MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 28|1969

Symposium on Celestial Mechanics

17.8. bis 23.8.1969

The conference on celestial mechanics of 1969 has been the third meeting at Oberwolfach on this subject within six years. All three meetings were directed by Professor E. Stiefel, Zurich.

The main domains on which papers have been delivered and which led, in most cases, to extremely stimulating discussions have been the following:

- Regularization of the differential equations of motion.
- The problem of three bodies.
- Optimization problems.
- Stellar dynamics; the problem of N bodies.
- Resonance problems.
- Stability of motion.
- Numerical integration of the differential equations of celestial mechanics.

The proceedings of the conference have been published in the journal "Celestial Mechanics", Volume 2, No. 3, October 1970.

List of participants

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C.A. Burdet, ETH Zürich
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R. Wielen, Heidelberg
G.A. Wilkins, Hailsham

Abstract of lectures

E. Stiefel: Remarks on numerical integration of Keplerian orbits.

The classical differential equations of the two-body motion are unstable in the sense of Lijapounov and suffer thus from a very bad numerical error propagation especially in the case of highly eccentric orbits. A stabilization is offered characterized by the use of the eccentric anomaly as the independent variable and the Levi-Cività coordinates as the dependent variables. (For three-dimensional motion the appropriate generalization of the Levi-Cività variables are the KS-variables.) In case of the eccentricity .9 for example, the errors are thus reduced by a factor 12000.

D.G. Bettis: Stabilization of finite difference methods of numerical integration.

To examine the stabilizing effects of a modification of the classical finite difference methods of numerical integration the differential equations of perturbed Keplerian motion are integrated for two examples: an artificial satellite of the Earth, and Hill's variation orbit. The modified methods remove much of the instability that is inherent to the classical methods.

G. Scheifele: Canonical theory of threedimensional regularization.

Generalizations in the canonical theory of dynamics are made; at first transformations which augment the number of canonical variables, and secondly differential transformations of the independent variable are outlined. This is applied to the perturbed two-body problem. The results are canonical systems using independent variables other than time. The application of the theory to the KS-transformation yields a completely regular canonical system in a 10-dimensional phasespace, using the eccentric anomaly as independent variable. Subsequently sets of 10 regular canonical elements are introduced.

J.M. Lewallen: Recent applications of regularization theory to trajectory optimization problems.

The regularized trajectory optimization problem is formulated and application is made to the optimal, low-thrust, Earth escape spiral and the optimal, low-thrust, Earth-Jupiter transfer. These examples are implemented in both rectangular Cartesian and polar cylindrical coordinates. The numerical accuracy achieved and the computer time required are compared for all cases using various numerical integration error bounds. The results obtained indicate that for space vehicles which experience wide variations in the gravitational force magnitude, significant reductions in computing time can be obtained by using the regularized equations. In some cases, the computing time is reduced by a factor of three if regularized variables are used. Furthermore, use of the polar coordinates consistently results in more favorable computer times than when rectangular coordinates are used.

J.M.A. Danby: Matrix perturbation methods using regularized coordinates.

Matrix methods for computing perturbations of non-linear perturbed systems, as formulated by Alexeev, involve an expression for the full solution of the first variational equations of the system evaluated about a reference orbit. These cannot be immediately applied to a regularized system of equations where perturbations about Keplerian motion are considered since the solution of the variational equations of regularized Keplerian motion does not in general correspond to the solution of the variational equations of the unregularized equations. But, as Kustaanheimo and Stiefel have pointed out, the regularized equations of Keplerian motion should be excellent for the initiation of a perturbation theory since they are linear in form. The lecture describes a method for applying

Alexeev's theorem to a regularized system where full advantage is taken of the basic linear form of the unperturbed equations.

E. Rabe: A method for stability determinations in the elliptic restricted problem.

Periodic solutions of the elliptic restricted problemexist for certain commensurabilities between the basic orbital period of the two primaries and the period of the corresponding periodic motion of the body of negligible mass in the circular restricted problem. For such periodic solutions, the infinite determinant approach to the determination of the characteristic exponents and thus of first order stability can be extended to the elliptic restricted problem. The resulting method is applied to small librational motions of long period (for appropriate values of the mass ratio μ of the primaries), and it is found that their stability in the elliptic problem depends on two different sets of (long- and short-period) exponents, in contrast to the one set in the ordinary restricted problem.

H.J. Sperling: The real singularities of the N-body problem in celestial mechanics.

In the first part some basic concepts and results are discussed. A brief survey of the present status and some recent progress in the problem of the real singularities is given. In the second part a more detailed discussion is presented of the proof of the following Theorem: Let the N-body motion be holomorphic on $[t_o, t^*]$ and $\lim J < \infty$ as $t \to t^*$. Then the motion remains holomorphic or there is a collision singularity at t^*. J is the polar inertia momentum of the N bodies; a collision singularity is such that the N bodies separate into distinct "clusters" with all bodies in each cluster colliding at the instant of the singularity, while the (centers of mass of the) clusters remain apart from each other.

V. Szebehely: Poincaré's hydrodynamic analogy in celestial mechanics.

The formal similarity and analogy between the differential equations describing two degrees of freedom non-integrable, irreversible dynamical systems on the one hand and two and three dimensional flow of incompressible or compressible fluids on the other hand is investigated in some detail.

D.G. Saari: Separation of clusters in the problem of N bodies.

The problem of N bodies is classified according to motion. Under certain conditions, this turns out to be a direct generalization of the two body problem. These classifications are then exploited to give new results on the behavior of the N-body problem as $t \to \infty$.

R. Wielen: Dynamical evolution of star clusters as an N-body problem in celestial mechanics.

The dynamical evolution of star clusters has been studied by numerically integrating the equations of motion of all the stars as an N-body problem. The method under consideration of integration uses time steps which vary with time and from star to star. The results of eleven star cluster models are presented. Each model contains 100 stars. A realistic spectrum of stellar masses is used. In some of the models the galactic tidal field, a mass loss of evolving stars, and an overall rotation of the cluster are taken into account. The initial conditions of most of the cluster models correspond to a stationary and stable solution of the encounterless Liouville equation (Plummer's model). Hence the dynamical evolution of the cluster models is mainly caused by the effects of relaxation.

The results of the star cluster model computations can be used to explain the observed distribution of ages of galactic star clusters as a consequence of the dynamical dissolution of the clusters due to the escape of stars. The empirical values for the total lifetimes of open clusters vary between 10^8 years and some 10^9 years; 50% of the open clusters evaporate within 2×10^8 years. The theoretical times of dissolution, based on an extrapolation of the escape rates of the star cluster models, can explain the observed range of lifetimes of the galactic clusters as due to different radii and total masses (or total numbers of stars) of the clusters. However, for the typical lifetime of a galactic cluster we predict from the models a value of 1×10^9 years, which is somewhat longer than observed. The main reason for this discrepancy may be the fact that we have neglected so far the disrupting effect of passing star clouds and of interstellar gas clouds in our star cluster models.

J. Moser: On the boundeness of the solution and the singularity of the Störmer problem.

This is a report on a joint work with Martin Braun (Brouwn University) on various results about the motion of a charged particle in a dipole field, the so-called Störmer problem. This problem leads to a Hamiltonian system of two degrees of freedom with a complicated singularity at the position of the dipole. The topological nature of the flow near this singularity can be described completely by transforming it into a simpler model system. In particular, an old question concerning the uniqueness of asymptotic orbits approaching the singularity can be answered affirmatively. Furthermore the existence of quasi-periodic motions and the stability of periodic motions are discussed.

O. Volk: Bemerkungen zur Geschichte der Himmelsmechanik (Remarks about the history of celestial mechanics).

Veranlasst durch eine Bemerkung von Jaques Levi in einem Vortrag, den er fast genau vor zwei Jahren, anlässlich der Feier des 300. Anniversariums der Grundsteinlegung des Pariser Observatoriums gehalten hat, mit dem Thema:"Der französische Beitrag zur Entwicklung der Himmelsmechanik im Laufe der letzten drei Jahrhunderte", wird den Gründen nachgespürt, die für das grosse Spatium von fast 70 Jahren verantwortlich sind, das verstreichen musste, bis Newtons "Philosophiae naturalis principa mathematica" auch in Frankreich voll zur Anerkennung kamen. Es wurde insbesondere auf die entscheidende Rolle eingegangen, die Maupertuis, Voltaire und die Marquise du Châtelet, die "Diva Emilia" von Voltaire, gespielt haben.

A.T. Sinclair: Periodic solutions in the commensurable three-body problem.

A proof of the existence of families of periodic solutions in the problem of three bodies close to a commensurability in mean motions is given. The method used is to eliminate short period terms from the disturbing function using a von Zeipel transformation, and to look for solutions of the resulting equation of motion in which the variables remain constant. The form of the equations is obtained by making use of the d'Alembert property of the disturbing function. Families of periodic solutions are shown to exist in the non-planar circular restricted problem, the planar non-restricted problem, and the planar elliptic restricted problem.

C.A. Burdet: Keplerian motion and harmonic oscillators.

It is well known that the three-dimensional classical equations of motion for the perturbed problem of two bodies can be brought into the form of harmonic oscillators which, in these cases, have varying frequencies. Transformations of dependent and independent variables as well as the introduction of elements of the motion are needed. Two further ways which lead to this aim are presented. In one case the energy and a Laplace vector are used, and in the other case the parameter and the inverse of the distance. Subsequently, the method of variation of constants is applied to the equations. The results are perturbation equations of the order fourteen, but it is possible to reduce slightly this total order.

B. Thüring: Numerische Untersuchungen über nichtperiodische Transtrojaner-Bahnen (Numerical explorations of non-periodic Transtrojan orbits).

For 14 values of the mass parameter μ (from 0.0010 until 0.0150) the non-periodic Transtrojan orbits (around L_4 an L_5) are investigated, which on the plane restricted problem of three bodies pass the point situated opposite to the body μ (looking from the main mass) with zero velocity in the rotating coordinate system. Results: The Transtrojan state contains a finite number of 'double librations' (around L_4 and L_5); this number decreases with growing value of the mass parameter. Above a value of mass parameter between 0.010 and 0.015 no further double-libration takes place. Certain topologic properties of the Transtrojan state are found; for example this state has a phase of narrowing and a phase of widening of the single librations; thereby the amplitues of the librations fluctuate in a characteristic manner.

B.D. Tapley: Regularization and the computation of optimal trajectories.

A completely regular form for the differential equations governing the three-dimensional motion of a continuously thrusting space vehicle is obtained by using the Kustaanheimo-Stiefel regularization. The differential equations for the thrusting rocket are transformed using the K-S transformation and an optimal trajectory problem is posed in the transformed space. The canonical equations for the optimal motion in the transformed space are regularized by a suitable change of the independent variable. The transformed equations are regular in the sense that the differential equations do not possess terms with zero divisors when the motion encounters a gravitational force center. The resulting equations possess symmetry in form and the coefficients of the dependent variables are slowly varying quantities for a low-thrust space vehicle.

B. Garfinkel: On the ideal resonance problem.

The ideal resonance problem is soluble in a power series in $\varepsilon^{1/2}$, where ε is the small parameter of the problem. The zeroth order solution is given by the motion of a simple pendulum. The perturbed solution to any order is expressible in terms of elliptic functions, and is free of singularities and mixed secular terms. This solution provides a general theoretical framework for an attack on resonance problems in celestial mechanics when the latter are reducible to the ideal form. Such a reduction is feasible in the problems of the critical inclination and in that of the tesseral harmonics resonance in the artificial satellite theory.

P.J. Message: On linear equations of variation in dynamical problems.

The linear equations of variation, associated with a motion of a particle moving in a plane under a field of force which admits a first integral of the motion of any form, are drawn up in terms of the tangential and normal displacements. The existence of the first integral implies that the normal displacement satisfies a single second-order differential equation, the tangential displacement being given from the solution of this by a single quadrature. The special cases are examined in which the integral is one of energy, and in which it is one of angular momentum. The extension is made to the motion of two particles moving in a plane under a conservative force-field depending on their positions, which admits also an integral of angular momentum. (The study of the relative motion in the gravitational problem of three bodies in the plane may be put into this form by Jacobi's formulation.) An equation is given for finding the non-zero characteristic exponents of a periodic solution of this second problem.

H.A. Kellner: <u>The local invariants under rotations about an axis and their applications.</u>

The three possible scalar products formed from the position-speed state vector lead to Stumpff's local invariants. The assumption of a distinguished axis (001) gives rise to the 'axial' local invariants z and $\dot{z}$ under the rotations about this axis. - If a satellite moves in the field of a rotationally symmetric central body, then there will be an appropriate set of invariants for which a regular, non-linear system of differential equations holds. It can be used to derive recurrence relations for time series expansions and special perturbation methods, which are characterised by the occurence of the scalar products of the perturbational acceleration with the position and speed vectors.

H.G. Walter: <u>Association of spherical and ellipsoidal gravity coefficients of the earth's potential.</u>

A comparison is drawn between the expansion of the potential in spherical harmonics on the one hand and in ellipsoidal harmonics on the other, with the objective of associating the spherical and ellipsoidal gravity coefficients of the Earth's potential.

For this purpose the properties of orthogonality of the Lamé functions of the first kind have been tailored to this subject of investigation and become instrumental in establishing the mathematical expressions which relate the two classes of gravity coefficients to each other. In deriving the elements of the transition matrices elliptic integrals have been encountered whose reduction to the three kinds of canonical elliptic integrals is discussed.

G.A. Wilkins: The improvement of the lunar ephemeris.

At present the fundamental lunar ephemeris is based on Brown's theory of the motion of the Moon with improvements based on the bypassing of Brown's Tables, the removal of the great empirical term, the substitution of the relevant constants of the IAU system of astronomical constants and the retransformation of Brown's series in rectangular coordinates to spherical coordinates. Even so this ephemeris does not represent adequately the recent range and range-rate radio observations, and it will be inadequate for use in the analysis of laser observations of corner reflectors on the Moon. Numerical integrations for these purposes have already been made at the Jet Propulsion Laboratory, but improved theoretical developments are also required; new solutions of the main problem are in hand elsewhere. Work at H.M. Nautical Almanac Office is aimed at obtaining improved values of the constants of the lunar orbit by a rediscussion of occulation observations made since 1943 and at the redevelopment of the series for the planetary perturbations using more precise theories of the motion of the Sun and planets. The techniques and preliminary results of exploratory numerical integrations were briefly described.

E. Hölder: Navigationsformel zu A. Busemann's Variationsproblem der Raumfahrt (Transition of minimum fuel consumption between Kepler ellipses in the plane).

The indicatrix of the variation problem is given, first by the perturbation differential equation for the semi-major axis and eccentricity, and secondly by a relation between the momentum and eccentric anomaly. The Hamiltonian equations of an extremal solution, therefore, reduce to a navigation equation. The remaining perturbation equations for the longitude for perihelion and the mean longitude at epoch, finally, yield the time dependence that gives the most economic fuel consumption.

R.A. Broucke: Solution of equations of motion with computerized series expansions.

Modern computer techniques have been investigated in detail for the purpose of computing perturbations in the form of series expansions (general perturbations). Work has been done principally in two areas:

- Automatic manipulation of Poisson series with a computer. In particular the classical expansions of the two-body problem with Bessel functions.
- Construction of new variation of parameter methods, mainly with rectangular coordinates, and especially suited for solutions with iterative series expansions on a computer.

E.A. Roth: Launch window study for the highly eccentric orbit satellite HEOS-1.

A satellite with a high eccentricity $e \simeq 0.95$ is strongly perturbed by the sun and the moon. This fact and mission constraints restrict considerably the possible launch times for such a satellite. The launch window calculations can be performed in two steps in order to save computing time. An approximate analytical solution provides a general survey of the launch opportunities. An accurate numerical approach is the necessary for the exact definition of the launch window. In the case of the orbit of HEOS-1, moreover the consideration of the injection errors has been of great importance.

H.G. Hertz: Comparison of Brouwer's theory with numerical integration.

Comparisons were made between Brouwer and modified Brouwer orbits and $1^m.0$ numerical integration orbits over 10^d and also 60^d. The differences between an unmodified Brouwer orbit and a numerical integration orbit (both orbits using the same initial conditions) were appreciable and much larger than the differences between the $1^m.0$ and $0^m.5$ numerical integration orbits which makes it less likely that the differences are due to errors in the numerical integration than in Brouwer's theory. If the computation of the short-period terms using the secular portions e",I" as suggested by Brouwer (unmodified orbit) is replaced by the computation using the long-period portions e',I' as is more reasonable on theoretical grounds the deviations from the numerical integration surprisingly become larger. However, they are greatly reduced if the second order short-period terms in the semi major axis are included and they are further reduced by reducing the mean motion of the mean anomaly to a value based on Kozai's theory. Further reduction is possible by adding empirical Fourier series to the elements but the practical value is low.

G. Scheifele, ETH Zurich