MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Ringe, Moduln und homologische Methoden

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Die Leitung der Tagung wurde wie in den letzten Jahren von F.Kasch (München) und A.Rosenberg (Ithaca) durchgeführt. Von den zahlreichen europäischen und außereuropäischen Teilnehmern konnte ein großer Teil in interessanten Vorträgen über die neuesten Ergebnisse auf dem Gebiet der Ringe, Moduln und der homologischen Algebra berichten.

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Vortragsauszüge

T. A L B U : Decomposition of Torsion Modules

We establish some theorems concerning the S-primary decomposition of torsion modules in the Dickson's sense (Math. Z., 104, 349-357, (1968)), the classical primary decomposition of these torsion modules and the connections between these two kinds of decompositions.

M. A N D R É: Nonnoetherian complete intersections

By Tate-Zariski, a commutative noetherian local ring A with residue field K is a complete intersection if and only if the integers $\beta_i = \mathrm{Tor}_i^A(K,K)$ appear in an equality of formal series

$$\Sigma\, \beta_i t^i = (1+t)^z/(1-t^2)^s$$

The same situation is studied in the nonnoetherian case by means of commutative homology theory. A criterion is given for characterizing the rings for which $\mathrm{Tor}_*^A(K,K)$ is a free algebra with divided powers having generators in degrees 1 and 2 only.

Example: $A = K \otimes_k K$ for any field extension K/k.

G. B E T S C H : Sheaf representation of near-rings

Dauns and Hofmann developed general methods to represent a ring with sufficiently many central idempotents as a ring of global sections in a suitable sheaf of rings. (Cf. Memoirs AMS No. 83, 1968). We discuss adaptations of the Dauns-Hofmann methods to suitable near-rings. Particulary, the main theorem on the representation of biregular rings (Math. Zeitschr. 91 (1966), 103-123) can be generalized to "biregular" near-rings.

I. B U C U R : Local rings. Divisibility properties. Some applications and conjectures

Proposition. Let R be a discrete valuation ring, m its maximal ideal, k = R/m the residue field, f : R ⟶ R an endomorphism of R and G a finite group of automorphisms, of R which induces the identity on k.

We assume there exists an endomorphism

$$\Phi : G \longrightarrow G$$

of G such that

$$f(gx) = \Phi(g)f(x) \ \forall g\in G \ \forall x\in R$$

and let be $\nu(f) = \nu(f(\pi) - \pi)$ $(\nu(\pi) = 1)$.

If we denote for every $g\in G$,

$$c^{\Phi}(g) = \mathrm{Card}\{x\in G / \Phi(x)gx^{-1} = g\}$$

and if we decompose $c^{\Phi}(g)$ in the form:

$$c^{\Phi}(g) = p^r q \ , \ (p,q) = 1, \ p = \mathrm{char}(k)$$

then $q \mid (\nu(g^{-1}f) - 1)$.

<u>Proposition.</u> There exists a formal series $\mathfrak{Z}_A(f, u_M, G) = 1 + b_1T + b_2 T^2 + \dots + b_n T^n + \dots$, s. that

$$T \frac{\mathfrak{Z}'_A}{\mathfrak{Z}_A} = \sum_{n\geq 1} \mathfrak{Z}_A(f^n, u_M^n, G)\, T^n$$

where M is a projective and of finite type A-module, which is a G-module, $u_M : M \longrightarrow M$ is an endomorphism and

$$\mathfrak{Z}_A(f^n, u_M^n, G) = \sum_{g\in G_{\Phi\varphi}} \left(\frac{\nu(g^{-1}f)-1}{c^{\Phi}(g)} \mathrm{Tr}_A(u_M g_M^{-1}) \right).$$

S.U. C H A S E : <u>Theory and Forms of Algebras</u>

Let k be a field of characteristic $p \neq 0$, K be a finite purely inseparable field extension of k of degree n, and A be a k-algebra containing K as a subalgebra, such that $[A:k] < \infty$. Finally, let B be the centralizer of K in A. We presented the following theorem which is related to the Galois theory of finite purely inseparable field extensions: Assume that, given any x in K, not in k, there is a commutative k-algebra T and an invertible element u of $A \otimes T$ such that

(i) $u(K \otimes T)u^{-1} = K \otimes T \quad (\subseteq A \otimes T)$

(ii) $u(x \otimes 1)u^{-1} \neq x \otimes 1$

Then $A \otimes_k K \cong M(n\times n, B)$ as K-algebras, where $M(n\times n, B)$ is the algebra of all $n\times n$ matrices over K.

The proof of the theorem, which uses both module theory and the theory of group schemes, was briefly outlined.

F. E C K S T E I N : Im engeren Sinne linear kompakte, kommutative Ringe, die sich als offene Unterringe in halbeinfache Ringe ohne offene Linksideale einbetten lassen.

Der Beweis des folgenden Satzes soll kurz skizziert werden.

Satz: Ein im engeren Sinne linear kompakter, kommutativer Ring R ist offener Unterring eines halbeinfachen Ringes A ohne offene Linksideale genau dann, wenn R topologisch isomorph ist zu einem $\Pi\{R_i : i \in I\}$ von vollständigen, lokalen, Noetherschen, semiprimen Ringen R_i mit Krull Dimension eins.

Der Ring A ist kommutativ und topologisch isomorph zum lokalen Produkt $\Pi\{Q_*(R_i).R_i; i \in I\}$ und jeder Ring $Q_*(R_i)$ ist ein endliches Produkt von Körpern.

R.M. F O S S U M : Trivial Extensions of Abelian Categories.

Suppose $F : \underline{A} \longrightarrow \underline{A}$ is and endofunctor (additive) on the abelian category $\underline{A}$. The trivial extension category $\underline{A} \times F$ is defined. It is abelian if F is right exact. Example: If $\underline{A}$ = R-mod and $F = {}_RM \otimes_R —$, then $\underline{A} \times F$ = RxM - mod.

If A is an object in $\underline{A}$, then the object T A in $\underline{A} \times F$ is the morphism $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} : F A \oplus F^2A \longrightarrow A \oplus F A$.

Proposition: An object π in $\underline{A} \times F$ (F right exact) is projective if and only if $\pi \cong T$ (coker π) and coker π is projective in $\underline{A}$.

Theorem: Suppose $\underline{P}$ is a full additive subcategory of $\underline{A}$ whose objects are projective in $\underline{A}$. Then $T(\underline{P})$ has the same proporties in $\underline{A} \times F$. Furthermore $T(\underline{P})$ is coherent if and only if $\underline{P}$ is coherent and for every $\underline{P}$-coherent object A in $\underline{A}$ the objects $L_iF(A)$ are P-coherent for all $i > o$, the object FA is pseudo-$\underline{P}$-coherent and if B is a $\underline{P}$-coherent subobject of FA, then FB is of finite P-type. Applications follow.

A.W. G O L D I E : Rings with finite Krull Dimension

Let R be any ring and M an R-module. M has Krull dimension α (ordinal) if any descending sequence $M_1 > M_2 > \dots$ of submodules of M has all factors M_n/M_{n+1} of Krull dimension $< \alpha$ for large n. M has Krulldimension zero when it satisfies d.c.c. for submodules. The Krulldimension of a ring R is that of R_R and is denoted by $|R|$. The classical Krull dimension (prime ideal chains) is denoted by $cl|R|$.

The following theorems hold:

Theorem 1: Let $|R|$ exist then $cl|R|$ exists (ordinal) and $|R| \geq cl|R|$. Note: $cl|R|$ may exist without $|R|$ existing.

Theorem 2: Let $|R|$ exist and the nilradical N of R be zero. Then R is a right order in a semi-simple artinian ring. Let $N \neq 0$ then the powers of N are distinct.

Theorem 3: Let R be a commutative ring, let $|R|$ exist and $cl|R|$ be finite. Then N is nilpotent.

The research was jointly with Prof. Lance Small.

P. G R I F F I T H : Applications of Trivial Extensions of Abelian Categories to Gorenstein Modules.

The following results on trivial extensions of rings combined with Bass' theory of Gorenstein rings provide a straightforward treatment of Gorenstein modules [References: Sharp, Foxby, Herzog-Kunz, Reitun]. A f.g. module M over a commutative Noetherian ring A will be called Gorenstein if, for $p \in \operatorname{Spec} A$, $\mu_i(p,M) \neq 0 \Longleftrightarrow \operatorname{ht} p = i$, [In the sense of Bass]. If, in fact, $\mu_i(p,M) = 1 \Longleftrightarrow i = \operatorname{ht} p$, then M is called a cannonical module.

A x M denotes the trivial extension of A by M [= Nagata's Principle of Idealization in this case].

Theorem 1: If X is a f.g. A x M-module with $id_{A \times M}(X) = n < \infty$, then

1) $id_A \overline{X} = n$, where $\overline{X} = Hom_{AxM}(A,X)$

2) If $X \longrightarrow I^{\bullet}$ is an AxM- (minimal) injective resolution of X, then $\overline{X} \longrightarrow Hom_{AxM}(A,I^{\bullet})$ is a (minimal)A-injective resolution of $\overline{X}$.

3) There is a natural isomorphism $Ext^i_{AxM}(-,X) \cong Ext^i_A(-,\overline{X})$ on Mod A.

4) If A is local, n = depth(AxM) = depthA = $depth_A M$

<u>Theorem 2:</u> If A x M is Gorenstein, then

a) A is Cohen-Macaulay

b) $Ext^i_A(M,M) = 0$ for $i > o$

c) $A \cong B \times End_A M$ (Ring direct product), where $B = ann_A M$.

d) B is a Gorenstein ring.

<u>Theorem 3:</u> If SpecA is connected and $M \neq 0$, then A x M is Gorenstein $\Longleftrightarrow$ AS Cohen-Macaulay and M is a cannonical A-module.

T. JÓZEFIAK: <u>Derivation and differential functors for commutative graded algebras and their derived functors</u>

The purpose of this lecture is to announce extensions of the André-Quillen homology and cohomology of commutative algebras to commutative graded algebras (graded algebra A is commutative if $ab = (-1)^{ij} ba$ for $a \in A_i$, $b \in A_j$, $a^2 = 0$ for $a \in A_i$, i odd).
For two commutative graded R-algebras A and B, an R-homomorphism $\varphi: A \longrightarrow B$ and graded B-module M the graded B-modules of derivations Der(A,B,M) and differentials Dif(A,B,M) are defined. The derived functors of Der and Dif are studied and applications to classical homological algebra and to topology are indicated.

M. K N E B Ü S C H : Real Closures of Semi-local Rings

All rings in this lecture are commutative and with 1. For any ring K we denote by W(K) the Wittring of non degenerate symmetric bilinear forms over K.

Definition 1. A signature σ of K is a ring homomorphism from W(K) to $\mathbb{Z}$.

Remark. If K is a field the signatures of K correspond uniquely with the orderings of K (Harrison, Leicht-Lorenz).

We consider pairs (K,σ) with K a connected ring and σ a signature of K. There is an obvious notion of a homomorphism $(K,\sigma) \longrightarrow (L,\tau)$ between pairs. We say that a homomorphism $\alpha : K \longrightarrow L$ of rings is a (connected) covering, if α is the inductive limit of finite etale connected extensions of K, as studied in Galois theory. We say that a homomorphism $\alpha : (K,\sigma) \longrightarrow (L,\tau)$ is a covering, if $K \longrightarrow L$ is a covering.

Definition 2. A real closure of a pair (K,σ) is a covering $\alpha: (K,\sigma) \longrightarrow (R,\rho)$ such that (R,ρ) does not admit any coverings except isomorphisms. By Zorn's lemma any pair (K,σ) has at least one real closure.

Theorem 1. Assume $\alpha: (K,\sigma) \longrightarrow (R,\rho)$ is a real closure of a pair (K,σ) with K semi-local. Let K_s denote the universal covering (= separable closure) of K.

1) For any other real closure $\alpha' : (K,\sigma) \longrightarrow (R',\rho')$ there exists an isomorphism $\beta : (R,\rho) \xrightarrow{\sim} (R',\rho')$ with $\alpha' = \beta \circ \alpha$.

2) There does not exist any automorphism of (R,ρ) leaving all elements of K fixed except the identity

3) The Galois group of K_s/R is a 2-group
Assume in addition that 2 is a unit in A.
Then even the following statements are true:

3a) $K_s = R(\sqrt{-1})$.

4) If R'/K is any covering such that $[K_s:R'] = 2$, then $K_s = R'(\sqrt{-1})$ and $W(R') \cong \mathbb{Z}$. This R' has a unique signature ρ'.

Consequence: If K is semi-local with 2 a unit, then the signatures of K correspond uniquely with the conjugacy classes of involutions in the Galois group of K.

Remark. If K is a Dedekind domain at least statement 1) of Th. 1 remains true and $[K_s:R] \leq 2$.
The statements 1) and 2) of Theorem 1 are closely related to the following

Theorem 2. Let L be a finite covering of the semi-local ring K and let $Tr^*: W(L) \longrightarrow W(K)$ denote the transfer map induced by the regular trace $Tr = Tr_{L/K}$ (Scharlan, Invent. math. 6, 1969). Then for any signature σ of K and any z in W(L)

$$\sigma\,(Tr^*(z)) = \sum_{\tau/\sigma} \tau(z),$$

where τ runs through all signatures of L lying over σ, with the convention that the sum is zero if there are no such τ.

The proofs of these theorems strongly depend on results obtained jointly with A. Rosenberg and R. Ware. They seem to be of interest even over fields, since they avoid Sturm's theorem and other theorems about zeros of real polynomials.

M.A. K N U S und
M. O J A N G U R E N : Descent and Brauer Group
(Vortragender: M. Ojanguren)

Let A be an R-Azumaya algebra of constant rank n^2 and $[A]$ its class in the Brauer group Br(R) of R. Using nonabelian cohomology, Grothendieck proved that $[A]^n = 1$. We give another proof of this result using faithfully flat descent.
With the same descent technique, together with results of Shuen Yuan (Trans.AMS, january 71) we prove that for "good" purely inseparable Extensions $R \subset K$ of integral domains the induced map $Br(R) \to Br(K)$ is surjective. For field extensions this result is due to Hochschild.

E. K U N Z : Almost Complete Intersections are not Gorenstein Rings

Let R be a commutative noetherian local ring, $\hat{R}$ its completion. Write $\hat{R} = S/\mathfrak{a}$, where S is a regular local ring and define $d(R) = \mu(\mathfrak{a}) - (\dim S - \dim R)$, where μ means the minimal number of generators. d(R) is an invariant of R. R is a complete intersection, if d(R) = 0 and an almost complete intersection, if d(R)=1. While complete intersections are always Gorenstein rings, it can be shown that almost complete intersections are never Gorenstein rings.

J. L A M B E K : Noncommutative Localization

Let I be an injective right R-module, consider the endofunctor $S = I^{(-,I)}$ of Mod R, and construct the equalizer of the two obvious maps $S \to S^2$. Then Q is left exact and idempotent, it is the localization functor of Gabriel. It gives rise to a reflective subcategory of Mod R, which is the limit closure of I and consists of the so-called I-torsionfree I-divisible modules. Joint work with Michler discusses the case when R is right noetherian and I is the injective hull of R/P, P a prime ideal. Then R satisfies the right Ore condition with respect to Goldie's multiplicative set $C(P) = \{r \in R \mid rs \notin P$ for all $s \notin P\}$ if and only if the Jacobson radical of Q(R) is $PQ(R) = M$ and Q(R)/M is simple Artinian. If moreover for each right ideal E there exists n such that $E \cap M^n \subseteq EM$ then the M-adic and I-adic topologies on Q(R) coincide and the completion of Q(R) is a full matrix ring over a complete local ring.

H. L E N Z I N G : Es soll im wesentlichen ein einfacher kategorieller Beweis für den folgenden auf Stephenson zurückgehenden Satz gegeben werden:

Satz. A und B seien assoziative Ringe mit 1. $M_I(A)$ und $M_J(B)$ seien die Ringe der spaltenendlichen IxI-Matrizen über A (bzw. JxJ-Matrizen über B) für unendliche Mengen I und J. Dann gilt

$$M_I(A) \simeq M_J(B) \iff |I| = |J| \text{ und } A \text{ und } B \text{ sind Morita-äquivalent.}$$

L.S. L E V Y : Diagonalizing Matrices over PIR's, Lifting and Straightenning Direct Sums

(joint research with I.C. Robson)

Two matrices A and B over a ring R are equivalent (A~B) if B = PAQ for invertible matrices P and Q over R. Consider the theorem: Every matrix over R is equivalent to some diagonal matrix. For R = the ring of integers, this was proved in 1861 by H.J.S. Smith. It was extended successivley by Dickson, Wedderburn, Ore, and Jacobson; and finally by Teichmüller who proved it in 1937 for any prinicipal ideal domain (=PID) (commutative or not). It is also known for any PIR mit DCC.

The authors have further extended the above results by proving the theorem for any PIR (= ring in which every I-sided ideal is principal).

In order to discuss uniqueness of the diagonal form of an mxn matrix A, we view A as the left multiplication map : $R^{(n)}_R \longrightarrow R^{(m)}_R$, elements of $R^{(n)}$ being written as columns, and define

$$\text{coker } A = R^{(m)}_R / AR^{(n)}_R$$

The classical uniqueness theorem for commutative PID's is

(*) $\quad \text{coker } A \cong \text{coker } B \Longrightarrow A \sim B$

(This is merely a module-theoretic restatement of the Invariant Factor Theorem.) The theorem (*) is no longer valid for non-commutative PID's; and the authors of the 1930's were not able to solve the problem of uniqueness. We have the following partial result, which shows that coker A nearly determines the equivalence class of A.

Theorem. Let coker A = $R/d_1R \oplus \ldots \oplus R/d_nR$ (A being nxn and $d_1 \neq 0$).

Then there is an χ in R such that

$$A \sim \text{diag}(\chi, d_2, d_3, \ldots, d_n) \text{ and } R/\chi R \cong R/d_1R$$

If, for some i, $0 \neq d_iR = Rd_i$, then we can take $\chi = d_1$. When R is commutative, this becomes the Invariant Factor Theorem (*).

In order to state the abstract theorem from which the above results follow, we define 2 (R-module) homomorphisms $f_i : M_i \longrightarrow U_i$ $(i = 1, 2)$ to be equivalent $(f_1 \sim f_2)$ if there exist isomorphisms such that the following square commutes

$$\begin{array}{ccc} M_1 & \xrightarrow{f_1} & U_1 \\ \vdots & & \vdots \\ \downarrow & & \downarrow \\ M_2 & \xrightarrow{f_2} & U_2 \end{array}$$

(To recover the definition of $A_1 \sim A_2$ for matrices, let $f_i : R^{(n)} \longrightarrow R^{(m)}$ be left multiplication by A_i.)

Lifting and Straightenning Theorem. Consider an R-module epimorphism $M = M_1 \oplus \ldots \oplus M_n \xrightarrow{f} U = U_1 \oplus \ldots \oplus U_n$

where M is projective, U is finitely generated, and $U/\text{rad}\, U$ is semi-simple. Suppose that there exist epimorphisms

$$g_i : M_i \longrightarrow\!\!\!\!\rightarrow U_i \quad \text{and} \quad g_{ni} : M_n \longrightarrow\!\!\!\!\rightarrow U_i .$$

Then the given decomposition of U can be "lifted" to a decomposition $M = M_1' \oplus \ldots \oplus M_n'$ for which each $M_i' \cong M_i$ and $f(M_i') = U_i$ and "straightened" so that

$$(f : M_i' \longrightarrow\!\!\!\!\rightarrow U_i) \sim g_i$$

for each i except possibly $i = n$.

The lifting of the decomposition of U above corresponds to diagonalizing a matrix, and the lack of straightenning in the last summand is what is responsible for the element "x" in the unigueness theorem.

To prove the diagonalization theorem, we first reduce to the case R = a prime ring by means of a theorem of Goldie, which states that every PIR is the direct sum of prime PIR's and a PIR with DCC. We then make use of the fact that a prime PIR is a Dedekind prime ring, by applying (a suitably extended form of) the module structure theorems of Eisenbud and Robson for finitely generated modules over such rings, and then applying the Lifting and Straightenning Theorem to presentations of such modules.

F.W. L O N G : The Brauer Group of Dimodule Algebras

We consider a finite abelian group Γ and a commutative ring R with 1. An R-algebra A is a Γ-dimodule algebra if it is graded by Γ and also acted on by Γ so that the action preserves the grading, i.e. ${}^{\gamma}(A_\delta) \subset A_\delta$ for all $\gamma, \delta \in \Gamma$. Given two such algebras A,B we put a multiplication on the module $A \otimes_R B$ by $(a_1 \otimes b_1)(a_2 \otimes b_2) = a_1({}^{\gamma}a_2) \otimes b_1 b_2$ where b_1 is homogeneous of grade γ, and we denote the R-algebra so obtained by $A \# B$.
After the necessary generalizations of the concepts of "opposite algebra" and "Azumaya" etc. we are able to define a Brauer group of Γ-dimodule algebras denoted $BD(R,\Gamma)$. (This generalizes the Brauer-Wall group, the Brauer group of Knus and Childs, Garfinkel and Orzech and the equivariant Brauer groups of Fröhlich and Wall).
Explicit calculations give the following results:
1. K separably closed, char $\neq$ p, $BD(K,\mathbb{Z}/p\mathbb{Z}) \cong D_{p-1}$.
2. K algebraically closed, char = p, $BD(K,\mathbb{Z}/p\mathbb{Z}) \cong \mathbb{Z}/(p-1)\mathbb{Z}$.
3. $BD(\mathbb{R},\mathbb{Z}/2\mathbb{Z}) \cong D_8$.
It is possible to replace Γ by a commutative, cocommutative Hopfalgebra H and to define H-dimodule algebras and Group BD(R,H). Here, Γ-acting corresponds to H-modules and Γ-grading to H-comodules; and in the group case $H = R[\Gamma]$.

M.P. M A L L I A V I N : Structure of Regular Local Rings and Conjectures of Serre

On donne une démonstration pour la conjecture de Serre sur $\chi_0^R(M,N)$ dans le cas où M et N sont annulés par la caracteristique residuell de R. (R local regulier complet).
A l'aide de condition sur l'anneau $A = R \otimes_k R$ (k anneau de coefficients de R) on caractérise les anneaux R de la forme: $R[[z_{m+1},\dots,z_n]]$ (R local regulier). On retrouve le résultat de J.P. Serre dans le cas où $R = V[[z_1,\dots,z_n]]$ (V anneau de valuation discret).

W.S. M A R T I N D A L E : Rings with Involution and Generalized Polynomial Identities

Let R be a prime ring, C its extended centroid, and A = RC + C its central closure. Theorem. If R is either (1) a prime ring satisfying a generalized polynomial identity (G.P.I.) over C or (2) a prime ring with involution whose symmetric elements S are G.P.I. over C, then A is a primitive ring with a minimal right ideal eA such that eAe is a finite dimensional division algebra over C. The approach used is to embed A in a primitive algebra B such that G.P.I.'s and involutions can be lifted to B, and then to invoke Amitsur's results. (1) was previously proved using different methods by the author; (2) is new.

E. M A L T I S : Divisible Artinian Modules

We will present a complete structure theory for divisible Artinian modules over a 1-dimensional noetherian local domain. This theory provides a Jordan-Hölder theorem for divisible Artinian modules over such a domain and has strong analogues with the theory of modules of finite length and the theory of primary decompositions of modules. There is a one to one correspondence between certain extension rings of the domain and certain divisible modules that sheds new light on both classes of objects.

J.C Mc C O N N E L L : Homomorphisms and Extensions of Modules over the Weyl Algebra A_1

Let k be a field of char 0, A the ring of formal differential operators with coefficients in the polynomial ring k[y] and B the ring of formal differential operators with coefficients in the field k(y). So A is generated as a k-algebra by x,y with xy - yx = 1 and B is a partial quotient-ring of A.

1. If M is an (A or B) module of finite length then M is cyclic.
2. If I,J are non zero right ideals of A then Hom(A/I,A/J) is finite dimensional over k.
3. If I,J are non zero right ideals of B then Hom(B/I,B/J) is finite dimensional over k.
4. If I,J are non zero right ideals of A then Ext(A/I,A/J) is finite dimensional over k.
5. If I,J are non zero right ideals of B then Ext(B/I,B/J) is infinite dimensional over k.

These results enable us to construct counter-examples to some conjectures on modules over Dedekind prime rings and arbitrary hereditary noetherian prime rings.

G. M I C H L E R :

Block Theory over Arbitrary Fields

A review was given on the main theorems on blocks $B \longleftrightarrow e \longleftrightarrow \lambda$ of group algebras FG of finite groups G over arbitrary fields F of characteristic $p > 0$, which can be proved by means of the ring theoretical study of the center ZFG of FG. As a new application we then proved: Theorem. Let $B \longleftrightarrow e \longleftrightarrow \lambda$ be a block of FG with defect groups $\delta(B) \underset{G}{=} D$. Then the exponent of nilpotency of the radical J(ZB) of the center ZB of B is greater or equal to the exponent of $\mathfrak{z}(D)$, where $\mathfrak{z}(D)$ denotes the center of D.

B.J. M Ü L L E R :

Morita contexts

A Morita context consists of two bimodules $_SP_R$ and $_RQ_S$ together with two pairings (,) : (,) : $Q \times P \longrightarrow R$ and [,] : $P \times Q \longrightarrow S$ satisfying associativity conditions. If the pairings are nondegenerate and the modules faithful, the context will be called nondegenerate. Two nondegenerate contexts between rings R,S and S,T may be composed to give a nondegenerate contexts between R,T; hence the existence of nondegenerate contexts constitutes an equivalence relation for rings. Examples: A ring is equivalent to a division ring iff it is primitive with minimal one-sided ideals. A ring is equivalent to a right Ore domain iff its maximal right quotient ring is a full linear ring iff it is prime with right singular ideal zero and a uniform right ideal. - An arbitrary Morita context induces a category equivalence between quotient categories of the categories of R- and S-right (or left) modules, descriable in terms of the trace ideals only. There exists am induced context between the corresponding quotient rings which is "normalized". If the original context was nondegenerate, there are induced contexts between the maximal one-sided and two-sided quotient rings. An equivalence relation leading to a classification of non-degenerate contexts is discussed, and some applications are indicated.

U. O B E R S T und H.-J. S C H N E I D E R :

Über Untergruppen und Darstellungstheorie formeller und endlicher algebraischer Gruppen

(Referent: U. Oberst)

Seien zunächst k ein Körper, G eine formelle k-Gruppe und G' eine Untergruppe von G. Seien $H' \subset H$ die zu $G' \subset G$ gehörigen Hopfalgebren, d.h. die dualen Hopfalgebren der linear kompakten, affinen Algebren von G' bzw. G. Sei $H/\!\!/H$ entsprechend die dem homogenen Raum G/G assoziierte Coalgebra.

Satz 1_f : Ist $[G : G'] := |H/\!\!/H' : k| < \infty$, so ist $H' \subset H$ eine projektive Frobeniuserweiterung zweiter Art im Sinne von Kasch und Nakayama-Tsuzuku. ⫽

Der vorige Satz ist für konstante Gruppen G aus der Darstellungstheorie abstrakter Gruppen

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wohlbekannt und trivial zu beweisen. Den wichtigen Spezialfall "$[G : 1] < \infty$, $G' = 1$" bewiesen Larsen-Sweedler. Der Beweis von Satz 1_f verwendet wesentlich die Strukturtheorie formeller Gruppen (Dieudonné-Cartier-Gabriel) und die beiden folgenden Sätze.

Satz 2_f : Sind G endlich oder G/G' infinitesimal oder k algebraisch abgeschlossen, so gibt es einen H'-linearen, H//H'-colinearen, unitären, augmentierten Isomorphismus $H/\!/H' \otimes H' \cong H$. //

Satz 3_f : Sind $[G : G'] < \infty$ und G separabel oder die Charakteristik ch(k) von k gleich 0 oder ch(k) > 0 und G infinitesimal von endlicher Höhe, so enthält G' einen Normalteiler von G von endlichem Index. Im allgemeinen ist diese Aussage falsch.

Seien jetzt k ein beliebiger kommutativer Ring und $G' \subset G$ endliche (lokalfreie, affine) k-algebraische Gruppen. Wieder seien H' bzw. H die dualen Hopfalgebren der affinen Algebren von G' bzw. G. Seien B die affine Algebra von G/G' und B^B das Ideal der Fixelemente der augmentierten Algebra B.

Satz 1_a : Der k-Modul B^B ist projektiv vom Rang 1. Ist er frei (z.B. k semilokal, k Hauptidealring), so gilt die zu Satz 1_f analoge Aussage. //

Satz 2_a : Ist k semilokal, so gilt die zu Satz 2_f analoge Aussage. //

T. O N O D E R A :

Linear kompakte Moduln und Kogeneratoren

Ein R-Linksmodul $_RM$ heißt linear kompakt nach B. Müller, wenn jedes endlich auflösbare System von Kongruenzen $\chi \equiv a_\alpha \pmod{M_\alpha}$ $\alpha \in I$, auflösbar ist. Sei $_RQ$ ein Kogenerator in der Kategorie der R-Linksmoduln. Dann, als eine Kennzeichnung von linear kompakten Moduln, haben wir folgenden Satz:

Ein R-Linksmodul $_RM$ ist dann und nur dann linear kompakt, wenn $_R M$ Q-reflexiv und Q_S M^*-injektiv sind, wobei S der Endomorphismenring von $_R Q$ und $M^*_S = Hom_R(M,Q)$ das Q-Duale von $_R M$ sind. Weiter werden einige Eigenschaften von linear kompakten Moduln besprochen.

J.-E. R O O S :

On the Homological Theory of Non-commutative Noetherian Rings

Recall that Auslander has introduced a notion of non-commutative Gorenstein ring: R (supposed here to be left-right noetherian) is left Gorenstein iff for all finitely generated left R-modules M and all submodules $V_R \subset Ext^j_R({}_R M, {}_R R_R)$ we have $Ext^i_R(V_R, R_R) = 0$ for $i < j$. This is left-right symmetric and coincides with the ordinary Gorenstein rings in the commutative case.

Theorem 1: Let R be a filtered ring ($F_o R \subset F_1 R \subset \ldots, \bigcup F_n R = R$) such that the graded associated ring Gr(R) is Gorenstein (we forget the grading). Then R is Gorenstein.

Corollary 1: The enveloping algebra of a finite dimensional Liealgebra is Gorenstein.

Corollary 2:

The ring $A_n(K) = K[x_1,\ldots,x_n,\delta/\delta x_1,\ldots,\delta/\delta x_n]$ of formal differential operators with polynomial coefficients is a Gorenstein ring. (We also have the same result when we take formal power series as coefficients or when we consider the ring of differential operators with analytic coefficients in a polydisc.

Corollary 3: Let G be a finite dimensional Lie algebra over $\mathbb{C}$ (say) andlet U(G) be the enveloping algebra. If P is a primitive ideal in U(G), then U(G)/P is a Gorenstein ring in case

a) $G = Sl(2,\mathbb{C})$

b) G nilpotent

c) G semisimple and P minimal

Conjecture: U(G)/P is Gorenstein for all G and all P.

Theorem 2: Let R be a Gorenstein ring, then:

(*) $\dim_{Kr} R \leqq \text{gldim } R$ (even $\text{injd}_R R$)

More generally:

(**) $\dim_{Kr} \text{Ext}^i_R({}_R M, {}_R R_R) \leqq \text{gldim } R - i$.

The proof of (*) follows from (**) by descending induction on i and a biduality spectral sequence argument. The proof of theorem 1 follows from a spectral sequence relating graded Ext to ungraded ones.

Finally we introduce the notion of a non-commutative regular ring (finite global dimension + Gorenstein + biduality filtration = Gabriel filtration)

Main Conjecture: $A_n(K)$ is regular. For $A_1(K)$ this is easy. For $A_2(K)$ we enter into questions that also have been studied by Mc Connell in another form.

L. S Z P I R O :

Linked Ideals and Varieties of codim 2

Deux variétés projectives plongées dans $\mathbb{P}^n_{\mathbb{C}}$, X et Y, sont dites liées si $X \cup Y$ est une intersection complète. On démontre un théorème de Choderlos de Laclos et que, partout de X à cône Cohen Mc Caulay (resp. X loc. intersection complète, resp. X normale et car k = o) on peut trouver, quand codim X = 2, une chaine $X_1, \ldots X_s$ de variétés ayant les propriétés citées de X telles que $X = X_1$, X_i et X_{i+1} soient liées et X_s soit une intersection complète.

J.R. S T R O O K E R :

Higer F-Functors

During the last few years, several authors have proposed higher functors K_n to continue the exact sequence of algebraic K-theory. The relationship between the various theories constitutes an interesting set of questions which has only partly been clarified.

We shall here describe several of these from a common point of view: cotriple resolutions on the category of rings to which we apply the general linear group. It turns out that the K_2's of these theories are connected with a "universal covering group" of Gl attached to such a cotriple.

E.J. T A F T :

On the Antipode of a Finite-dimensional Hopfalgebra

We give examples of finite-dimensional Hopf algebras having antipodes of arbitrary even order. Let H be a finite-dimensional Hopf algebra with antipode S. If A is pointed as a coalgebra and if the base field has characteristic $p > 0$, then S has finite order.

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