MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 33/1970

Topologie

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Vortragsauszüge

T. TOM DIECK: Partitions of unity in homotopy theory

The main theorems that we proved are the following:

Theorem 1. Let $(X_\alpha | \alpha \in A)$ be a numerable covering of X and $(Y_\alpha | \alpha \in A)$ be a numerable covering of Y. Let $f : X \to Y$ be a map with $f(X_\alpha) \subset Y_\alpha$. Assume that for finite (non-empty) $\sigma \subset A$ the induced maps $f_\sigma : \bigcap_{\alpha\in\sigma} X_\alpha \to \bigcap_{\alpha\in\sigma} Y_\sigma$ are homotopy equivalences. Then f is a homotopy equivalence. A similar result holds in the category of spaces over a fixed space.

Theorem 2. Let $(E_\alpha | \alpha \in A)$ be a family of subsets of E and $(B_\alpha | \alpha \in A)$ be a numerable covering of B. Let $f : E \to B$ be map with $f(E_\alpha) \subset B_\alpha$. Assume that for finite $\sigma \subset A$ the map $\bigcap_{\alpha\in\sigma} E_\alpha \to \bigcap_{\alpha\in\sigma} B_\alpha$ is shrinkable. Then f has a section.

Theorem 3. Let $(E_\alpha \mid \alpha \in A)$ be a numerable covering of E. Let $f : E \to B$ be a map such that for finite $\sigma \subset A$ the map $f_\sigma : \bigcap_{\alpha\in\sigma} E_\alpha \to B$ is a fibration (an h-fibration, shrinkable). Then f is a fibration (an h-fibration, shrinkable).

CH.H. GIFFEN: Zeta functions and Reidemeister torsions of circle group actions

If S^1 acts PL on a finite polyhedral pair (X,Y), then there is defined a meromorphic function $\rho_{X,Y}(s)$, called the (exponential) zeta function of the S^1 pair (X,Y). It is an equivariant homotopy type invariant, closely related to the Smale zeta function of a flow. A periodic PL map h induces a PL S^1 action on its mapping torus T(h), and $\rho_{T(h)}(s)$ is canonically determined by the Weil zeta function $\xi_h(t)$ of h. If the cyclic subgroup T_m of order m acts freely on $X \setminus Y$, then for each $k|m$ a Reidemeister torsion $D_{m:k}(X,Y)$ is defined corresponding to the complex representation $T_m \hookrightarrow S^1 \longrightarrow S^1 = U(1)$ where $z \longmapsto z^k$. These Reidemeister torsions are determined by the exponential zeta function (and conversely) via the formula $\rho(2\pi ibk/m) \cdot D_{m:k} = 1$ where $b \equiv 1 \pmod{m}$ and $b \equiv 0 \pmod{q}$ for every q such that $\rho(2\pi iq) = 0$ or ∞. Some examples are discussed, and applications to non-imbedding of free finite cyclic actions in S^1 actions are given.

P.E.CONNER: Complexes with few cells

An odd prime is fixed, p, together with a pair of integers (s,r) with $s \leq r$. A third integer $i \equiv 0 \bmod (p-1)$ is then chosen so that $s \leq \nu_p(i)$ for a large value of k there is a cell complex $W = S^{2k} \cup_{p^{r+1}} e^{2k+1} \cup e^{2(k+i)+i}$ such that if $\sigma \in \tilde{\Omega}^U_{2k}(W)$ is the spherical bordism class given by inclusion $S^{2k} \subset W$ then the annihilator ideal $A(\sigma) \subset \Omega^U_*$ is $(p^{r+1}, p^{r-s} CP(p-1)^{i/p-1})$. The problem is to characterize those bordism classes for which $V \cdot \sigma$ is spherical. The basic result asserts that in such a case the order of $V \cdot \sigma$ is equal to the order of $Td(V)$ in $Z/p^{r-s}Z$. As a corollary it follows that if Y is obtained from W by attaching a cell killing $V \cdot \sigma$, then hom dim $\Omega^U_*(Y)=2$. Among several corollaries to this follow some standard results about e_c.

C.B. THOMAS: <u>Structures on manifolds defined by cross-sections</u>

Let Y_π be a manifold of constant positive curvature with soluble fundamental group π, $\underset{\sim}{X}$ denote some structure on Y_π, and ρ be generated by Sylow subgroups $\pi_2 \& \pi_3$. Suppose further that $\underset{\sim}{X}$ corresponds to a cross-section of some bundle ξ over Y_π with fibre F.

<u>Theorem</u>. Suppose that for $i \leq \dim Y$, (i) $|\pi_i F| = \infty$ implies i even, and (ii) $|\mathrm{Tor}\,\pi_i F|$ is divisible only by 2 and 3. Then Y_π admits an $\underset{\sim}{X}$-structure if and only if Y_ρ does.

Among applications are the existence of normal invariants ($F=G/Top$) and of immersions ($F=V_{n,n-k}$, $k < 6$), for Y_π in terms of the covering space Y_ρ.

F. HIRZEBRUCH: Free involutions and some elementary number theory

Using the Atiyah-Bott-Singer fixed point theorem (G-signature theorem) the Browder-Livesay invariant of certain involutions on lens spaces is calculated.

Theorem. Consider the lens space $L(q;1,p_2,\ldots,p_n)$ for p_j and 2q relatively prime and n even. It admits a free involution T whose orbit space is $L(2q;1,p_2,\ldots,p_n)$. The Browder-Livesay invariant of T is

$$\# \left\{x=(x_2,\ldots,x_n)\in \mathbb{Z}^{n-1} \mid 0<x_j<q \wedge 0<\sum_{j=2}^{n} \frac{x_j p_j}{q} <1 \bmod 2\right\}$$

$$- \# \left\{x=(x_2,\ldots,x_n)\in \mathbb{Z}^{n-1} \mid 0<x_j<q \wedge 1<\sum_{j=2}^{n} \frac{x_j p_j}{q} <2 \bmod 2\right\}.$$

For n = 2 the Browder-Livesay invariant of the involution on $L(q;1,p)$ is denoted by $c_{p,q}$, a number studied in the dissertation of W. Neumann (Bonn). If p,q are relatively prime and both odd, then

$$c_{p,q} = -4\,N_{p,q} + q-1$$

where $N_{p,q}$ was used by Gauß (proof of the quadratic reciprocity law, Gauß lemma), namely

$N_{p,q}$ = number of integers x with $1 \leqq x \leqq (q-1)/2$ for which xp has modulo q a remainder of smallest absolute value which is negative.

H. HERRLICH: Between compactness and complete regularity

Let $\mathfrak{A}$ and $\mathfrak{B}$ be full subcategories of $\mathfrak{C}$ with $\mathfrak{A} \subset \mathfrak{B}$. Then $\mathfrak{B}$ is called $\mathfrak{A}$-fitting in $\mathfrak{C}$ provided that whenever $\begin{array}{ccc} P & \rightarrow & A \\ \downarrow & & \downarrow \\ B & \rightarrow & C \end{array}$ is a pullback in $\mathfrak{C}$ with A in $\mathfrak{A}$ and B in $\mathfrak{B}$ then P in $\mathfrak{B}$.

Theorem. Let $\mathfrak{C}$ be complete, wellpowered, and cowellpowered, let $\mathfrak{M}$ be a class of monomorphisms in $\mathfrak{C}$ which is closed under composition, intersection, and pullbacks, is left-cancellative (i.e. $f \cdot g \in \mathfrak{M} \Rightarrow g \in \mathfrak{M}$), and has the property that $f \cdot g$ epi and $f \in \mathfrak{M}$ implies g epi, and let $\mathfrak{A}$ be $\mathfrak{M}$-reflective in $\mathfrak{C}$. Then:

a) equivalent are (1) $\mathfrak{B}$ is reflective in $\mathfrak{C}$, (2) $\mathfrak{B}$ is $\mathfrak{M}$-reflective in $\mathfrak{C}$. (3) $\mathfrak{B}$ is epireflective in $\mathfrak{C}$ (4) $\mathfrak{B}$ is closed under products and extremal subobjects. (5) $\mathfrak{B}$ is $\mathfrak{A}$-fitting and strongly closed under intersections, (6) $\mathfrak{B}$ is strongly closed under finite products and arbitrary intersections.

b) If $\mathfrak{B}$ is $\mathfrak{A}$-fitting and $\tilde{\mathfrak{B}}$ is the reflective hull of $\mathfrak{B}$ in $\mathfrak{C}$ then for each $\mathfrak{C}$-object X the $\tilde{\mathfrak{B}}$-reflection of X can be obtained as the intersection of all $\mathfrak{M}$-subobjects of the $\mathfrak{A}$-reflection of X which belong to $\mathfrak{B}$ and contain X.

F. TAKENS: On the tolerence stability conjecture of Zeeman

According to Zeeman's tolerence stability conjecture, the map which assigns to each vectorfield (= R action) , on a manifold M, its "orbit structure" should be continuous on a residual subset of the set of all vectorfields. Some weakened forms of this conjecture can be proved for the case that M is compact. The strongest results are obtained for the case where one restricts to conservative systems.

R.VOGT: On infinite loop spaces

The classifying spaces BO, BU, BF etc. are of great importance in algebraic topology. As H-spaces, they have similar structures: They have homotopy-associative and homotopy-commutative multiplications which satisfy all coherence conditions. Such an H-structure will be called an E-structure. We give a recipe for how to obtain topological spaces X with an E-structure, and show that they are infinite loop spaces, i.e. there exists a sequence $\{X_n\}, n=0,1,2,\ldots$ of based topological spaces such that $X=X_o$ and X_n has the homotopy type of the space ΩX_{n+1} of based loops on X_{n+1}. The proof of the last part will be rather sketchy. The results were obtained in joint work with J.M. Boardman.

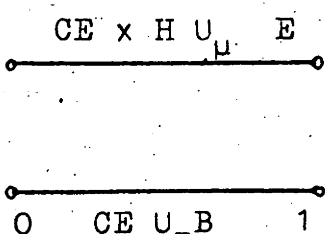
M. FUCHS: A modified Dold-Lashof construction that does classify H-principal fibrations.

Let H be a strictly associative H-space with strict unit element and a homotopy inverse ν such that

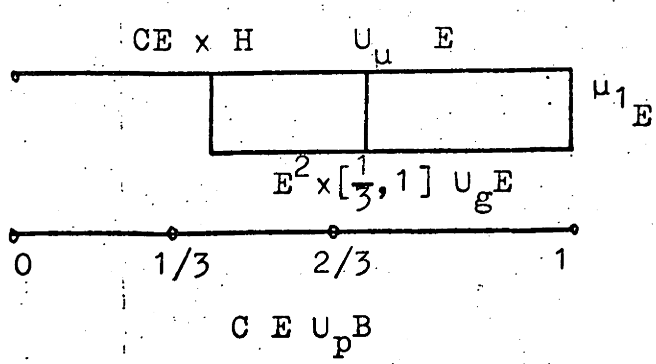
$$H \xrightarrow{\Delta} H \times H \xrightarrow{1\times\nu} H \times H \xrightarrow{\mu} H$$

is homotopic to id_H. Modify the Dold-Lashof construction in the following way

Dold-Lashof



modification



where
$$\begin{array}{ccccc} E \times H & \xrightarrow{F} & E^2 & \xrightarrow{g} & E \\ & & \downarrow & & \downarrow p \\ & & E & \xrightarrow{p} & B \end{array}$$
and g is the canonical map from a

pull back $F : E \times H \to E^2$;
$F(y,h) = (y,yh)$ is a fibre homotopy
equivalence.

Use a telescope to obtain $E_H \xrightarrow[p_H]{} B_H$, which is a loc.-h-trivial principal fibration. E_H is contractible. Classify as for G-principal bundles.

M. ANDRÉ: On the structure of Hopf algebras with divided powers

Over a field of positive characteristic, a cocommutative graded Hopf algebra (connected) is the enveloping algebra of a graded Lie algebra (not restricted) if and only if, roughly speaking, its dual Hopf algebra is a Hopf algebra with divided powers. Such an algebra is an algebra with divided powers A plus a coassociative comultiplication $\Delta : A \to A \otimes A$ which is a homomorphism of algebras with divided powers. First application: short computation of Eilenberg-MacLane algebras. Second application: A is a local ring (commutative but not necessarily Noetherian) with maximal ideal I, if $Tor_n^R(R/I,R/I)$ is equal to 0 for at least one n, then the commutative graded algebra $\Sigma\, I^k/I^{k+1}$ over A/I is a symmetric algebra.

F. WALDHAUSEN: On Whitehead torsion

Let a space with n-fold end structure consist of a locally finite CW complex X together with (inverse) systems of subspaces $\{Y_k^m\}_{k\in I_m}$ $1\leq m\leq n$, such that for any n-tupel $(k_1,..,k_n) \in I_1 x..x I_n$, the union $Y_{k_1}^1 \cup .. \cup Y_k^n$ is cofinite. Example: A product $X_1 x...x X_n$ of locally finite CW complexes with the obvious n-fold end structure. A homotopy theory can be built out of these devices, generalizing "proper" homotopy. There is a corresponding "simple homotopy"theory, generalizing Siebenmann's "infinite simple homotopy type". Mimicking Siebenmann's techniques, one can analyze the "Whitehead group" of the theory and express it in terms of ordinary algebraic K-functors. The theory sometimes makes it possible to"localize" Whitehead torsion over open subspaces (with suitable end structure). This has implications on the topological invariance problem.

J.F. KRAUS: Homotopy friendly diagrams

A diagram $\mathfrak{D}$ of topological spaces over an 1-connected diagram scheme is called h-friendly, if there exists a space $\mathfrak{D}_*$ in $\mathfrak{D}$, such that all arrows which point away from $\mathfrak{D}_*$ are h-cofibrations. Examples are $\mathfrak{D}_* \leftarrow \mathfrak{D}_0 \subsetneq \mathfrak{D}_1$ and $\mathfrak{D}_* \subsetneq \mathfrak{D}_1 \subsetneq \mathfrak{D}_2 \subsetneq \dots$, where $\subset$ are h-cofibrations.

Theorem 1: If $\mathfrak{D}$ is h-friendly, then the canonical map $\mathfrak{D}_* \to \varinjlim \mathfrak{D}$ is an (induced) h-cofibration.

Theorem 2: If the h-friendly diagrams $\mathfrak{D}_1$ and $\mathfrak{D}_2$ have the same homotopy type (i.e. they are equivalent as diagrams in Top/homotopy), then $\varinjlim \mathfrak{D}_1$ and $\varinjlim \mathfrak{D}_2$ have the same homotopy type.

Theorem 3: If $\mathfrak{D}$ is a h-friendly diagram of pointed spaces and Y is a pointed space, then there exists a short exact sequence

$$* \to \varprojlim{}^1[\mathfrak{D},\Omega Y] \to [\varinjlim \mathfrak{D},Y] \to \varprojlim[\mathfrak{D},Y] \to *$$

of pointed sets, [-,-] denotes pointed homotopy classes.

These theorems (and their proofs) are formal, hence dual theorems are true. Replace point away from, h-cofibration,

$\varinjlim$ and $[-,Y]$ by point toward, h-fibration, $\varprojlim$ and $[Y,-]$.

Theorem 4: If $X_o \to X_1 \to \dots$ and $Y_o \to Y_1 \to \dots$ are h-cofibrations, then the canonical map $\varinjlim(X_n x Y_n) \to \varinjlim X_n \times \varinjlim Y_n$ is a homotopy equivalence.

M.K. AGOSTON: Surgery on maps of degree d and the reducibility of Thom complexes

The basis geometric problem which is considered is the following: given a normal map $f : V^n \to M^n$ of degree $d > 1$, when can one do surgery on V to end up with something of the homotopy type of the d-fold connected sum $M^n \# \dots \# M^n$? There are obstructions to doing this. This obstruction theory is best phrased in terms of an obstruction theory for the reducibility of the Thom complex of a vector bundle over a manifold. Applications are made to imbedding manifolds in the metastable range.

R. FRITSCH: Charaktersisierung semisimplizialer Mengen durch Unterteilung und Graphen

Es wird ein Standard-Unterteilungsfunktor G (im Sinne von Math.Z. 108/1968-1969, 329-367) definiert, so daß die folgenden Sätze gelten:

a) Sind X und Y semisimpliziale Mengen, so gibt es zu jedem semisimplizialen Isomorphismus $h : GX \to GY$ genau einen semisimplizialen Isomorphismus $h^{\#}: X \to Y$ mit $h = Gh^{\#}$.

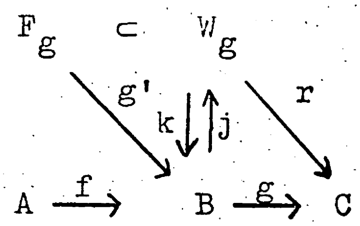
b) Sind X und Y semisimpliziale Mengen mit $(GX)^1 \cong (GY)^1$ und "beschränkten" Entartungen, so sind sie isomorph.

(Vermutung: b gilt auch ohne Voraussetzung über beschränkte Entartungen).

Entsprechende Sätze gelten auch für die Normal-Unterteilung simplizialer Komplexe, wie R.L. Finney (Mich.Math.J.12/1965, 263-27) und J. Segal (BAMS 71/1965, 571-572) gezeigt haben.

D. PUPPE: Fibrations up to homotopy and the James theorem on $\Omega\Sigma X$

For given (1) $A \xrightarrow{f} B \xrightarrow{g} C$ and $\sigma \in C$ we consider



where $W_g = \{(b,w) \mid g(b) = w(1)\} \subset B \times C^I$

$r(b,w) = w(0)$

$F_g = r^{-1}(\sigma)$

$j(b) = (b,$ constant path at $g(b))$

$k(b,w) = w$

$g' = k|F_g$.

(1) is called exact if there exists a homotopy equivalence $h : A \to F_g$ such that $g'h = f$; and (1) is called strictly exact if $gf(A) = \sigma$ and the map $j_o : A \to F_g$ induced by j is a homotopy equivalence.

These notions are used to improve the statement and the proof of the "relative James theorem" given in my paper "Some well known weak homotopy equivalences are genuine homotopy equivalences", to appear in Symposia Mathematica V, Istituto di Alta Mathematica Rôma.

H.A. HAMM: Singuläre Punkte von vollständigen Durchschnitten in $\mathbb{C}^m$

Seien $f_1,\ldots,f_k : \mathbb{C}^m \to \mathbb{C}$ Polynome, $X := \{z \in \mathbb{C}^m \mid f_1(z)=\ldots=f_k(z)=0\}$, $0 \in X$, $X - \{0\}$ regulär, $\dim X = m-k$. Σ sei der Durchschnitt von X mit einer Sphäre um 0 mit genügend kleinem Radius; Σ ist dann eine C^∞-Mannigfaltigkeit. Frage: Wann ist Σ eine topologische Sphäre (dann ist nämlich 0 regulärer Punkt von X)?

Satz: In jedem Fall ist Σ $(m-k-2)$ - zusammenhängend. Mit Hilfe einer Picard-Lefschetz-Monodromie läßt sich eine hinreichende Bedingung dafür angeben, daß Σ eine topologische Sphäre ist.

Für das Beispiel $f_\mu = \Sigma\, \alpha_{\mu\nu}\, z_\nu^{a_\nu}$, $\alpha_{\mu\nu} \in \mathbb{C}$, $a_\nu \geq 2$, läßt sich mit Hilfe dieser Methode eine explizite Bedingung an $a_1,\ldots,a_m$ angeben, wann Σ eine topologische Sphäre ist.
(Für den Fall k=1 s. Milnor, Singular Points of Complex Hypersurfaces).

N. STEENROD: Cohomology automorphisms and characteristic classes

The method of formal power series in chomomology operations and characteristic classes, introduced by F.Hirzebruch and used extensively, is inadequate when treating characteristic classes of odd degrees over a coefficient ring other than Z_2. By reformulating and generalizing the method, the inadequacy can be removed. If L is a graded coalgebra and H a graded algebra, then Hom(L,H) is a graded algebra. Its elements of degree zero, that preserve the unit, form a multiplicative subgroup G(L,H). When L is the dual of a polynomial algebra on one variable, then G(L,H) consists of power series in elements of H with leading coefficient 1. If A is a graded, connected Hopf algebra (of cohomology operations), and L is cocommutative, then the coalgebra mappings $L \longrightarrow A$ form a subgroup $G'(L,A)$ of $G(L,A)$. If H is a commutative algebra over A, then $G'(L,A)$ is a group of (cohomology) automorphisms of $\mathrm{Hom}(L,H)$.

M. KLINGMANN: <u>Morsetheorie und Kohomologieoperationen</u>

Sei ϕ eine Kohomologieoperation $H^{r_1}(X,Y;K)\times\ldots\times H^{r_k}(X,Y;K) \supset \mathrm{Def}\,\phi \xrightarrow{\phi} H^r(X,Y;K)$ / Indet ϕ höherer Ordnung mit Koeffizienten in einem Körper K. Wir bezeichnen ϕ als ein verallgemeinertes Produkt, falls gilt: Sind $Y_\lambda \subset X$ offene Untermengen und $u_\lambda \in j^*H^{r_\lambda}(X,Y_\lambda;K) \subset H^{r_\lambda}(X;K)$ und ist $j^*: H^*(X,\cup Y_\lambda;K) \to H^*(X;K)$ injektiv für alle Vereinigungen von Y_λ, so ist $\emptyset\,(u_1,\ldots,u_k) \cap j^*H^r(X,\bigcup_{\lambda=1}^{k} Y_\lambda;K) \neq \emptyset$. Unter diesen Begriff fallen z.B. alle einstelligen Kohomologieoperationen, Masseyprodukte, Cupprodukte usw. Sind $u_1,\ldots,u_k$ Kohomologieklassen mit Körperkoeffizienten auf einer kompakten differenzierbaren Mannigfaltigkeit M^n und existiert ein ϕ mit $\phi\,(u_1,\ldots,u_k) \not\ni 0$, so nennen wir je zwei dieser Kohomologieklassen verkettet durch ein verallgemeinertes Produkt. Ist f eine minimale Morsefunktion auf M^n und sind u,v mit $\dim u + \dim v < n$ verkettet, so ist der untere kritische Wert von u kleiner als der obere kritische Wert von v. Verkettung durch verallgemeinerte Produkte ist also ein Hindernis gegen das Zusammenfassen von kritischen Werten einer minimalen Morsefunktion. In manchen Fällen ist das Verschwinden bestimmter verallgemeinerter Produkte sogar hinreichend für diese Zusammenfassung.

V. PUPPE: Komplexe K-Theorie von Produkträumen

Analog zu den Ergebnissen von F.P. Palermo für singuläre Kohomologie von Produkträumen (s. Trans. AMS 86(1957), S. 174 -196) gilt der

Satz: Für endliche CW-Räume X und Y (mit Grundpunkt) ist das [multiplikative] $\tilde{K}^*$-Bocksteinspektrum des smash-Produktes $X \wedge Y$ von X und Y in natürlicher Weise durch die [multiplikativen] $\tilde{K}^*$-Bocksteinspektren der Faktoren X und Y bestimmt.

Dabei besteht das [multiplikative] $\tilde{K}^*$-Bocksteinspektrum von X aus allen [Ringen] $\tilde{K}^*(X,\mathbb{Z}_n)$, n = 0,1,2... zusammen mit allen Bockstein- und Koeffizientenhomomorphismen.

F. SIGRIST: On H-spaces which are finite complexes

A survey of recent results on finite complexes which are H-spaces is presented. If we call rank of an H-space the number of odd-dimensional spheres occurring in Hopf's theorem: $H^*(X;\mathbb{Q}) \approx H^*(S^{r_1} \times S^{r_2} \times \ldots \times S^{r_n};\mathbb{Q})$, we conjecture that there are finitely many H-spaces (up to homotopy type) of given rank.
An essential step towards this conjecture is the Curjel-Douglas theorem: there are finitely many H-spaces of given dimension. As for homomorphisms, or H-maps, one approach consists in asking, for a given map $f : X \to Y$, that X and Y possess suitable multiplications making f into an H-map.
This problem can be completely solved for the spheres S^1, S^3, S^7 and maps beween them. One part of the proof uses various results

on nilpotency and non-commutativity of S^3 and S^7, the other uses a complete analyse of the Hopf construction applied to a map $S^7 \to S^3$, and shows in particular that this essential map cannot be an H-map.

K. LAMOTKE: Almost free G-manifolds

Assume: X is a smooth closed oriented G-manifold, G a compact connected Lie group. The fixed point set $F \subset X$ is finite and non-empty. G acts freely on X-F. The orbit space X/G is a manifold.

Then:

$G = U(1) = S^1$ and dim X = 4 or $G = SU(2) = S^3$ and dim X = 8.

The number of fixed points equals the Euler characteristic of X, and is even. Given a closed, oriented (p+2)-dim. manifold N, given a number r, there is a S^p-manifold X satisfying the assumptions with 2r fixed points and X/G diffeomorphic to N, p=1 or 3.

Let the S^p-manifolds X and X' satisfy the assumptions. Assume furthermore: The number of fixed points is the same, and X/S^p and X'/S^p are diffeomorphic manifolds. Then for p = 1: X and X' are equivariantly diffeomorphic. For p=3: X and X' are equivariantly homeomorphic, and are (may be not equivariantly) diffeomorphic.

H. Klopfer (Heidelberg)