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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1972

Differentialgeometrie im Großen

16.7. bis 22.7.1972

Die alle vier Jahre stattfindende Tagung "Differentialgeometrie im Großen" stand dieses Jahr unter der Leitung von M.Barner, S.S.Chern und W.Klingenberg. Es war dies die letzte Tagung, deren Vorträge im alten Lorenzenhof stattfanden. Am Vormittag wurden vier Vorträge mit neuen Forschungsergebnissen gehalten und am Nachmittag zwei Übersichtsvorträge.
Bei allem fehlte der traditionelle Spaziergang am Mittwoch nachmittag natürlich nicht, der zum Glaswaldsee und Palm Springs Gasthaus führte.

Teilnehmer

M.Barner, Freiburg
W.Barthel, Würzburg
L.Berard-Bergery, Paris
J.P.Bourguignon, Paris
E.Calabi, Philadelphia
S.S.Chern, Berkeley
P.Dombrowski, Köln
P.Eberlein, Bonn
F.Flaherty, Bonn
S.Goldberg, Haifa
D.Gromoll, Stony Brook
K.P.Grotemeyer, Bielefeld
K.Grove, Aarhus
E.Heintze, Bonn
C.C.Hsiung, Bethlehem/Pa.
H.C. Im Hof, Basel
H.Karcher, Bonn
J.L.Kazdan, Philadelphia
J.Kern, Bonn
W.Klingenberg, Bonn
T.Klotz-Milnor, Rutgers
K.Leichtweiß, Stuttgart
J.Lelong-Ferrand, Paris
W.T.Meyer, Stony Brook
R.Michel, Marseille
H.F.Münzner, Bremen
H.Nakagawa, Bonn
J.C.Nash, Bonn
E.Ruh, Bonn
U.Simon, Berlin
G.Stanilov, Sofia
G.Tsagas, Patras
R.Walden, Berlin
R.Walter, Dortmund
B.Wegner, Berlin
T.Willmore, Durham
W.Ziller, Bonn

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L.Berard-Bergery : Laplacian and closed geodesics on compact hyperbolic space forms

Let M ba a compact locally symetric Riemannian manifold of negative curvature. The analysis of the Selberg trace formula in this setting leads to a relation between the spectrum of the laplacian of M and the length and holonomy of the closed geodesics on M. This generalizes some previous results of M.Huber. As an application of the theory of Dirichlet series one gets an asymptotic formula for the number of closed geodesics of length smaller than t.

J.P.Bourguignon : Deformation of metrics

The work of D.G.Ebin points out that it is necessary to examine the action of the group of diffeomorphisms D to study geometric deformation of metrics. By using the techniques developed by H.Berger-D.G.Ebin , we obtain some results on the H.Hopf conjecture on the product of two manifolds.Some other applications can be made to the space of metrics M on a compact manifold, especially to properties of the projection $\pi: M \to M/D$.

E.Calabi : Integral invariants of convex cones (Research by S.Gigena)

We consider in a convex cone K in an (n+1)-dimensional Euclidean space the envelope of the hyperplanes that cut K into two parts, one of which is bounded and has constant volume c. This hypersurface has a natural measure, invariant under the group SL(n+1,R) acting on the enveloping vector space as well as a Riemannian structure. The relative pointwise defined ratio between the natural measure and that induced by the Riemannian structure is related to integral invariants of m-dimensional convex bodies, called the mean squared fractional volume, this is the average of the square of

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the fractional volume in a convex body C of a simplex contained in C. Further applications and relations of this invariant are included.

S.S.Chern : Characteristic classes and elliptic operators

A summary was given of the developments in the above topic including the following:

Curvature definition of Chern classes including the character and the Todd class. Elliptic operators, its topological and analytical index and the Atiyah-Singer Index Theorem, stating the equality of both. The ζ-function of an elliptic operator and the secondary characteristic classes according to Chern-Simons. Finally the operator $D = \pm d * \pm * d$ and conjectures of Atiyah-Singer relating the η-function and the secondary invariants were discussed.

P.Eberlein : The fundamental group of a manifold with nonpositive curvature

A summary was given of the known results in the above topic including:

(1) every element of $\pi_1(M)$ has infinite order

(2) every abelian subgroup of $\pi_1(M)$ is infinite cyclic (M compact, $K < o$), more general every solvable subgroup of $\pi_1(M)$ is infinite cyclic and so is every monic subgroup

(3) Center Theorem of Wolf

(4) $\pi_1(M)$ solvable implies M flat (M compact, $K < o$)

(5) either $\pi_1(M)$ has exponential growth or M is flat (M compact)

F.Flaherty : Curvature measures of polyhedral manifolds

The p-th curvature measure σ_p of an oriented finite polyhedral manifold

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K^m in R^n is defined by: $\sigma_p(K,A) = \sum_{\text{all p-cells}} H^p(A \cap s_p) \cdot \int_{\bar{s}_p} d\, H^{n-p-1}$

for all bounded Borel sets $A \subset R^n$, where H^p is the p-dimensional Hausdorff measure and $\bar{s}_p$ is the chain corresponding to the outer normal of s_p.

The σ_p are stable in the sense that $\int \sigma_j(L \cap K)\xi(n,k) = b_j\, \sigma_{n-k+j}(K)$ where L is a k-dimensional affine subspace of R^n, $\xi(n,k)$ the volume element of the space of such subspaces and b_j depends only on n,k,j.

A Radon measure φ on R^n is called a geometric measure of dimension m iff φ is stable, has support in sets of dimension $\leq n$, is rigid motion invariant and φ_o is a topological invariant. One obtains the following result:

Theorem: Let φ be a geometric measure of dimension m in R^n, then there exist $a_i \in R$ such that $\varphi(K) = \sum_{i=o}^{m} a_i \sigma_i(K)$ for any oriented finite polyhefral manifold K.

S.Goldberg : A Schwartz lemma for a class of harmonic mappings

Let $f: M^n \to \bar{M}^n$ be a harmonic mapping between Riemannian manifolds.

A formula relating the laplacian of the ratio of volume elements of M and $\bar{M}$ with scalar and Ricci curvature $\bar{R}, \bar{\mathrm{Ric}}$ of $\bar{M}$ is obtained which is useful in deriving:

Theorem: (1) f totally geodesic, $R > o$, $\bar{\mathrm{Ric}} \leq o$ (resp. $R < o$, $\bar{\mathrm{Ric}} \geq o$)

$\Rightarrow$ f is totally degenerate.

(2) f totally geodesic, $R \neq o$, $\operatorname{sign} \bar{R} = \operatorname{sign} R$, $|\bar{R}| \geq |R|$

$\Rightarrow$ f is volume decreasing. If in addition f is not totally degenerate and $|\bar{R}| \geq n|R|$ then f is distance decreasing.

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(3) There exists a certain class of harmonic mappings of hyperbolic space into Einstein space with $R \leq -6\, n^2$ which are volume decreasing.

D.Gromoll : Manifolds of nonnegative Ricci curvature

The recent results obtained in this area were discussed. A structure theorem for the fundamental group of such a manifold, obtained by Cheeger and Gromoll, states that there exists a finite invariant subgroup $\Phi \subset \pi_1(M)$ such that $\pi_1(M)/\Phi$ is a Bieberbach group, i.e. an extension of a finite group by a lattice $\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z} \cdots \oplus \mathbb{Z}$ (k times) and $\tilde{M}$ is isometric to $\bar{M} \times \mathbb{R}^k$, where $\bar{M}$ is compact. This result is derived from a generalization of Toponogov's splitting theorem, i.e. every line splits of as a riemannian factor $\mathbb{R}$. It was also mentioned that the only other known topological results are that the $\hat{A}$- genus of a compact 4k-dimensional simply connected spin manifold with $Ric \neq 0$ vanishes and that by a theorem of Aubin a metric on M with $Ric \geq 0$ and $Ric_p > 0$ for some p can be approximated by a metric with $Ric > 0$. The examples of Cheeger, i.e. certain Brieskorn varieties with $Ric > 0$ some of which are exotic spheres, and a larger class of modified Brieskorn varieties with $Ric > 0$ by Hernandez were given, which are not diffeomorphic to homogenous spaces.

K.Grove : Isometry invariant geodesics on compact Riemannian manifolds

The question was considered if there exist A-invariant geodesics where A is an isometry of a compact manifold M. By verifying the Palais-Smale condition for the energy function E on the Hilbert manifold $\Lambda_{G(A)}(M) = \{\sigma \in H_1(I,M) \mid \sigma(1) = A(\sigma(0))\}$

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and applying infinite dimensional critical point theory one obtains, since the critical points of E are the A-invariant geodesics, that if there are no A-invariant geodesics on M then the inclusion $Fix(A) \to \Lambda_{G(A)}(M)$ is a homotopy equivalence. This implies:

Theorem: There exists an A-invariant geodesic on M if one of the following conditions is satisfied:

(1) A is homotopic to Id_M

(2) M is 1-connected, A has not exactly one fixed point and

$Id - A_*: \pi_q(M) \to \pi_q(M)$ is not an isomorphism for all q

(3) $\pi_1(M)$ is finite and A^{p^n} is homotopic to Id_M, p a prime

(4) $\operatorname{rank}(\pi_q(M))$ is odd for some $q \geq 2$

C.C.Hsiung : A remark on the signature of compact manifolds with boundary

Let M^{4k} be an oriented compact C^∞ manifold with boundary B^{4k-1}. The oriented B is called a reflecting boundary of M if it admits an orientation reversing involution. There is presented a generalization, to maifolds M with reflecting boundary B, of the Hirzebruch signature theorem, Atiyah-Singer's G-signature formula and other theorems on the G-signature, and Hirzebruch's signature defect.

H.C.Im Hof : The group of isometries of a compact manifold of negative curvature

A geometric proof is given of the fact that isometry group of a compact manifold of negative curvature is finite, which also gives an upper bound

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for the order of $I(M)$ depending on dim M, diam M, bounds of curvature, and of the length of a certain geodesic loop. By an explicit computation in the hyperbolic space $(H, \bar{g})$, the universal riemannian covering of M, and a study of translations and their invariant geodesics together with Rauch's comparison theorem the result is obtained.

J.L.Kazdan : A converse of the Gauß-Bonnet theorem

The following question is considered: Which functions K on a manifold M^n are the Gauß curvature ($n = 2$) or scalar curvature ($n \geq 3$) of some metric on M. By proving new existence and non-existence theorems for a certain class of non-linear elliptic partial differential equations involved in this problem, the following is proved:

Theorem: (1) M compact, dim $M = 2$. The necessary sign condition on K given by the Gauß-Bonnet theorem is also sufficient if $\chi(M) \leq 0$

(2) M compact, dim $M \geq 3$. Any K that is negative somewhere is a scalar curvature. For certain manifolds this condition is also necessary.

(3) Many open manifolds admit any function as curvature function

(4) $M = \mathbb{R}^2$, K is Gauß curvature of a complete metric if and only if

$$\liminf_{|x| \to \infty} K(x) \leq 0 .$$

(This is joint work with F.W.Warner)

W.Klingenberg : Manifolds with a geodesic flow of Anosov type

(The last lecture in the old Lorenzenhof)

The following consequences of the property that the geodesic flow φ_t of a compact Riemannian manifold M is of Anosov type were considered:

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(1) φ_t is ergodic, i.e. $\operatorname{meas}\{ X \in T_1M \mid \varphi_t X \text{ is dense} \} = \operatorname{meas} T_1M$

(2) $\{ X \in T_1M \mid \varphi_t X \text{ is periodic} \}$ is dense

(3) M has no conjugate points and $\exp_p$ is a covering

(4) $\pi_1(M)$ has exponential growth

(5) every abelian subgroup of $\pi_1(M)$ is infinite cyclic

(6) the isometry group of M is finite

Since the geodesic flow of a manifold with negative curvature is of Anosov type, (1)-(6) are generalizations of well known theorems for these manifolds.

T.Klotz-Milnor : Some restrictions on the smooth immersion of complete surfaces in E^3

Results related to the following conjecture by John Milnor were discussed:

If S is a complete, umbilic free surface C^2 immersed in E^3 with $|k_1| + |k_2| \geq \text{const.} > 0$ then either K changes sign, or else $K \equiv 0$.

This conjecture has been varified in the following special cases:

(1) for surfaces of revolution by John Milnor (unpublished)

(2) for surfaces with $K < \text{const.} < 0$ by Efimov

(3) for surfaces with $K \geq 0$ and $0 < c < |H| < 2c$ by Pogorelov

(4) for C^3 immersed surfaces with $(k_1 - c)(k_2 - c) \leq 0$ by Münzner

(5) for surfaces with $K \leq 0$ on which $Q = \frac{1}{4} \{ L - N - 2iM \}$ is bounded in terms of some global osothermal coordinates on the universal covering of S by the speaker

Various other results suggested by a theorem of Hawley were stated.

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K.Leichtweiß : On the integral formula method in surface theory

The integral formula method consists in the transformation of surface integrals with nonnegative integrand into such integrals with nonpositive integrand. These transformations have been made by applying tensor calculus as well as Cartan's calculus. It was shown that the mixed calculus introduced by K.Voss is of special advantage for the integral formula method. Two general integral formulas for surfaces in E^3 were proofed and as special cases the translation theorems of Christoffel, Minkowski, and Voss and the congruence theorems of Herglotz and Grove were obtained. Infinitisemal versions of these theorems are announced.

J.Lelong-Ferrand : Global conformal invariants on Riemannian manifolds

A Riemannian manifold M^n $(n \geq 2)$ is called of type H_o iff $H^*(M)$ separates points of M where $H^*(M) = \{ u \in H(M) \mid \sup_{x \in \partial D} u(x) = \sup_{x \in D} u(x)$, $\inf_{x \in \partial D} u(x) = \inf_{x \in D} u(x)$ for every conditionally compact domain $D \subset M \}$ and $H(M) = \{ u: M \to \mathbb{R}$ continous $\mid I(u,M) = \int |du|^n * 1 < \infty \}$.

If M is of type H_o then $\delta_M(x,y) = \sup_{u \in H^*(M)} \frac{|u(x)-u(y)|}{(I(u,M))^{1/n}}$ is a conformally invariant distance on M. For M non-compact $M\backslash\{p\}$ for any $p \in M$ is of type H_o and for M compact $M\backslash\{p,q\}$ is of type H_o . These conformal invariants $i(p,x,y) = \delta_{M\backslash\{p\}}(x,y)$, $j(p,q,x,y) = \delta_{M\backslash\{p,q\}}(x,y)$ can be expressed in terms of conformal capacities. Their existence gives some information on the conformal group $C(M)$. In the compact case it allows one to give a new proof of the fact that $C(M)$ is compact if M is not conformally equivalent to the sphere.

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W.Meyer : Manifolds of nonnegative sectional curvature

A summary of the known results in this area was given including:

(1) The soul of a non compact manifold with $K \geq o$, i.e. a totally geodesic compact submanifold of M with $K \geq o$ whose normal bundle is diffeomorphic to M (Cheeger, Gromoll). If $K > o$ then M is diffeomorphic to $\mathbb{R}^n$ (Gromoll and the author).

(2) The δ-pinching theorem for compact simply connected manifolds.

(3) Besides the known examples of locally homogenenous spaces with $K \geq o$ a new example due to Gromoll and the author, i.e. an exotic 7-sphere with $K \geq o$, was constructed, by using the fact that riemannian submersions (esp. projections $\pi: M \to M/G$) weakly increase curvature. The example comes from an appropriate action of Sp(1) on Sp(2) and is shown to be an exotic sphere by describing it as an S^3-bundle over S^4 and using Milnor's classification. Also an example was given, using this method, of a metric on TS^n such that it's soul S^n does not split isometrically.

R.Michel : Geometric analysation in $P_n(\mathbb{K})$ in relation with the generalized conjecture of Blaschke

The generalized conjecture of Blaschke states that the canonical metric structure g on $P_n(\mathbb{R})$ is the only one such that every geodesic is simply closed of length π. This is known to be true for $P_2(\mathbb{R})$ by a theorem of Green. Let Φ be the set of metrics that satisfy the above property. Let $O(g)$ be the orbit of g by the group of diffeomorphisms acting on the

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space of metrics by pullback. An infinitisemal version of Blaschke's conjecture turns out to be true:

Theorem: Every C^∞ variation of g in Φ is tangent to $O(g)$ at g.

For the proof one needs:

Theorem: The space F of symetric tensors K of degree 2 such that $\int_\gamma K(\dot\gamma(t),\dot\gamma(t))dt = 0$ for each " great circle " on $(P_n(\mathbb{R}),g)$ coincides with $T_g O(g)$.

For $P_n(\mathbb{K})$, $\mathbb{K} = \mathbb{R},\mathbb{C},\mathbb{H},\mathbb{C}a$, a weak generalization of the conjecture is true:

Theorem: The space $F_{\mathbb{K}}$ of symetric tensors K of degree 2 with $i^*K = L_X(i^*g)$ for each projective line $i: S \to P_n(\mathbb{K})$ and some vectorfield X along S is equal to $T_g(O(g))$.

H.F.Münzner : Isoperimetric hypersurfaces in spheres

The question was considered if a hypersurface M of S^n with constant principal curvatures is part of an orbit of a group $G \subset SO(n+1)$. M is then defined through an algebraic equation. It was shown with topological methods that it's degree can only take on the values 1,2,3,4,6 . Calculations have shown that for homogenous hypersurfaces these constants do appear, so that the result is sharp.

E.Ruh : Curvature and differentiable structures on spheres

A sketch of the proof of the following theorem was given:

Theorem: A complete simply connected Riemannian manifold of dimension n, with 0.80 -pinched sectional curvature is diffeomorphic to S^n.

The main idea of the proof is to model a diffeomorphism $f: M \to S^n$ on

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the Gauß map. To this effect we define a connection ∇' on the bundle $T(M) \oplus \varepsilon(M)$ where $\varepsilon(M) = M \times \mathbb{R}$. We utilize ∇' to define trivializations u_o and u_1 of the bundle $T(M) \oplus \varepsilon(M)$, restricted to the nothern and southern hemisphere of M. The trivializations u_o and u_1 give rise to Gauß maps f_o and f_1. We define $f: M \to S^n$ as a certain average of f_o and f_1. Estimates on the curvature of ∇' imply that f is a diffeomorphism.

G.Stanilov : A generalization of Riemann curvature and some applications

A generalization of the sectional curvature of a Riemannian manifold is considered, giving a curvature $K(E^m, E^k)$ in the direction of two arbitrary linear subspaces E^m, E^k of M_p. This also includes as a special case the Ricci curvature. A characterisation in terms of this is given of the Riemannian manifolds of constant curvature and Einstein manifolds. For complete Riemannian manifolds a generalization of the theorem of Bonnet and Myers on the diameter of a maifold of positive curvature is given.

G.Tsagas : The cohomology structure of a k-symetric space

A Riemannian manifold M with isometry group I(M) and a map $f: M \to I(M)$ such that x is an isolated fixed point of $f(x)$ is called a symetric space. It is called k-symetric if $(d(f(x))_x)^k = Id$.

Theorem: Let M be symetric. Then M is k-symetric for some k and M is a homogenous space. If M is compact symetric then

$$H^*(M,T) = Tor_{H^*(BG,T)}(T, H^*(BH,T)) \text{ where T is a field.}$$

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B.Wegner : Codazzi tensors and a characterisation of spherical immersions

A Codazzi tensor on (M,g) is a self adjoint tensor A of type $(1,1)$ such that $(D_X A)(Y) = (D_Y A)(X)$. If A is a Codazzi tensor and $\det A \neq o$ A^{-1} is a Codazzi tensor with respect to $\bar{g} = A^* g$. The following formula for Codazzi tensors was proved:

$$\frac{1}{2} \Delta \operatorname{Sp} A^2 = \sum_{i<j} K(x_i, x_j)(\lambda_i - \lambda_j)^2 + \| DA \| - \| \operatorname{grad} \operatorname{Sp} A \|^2 + dv(A(\operatorname{grad} \operatorname{Sp} A))$$

where the λ_i are the eigenvalues of A with eigenvectors x_i and K is the sectional curvature. The second fundamental form A_ξ of an isometric immersion of M into a space form with respect to a normal ξ without zeroes is a Codazzi tensor. In the case of nonnegative sectional curvature one obtains a characterisation of spherical immersions if $\operatorname{Sp} A_\xi$ or $\operatorname{Sp} A_\xi^{-1}$ is constant.

T.Willmore : Stable riemannian immersions

Let M^m ($m > 1$) be a compact oriented immersed hypersurface of $\bar{M}^{m+1}$ with mean curvature μ . The immersion is said to be stable if $\delta \int \mu^m * 1 = o$

The corresponding Euler equation is:

$$\Delta \mu^{m-1} + m(m-1)\mu^{m+1} + \mu^{m-1}(R - \bar{R} - \bar{R}_{\alpha\beta} n^\alpha n^\beta) = o$$

from which it follows that $m(m-1)\mu^2 + R - \bar{R} \geq o$ and also:

Theorem: Let M^{2k-1} be a compact oriented immersed hypersurface of $\bar{M}^{2k}$ whose Ricci curvature is negative definite. Then if M is stable it is necessarily a minimal hypersurfave.

W.Ziller (Bonn)

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