent measurements in high-speed telecommunications networks have revealed the occurrence of traffic conditions that exhibit long-range dependence and burstiness over an extremely wide range of time scales. This has triggered research on queues with heavy-tailed distributions: fluid queues fed by on-off sources with heavy-tailed on- and/or off distributions, but also the classical $GI/G/1$ queue with heavy-tailed service and/or interarrival time distribution.
In this talk we discuss the latter $GI/G/1$ queue. The service- and/or interarrival time distribution is assumed to be regularly varying of index $-\nu \in (-2,-1)$. The emphasis will be on the $M/G/1$ queue. Several service disciplines will be considered: FCFS, LCFS-PR, and processor sharing.
Two types of results will be presented. Firstly, the tail behaviour of waiting times and workloads is investigated; in several cases, these tails also turn out to be regularly varying, the index depending on the service discipline. Secondly, we present heavy-traffic limit theorems for the distributions of several quantities, like the stationary waiting time $\mathbf{W}$ and the residual busy period $\hat{\mathbf{P}}$ in the $M/G/1$ queue. Appropriate 'contraction factors' $\Delta_W(\rho)$ and $\Delta_P(\rho)$ are identified, so that $\Delta_W(\rho)\mathbf{W}$ and $\Delta_P(\rho)\hat{\mathbf{P}}$ converge in distribution when the traffic load ρ approaches one. The corresponding limiting distributions are identified and discussed.

SOME STOCHASTIC MODELS IN TELECOMMUNICATIONS

BRANDT, A.
HUMBOLDT UNIVERSITY OF BERLIN, GERMANY

We present some stochastic models and their analysis. In particular we give a detailed analysis of a call center with an integrated voice mail server. The corresponding mathematical model is a two-queue priority system with impatience, being a generalization of the M(n)/M(n)/s+GI system. We derive the balance equations for the density of the stationary state process in terms of integral equations. Applying recent results for those equations we can give an analytical solution for the first queue. For obtaining performance measures for the second queue (in the application the voice mail server) we construct a system approximation based on fitting impatience intensities. The results are specialized for the call center application. Numerical results show that the approximation works well. Further, for an important special case a stochastic decomposition is derived illuminating the connection to the dynamics of the M(n)/M(n)/s+GI system.

[4] The processor sharing results are based on joint work with A.P. Zwart; the heavy-traffic limit theorems are based on joint work with J.W. Cohen.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

HAWKES BRANCHING PROCESS WITHOUT ANCESTORS AND LONG-RANGE DEPENDENCE

BREMAUD, P.[5]
CNRS, GIF SUR YVETTE , FRANCE

Ancestors are born at the times of a point process (Poisson, intensity ν). They give birth to children, which in turn ... Each member of the community including the ancestors has the following reproduction pattern. When born at time s, the individual produces children acording to a Poisson pattern, with intensity $h(t-s)$. The result is a Hawkes branching process of stochastic intensity

$$\lambda(t) = \nu + \int_{-\infty}^{t} h(t-s)N(ds) \quad , \nu > 0 .$$

If $\int h < 1$, there is a stationary state, and the average rate of birth is of course $\lambda = \nu + (\int h)\lambda$. What happens when $\nu \to 0$, keeping λ constant? More seriously, what happens when $\nu = 0$? (no ancestors). In other words, does a point process, stationary, exist with the intensity

$$\lambda(t) = \int_{-\infty}^{t} h(t-s)N(ds) .$$

Of course, if we want that $\lambda < \infty$, then necessarily $\int h = 1$ (critical case). We show existence when $h(t) \sim 1/t^{1+\alpha}$, $0 < \alpha < 1/2$, as $t \to \infty$. Of course there is no uniqueness. The resulting process is long-range dependent, with a spectral density which is $\sim 1/\omega^{2\alpha}$ at the right.

CORRELATION THEORY FOR EXPONENTIAL NETWORKS: END-TO-END-DELAY APPROXIMATIONS

DADUNA, H.
UNIVERSITY OF HAMBURG, GERMANY

For exponential multiserver networks Jackson's theorem states joint queue length vectors in equilibrium having independent marginals. Despite of this the space-time correlation structure of Jacksonian networks exhibits complex behaviour and being almost unknown: especially, customers traversing the network experience in most cases complex interactions of their successive sojourn times.
On overtake-free paths in equilibrium a customer's successive sojourn times behave as independent quantities. Especially, every 2-stations walk of different nodes is overtake-free. Consequently an appealing general approxiamtion is Independent-Flow-Time-Approximation (IFTA): Compute individual node sojourn times and **assume** independence. This works well for feed-forward networks, but fails e.g. for acyclic networks.
Association- and Correlation-Theorems should yield bounds for joint sojourn times vector probabilities in terms of their independent versions, thus providing information about the goodness of IFTA. We prove: On any 3-station walk of distinct

[5] Joint work with L. Massoulié.

nodes the successive sojourn times are positive upper orthant dependent (PUOD): So in the case of 3-stations walk IFTA yields exact lower bounds for the desired quantities.

HEAVY TRAFFIC LIMITS FOR SOME QUEUEING NETWORKS

DAI, J.[6]
GEORGIA INSTITUTE OF TECHNOLOGY, ATLANTA, USA

The Brownian limits of queueing networks, known as heavy traffic limits, are a topic of continuing interest. We discuss a recent framework of M. Bramson and R. Williams for proving heavy traffic limit theorems. Old and new heavy traffic limit theorems based on this approach will be presented.

LONG RANGE DEPENDENCE OF INPUTS AND OUTPUTS OF CLASSICAL QUEUES

DALEY, D.
AUSTRALIA NATIONAL UNIVERSITY, CANBERRA, AUSTRALIA

Input and output processes of some classical queueing systems are studied as stationary point processes with finite second moments. Specifically, for the stationary arrival and departure processes denoted N_{arr} and N_{dep}, I consider their long range dependence (LRD) properties, meaning, counting processes N, that $\mathrm{E}[(N(0,t])^2] < \infty$ for finite t and $\limsup_{t\to\infty} \frac{\mathrm{var}N(0,t]}{t} = \infty$. The Hurst index of an LRD point process is defined by $H = \inf\{h : \limsup_{t\to\infty} t^{-2h}\mathrm{var}N(0,t] = \infty\}$.
A stationary renewal process whose generic lifetime random variable X has moment index $\kappa \equiv \sup\{k : \mathrm{E}(X^k) < \infty\}$, is LRD if and only if $\mathrm{E}(X^2) = \infty$, and it has Hurst index $\frac{1}{2}(3-\kappa)$. There exist renewal processes for which $0 = \liminf_{t\to\infty} t^{-2H}\mathrm{var}N(0,t] < \limsup_{t\to\infty} t^{-2H}\mathrm{var}N(0,t] = \infty$. In other words, there need not be any limit associated with the Hurst index.
In all the examples of queueing systems that are given, the generic service time random variable S has finite first moment, and the systems are stable and stationary.

1. Fourier techniques show that in a G/GI/∞ system, N_{dep} is LRD if and only if N_{arr} is LRD.

2. A consistency property for inputs and outputs shows that in a GI/M/k system with a finite number of servers k, N_{dep} is LRD if and only if N_{arr}, and they have the same Hurst index $\frac{1}{2}(3-\kappa_T)$ where κ_T is the moment index of a generic interarrival time random variable T.

3. In a pure loss GI/GI/1/0 system with LRD input or $\mathrm{E}(S^2) = \infty$, the output is LRD and $H_{\text{dep}} = \max(H_{\text{arr}}, \frac{1}{2}(3-\kappa_S))$.

4. In a stable M/GI/1 system with $\mathrm{E}(S^2) < \infty$, and moment order in $(2,3)$, N_{dep} is LRD.

5. In a stable GI/GI/k system with $\mathrm{E}(S^2) < \infty$, N_{dep} cannot be LRD if the system is stable with only $k-1$ servers unless N_{arr} is LRD.

[6] Joint work with M. Bramson.

6. Decompose the output of a stationary M/GI/∞ system (and, this output is Poisson), into those departures leaving no other customers in the system, and all other departures: $N_{\text{dep}} = N_{\text{empty}} + N_{\text{non-empty}}$, say. Then the points of N_{empty} are those of a renewal process which, if $E(S^2) = \infty$, is LRD (and $N_{\text{non-empty}}$ is LRD, too).

PHI-RENOVATION, BACKWARDS COUPLING, AND PERFECT SIMULATION

FOSS, S.
NOVOSIBIRSK STATE UNIVERSITY, RUSSIA

We introduce a concept of the Phi-renovation and give a criterion for its existence. The Phi-renovation may take place when a Markov chain (X_n) is not Harris ergodic itself but some functional $\phi(X_n)$ converges to a limit in the total variation norm. A proof of the criterion is based on the backward coupling construction (due to Loynes), which allows us to propose a way for the perfect (exact) simulation for an unknown stationary distribution of $\phi(X_\infty)$. An example of a random test graph is considered.

FLUID APPROXIMATION OF CONTROLLED MULTICLASS TANDEM NETWORKS

GAJRAT, A.[7]
MOSCOW STATE UNIVERSITY, RUSSIA

The asymptotically optimal policy is constructed for a .nulticlass tandem network. This is achieved in two steps. First, the corresponding fluid optimal control is solved. The second step is the construction of a policy for the stochastic queueing network which has as fluid limit the optimal control of the fluid problem.

MULTIMODULARITY AND ADMISSION CONTROL IN (max, +)-LINEAR SYSTEMS

GAUJAL, B.[8]
INRIA, LORRAINE, FRANCE

We show that quantities of interest such as expected waiting time or expected workload are multimodular functions with respect to the arrival stream. Using the convexity property of such functions, we can derive convex ordering properties or cone ordering comparisons. We can also show that multimodular functions are sturmion sequences. This is then used to solve particular cases of routing problems as well as polling problems when the rates are balanceable.

[7] Joint work with A. Hordijk.
[8] Joint work with E. Altman and A. Hordijk.

VARIANCE REDUCTION VIA AN APPROXIMATING MARKOV PROCESS

GLYNN, P.
STANFORD UNIVERSITY, USA

In many settings, one may wish to simulate a stochastic process for which there exists a corresponding Markov process which is believed to be a good approximation to the process of interest. For example, in the context of the single-server queue, an approximation can be obtained from reflecting Brownian motion. In this talk, we discuss a comprehensive framework for developing martingale control variates and then take advantage of the presence of such approxiamtions. When computing steady-state expectations, the martingale is constructed by exploiting the solution Poisson's equation for the approximating Markov process; the better the approximation, the greater the degree of variance reduction. We further show how to extend the methodology to transient expectations and many others. We conclude the talk with some special issues that arise in the context of an approximating process that is a diffusion which exhibits "state space collaps" relative to the original process.

A POINT PROCESS MODEL FOR MINIMAL REPAIRS

JENSEN, U.[9]
UNIVERSITY OF ULM, GERMANY

Minimal repairs have been given considerable attention in the reliability literature. Instead of replacing a failed system by a new one such a minimal repair restores the system to the state it had just before failure. But the state just before failure depends on the information which is available about the system. Different information levels are possible. In the talk a general definition is given characterizing point processes which describe time points of minimal repairs with respect to a certain information level, i.e., with respect to a certain filtration. Some examples demonstrate the wide range of applications.

A MULTI-DIMENSIONAL MARTINGALE FOR MARKOV ADDITIVE PROCESSES AND ITS APPLICATIONS

KELLA, O.[10]
THE HEBREW UNIVERSITY OF JERUSALEM, ISRAEL

We establish new multidimensional martingales for Markov additive processes and certain modifications of such processes (e.g., such processes with reflecting barriers). These results generalize corresponing onedimensional martingale results for Lévy processes. Various examples of storage processes, queues and Brownian motion models are given which demonstrate the applicability of these martingales.

[9] Joint work with T. Aven.
[10] Joint work with S. Asmussen.

SAMPLING AT SUBEXPONENTIAL TIMES, WITH QUEUEING APPLICATIONS

KLÜPPELBERG, C.
TECHNICAL UNIVERSITY OF MUNICH, GERMANY

We study the tail asymptotics of the random variable $X(T)$, where $(X(t))_{t\geq 0}$ is a stochastic process with a linear drift, satisfying some regularity conditions like a central limit theorem, and T is an independent random variable with a subexponential distribution. We find that the tail of $X(T)$ is sensitive to whether or not T has a heavier or lighter tail than a Weibull distribution with tail $e^{-\sqrt{x}}$. This leads to two distinct classes, heavy-tailed and moderately heavy-tailed. Also the light-tailed case can be covered. The results can be applied via distributional Little's law to establish tail asymptotics for steady-state queue lenght in GI/GI/1 queues with subexponential service times.

REPRESENTAION AND PERFORMANCE OF STOCHASTIC FLUIDS

KONSTANTOPOULOS, T.
UNIVERSITY OF TEXAS AT AUSTIN, USA

Consider a single stochastic fluid, defined on the one-dimensional Skorohod reflection of a stochastic process $\{X_t, t \geq 0\}$ with càdlàg paths, viz., $Q_t = X_t + L_t$, $L_t = -\inf_{0\leq s\leq t}(X_s \wedge 0)$. We are interested in the case where $X_t - X_s = A(s,t] - B(s,t)$, $s \leq t$, for two locally finite random measures A, B. When A is continuous (with respect to the Lebesgue measure), B proportional to the Lebesgue measure, and $Q_0 = 0$, it can be shown that Q admits the alternative representation $Q_t = \int_0^t 1(s < c^{-1}Q_s)A(ds)$, where $c = B(dt)/dt$. Under stationarity and integrability assumptions this leads to Little's law $\mathrm{E}Q_0 = \alpha \mathrm{E}_A Q_0$, where $\alpha = \mathrm{E}A(0,1)$, and P_A is the Palm probability of the underlying stationary probability P, with respect to A. We show in this talk that the above can be generalized to cover a very general case, namely when A, B are jointly stationary, and Q_0 arbitrary. The alternative integral representation is completely equivalent to the reflection mapping representation of the stochastic fluid. Analogously, we obtain a general Little's law. Under some conditions, we obtain a distributional version of this law which can be used to give short proofs of formulae when A is a Lévy process. The proofs are probabilistic in contrast to analytical (Wiener-Hopf) approaches. Finally, we define a "dual" process Q^* that admits queueing interpretation and helps establish Little's law from the departures' point of view. The talk is motivated by the desire to explain and generalize some well-known results in communication networks performance, but also by the need to understand certain aspects of Lévy processes.

ON THE VALUE FUNCTION OF A PRIORITY QUEUE WITH AN APPLICATION TO A CONTROLLED QUEUEING MODEL

KOOLE, G.[11]
VRYE UNIVERSITEIT AMSTERDAM, THE NETHERLANDS

For a two-queue system with Poisson arrivals and exponential service times operated under the preemptive priority policy we computed the expected discounted holding and switching costs. Applying one step of policy improvement to this value function gives a policy that is nearly optimal for the system in which one has to decide when to switch.

ON THE USE OF LYAPUNOV-FUNCTIONS IN RENEWAL THEORY

LAST, G.[12]
TU BRAUNSCHWEIG, GERMANY

We consider a renewal process N, where the underlying distribution function F is absolutely continuous and has a finite first moment μ. The age process A_t, defined as the distance from t to the last renewal point of N, is a homogeneous Markov process with invariant probability measure $\pi(dx) = \mu^{-1}(1 - F(x))dx$. We introduce a Lyapunov function V satisfying the drift condition $\mathcal{A}V(x) \leq -\varepsilon(r(x) + 1)$ for all sufficiently large x, where $\mathcal{A}$ is the (extended) infinitesimal generator of (A_t), $\varepsilon > 0$, and $r(x)$ is the hazard rate of F. We then use well-known ergodic theorems to give a new proof of the uniform version of Blackwell's renewal theorem. In the second part of the talk we elaborate on this idea and assume the existence of the moment of order $\alpha \geq 1$. In this case the convergence in Blackwell's theorem takes place in a stronger sense and/or at a polynomial rate.

INFINITE TANDEM QUEUEING NETWORKS

MAIRESSE, J.[13]
LIAFA, PARIS, FRANCE

Consider $\cdot/GI/1/\infty$ FIFO queues interconnected in such a way that the interdepartures from a queue form the interarrivals in the next queue. We denote respectively by $s(n,k)$, $t(n,k)$ and $v(n,k)$ the service time of customer n at station k, the interarrival time between customers n and $n+1$ at station k, and the sojourn time of customer n at station k. We first consider a model with a one-sided infinite stream of customers (numbered by $\mathbb{N}$) sent through a one-sided infinite tandem of stations (numbered by $\mathbb{N}$). If the services $\{s(n,k); n \in \mathbb{N}, k \in \mathbb{N}\}$ are i.i.d. and if the process $\{v(0,k); k \in \mathbb{N}\}$ is ergodic and verifies $E[v(0,0)] > E[s(0,0)]$, then the interdeparture processes $T(k) = \{t(n,k); n \in \mathbb{N}\}$ converge weakly to a limit

[11] Joint work with P. Nain.
[12] Joint work with T. Konstantopoulos.
[13] Joint work with F. Baccelli, A. Borovkov and B. Prabhakar.

distribution.
Assume now that we consider a bi-infinite stream of customers (numbered by $\mathbb{Z}$) going through a one-sided infinite tandem of stations. Under stability conditions, Loynes' theorem states that if the interarrival process is ergodic then the interdeparture one is also ergodic of the same mean. Hence a station can be viewed as an operator on the set of stationary measures in $\mathbb{R}_+^{\mathbb{Z}}$. If we have $E[s(0,0)^{3+a}] < +\infty$, $a > 0$, we prove that there exist means $\alpha > E[s(0,0)]$ for which the operator admits fixed points which are ergodic and of mean α. For these means, if one starts from an interarrival process which is ergodic there is weak convergence to the fixed point. This extends results which were previously known for exponential service times.

RANDOM GRAMMARY

MALYSHEV, V.
MOSCOW STATE UNIVERSITY, RUSSIA

After briefly introducing physical terminology, it will be shown a drastic analogy between one queue models from one side and quantum particle and one-dimensional quantum gravity from another side.
After this more general models will be introduced providing relations between computer science and modern physics.

A CONSTRUCTIVE LARGE DEVIATION THEORY APPLIED TO THE JOIN-THE-SHORTEST-QUEUE PROBLEM

McDONALD, D.
UNIVERSITY OF OTTAWA, USA

Consider two queues with exponential service times which serve Poisson arrivals of two types. Some customers are dedicated to a particular queue while some customers join the shorter queue. The manner in which this system overloads is used to characterize the efficiency of the join the shorter queue protocol where efficiency means approximating a pooled system which acts like an M/M/1 queue. If enough discretionary customers are present then pooling occurs and the two queues are approximately of the same length. Surprisingly a weak pooling regime may occur where the system does act as an M/M/1 queue but one queue is a proportion of the other!

QUEUEING NETWORKS WITH TRACTABLE STATIONARY DISTRIBUTIONS

MIYAZAWA, M.
SCIENCE UNIVERSITY OF TOKYO, JAPAN

Three types of network models are considered: single stage transition, multi-stage transition, and concurrent movements. All may have multiple classes of arrivals and departures. The single stage transition means that entities in a network are routed from one node to another only once at a time, while the multi-stage

transition means that they are routed repeatedly with given triggering probabilities. The states of these networks are countable, but can be very general. On the other hand, the concurrent movement means that departures are simultaneously generated at nodes, and instantaneously transferred to nodes as arrivals. For the single transition network, necessary and sufficient conditions are derived for the stationary distribution to be product form, i.e., the product of its marginal distributions at nodes. These conditions immediately yields that quasi-reversibility implies the product form. The latter is also true for the multi-transition network, and we also have a so called biased local balance, called "cross", under some extra conditions. The concurrent movement network has a tractable stationary distribution if the traffic equations are linear with respect to micro level of states, i.e., network-state level. Finally it is noted that, using the same idea as for the network with the linear traffic equations, quasi-reversible networks can be extended so as to have network-state dependent arrival effects and departure rates, while keeping tractable stationary distributions.

QUASISTATIONARITY IN CONTINUOUS-TIME MARKOV CHAINS WITH POSITIVE DRIFT

POLLETT, P. [14]
UNIVERSITY OF QUEENSLAND, BRISBANE, AUSTRALIA

We shall consider continuous-time Markov chains on Z_+, which are both irreducible and transient, and which exhibit discernable stationarity before drift to infinity 'sets in'. After demonstrating this quasistationary behaviour with reference to several examples of birth-death chains, we will show how it can be modelled using a limiting conditional distribution: specifically, the limiting state probabilities conditional on not having left 0 for the last time. By defining a dual chain, obtained by killing the original process on last exit from 0, we can invoke standard theory on quasistationarity for absorbing Markov chains, thus obtaining new results on the existence of limiting conditional distributions for irreducible transient chains.

THE CRITICALLY LOADED MULTICLASS GI/PH/N QUEUE

REIMAN, M.[15]
BELL LABS, MURRAY HILL, NJ, USA

We consider the GI/PH/N queue (renewal arrivals, "phase-type" service time distribution, and N servers) with several customer classes, where a customer class is distinguished by its priority level in the queue and its first phase of service. We examine the behaviour of this system in the "critically loaded" regime: $N \to \infty$ and $\rho_N \to 1$ with $\sqrt{N}(1-\rho_N) \to \beta$, $-\infty < \beta < \infty$, where ρ_N is the traffic intensity in the N server system. We prove that, properly normalized, the queue length process converges to a diffusion process in $I\!R^K$, where K is the number of phases in the service time distribution. We also show that waiting times (properly normalized) converge to a simple functional of the limit diffusion for the queue length.

[14] Joint work with P. Coolen-Schrijner and A. Hart.
[15] Joint work with T. Puhalskii.

RECENT RESULTS ON ROSS'S CONJECTURE

ROLSKI, T.
UNIVERSITY OF WROCLAW, POLAND

Ross '78 asked whether higher fluctuation of the arrival processes worsens performance characteristics. In this context he posed a conjecture in the set up of a Cox/GI/1 queue with random intensity function $\lambda(t)$ such that $\frac{1}{t}\int_0^t \lambda(v)dv \to \overline{\lambda}$, that the expected mean waiting time in the Cox/GI/1 system is bigger than in the associated M/GI/1 queue with arrival rate $\overline{\lambda}$ and same service times.
In this talk I show two new results dealing with Markov-modulated queues. The first result gives the strong stochastic ordering between the waiting times in the Markov-modulated queue and the associated M/GI/1 queue. The second result deals with a family of Markov-modulated systems parameterized by $c > 0$, where $Q_c = cQ$ is the transition intensity matrix of the environmental process $J(t)$, and says the $\leq_{\mathrm{icx}}$ monotonocity of work-load V_c, $c > 0$.

ASYMPTOTICS OF STOCHASTIC NETWORKS WITH SUBEXPONENTIAL SERVICE TIMES

SCHLEGEL, S.[16]
UNIVERSITY OF ULM, GERMANY

We analyse the tail behaviour of stationary response times in the class of open stochastic networks with renewal input admitting a representation as (max, +)-linear systems.

For a K-station tandem network of single server queues with infinite buffer capacity, which is one of the simplest models in this class, we first show that if the tail of the service time distribution of one server, say server $i_0 \in \{1, \dots, K\}$, is subexponential and heavier than those of the other servers, then the stationary distribution of the response time until the completion of service at server $j \geq i_0$ asymptotically behaves like the stationary response time distribution in an isolated single-server queue with server i_0. Similar asymptotics are given in the case when several service time distributions are subexponential and asymptotically tail-equivalent.

This result is then extended to the asymptotics of general (max, +)-linear systems associated with i.i.d. driving matrices having one (or more) dominant diagonal entry in the subexponential class. In the irreducible case, the asymptotics are surprisingly simple, in comparison with results of the same kind in the Cramér case: the asymptotics only involve the excess distribution of the dominant diagonal entry, the mean value of this entry, the intensity of the arrival process, and the Lyapunov exponent of the sequence of driving matrices. In the reducible case, asymptotics of the same kind, though somewhat more complex, are also obtained.

[16] Joint work with F. Baccelli and V. Schmidt.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

AN EXTENDED LÉVY FORMULA FOR MARKOV PROCESSES

SERFOZO, R.
GEORGIA INSTITUTE OF TECHNOLOGY, ATLANTA, USA

Consider a continuous-time Markov process in which a value (cost or utility) is associated with each transition of the process. Lévy's formula is an expression for the expectation of these values in a time interval. Many applications of Markov processes, such as those involving travel times, are based on more general values of the process at transition times. We present an extended Lévy formula for the expectation of such functions. This formula provides a framework for characterizing Palm probabilities for stationary Markov processes. The key idea is to formulate events of interest as the "entire" sample path of the process being in certain subsets of all sample paths. An example is given for the expected time it takes a Markov process in equilibrium to travel from one set to another.

LARGE DEVIATIONS FOR PERFORMANCE ANALYSIS

SHWARTZ, A.
TECHNION, HAIFA, ISRAEL

We consider pure-jump Markov processes. We make the general observations about structure of local state functions in typical queues, state of the art in boundary theory, and "useful" soft arguments for solving the variational problem.
We then treat two applications: serve the longer queue with asymmetric service rates, and AMS model with arrivals, departures and "off" states. In each case we find the optimal (most likely) path to overflow and calculate its probability.

CHANGE ANALYSIS IN QUEUEING SYSTEMS

STEINEBACH, J.[17]
PHILIPPS-UNIVERSITY, MARBURG, GERMANY

We are interested in testing *changes* of certain characteristics of queueing systems which are often determined as the asymptotic *means* or *variances* of corresponding processes describing the model. On assuming that the latter processes satisfy a *weak invariance principle* with a suitable rate, we derive some *asymptotic CUSUM tests* for detecting changes in these characteristics. Conditions for the consistency of these tests are also discussed.

[17] Joint work with L. Horváth.

STRONG CONSERVATION LAWS ON SAMPLE PATHS: APPLICATIONS TO SCHEDULING QUEUES AND FLUID MODELS

STIDHAM, S., JR.
UNIVERSITY OF NORTH CAROLINA AT CHAPEL HILL, USA

The achievable-region approach, based on strong conservation laws, has most often been applied to stochastic scheduling and other control problems in the context of performance measures that are steady-state expected quantities. For some problems, however, strong conservation laws hold for performance measures at every point in time on every sample path. We exploit this property to study optimal control for certain queueing systems on a sample-path basis. Examples include preemptive scheduling to minimize a weighted sum of work in the system in each class, non-preemptive scheduling to minimize a weighted sum of the number of customers in each class (when all classes have the same service-time distribution), and scheduling processing of fluid in a multi-class fluid system operating in a random environment. The last problem is solved by considering the related Skorohod problem and its minimal solution.

JOB-SHOP SCHEDULING AND CONTROL OF MULTI-CLASS QUEUEING NETWORKS VIA MULTI-CLASS FLUID NETWORKS

WEISS, G.
UNIVERSITY OF HAIFA, ISRAEL

We discuss "flúid heuristics" for the scheduling and control of manufacturing systems. Such systems can be modeled as job-shops or as multi-class queueing networks. We point out the similarities and differences between the two types of models, and re-formulate them with a stress on the common features. We define multi-class fluid networks, which approximate the system, and are obtained through scaling and limiting process from the multi-class queueing model, or through relaxation from the job-shop model. We then propose the following scheme for heuristic scheduling and control: Formulate a multiclass fluid network problem. Solve the fluid problem. Obtain a scheduling heuristic which imitates the fluid solution. Evaluate the performance of this heuristic. We discuss the various steps in general and illustrate them in examples. First for the solution of fluid scheduling problems, we solve the fluid minimum makespan problem. The solution is very simple, it consists of constant flows which reduce all bufers at the same rate until they reach zero at a time which is easily seen to be lower bound for both the fluid and the original problem. We also discuss minimization of flowtime, which is a separated continuous linear program of the kind solved in general by Pullan, and for which we have developed an efficient algorithm in the case of two machine re-entrant lines. Next we discuss heuristic scheduling rules. We show how to construct an on-line heuristic (which starts jobs in order of arrival, in non-predictive fashion), which with the use of safety stocks keeps the bottleneck busy and the other machines keep pace with the bottleneck. We then perform a probabilistic analysis of this heuristic to show that for a problem of size N its suboptimality does not exceed $c\log(N)$ with a probability of $1 - 1/N$.

NONLOCAL AND NONLINEAR DIFFUSION EQUATIONS AND THEIR APPLICATIONS TO QUEUEING NETWORKS

WOYCZYNSKI, W.
CASE WESTERN RESERVE UNIVERSITY, CLEVELAND, OH, USA

I will discuss our recent work on stochastic flows governed by nonlocal and nonlinear integro-differential equations of "parabolic" type and their applications to the analysis of queueing networks. That would include some scaling limit results as well as a McKean-style "propagation of chaos" approximation results. The typical queueing application is the queueing serial network where the hydrodynamic limit for the related asymmetric exclusion interacting particle system leads to the Burgers equation. Another example is the $(SN)^2$-(lack of) discipline network which leads to other conservation laws. $(SN)^2 = SNSN$ = Serial Networking Secretary Network forbids service unless there is an extra customer waiting in line.

Author of the report: Sabine Schlegel, Ulm

E-Mail Addresses and Web-Pages of Participants

Eitan Altman	Eitan.Altman@sophia.inria.fr http://www.inria.fr/mistral/personnel/Eitan.Altman
Venkat Anantharam	ananth@vyasa.eecs.berkeley.edu http://www.eecs.berkeley.edu/~ananth
Søren Asmussen	asmus@maths.lth.se http://www.maths.lth.se/matstat/staff/asmus
François Baccelli	Francois.Baccelli@ens.fr http://www.dmi.ens.fr/~mistral/mcr.html
Nicholas Bambos	bambos@leland.stanford.edu http://pocari.stanford.edu/telecom/faculty/bambos.html
Bartłomiej Blaszczyszyn	blaszcz@math.uni.wroc.pl http://www.math.uni.wroc.pl/~blaszcz
Alexander Borovkov	borovkov@math.nsc.ru http://www.math.nsc.ru/LBRT/v1/borovkov/borovkov.html
Sem Borst	sem@cwi.nl
Onno Boxma	boxma@win.tue.nl http://www.win.tue.nl/math/bs/stoch_opt/boxma
Andreas Brandt	brandt@wiwi.hu-berlin.de http://www.wiwi.hu-berlin.de/institute/or/institut/brandt.shtml
Pierre Brémaud	bremaud@lss.supelec.fr
Hans Daduna	daduna@math.uni-hamburg.de http://www.math.uni-hamburg.de/home/daduna/
Jim Dai	dai@isye.gatech.edu http://www.isye.gatech.edu/faculty/dai/
Daryl Daley	daryl@maths.anu.edu.au
Serguei Foss	foss@math.nsc.ru http://www.nsu.ru/mmf/tvims/foss/foss.html
Sasha Gajrat	gajrat@facet.inria.msu.ru
Bruno Gaujal	Bruno.Gaujal@loria.fr http://www.inria.fr/sloop/personnel/Bruno.Gaujal/moi.html
Peter Glynn	glynn@leland.stanford.edu http://www.stanford.edu/~glynn
Arie Hordijk	hordijk@wi.leidenuniv.nl
Uwe Jensen	jensen@mathematik.uni-ulm.de http://www.mathematik.uni-ulm.de/stochastik/allgemeines/jensen.html
Offer Kella	mskella@mscc.huji.ac.il http://olive.mscc.huji.ac.il/~mskella/kella.html
Claudia Klüppelberg	cklu@ma.tum.de http://www.ma.tum.de/stat/

Takis Konstantopoulos	takis@alea.ece.utexas.edu http://www.ece.utexas.edu/ece/people/profs/#K
Ger Koole	koole@cs.vu.nl http://www.cs.vu.nl/~koole
Günter Last	G.Last@tu-bs.de http://fb1.math.nat.tu-bs.de:80/~schueler/mitarb/last.htm
Jean Mairesse	mairesse@liafa.jussieu.fr http://www.liafa.jussieu.fr/~mairesse
Vadim Malyshev	malyshev@vertex.inria.msu.ru
David McDonald	dmdsg@omid.mathstat.uottawa.ca http://www.uottawa.ca/~dmcdonal
Masakiyo Miyazawa	miyazawa@is.noda.sut.ac.jp http://queue5.is.noda.sut.ac.jp/~miyazawa
Phil Pollett	pkp@maths.uq.edu.au http://abacus.maths.uq.edu.au/~pkp
Martin Reiman	marty@research.bell-labs.com http://cm.bell-labs.com/who/marty
Ulrich Rieder	rieder@mathematik.uni-ulm.de http://www.mathematik.uni-ulm.de/or/allgemeines/rieder.html
Tomasz Rolski	rolski@math.uni.wroc.pl http://www.math.uni.wroc.pl/~rolski
Manfred Schäl	schael@uni-bonn.de
Rolf Schassberger	R.Schassberger@tu-bs.de http://fb1.math.nat.tu-bs.de/~schueler/home-ms.htm
Sabine Schlegel	schlegel@mathematik.uni-ulm.de http://www.mathematik.uni-ulm.de/stochastik.allgemeines/schlegel.html
Volker Schmidt	schmidt@mathematik.uni-ulm.de http://www.mathematik.uni-ulm.de/stochastik.allgemeines/schmidt.html
Richard Serfozo	rserfozo@isye.gatech.edu http://www.isye.gatech.edu/people/homepages/richard_serfozo.html
Adam Shwartz	adam@ee.technion.ac.il http://www.ee.technion.ac.il/~adam
Flos Spieksma	spieksma@wi.leidenuniv.nl
Josef Steinebach	jost@mathematik.uni-marburg.de
Shaler Stidham, Jr.	sandy@or.unc.edu http://www.or.unc.edu/faculty.html#stidham
Gideon Weiss	gweiss@stat.haifa.ac.il http://rstat.haifa.ac.il/~gweiss
Wojbor A. Woyczynski	waw@po.cwru.edu http://sun.stat.cwru.edu/~Wojbor

Tagungsteilnehmer

Dr. Eitan Altman
INRIA
Unite de Recherche
Sophia Antipolis
BP 109

F-06561 Valbonne Cedex

Prof.Dr. Venkat Anantharam
Department of Electrical
Engineering and Computer Sciences
University of California

Berkeley , CA 94720
USA

Prof.Dr. Soren Asmussen
Institut for Matematisk Statistik
Lunds Universitet
Box 118

S-221 00 Lund

Dr. Francois Baccelli
INRIA
Unite de Recherche
Sophia Antipolis
BP 109

F-06561 Valbonne Cedex

Prof.Dr. Nicholas Bambos
Department of Operations Research
Stanford University

Stanford , CA 94305-4022
USA

Prof.Dr. Bartlomiej Blaszczyszyn
Instytut Matematyczny
Uniwersytet Wroclawski
pl. Grunwaldzki 2/4

50-384 Wroclaw
POLAND

Prof.Dr. Alexander Borovkov
Institute of Mathematics SO RAN
Koptjug pr. 4

Novosibirsk, 630090
RUSSIA

Prof.Dr. Simon C. Borst
CW - Centrum voor Wiskunde en
Informatica
Postbus 94079

NL-1090 GB Amsterdam

Dr. Onno J. Boxma
Centrum voor Wiskunde en
Informatica
Kruislaan 413

NL-1098 SJ Amsterdam

Prof.Dr. Andreas Brandt
Institut für Operations Research
Wirtschaftswissenschaftliche Fak.
Humboldt-Universität Berlin
Spandauer Str. 1

10178 Berlin

Prof.Dr. Pierre Bremaud
Laboratoire des Signaux & Sytemes
Ecole Superieure d'Electricite
CNRS
Plate. u de Moulon

F-91192 Gif-sur-Yvette Cedex

Prof.Dr. Hans Daduna
Institut für Mathematische
Stochastik
Universität Hamburg
Bundesstr. 55

20146 Hamburg

Prof.Dr. Jim Dai
Industrial and Systems Engineering
Georgia Institute of Technology

Atlanta , GA 30332-0205
USA

Prof.Dr. Daryl J. Daley
CMA
School of Mathematical Sciences
Australian National University

Canberra ACT 0200
AUSTRALIA

Prof.Dr. Serguei Foss
Institute of Mathematics SO RAN
Akademic Koptyug prospect 4

630090 Novosibirsk 90
RUSSIA

Prof.Dr. Sasha Gajrat
Department of Mathematics and
Computer Science
P.O.Box 9512

NL-2300 RA Leiden

Prof.Dr. Bruno Gaujal
INRIA Sophia Antipolis
B.P. 93
2004 Route des Lucioles

F-06902 Sophia Antipolis Cedex

Prof.Dr. Peter Glynn
Department of Operations Research
Stanford University

Stanford , CA 94305-4022
USA

Prof.Dr. Arie Hordijk
Department of Mathematics and
Computer Science
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Uwe Jensen
Institut für Angewandte Mathematik
und Statistik
Universität Hohenheim

70593 Stuttgart

Prof.Dr. Offer Kella
kepartment of Statistics
The Hebrew University of Jerusalem
Mount Scopus

Jerusalem 91905
ISRAEL

Prof.Dr. Jean Mairesse
LITP Paris
7 Denis Diderot
4 place Jussieu

F-75252 Paris Cedex 05

Prof.Dr. Claudia Klüppelberg
Zentrum Mathematik
TU München
Arcisstr. 21

80333 München

Prof.Dr. Vadim A. Malyshev
INRIA
Rocquencourt
B.P. 105

F-78153 Le Chesnay Cedex

Prof.Dr. Takis Konstantopoulos
Dept. of Electrical and Computer
Engineering
University of Texas at Austin

Austin , TX 78712
USA

Prof.Dr. David McDonald
Department of Mathematics
University of Ottawa
585 King Edward

Ottawa , Ont. K1N 6N5
CANADA

Prof.Dr. Ger Koole
Mathematisch Instituut
Vrije Universiteit
De Boelelaan 1081 a

NL-1081 HV Amsterdam

Prof.Dr. Masakiyo Miyazawa
Department of Information Sciences
Faculty of Science and Technology
Science University of Tokyo
Noda City

Chiba 278
JAPAN

Dr. Günter Last
Institut für Mathematische
Stochastik der TU Braunschweig
Pockelsstr. 14

38106 Braunschweig

Prof.Dr. Philip K. Pollett
Department of Mathematics
University of Queensland

Brisbane, Queensland 4072
AUSTRALIA

Prof.Dr. Martin Reiman
Lucent Technologies
Bell Laboratories
700 Mountain Avenue

Murray Hill , NJ 07974-0636
USA

Sabine Schlegel
Abteilung Stochastik
Universität Ulm

89069 Ulm

Prof.Dr. Ulrich Rieder
Abteilung für Mathematik VII
Universität Ulm

89069 Ulm

Prof.Dr. Volker Schmidt
Abteilung Stochastik
Universität Ulm

89069 Ulm

Prof.Dr. Tomasz Rolski
Instytut Matematyczny
Uniwersytet Wroclawski
pl. Grunwaldzki 2/4

50-384 Wroclaw
POLAND

Prof.Dr. Richard F. Serfozo
School of Industrial Engineering
Georgia Institute of Technology

Atlanta , GA 30332-0205
USA

Prof.Dr. Manfred Schäl
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 6

53115 Bonn

Prof.Dr. Adam Shwartz
Dept. of Electrical Engineering
TECHNION
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof.Dr. Rolf Schaßberger
Institut für Mathematische
Stochastik der TU Braunschweig
Pockelsstr. 14

38106 Braunschweig

Prof.Dr. Flora M. Spieksma
Department of Mathematics and
Computer Science
Rijksuniversiteit Leiden
Postbus 9512

NL-2300 RA Leiden

Prof.Dr. Josef Steinebach
Fachbereich Mathematik
Universität Marburg

35032 Marburg

Dr. Shaler Stidham, Jr.
Dept. of Operations Research
University of North Carolina
CB 3180, 210 Smith Building

Chapel Hill , NC 27599-3180
USA

Prof.Dr. Gideon Weiss
Department of Statistics
University of Haifa

Haifa, 31905
ISRAEL

Prof.Dr. Wojbor A. Woyczynski
Dept. of Mathematics and Statistics
Case Western Reserve University
10900 Euclid Avenue

Cleveland , OH 44106-7058
USA