# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

## Tagungsbericht 32/1970

## Unendlich - Dimensionale Topologie

11.9. bis 17.9.1970

Im Frühjahr 1970 fand an der LSU in Baton Rouge eine Konferenz zum Thema "Infinite - Dimensional Topology" statt, die von zahlreichen namhaften Vertretern aus verschiedenen Gebieten der Topologie und Analysis besucht wurde. Sie behandelte unter anderem Fragen der topologischen Struktur des Hilbertwürfels und anderer unendlich - dimensionaler Kompakta wie auch Linearer Räume, Fixpunktsätze und Probleme der Differentialtopologie. Die Proceedings dieser Tagung erscheinen demnächst in der Reihe "Annals of Mathematics Studies".

In der Folgezeit wurden diese Fragen an verschiedenen Orten zum Teil mit guten Fortschritten weiter verfolgt, sodaß bald der Wunsch entstand, ein neues Internationales Treffen zu veranstalten, an dem Ergebnisse and Anregungen ausgetauscht werden könnten. Dieses fand nunmehr hier in Oberwolfach unmittelbar nach dem Kongress von Nizza unter der Leitung von R.D. Anderson (Baton Rouge) und G. Neubauer (Konstanz) statt.

Die Vorträge behandelten die verschiedensten Gebiete, doch stand die Theorie der unendlich - dimensionalen Mannigfaltigkeiten im Vordergrund. Als besonders fruchtbar erweisen sich Diskussionssitzungen, in denen Probleme gestellt und besprochen wurden. Bemerkenswert war vor allem die intensive Arbeitsatmosphäre, die während der ganzen Tagung herrschte. Sie

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brachte sogar ein konkretes Ergebnis:
Herrn R.Y. Wong (Sta. Barbara) gelang es, gestützt auf Ergebnisse, die ihm von anderen Teilnehmern mitgeteilt wurden, ein seit über dreissig Jahren anstehendes und in den letzten Jahren wiederholt aufgegriffenes Problem zu lösen. (siehe zweiter Vortrag R.Y. Wong) Ein Teilnehmer übernahm es, die während der Tagung erarbeiteten oder von anderer Seite beigesteuerten Probleme zu sichten und zu redigieren. Diese Problemsammlung soll ebenfalls allen Teilnehmern der Tagung zugesandt werden.

Außerdem fand eine ausgedehnte Diskussionssitzung über Fragen des Unterrichts in Unendlich - Dimensionaler Topologie statt. Eine kurze Zusammenfassung der Ergebnisse ist den Vortragsauszügen unten angefügt.

<u>Teilnehmer</u>

J. Aarts, Delft
R.D. Anderson, Baton Rouge
und Amsterdam
W. Barit, Baton Rouge
C. Bessaga, Warschau
T.A. Chapman, Amsterdam
P. Enflo, Stockholm
D. Elworthy, Warwick
P. Flor, Wien
K. Gęba, Gdansk
J. de Groot, Amsterdam
P. de la Harpe, Warwick
D.W. Henderson, Cornell
G. Jameson, Warwick
K. Jänich, Regensburg

F.B. Jones, Riverside
G. Neubauer, Konstanz
R. Ramer, Amsterdam
H.Ch. Reichel, Wien
R.M. Schori, Baton Rouge
und Amsterdam
A. Szankowski, Aarhus
H. Torunczyk, Warschau
K.K. Uhlenbeck, Berkeley
J.E. West, Cornell
R.Y. Wong, Sta. Barbara
S.F. Wong, Warwick

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## Vortragsauszüge

### R.D. ANDERSON: Problems still open in infinite - dimensional topology

The author gives a brief review of the recent development of set-theoretic infinite-dimensional topology in the context of recent results and their implications for future activity. Among classes of significant open problems discussed were those dealing with the factors of the Hilbert cube and of $\ell_2$, with added structures on manifolds, with more general characterizations of $\ell_2$ und of the Hilbert cube, with spaces of homeomorphisms of finite manifolds, and with closed submanifolds of local co-dimension 2 or more.

### C. BESSAGA: Different notions of the "interior" of convex compacta. Generalizations of Keller's theorem and their applications

Let K and L be infinite-dimensional compact convex subsets of the Hilbert space $\ell_2$. By Keller's theorem, $K \underset{\text{top}}{=} L$. This report is concerned with defining in terms of the affine-topological structure a notion of "interior" in order to generalize Keller's theorem as follows:

$$(K, \text{"interior" of } K) \underset{\text{top}}{=} (L, \text{"interior" of } L).$$

Combining this result with Anderson's Z-sets technique A. Pełczyński has obtained:

1. If X is a seperable complete metric space with card X ≥ 2, then the space $M_X$ of equivalence classes of X-valued measurable functions of a real variable is homeomorphic to $\ell_2$.

2. Every separable metric group G is isomorphic to a subgroup of the group $M_G$ which is homeomorphic to $\ell_2$.

A particular case of (1) for X being either an interval or a two-point set have been obtained by Bessaga and Pełczyński. (Scand. Math.)

## T.A. CHAPMAN: Hilbert Cube Manifolds

A Hilbert cube manifold, or Q-manifold, is a separable metric manifold modeled on the Hilbert cube $I^\infty$. A number of recent results that have been obtained for Q-manifolds are cited. The following is a partial list:

Theorem 1. If X and Y are Q-manifolds having the same homotopy type, then $X \times [0,1)$ and $Y \times [0,1)$ are homeomorphic.

Theorem 2. If X is a Q-manifold, then X can be written as the union of two open sets, each one of which can be embedded as an open subset of $I^\infty$.

Theorem 3. If X is a Q-manifold, then there is a countable locally-finite simplicial complx K such that $X \times [0,1)$ and $|K| \times I^\infty$ are homeomorphic.

Theorem 4. If X is a Q-manifold, then $X \times [0,1)$ can be embedded as an open subset of $I^\infty$.

Theorem 5. A compact Q-manifold is homeomorphic to $I^\infty$ if and only if it is homotopically trivial.

## D. ELWORTHY: Fredholm Filtrations of Smooth Banach Manifolds

A separable $C^\infty$ Banach manifold may often be expressed as a union of tubular neighbourhoods of an increasing sequence of compact submanifolds. This is a major tool for the diffeomorphism classification of such manifolds, and for the isotopy classification of closed embeddings.

D. ELWORTHY: Remarks on the present state of differentiable Banach manifold theory

There remain many problems in this field, particularly in the situation where smooth partitions of unity do not exist, or in the non-separable situation where their existence is open. For example it is not known whether every separable metrizable $C^r$-manifold admits any non-trivial $C^r$ function ($r > 0$).

It is expected that the only invariants arise from the homotopy type and from the tangent bundles, and that manifolds would be classified by their tangential homotopy type. This is true for $C^\infty$-smooth separable metrizable manifolds with trivial tangent bundles, under mild conditions on their model spaces.

Another direction has been in investigating notions of topological degree. Possibly there are some interesting problems here in the topological category as well.

P. ENFLO: Uniform topology in linear spaces and groups

For many questions concerning the topological structures of linear spaces or manifolds, the same or similar questions may be asked concerning the uniform structures. However, the answers are often very different. It turns out that for linear spaces there is a strong connection between the uniform and the linear structures. For instance we have the following theorem: If a Banach space is uniformly homeomorphic to a Hilbert space, then it is linearly isomorphic to the Hilbert space; a theorem in strong contrast to the theorem of Kadec, that all separable Banach spaces are homeomorphic. If we consider instead local uniform structures of linear spaces, there are theorems which show that the situation is in some cases the same as for topological structures, in other cases different. There does not seem to exist as yet any manifold theory.

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K. GĘBA: Fredholm maps and framed bordism theory

Let E be an infinite dimensional Banach space, let

$$E = E_0 \supset E_1 \supset \dots \supset E_n \supset \dots$$

be a decreasing sequence of closed linear subspaces, codim $E_n = n$. Let X be a closed and bounded subset of E. Consider a $\sigma$-proper $C^r$ Fredholm map, $r > 1 + \mathrm{ind} f$, $f: E \to E_n$ such that $0 \notin f(X)$. Using the Smale-Sard theorem one can choose a regular value $x_0$ of f close to 0. Then $M = f^{-1}(x_0)$ is a finite dimensional submanifold of E - X. Using Neubauer's methods one can define a normal bundle $\nu(M)$ together with a trivialisation $\alpha: \nu(M) \to M \times E_n$ induced by df. This gives rise to an assignment ind: $f \mapsto$ a bordism class of the pair $(M,\alpha)$ (= framed submanifold).

J. de GROOT: Topological axioms for the Hilbert cube

Definitions. Two subsets of a set are called comparable if one of them is contained in the other. A family of subsets of a set is called comparable, if two elements of the family are comparable whenever there exists a third element in the family which together with each of the elements covers the set.

A family of subsets of a set is called binary if each cover of the set by elements of the family contains a subcover by two elements.

Results. It follows from a theorem by Joel O'Connor that a topological space is a metrizable continuum, iff it is Hausdorff, has a countable (sub)base, has a binary subbase and is connected.

The speaker proves that a space is a metric cube, iff it is $T_1$, has a countable (sub)base, has a binary, comparable subbase and is connected. Hence, adding e.g. either the property of infinite dimensionality or of homogeneity characterizes topologically the Hilbert cube.

P. de la HARPE: Symmetric spaces and smooth Hilbert manifolds

Many problems concerning complete smooth Riemannian manifolds have been solved in finite dimensions with the help of Elie Cartan's classification of symmetric spaces. Similar infinite dimensional objects can hopefully be used in the same way. One expects their classification to result from three steps:

1.- classification of relevant simple complex Lie algebras;
2.- classification of their real forms.
3.- classification of corresponding "symmetric spaces".

Step one has been achieved by J.R. Schue (Trans.AMS 95+96); step two follows essentially as in finite dimensions; so far, step three is conjectural only.

One of the typical groups given by step two is $U(\ell_2;L^2C)$, i.e. the group of those unitary operators in $\ell_2$ of the form (id. + Hilbert-Schmidt). Maximal tori, Borel subgroups and other subgroups, and many associated homogeneous spaces work essentially as for SU(r). In particular, the associated Grassmannians give smooth manifolds as models of classifying spaces for the classical groups.

Hopefully, one of the outputs of that study will be a better understanding of the behaviour of geodesics in infinite dimensional geometry.

D.W. HENDERSON: What should infinite-dimensional topologists do now?

Since the theory of ($C^0$-) infinite-dimensional manifolds has been developed to a fairly complete state, I think that we infinite-dimensional topologists can very profitably look at applications of the theory. These applications in turn should give us ideas as to additional structures which could usefully placed on infinite-dimensional manifolds. As an example of applications to topology we have the following:

Theorem. Let X and X' be complete ANR's (for metric spaces) and let $A \subset X$ and $B \subset X'$ be closed subsets. If there are homotopy equivalences $f'\colon X \to X'$ and $f\colon A \to B$ such that f is homotopic to $f'|A$ in X', then the quotient spaces X/A and X'/B have the same homotopy type.

This theorem can be proved by well-known homotopy-theoretic techniques in the case that the pairs (X,A) and (X',B) have the homotopy extension property with respect to all spaces. However, if the homotopy extension property is not satisfied, then only infinite-dimensional manifold techniques appear to give a proof.

G. JAMESON: <u>Which topological linear spaces are normal</u>?

Some TLS are not normal, e.g. $R^A$, where A is uncountable. A Lindelöf TLS is normal (in fact, paracompact), since every TLS is regular. Two classes of Lindelöf TLS are:

(1) Spaces where the topology is smaller than a separable metric topolocy,

(2) Banach dual spaces with the weak$^*$-topology.

Let (X,Y) be a separated dual pair. Let $\sigma(Y)$ denote the weak topology induced on X by Y. If $X-\{0\}$ is $\sigma(Y)$-Lindelöf, then Y is $\sigma(X)$-separable. Proof: take a countable subcover from $\{x\colon y(x) \neq 0\}$ $(y \in Y)$. Hence if X is an inseparable locally convex space, then $X^*-\{0\}$ is not $\sigma(X)$-Lindelöf (though $X^*$ is, by (2) - hence points are not negligible in $(X^*,\sigma(X))$).

Corson (1961) showed that for a Banach space with its weak topology, paracompact is equivalent to Lindelöf. He also showed that m is not normal in its weak topology, but a more direct proof would be welcome (Suggestion: use Whitley's technique, Amer. Math. Monthly, 1966). Other problems: find a normal TLS that is not paracompact. What about products?

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## K. JÄNICH: Results of U. Koschorke on pseudo-compact subsets of infinite-dimensional manifolds

Definition (R. Palais): If $\mathfrak{A}$ is an atlas of a Banach manifold M, then the set $\psi(M,\mathfrak{A})$ of all finite unions of sets $\varphi^{-1}(A)$, where $\varphi: U \to \mathcal{O}$ is in $\mathfrak{A}$ and $A \in \mathcal{O}$ is closed and bounded in the model space, is called a $\psi$-structure for M. In many cases there is some natural choice for an atlas of a manifold and hence for a $\psi$-structure. The $\psi$-sets (or pseudo-compact sets) are playing in a certain sense the same rôle as the compact sets in the finite dimensional case. In particular, there is a sensible notion of "compactifying" a Hilbert $\psi$-manifold by realizing it as the interior of a $\psi$-manifold with boundary. Theorem 1 is a classification theorem for such mock "compactifications", roughly saying that a $\psi$-manifold with boundary is determined by its interior. Another interesting aspect of pseudo-compact sets is that some $\psi$-manifolds have a "pseudo-compacta deletion property" (PDP). Theorem 2 states that certain section-manifolds of finite dimensional bundles do have the PDP, a fact which seems to have some relevance in the theory of non-linear elliptic differential operators.

## G. NEUBAUER: Survey of recent results on the homotopy type of linear groups

In the past year considerable progress has been achieved in the investigation of the homotopy structure of linear groups in Banach spaces, mostly due to B.S. Mitjagin and his co-operators. (Bull. Ac. Pol. 18 (1970) 27-33, 213, Funkc. anal. i ego prilož. 4 (1870) 61-72, Uspehi Math. N. 25 (1970) no.5, 63-106) They have shown that a large number of different Banach spaces have contractible linear groups, among them $\ell^\infty$, $L^p(0,1)$ $(1 \leq p \leq \infty)$, $C(0,1)$, $S^p$(operator ideals of "$\ell_p$-trace class") etc. They have further given new examples of homotopically non-trivial linear groups, among these $GL(J^n)$ which has the homotopy type of $GL(n)$ (J the James space). The contractibility theorems make use of an abstract scheme modeled on the proof for $GL(\ell_p)$.

R. RAMER: "Infinite exterior products" and forms

It is possible to construct finite co-dimensional exterior products of certain types of Banach spaces. For infinite dimensional manifolds admitting a refined Fredholm structure of a certain type, exterior forms of the finite codimension can be defined.

H.-Ch. REICHEL: Über nicht-archimedisch topologische Strukturen

Der Vortrag geht von n.-a. normierten Räumen über n.-a. bewerteten Körpern aus, streift die Theorie der sogenannten V-Räume und behandelt schließlich die n.-a. Strukturen vom rein topologischen Standpunkt aus. Er gibt einen kurzen Überblick über die wichtigsten Eigenschaften n.-a. metrischer Räume und n.-a. Analoga zu den bekanntesten Metrisierungssätzen. Eine wichtige Verallgemeinerung ist die Theorie der n.-a. uniformen Strukturen. Diese werden topologisch durch den Satz von Monna charakterisiert. Die Frage nach topologischen (nicht metrischen) Eigenschaften, die die strenge Dreiecksungleichung kennzeichnen, wird durch diese Verallgemeinerung aber nicht gelöst. Es gibt n.-a. uniforme Räume, die metrisierbar aber nicht n.-a. metrisierbar sind. Hingegen läßt sich zeigen: ein metrisierbarer Raum ist genau dann n.-a. metrisierbar, wenn er eine Basis $\{M_\alpha\}$ besitzt mit folgender Eigenschaft $M_\alpha \cap M_\beta = \emptyset \vee M_\alpha \vee M_\beta$ für alle $\alpha, \beta$. Diese sogenannten n.-a. topologischen Räume werden im Vortrag kurz charakterisiert.

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## K. UHLENBECK: Intrinsically Bounded Sets in Manifolds of Maps

The concept of intrinsically bounded sets in Banach manifolds of sections of a fiber bundle extends the concept of bounded sets in linear spaces and is useful in applications. Let $\pi:E \rightarrow M$ be a finite dimensional fiber bundle, M compact, $i:\Gamma(M,R) \subseteq C^0(M,R)$ a space of functions, the imbedding i completely continuous. If $\Gamma$ satisfies the axioms of Palais, then $\Gamma(E)$ is a Banach manifold. If $f: E \rightarrow E'$ is a $C^\infty$ fiber preserving map, then composition by $f: \Gamma(f): \Gamma(E) \rightarrow \Gamma(E')$ is a $C^\infty$ map of Banach manifolds. Examples are $\Gamma = C^k, C^\alpha, L^p{}_k$ $(pk > \dim M)$.

Definition: $S \subseteq \Gamma(E)$ is intrinsically bounded (I.B.), if S is relatively compact in $C^0(E)$ and if for any open imbedding $\eta \subseteq E$, any subset of S which is bounded in $C^0(\eta)$ is bounded in $\Gamma(\eta)$. Several equivalent properties are given and the map $\Gamma(f)$ above maps I.B.sets to I.B.sets. If in addition, the unit ball of $\Gamma(M,R)$ is compact in the weak topology, the $C^0$ topology and the weak topology are the same on bounded sets in $\Gamma(\eta)$. This can be used to define a weak topology on $\Gamma(E)$ with the properties that $\Gamma(f)$ is weakly continuous and the closed I.B.sets are exactly the weak compacta.

## J.E. WEST: Factors of the Hilbert cube and Hilbert space

Four new theorems were announced, and proofs of the last two below were discussed in outline. (Each factor of the Hilbert cube is a factor of Hilbert space).

Theorem 1: If $f:X \rightarrow Y$ is a map between Hilbert cube factors, its mapping cylinder $M_f$ is also a Hilbert cube factor. Moreover, the natural retraction $g:M_f \rightarrow Y$ has the property that $g \times id:M_f \times Q \rightarrow Y \times Q$ is a uniform limit of homeomorphisms between $M_f \times Q$ and $Y \times Q$. (Here Q is the Hilbert cube).

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Theorem 2: The space of subcontinua of a dendron D is a Hilbert cube factor which is a Hilbert cube if the branch points of D are dense in D.

Theorem 3: The space of all non-void closed subsets of the unit interval [0,1] is a factor of the Hilbert cube.

Theorem 4: If X is the space of countably many circles in $R^2$ converging to their common point $x_o$ of tangency, then the space of closed subsets of X containing $x_o$ is a Hilbert cube. (In (3) and (4), the topology is given by the Hausdorff metric).

### R.Y. WONG: Homemorphisms of Infinite-dimensional Bundles

We study various properties of fibre-preserving homeomorphisms (diffeomorphisms) on a bundle $\xi: E \to B$, of which two major aspects are considered: Isotopy deletion of subsets and extension of fibre preserving homeomorphisms on E.

Theorem. Let $\xi: E \to B$ be a bundle over base space B a locally finite simplicial complex with fibre $F = R^\infty$ (countable infinite product of reals). Then a closed set K in E can be strongly deleted from E at the end of a $\xi$(bundle) - isotopy if and only if $K \cap \xi^{-1}(b)$ is a Z-set in each $\xi^{-1}(b)$.

Theorem. (Smooth deletion). Let $\xi: E \to B$ be a $C^\infty$-bundle over base space B a $C^\infty$-manifold with fibre F an infinite-dimensional $C^\infty$-Hilbert manifold. Then every locally closed, locally compact subset K of E can be strongly $C^\infty$-deleted from E at the end of a $\xi$-isotopy.

Theorem. (lifting). Let $\xi: E \to B$ be a bundle over Hausdorff space B with fibre F an AR. Let K be a closed subset of E such that $K \cap \xi^{-1}(b)$ is homotopy negligible in each $\xi^{-1}(b)$. Suppose (T,L) is a locally finite simplicial pair, and $f_o: |T| \to B$ is a map, then $f_o$ can be lifted to a map of $|T|$ into $E \setminus K$ if and only if $f_o|\,|L|$ can be lifted to a map of $|L|$ into $E \setminus K$. Various results on extending homemorphisms in $\xi$ are obtained

### R.Y. WONG: The Hyperspace of the closed unit interval is homeomorphic to the Hilbert cube

The hyperspace space of the closed unit interval is the space of all closed subsets of $I = [0,1]$ with the Hausdorff metric. West recently proved the following Theorem: Theorem 1: $I^\infty \times 2^I_{0,1} \cong I^\infty$ (homeomorphic to), where $2^I_{0,1} = \{K \in 2^I : \{0,1\} \subset K\}$. It follows from this and some known results of J.West that we have Theorem 2. $(2^I_{0,1})^\infty \cong I^\infty$. Using West's Interior Approximation Lemma, we show Theorem 3. $2^I_{0,1} \cong (2^I_{0,1})^\infty \cong I^\infty$. But since $2^I$ is the double cone of $2^I_{0,1}$, we have Theorem $2^I \cong Q$.

### S.F. WONG: The topological degree of A-proper maps

A class of maps which has proved very useful in the study of solutions of certain equations in functional analysis is the class of A-proper (approximation proper) maps introduced by W.V.Petryshyn. This class is a generalization of maps of the type I + Compact and in the linear case it turns out to be Fredholm maps of non-negative index.

If X and Y are Banach spaces each having filtration of oriented finite dimensional subspaces $\{X_n\}$ and $\{Y_n\}$ respectively and projections $p_n : Y \to Y_n$ then a map f from the closure of an open bounded set G to Y is A-proper if the existence of a sequence $\{x_n \in \bar{G} \cap X_n\}$ such that $p_n f x_n$ converges to an element y of Y implies the existence of a subsequence $\{x_{n_j}\}$ such that $\lim x_{n_j} = x$ and $fx = y$.

For this class of maps, a topological degree can be defined which coinsides with the Leray-Schauder degree when defined. Also, it seems to be somewhat related to D.Elworthy and A.Tromba's definition of degree for $\Phi_0$-proper maps.

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## K. JÄNICH: Report on a discussion on problems of teaching ∞-dimensional topology

Three different types of courses in ∞-dimensional topology have been discussed:

(a) Set theoretic ∞-dimensional topology

(b) Topology of ∞-dimensional $C^0$-manifolds

(c) ∞-dimensional differential topology

(a): R.D. Anderson reported on his course in set theoretic infinite-dimensional topology, which is directed towards research rather than covering a large area. He uses the method of R.L. Moore to present the material in a way in which the students can find all the proofs by themselves, thereby developing the necessary self-confidence and the techniques for own work in this field. Anderson starts with the Hilbert cube rather than the Hilbert space, emphasizing intuitive geometric thinking at first. The students are supposed to have a background of about one or two years of topology and to have had some non-sophisticated functional analysis.

(b): C.Bessaga gave an outline of a one-year course on $C^0$-infinite dimensional topology he plans to give at Warsaw in the coming academic year. Students are supposed to have a good background in functional analysis. Some topological tools (covers, partitions of unity, simplicial complexes, microbundle theory) will be included where needed in the course. The course starts with some intuitive geometry on Fréchet manifolds, Wong's "switching coordinates" and applications. Next follow the closed embedding theorem and related topics, then the stability theorem, the open embedding theorem and finally the classification of manifolds by homotopy. The rest of the time will be devoted to the topology of linear topological spaces and the Hilbert cube.

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(c) Mainly based on suggestions by D. Elworthy and K. Uhlenbeck, a tentative plan was set up for a one-semester course in $C^\infty$-infinite dimensional topology with applications. The topics could be: $\infty$-dimensional manifolds (function manifolds as examples), Morse theory and applications to geodesics, $G(E)$, $\Phi(E)$, $G_c(E)$, Leray-Schauder-degree and applications to non-linear equations.

The course (a) and (b) are one-year courses. They contain several of the more recent results and bring students to the frontier of knowledge. Course (c) is intended as a one-semester course and does not include research material of the last few years. It could be followed by a course covering these more recent developments.

G. Neubauer, Konstanz

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