MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 29/1969

Harmonische Analyse und Darstellungstheorie topologischer Gruppen

24. bis 30. August 1969

Im Mathematischen Forschungsinstitut in Oberwolfach fand erstmals eine Tagung über Harmonische Analyse und Darstelluhgstheorie topologischer Gruppen statt, die von den Herren H. Leptin (Heidelberg) und E. Thoma (Münster) geleitet wurde. Erfreulich war die große Zahl der ausländischen Gäste. 22 Vorträge gaben einen Überblick über neueste Ergebnisse und Forschungen, vor allem zur Dualitätstheorie und zur Darstellungstheorie von Liegruppen, doch wurden auch Fragen der harmonischen Analyse kommutativer und nicht-kommutativer Gruppen, der Strukturtheorie topologischer Gruppen, der von Neumann-Algebren, der Darstellungstheorie diskreter Gruppen sowie über mittelbare Gruppen behandelt.

Teilnehmer:

H. Behncke, Heidelberg
W. Beiglböck, Heidelberg
J. Carmona, Marseille
J. Cigler, Groningen
Derighetti, Zürich
F. Eckstein, Münster
J. Ernest, Santa Barbara
P. Eymard, Nancy
V. Flory, Heidelberg
F.P. Greenleaf, New York
Gerstner, Erlangen
S. Grosser, Ithaka
S. Helgason, Cambridge (Mass.)
R.W. Henrichs, Münster
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K.H. Hofmann, New Orleans
A. Hulanicki, Wroclaw
H. Johnen, Aachen
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M. Kneser, Göttingen
A. Kunze, Irvine
G.A. Lancaster, St. Louis
H. Leptin, Heidelberg
G. Macdonald, Oxford
M. Mebkhout, Marseille
Y. Meyer, Orsay
M. Moskowitz, New York
D. Poguntke, Berlin
H. Reiter, Utrecht
L. Robertson, Seattle
G. Schlichting, Münster
Schreiber, Paris
J. Stegeman, Utrecht
M. Takesaki, Philadelphia
N. Tatsuuma, Los Angeles
E. Thoma, Münster
G. van Dijk, Utrecht
T.W. Wilcox, Rochester
E.N. Wilson, St. Louis
G. Zumbusch, Münster

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Vortragsauszüge

H. BEHNCKE: Äußere Automorphismen von W*-Algebren.

Sei G eine unendliche diskrete Gruppe, deren Klassen bis auf $\{e\}$ alle unendlich sind. Dann besitzt die W*-Algebra der regulären Darstellung von G äußere Automorphismen, sofern G nicht komplett oder vollständig ist. Ist G ein freies Produkt $G = G_1 * G_2$, dann lassen sich Automorphismen σ_1 von G_1 und σ_2 von G_2, die nicht beide trivial sind, gleichzeitig zu einem äußeren Automorphismus der W*-Algebra der regulären Darstellung von G fortsetzen. Insbesondere läßt sich jede separable lokalkompakte Gruppe H als Gruppe von äußeren Automorphismen der Linksalgebra der freien Gruppe mit zwei Erzeugenden darstellen.

J. CARMONA: Représentations unitaires induites holomorphes et irreductibilité des représentations des groupes de Lie semi-simples connexes réels.

Dans tout ce qui suit, on désigne par G un groupe de Lie engendré par un voisinage compact de l'unité, $\mathfrak{g}$ son algèbre de Lie, Γ un sous-groupe fermé de G, γ son algèbre de Lie, $\mathfrak{E}_c$ le (respmt la) complexifié (e) de tout sous-espace (respmt sous-algèbre) $\mathfrak{E}$ de $\mathfrak{g}$, $X \to \bar{X}$ la conjugaison de $\mathfrak{g}_c$ définie par $\mathfrak{g}$, $\mathfrak{J}_c$ une sous-algèbre de $\mathfrak{g}_c$ verifiant les conditions suivantes:

(i) $\mathrm{Ad}\,\Gamma(\mathfrak{J}_c) \subseteq \mathfrak{J}_c$, où $x \to \mathrm{Ad}\,x$ est la représentation adjointe de G dans $\mathfrak{g}_c$,

(ii) $\mathfrak{J}_c \cap \bar{\mathfrak{J}}_c = \gamma_c$,

(iii) $\mathfrak{J}_c + \bar{\mathfrak{J}}_c = \mathfrak{s}_c$ complexifiée d'une sous-algèbre $\mathfrak{s}$ de $\mathfrak{g}$.

Etant données une représentation L de Γ dans un espace de Hilbert $\mathcal{V}$ et une représentation ν de $\mathfrak{J}_c$ dans l'algèbre $\mathcal{L}(\mathcal{V}^\infty)$ des opérateurs linéaires continus sur l'espace de Frechet $\mathcal{V}^\infty \subseteq \mathcal{E}(G,\mathcal{V})$ des vecteurs de $\mathcal{V}$ différentiables pour L, on suppose que les opérateurs adjoints $L(\gamma)^*, \gamma \in \Gamma$, et $\nu(X)^*$, $X \in \mathfrak{J}_c$, sont dans $\mathcal{L}(\mathcal{V}^\infty)$ et que les conditions suivantes sont vérifées:

(i) $\nu(\mathrm{Ad}\gamma(X)) = L(\gamma)\,\nu(X)\,L(\gamma)^{-1}$, $\gamma \in \Gamma$, $X \in \mathfrak{J}_c$,

(ii) la restriction de ν à γ est la différentielle de la représentation $\gamma \to \Delta_G(\gamma)^{-\frac{1}{2}}\,\Delta_\Gamma(\gamma)^{\frac{1}{2}}\,L(\gamma)$.

Une représentation induite holomorphe par le couple (L,ν) de $(\Gamma,\mathfrak{J}_c)$ sur G est une représentation unitaire obtenue par restriction de la représentation régulière droite r de G dans $\mathcal{L}_b(\mathcal{D}(G),\mathcal{V})$ à un sous-espace hilbertien invariant $\mathcal{H}$ forme de distributions vérifiant les conditions:

(1) $\ell(\gamma^{-1})T = \Delta_G(\gamma)^{-\frac{1}{2}} \Delta_\Gamma(\gamma)^{\frac{1}{2}} L(\gamma)_o T, \gamma \in \Gamma,$
(2) $\ell(X)T + \vartheta(X)_o T = 0, X \in \mathfrak{J}_c$,

où ℓ désigne la représentation régulière gauche de G dans $\mathcal{L}(\mathcal{D}(G),\mathcal{V})$. On caractérise les noyaux reproduisants de ces espaces hilbertiens, et on définit une injection de l'espace des opérateurs d'entrelacement de deux représentations de ce type dans un espace de distributions à valeurs vectorielles généralisant l'espace proposé par Blattner.
On applique ces résultats aux groupes de Lie semi-simples connexes réels en associant à chaque sous-algèbre de Borel $\mathfrak{J}_c$ de $\mathfrak{g}_c$, au sous-groupe résoluble maximal correspondant Γ de G et a tout caractère L de Γ une représentation unitaire induite holomorphe. On démontre des critères d'irréductibilité et d'équivalence de telles représentations ainsi que des résultats sur la détermination explicite de la structure hilbertienne de $\mathcal{H}$ et des opérateurs d'entrelacement.
Les exemples traites sont destinés à montrer que la théorie permet d'étudier comme cas particulier les groupes compact, les séries discrètes de G, les séries principales et complémentaires construites par F. Bruhat.

J. CIGLER: Normed ideals in $L^1(G)$.

Let G be a locally compact Abelian group and $L^1(G)$ the group algebra of G. An ideal N in $L^1(G)$ is called normed ideal if the following conditions hold:

N 1) N is dense in $L^1(G)$;

N 2) N is a Banach space under some norm $\|\cdot\|_N$ such that
$\|f\|_1 \leq \|f\|_N$ for all $f \in N$,

N 3) $\|h * f\|_N \leq \|h\|_1 \cdot \|f\|_N$ for all $h \in L^1(G)$ and $f \in N$.

Normed ideals may be regarded as generalizations of Segal algebras or of homogeneous Banach spaces. There definition is modelled after the corresponding fundamental notion in Hilbert space theory.
The structure of normed ideals is studied, Segal algebras are characterized in terms of normed ideals, and some examples and counter examples are given.

J. ERNEST: Duality in representation theory.

We discuss the duality principle as it appears in the unitary representation theory of locally compact groups, from its beginning with the Pontrjagin and Tannaka duality theorems. Formulations of duality for arbitrary locally compact groups, including the recent results of Tatsuuma, Takesaki, Katz and Ernest, are discussed, furthermore duality theorems for objects other than groups (including C*-algebras). The current and possible future direction of research in this field is considered. In particular, we examine the duality principle as it arises in the context of covariant representations of physical systems.

P. EYMARD: Moyennes invariantes sur les espaces homogènes et représentations quasi-régulières des groupes localement compacts.

Un théorème sur les rapports entre les deux propriétés suivantes pour un groupe localement compact G et un sousgroupe fermé H de G: (P_1) Sur l'espace des fonctions continues bornées $\mathcal{C}(G/H)$ définies sur l'espace homogène des classes à droite de G selon H, il existe une moyenne invariante pur l'action de G; (P_2) La représentation identité est faiblement contenue, au sens de J.M.G. Fell, dans la représentation quasi-régulière de G dans $L^2(G/H)$. Ce théorème généralise l'étude faite par H. Reiter et A. Hulanicki sans le cas des groupes, ou H est réduit a l'élément neutre de G.

F. GREENLEAF: Measure of powers C^p for sets in locally compact groups.

Let C be an open, relatively compact set in an abelian group. Then Emerson and Greenleaf have shown that the Haar measure $|C^p|$ has the growth property

$$|C^p| = A\, p^k + \mathcal{O}(p^{k-1}\log p) \quad \text{as } p \to \infty ,$$

and in particular $\dfrac{|C^{p+1}|}{|C^p|} \to 1$ (*)

The growth condition (*) makes sense in every locally compact group G and seems to be closely connected with amenability of the groups. This growth condition is very difficult to verify (or disprove) in general groups and there are many open problems. The weaker condition

$$\liminf_{p\to\infty} \left\{\frac{|C^{p+1}|}{|C^p|}\right\} = 1 \qquad (**)$$

for all open relatively compact sets is easier to verify and turns out to be true for 1. solvable discrete finitely generated groups with a nilpotent subgroup of finite index (J.A. Wolf: J. Diff. Geometry,1968), 2. nilpotent groups. The condition (**) fails for the solvable finitely generated groups which do not have a nilpotent subgroup of finite index: Compact open sets C exhibit exponential growth for $|C^p|$ (Milnor: J.Diff. Geometry, 1968). Emerson and I conjecture that in every group G we must have C^p getting "size regular" as $p \to \infty$, so that (*) iff (**). But this seems to be a difficult open problem.

S. GROSSER: Harmonische Analyse auf zentralen Gruppen.
Eine zentrale Gruppe ist eine lokal-kompakte Gruppe G, deren Zentrumsfaktorgruppe G/Z kompakt ist. Sei [Z] die Klasse dieser Gruppen. Verfasser hat zusammen mit M. Moskowitz in einer Reihe von Arbeiten die Theorie dieser Gruppen entwickelt. In bezug auf Strukturtheorie, Darstellungstheorie und harmonische Analyse erweisen sich [Z]-Gruppen als die natürlichste Verallgemeinerung der kompakten und der lokalkompakten abelschen Gruppen, dies sowohl in dem Sinne, daß viele der in diesen beiden Klassen auftretenden Phänomene erst unter diesem Gesichtspunkt zusammengefaßt werden können, als auch in dem, daß eine weitergehende Verallgemeinerung zu starkem Verlust an Detail führt. Die in der harmonischen Analyse zentraler Gruppen erzielten Resultate lassen sich unter folgende Rubriken einordnen: 1. Approximationssätze und eine Charakterformel, 2. Der $\#$-Operator und die starke Halbeinfachheit von $L^1(G)$, 3. Eine Charakterisierung maximalfastperiodischer Gruppen und Eindeutigkeitssätze für die Fourier-Transformation, 4. Die Plancherel-Formel für zentrale Gruppen, 5. Äquivalenzkriterien für endlich-dimensionale unitäre Darstellungen, 6. Die Struktur der Algebren $L^1(G)$ und $\mathfrak{Z}(L^1(G))$.

S. HELGASON: Radon transforms and group representations.
Let X be a symmetric space of the noncompact type, Y the space of horocycles in X, and G the identity component of the group of isometries of X. The semisimple group G is transitive on both X and Y, so that $X = G/K$, $Y = G/H$ for suitable subgroups K, H of G. In analogy with spherical functions on X we consider the H-invariant eigenfunctions of the G-invariant differential operators on Y, the socalled conical functions. Conical distributions on Y are defined similarly. Whereas the conical functions correspond explicitly to the irreducible finite-dimensional representations of G with a fixed vector under H, the conical distributions are intimately related to the complex principal series of infinite-dimensional representations of G. These distributions can thus be used to construct the intertwining operators, derive the irreducibility criteria and set up the rudiments of a highest weight theory for the representations in the complex principal series.

K.H. HOFMANN: Duality theories for compact semigroups and C*-bigebras.
We establish a complete duality between the category of compact topological semigroups and the category of commutative C*-bigebras

(C*-Hopf-algebras) with identity and derive from this general duality the duality theorems of Pontrjagin and Tannaka as well as lesser known duality theories for special classes of compact semigroups such as semilattices. In the course of the discussion the concept of a C*-bigebra has to be introduced properly, and the working of the duality in terms of bigebra ideals on one side and subsemigroups and semigroup ideals on the other is described in some detail. Particular emphasis is placed on connections with classical character or semi-character theory and representation theory with the present set-up.

A. HULANICKI: On ℓ_1-group algebra of a discrete group.

It is proved that if G is a nilpotent discrete group, then for any x in $\ell_1(G)$ such that $x^* = x$ we have

$$\lim_{n\to\infty} \sqrt[n]{\|x^n\|} = \|L_x\| ,$$

where $x \to L_x$ is the left regular representation of $\ell_1(G)$ on $\ell_2(G)$. This implies the symmetry of $\ell_1(G)$. (The details will appear in Studia Mathematica).

E. KANIUTH: Zur Struktur der regulären Darstellung diskreter Gruppen.
G sei eine diskrete Gruppe, G_f der Normalteiler aller Elemente in G, die zu endlichen Konjugationsklassen gehören, G_f' die Kommutatorgruppe von G_f und $Z(G_f)$ das Zentrum von G_f. Ferner bezeichne L die links-reguläre Darstellung von G.
Satz: (1) L ist vom Typ I genau dann, wenn $[G:Z(G_f)] < \infty$ ist.
(2) L ist vom Typ II_1 genau dann, wenn entweder $[G:G_f] = \infty$ oder $[G:G_f] < \infty$ und G_f' unendlich ist.
[erscheint in Math. Annalen].

J.H.B. KEMPERMANN: Functional equations over abelian groups.
We are interested in the equation

$$(1) \qquad \sum_{j=1}^{n} a_j f(x + T_j y) = 0 \qquad \text{for all } x \in X,\ y \in Y .$$

Here, X and Y are abelian groups, $T_j : Y \to X$ a homomorphism $(j=1,\dots,n)$, $f : X \to F$ with F a field and $a_j \in F$ $(j=1,\dots,n)$. The set of all solutions of (1) is denoted by ϕ. If A, B are abelian groups then $f : A \to B$ is called a polynomial of degree $\leq n$ $(f \in \mathfrak{F}_n(A,B))$, if $\Delta_{h_1} \dots \Delta_{h_n} f = 0$ for all $h_1,\dots,h_n \in X$. The composition of a polynomial $f : A \to B$ with a polynomial $g : B \to C$ is again a polynomial. If A is divisible and B has no elements of order $\leq n$ then $\mathfrak{F}_n(A,B)$

coincides with the class $\mathcal{F}^*_n(A,B)$ of all solutions of

$$\Delta^n_h f(x) = \sum_{j=0}^{n} (-1)^{n-j} \binom{n}{j} f(x+jh) = 0$$

If A,B are torsionsfree then $f \in \mathcal{F}_n(A,B)$ iff $f(x) \sim \sum_{\Gamma} \bar{b}_\gamma x^\gamma$.
Here Γ is the set of all $\gamma : I \to \mathbb{Z}^+$ with $\sum \gamma(\alpha) < n$, and I denotes the index set of a Hamel basis $\{\xi_\alpha | \alpha \in I\}$ for X over the rationals $\mathbb{Q}$: $x \sim \sum x_\alpha \xi_\alpha$, $x_\alpha \in \mathbb{Q}$. Finally $\bar{b}_\gamma$ denote essentially unique rational linear combinations of elements in B. For convenience suppose that p = Char F = 0 .

<u>Theorem:</u> $\phi \subset \mathcal{F}_N(X,F)$ for some $N \geq 1$ iff $X_\pi = X$ for each partition $\pi = \{I_1,\dots,I_\varrho\}$ of $I = \{0,\dots,n\}$ such that

$\sum_{j \in I_r} a_j = 0$ $(r=1,\dots,\varrho)$. Here X_π is the subgroup of X generated by the images $X_{i,j} = (T_i - T_j)Y$ with i,j in the same components I_r.

A. KUNZE: <u>Some Remarks on Uniformly Bounded Representations.</u>

Let G be a semi-simple Lie group with finite center, G = KAN an Iwasawa decomposition of G, M the centralizer of A in K, θ the Cartan involution of G, and $V = \theta(N)$. Let H be the group MAV and M' the normalizer of A in K. Then the restricted Weyl group M'/M is finite, and if P is a complete set of representatives for the cosets pM of M in M', every element of G lies in exactly one of the sets HpV $(p \in P)$. Since H is an amenable group, every uniformly bounded representation of G is similar to one whose restriction to H is unitary. Suppose that T is an arbitrary continuous representation of G which is unitary on H. Then, since G is the union of the sets HpV, it follows that T is uniformly bounded, in fact that the set of norms

$$\{\|T(y)\| : y \in G\}$$

is finite. This raises the following problem: Starting with an arbitrary unitary representation T of H, find all ways of assigning operators $\bar{T}(p)$ to the elements of P so that the map $\bar{T}$ defined on each HpV by

$$\bar{T}(hpv) = T(h)\,\bar{T}(p)\,T(v)$$

is a representation of G. Certain but probably not all solutions of this problem have been constructed by Kunze and Stein by analytically continuing an appropriate normalization $R(\cdot,\lambda)$ of the usual principal series for G.

I.G. MACDONALD: Spherical functions on $\mathfrak{p}$-adic groups.

Let G be the Chevalley group associated with a complex semisimple Lie algebra $\mathfrak{g}$ and a $\mathfrak{p}$-adic field K. With the topology its inherits from K, the group G is locally compact and contains a maximal compact subgroup U, the stabilizer in G of a Chevalley lattice $\mathfrak{o} \otimes \mathfrak{g}_{\mathbb{Z}}$, where $\mathfrak{o}$ is the ring of integers of K. We consider zonal spherical functions on G with respect to U, and obtain explicit formulas for their values and for the associated Plancherel measure. For details we refer to the note [1]. These formulas involve a function c(s), which is the $\mathfrak{p}$-adic analogue of Harish-Chandra's c-function. The analogy may be made precise as follows. For any local field K, Tate [2] has defined a meromorphic function ϱ_K (sometimes called the gammafunction of K). Then the factor $|c(s)|^2$ in the Plancherel measure may be written in the form

$$|c(s)|^2 = \prod_{\alpha} \varrho_K((\alpha^*, s))$$

(using the notation of [1], and this is valid for all local fields K (real, complex or $\mathfrak{p}$-adic).

[1] I.G. Macdonald: Spherical functions on a $\mathfrak{p}$-adic Chevalley group. Bull. Amer. Math. Soc. 74, 520-525 (1968).

[2] J.T. Tate: Fourier analysis in number fields and Hecke's zeta functions. (reprinted in "Algebraic number theory", ed. J.W.S. Cassels and A. Fröhlich, Academic Press 1967).

Y. MEYER: Pisot numbers and harmonic analysis.

We give a new proof of a celebrated theorem of Salem: let Θ be a real number greater than 2 and E the set of all (infinite) sums $\sum_{k\geq 1} \varepsilon_k \Theta^{-k}$ where $\varepsilon_k = 0$ or 1. Then E is a set of unicity for trigonometric development if and only if Θ is a Pisot-Vijayaraghavan number. This new proof depends on the following lemma: if Θ is a P.V. number, let n be the degree of Θ over $\mathbb{Q}$, h the homomorphism from $\mathbb{R}$ to Π^n defined by $h(t) = (\exp 2\pi\, it, \dots, \exp 2\pi\, i\, \Theta^{n-1} t)$. One can find a compact subset K of Π^n, of Lebesque measure zero, such that, for all $k \geq 0$, $h(\Theta^k E) \subset K$. An improvement of Salem's theorem is the following: let $\mathcal{K}$ be an algebraic field of degree n over $\mathbb{Q}$, $\mathcal{K} \subset \mathbb{R}$, let $(\Theta_k)_{k\geq 1}$ be a sequence of algebraic integers of $\mathcal{K}$, of degree n. If there are two real numbers α and β such that $\alpha > 2$, $0 < \beta < 1$ and for all k, $\Theta_k \geq \alpha$ and $|\Theta_{k,j}| \leq \beta$ ($\Theta_{k,j}$ are the conjugates of Θ_k different from Θ_k, $2 \leq j \leq n$) the symmetrical set E constructed with dissection ratio Θ_k^{-1} is a set of unicity.

H. REITER: Some remarks on locally compact groups.

Let G be a locally compact group. The property P_1 (cf. Reiter: Classical harmonic analysis and locally compact groups) and its equivalence with Godement's property, with the existence of a left-invariant mean on $C^b(G)$, with the 'fixed point property' and some other conditions is discussed. Further the equivalence of P_1 with the condition that the ideal $J^1(G) = \{ f_0 \in L^1(G) = \int f_0(x)dx = 0 \}$, considered as a Banach algebra of its own (with the L^1-norm), has bounded approximate units. Let H be a closed normal subgroup of G; there is a 'natural' homomorphism T_H of $L^1(G)$ onto $L^1(G/H)$. If H has property P_1, then

$$(*) \quad \inf_{\substack{y_n \in H,\ c_n > 0 \\ \sum c_n = 1}} \| \sum_n c_n A_{y_n} f \|_{L^1(G)} = \| T_H f \|_{L^1(G/H)} \quad \forall f \in L^1(G)$$

where $(A_y f)(x) = f(xy)\Delta(y)$. Applications of (*): If H has property P_1, then

1. $T_H(I)$ is a closed ideal, whenever I is a closed ideal.
2. The kernel of T_H, considered as a Banach algebra $J^1(G,H)$ of its own (with the $L^1(G)$-norm) possesses bounded approximate units.

L. ROBERTSON: Some structure theorems for groups with compactness conditions.

Let [Moore] denote the class of all locally compact groups such that every continuous irreducible unitary representation is finite dimensional. It is possible to characterize Moore groups by displaying the relationship to those maximally almost periodic groups such that the commutators generate a group with compact closure. Various splitting theorems involving direct products and semidirect products can be obtained. Let $[FC]^-$ denote the class of groups such that every conjugacy class has compact closure. Then there exists a compact neighborhood invariant under conjugation, and a characterization of $[FC]^-$ can be obtained. The proofs make repeated use of results due to S. Grosser, M. Moskowitz, and C.C. Moore.

G. SCHLICHTING: Über die Operatoralgebra der regulären Darstellung einer diskreten Gruppe.

Sei G eine diskrete Gruppe und $g \to U_g$ die reguläre Darstellung von $L_1(G)$ in $L_2(G)$, dann ist die Operatoralgebra $\mathcal{L}(G)$ der regulären Darstellung von G definiert als der Normabschluß von $\{U_g : g \in L_1(G)\}$ in der Algebra $\mathcal{B}(L_2)$ aller beschränkten linearen Transformationen von $L_2(G)$

in sich. Zu $x \in G$ sei $K_x = \{yxy^{-1} : y \in G\}$ und G_f der Normalteiler aller $x \in G$ mit K_x endlich. Dann gibt es zu jedem Extremalpunkt α der konvexen (und bezüglich der Topologie der punktweisen Konvergenz kompakten) Menge $K(G_f,G)$ aller positiv-definiten G-invarianten Funktionen $\varphi : G_f \to \mathbb{C}$ mit $\varphi(e) = 1$ einen Homomorphismus H_α des Zentrums $\mathfrak{Z}(G)$ von $\mathcal{L}(G)$ auf $\mathbb{C}$ mit $H_\alpha(U_g) = \sum_{x \in G_f} g(x)\alpha(x)$ für alle Klassenfunktionen $g \in L_1(G)$. Die Zuordnung $\alpha \to H_\alpha$ definiert eine Homöomorphie von $E(G_f,G)$ (= Menge aller Extremalpunkte von $K(G_f,G)$) auf den Strukturraum von $\mathfrak{Z}(G)$. Ist N ein endlicher Normalteiler in G und sind $\alpha_1,\ldots,\alpha_r \in E(G_f,G)$ mit $\alpha_i|N$ paarweise verschieden $(1 \leq i \leq r)$, so sei $p = \sum_{i=1}^{r} \left(\sum_{x \in N} |\alpha_i(x)|^2\right)^{-1} \cdot \alpha_i|N$: dann ist die triviale Erweiterung $\tilde{p}$ von p auf G ein zentrales Idempotent in $L_1(G)$, und jedes Idempotent in $\mathfrak{Z}(G)$ ist von der Form $U_{\tilde{p}}$.

J. STEGEMAN: Combinatorial methods in harmonic analysis.
(Bernstein type theorems for group algebras and tensor algebras).
It is well known (Bernstein) that there exists a function $f : T \to \mathbb{C}$ (T = one-dimensional torus) of Lipschitz type 1/2 ($f \in \Lambda_{1/2}(T)$) such that $f \notin A(T) \cong \mathcal{F}L^1(\mathbb{Z})$. The following theorem strengthens this result and gives analogous statements for the group algebras $A(T^n) \cong \mathcal{F}L^1(\mathbb{Z}^n)$ and the tensor algebras $V(T^n) = C(T) \hat{\otimes} \ldots \hat{\otimes} C(T)$ (n times, $n \geq 1$).

Theorem: $\lambda_{1/2}(T) \not\subset A(T)$; $\lambda_{1/2}(T^2) \not\subset V(T^2)$

$\Lambda_{1,\log_N}(T^2) \not\subset A(T^2)$; $\Lambda_{1,\log_N}(T^3) \not\subset V(T^3)$ (all $N \geq 1$)

$\bigcap_{\alpha < n/2} \Lambda_\alpha(T^n) \not\subset A(T^n)$; $\bigcap_{\alpha < n/2} \Lambda_\alpha(T^{n+1}) \not\subset V(T^{n+1})$ (alle $n \geq 1$)

For $\alpha > 1$ Λ_α is the space of $(\alpha) = \sup |\alpha - \varepsilon|$ times differentiable functions where the derivatives are of Lipschitz types $\Lambda_{\alpha-(\alpha)}$; $\Lambda_{1,\log_N}$ is the space of functions satisfying

$$|f(x) - f(x')| \leq C|x-x'| \max(1, \underbrace{\log\ldots\log}_{N \text{ times}} |x-x'|^{-1}) \text{ for all } x,x'.$$

Some parts of the proofs are sketched.

M. TAKESAKI: Group algebra and duality.
Theorem: To each locally compact group G, there corresponds uniquely an abelian involutive Hopf-von Neumann algebra $\{A(G), \delta_G, j_G\}$ with left invariant measure μ_G and a symmetric involutive Hopf-von Neumann algebra $\{M(G), \pi_G, \kappa_G\}$, and G is realized as the spectrum

of the resulting Banach algebra $M_*(G)$. Conversely, to each abelian involutive Hopf-von Neumann algebra $\{A, \delta, j, \mu\}$ with a left invariant measure, there corresponds uniquely a locally compact group G, whose associated Hopf-von Neumann algebra $\{A(G), \delta_G, j_G, \mu_G\}$ is isomorphic to the given one $\{A, \delta, j, \mu\}$. To each symmetric involutive Hopf-von Neumann algebra $\{M, \Pi, \varkappa, \tau\}$ with unimodular measure, there corresponds a unique unimodular group G, whose dual Hopf-von Neumann algebra $\{M(G), \Pi_G, \varkappa_G, \tau\}$ is isomorphic to the given one $\{M, \Pi, \varkappa, \tau\}$.

N. TATSUUMA: On a duality theorem for locally compact groups.
In a proof of the generalized Tannaka duality theorem for locally compact groups G, the operator
$W : (f_1 \otimes f_2)(g_1, g_2) \to f_1(g_1 g_2) f_2(g_2)$, $(f_1, f_2 \in L^2(G))$, on $L^2(G) \otimes L^2(G)$, plays an important role. W is a coordinatefree form of the mapping decomposing the Kronecker product of two right regular representations to a multiple of them. In this way, the (weak) duality theorem is stated as follows. Let G' be the group associated with the weak topology, consisted of all bounded operators T on $L^2(G)$ such that 1) $T \neq 0$,
2) $W(T \otimes T) = (I \otimes T)W$, then G' is isomorphic to G by the mapping: $g \to R_g$ (the operator of right regular representation). Here we consider the family G" of all closed operators satisfying the conditions 1) and 2), instead of G'. We shall determine the structure of operators in G". G" is considerable to give the C. Chevalley's complexification of G in the case of G being a compact Lie group.

T. WILCOX: On the structure of maximally almost periodic groups.
An investigation of "finitely orbited characters" yields the following result:
Theorem: Let G be a normal subgroup of a MAP-topological group K. Then (1) If G is a vector group, the centralizer of G has finite index in K; (2) If G is a locally compact abelian group, the centralizer of G contains the intersection of closed subgroups of finite index in K.
The first part was announced in Bull. Amer. Math. Soc. 73 (1967), 732 - 734, and the second part of the above theorem together with an investigation of semi-direct products of MAP-groups will appear in Math. Scand. 24 (1969).

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