MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35|1972

"Mathematical Methods in Celestial Mechanics"

August 27 to September 2, 1972

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Abstracts

K. Aksnes: Application of Hill Variables and the Hori-Lie Perturbation Method in Satellite Theory.

R.F. Arenstorf: Periodic Solutions of Elliptic Type in the Three-Body Problem and the Four-Body Problem.

An exposition of the main points and ideas for the proof of existence of the periodic motions referred to in the title will be given. These motions can be roughly described as follows: Consider in the first case the bodies P_1, P_2, P_3 in a fixed plane. Then the pair (P_1, P_2) performs nearly keplerian, precessing elliptic motion of arbitrarily given eccentricity; and their center of mass, P_{12} say, together with P_3 performs nearly circular, slowly precessing elliptic motion, such that the distance $\overline{P_1P_2}$ remains small compared to $\overline{P_3P_{12}}$, always. The positive masses of the three bodies can be assigned arbitrarily. In the second case (again for plane motion) we replace the body P_3 in the preceding description by two bodies P_3 and P_4 of arbitrary, but equal masses. Then P_1, P_2 perform precessing, nearly elliptic motions about P_{12}, as above; while P_{12} together with P_3 and P_4 form a rotating, nearly equilateral triangle of large dimension compared to $\overline{P_1P_2}$, in which each side executes a slowly precessing elliptic motion of small eccentricity. These

motions are strictly periodic in suitable, uniformly rotating coordinate systems (relative to an inertial system with origin at the fixed center of mass of all bodies), the speed of rotation depending on the individual solution. The solutions of the first case are applicable to the Lunar Theory.

J. Baumgarte: Numerical tabilization of all Laws of Conservation in the Many-Body Problem.

When a system of differential equations admits a first integral (e.g. the law of energy), the value of that integral may be used as a check during the numerical integration. Often this check is satisfied with poor accuracy since the existence of the first integral is unknown to the computer. The aim of the paper is to show how such a first integral can be satisfied with better accuracy and in a stabilized manner by adding an appropriate control term to the differential system. The accuracy of the numerical integration is thereby improved. This basic idea is applied to the problem of n bodies.

D.G. Bettis: A Runge-Kutta Nyström Algorithm.

A Runge-Kutta algorithm of order five is presented for the solution of the initial value problem where the system of ordinary differential equations is of second order and does not contain the first derivative. The algorithm includes the Fehlberg step control procedure.

R. Broucke: Periodic Solutions of a Spring-Pendulum System.

A study has been made of a dynamical system composed of a pendulum and a harmonic oscillator, in order to show the remarkable resemblance with many classical celestial mechanics problems, in particular the restricted three-body problem. It is shown that the well-known investigations of periodic orbits can be applied to the present dynamics problem.

J.M.A. Danby: The Evolution of Periodic Orbits Close to Homoclinic Points.

For conservative dynamical systems having two degrees of freedom Birkhoff has established the existence of two classes of periodic orbits. The first consists of stable-unstable pairs close to periodic orbits of the stable type, and the second of orbits having fixed points (in a suitable surface of section) close to homoclinic points. In this paper orbits of the latter type are listed, and their evolution followed as a function of the energy. For the energy at which they were first computed, all were unstable; but they evolved, with diminishing energy, into one orbit of the stable type which appears to be a member of the first class of orbits mentioned above.

C. Froeschlé: Numerical Study of a Four-Dimensional Mapping.

The study of dynamical systems with three degrees of freedom can be reduced to the study of a four-dimensional mapping using the method of surface of section.

On the other hand, a second and more important reason for studying a diffeomorphism problem is that in such a problem the phenomena of the qualitative theory of ordinary differential equations are present in their simplest form.

Therefore a mapping T of a four-dimensional torus $M_4 = (x\ y\ z\ t)$ (mod 2π) into itself has been studied numerically. The mapping T is defined by:

$$T \begin{cases} x_1 = x_o + a_1 \sin(x_o + y_o) + b \sin(x_o + z_o + t_o) \\ y_1 = x_o + y_o \\ z_1 = z_o + a_2 \sin(z_o + t_o) + b \sin(x_o + y_o + z_o + t_o) \\ t_1 = z_o + t_o \end{cases} \pmod{2\pi}$$

Two methods of a non graphical nature have been used:

1- The divergence of two initially close orbits
2- The variations of the two largest eigenvalues of the linear tangenital mapping of T^i.

Earlier results for a system of two degrees of freedom are confirmed. These methods appear to be good indicators of stochasticity. In particular, a new numerical method enables us to study the variation with i of the two largest eigenvalues in absolute magnitude of the linear tangential mapping T^{i*} of the mapping T^i. This variation appears to be a very sensitive indicator of stochasticity and shows that the diffeomorphism T seems to follow the general behaviour of a C-system in the "wild" zone (sometimes abusively called "ergodic").

On the other hand, the results for some non-integrable cases show a behaviour that is always purely "ergodic". [This phenomenon is studied further, using the visual methods, that is slice-cutting and perspective views. It is found that the points do not fill a broad wild zone but remain in the neighbourhood of some invariant manifold.]

A study of the effect of coupling using both types of methods shows that even for large coupling there are still integrable zones and in some cases the slice-cutting method shows the existence of small islands.

B. Garfinkel: <u>Global solution of the Ideal Resonance Problem.</u>

If a dynamical system of n degrees of freedom is reduced to the Ideal Resonance Problem, the Hamiltonian takes the form,

$$F = B(y) + 2\,\mu^2 A(y) \sin^2 x_1 , \qquad \mu \ll 1. \tag{1}$$

Here y is the momentum-vector y_k, $k = 1,2,\dots,n$, and x_1 is the <u>critical argument</u>. Such a reduction is possible if the resonant Hamiltonian of the original problem is <u>strongly</u> dominated by one of its trigonometric terms. Then (1), as a perturbed simple pendulum, furnishes a convenient and accurate reference orbit for an attack on the corresponding resonance problem, arising from the near-commensurability of two fundamental frequencies of the motion. In celestial mechanics such problems are illustrated by the critical inclination of an artificial satellite orbit, and by tesseral harmonics resonance, involving a satellite of a spinning primary.

The purpose of this Note is to summarize a first-order solution of the problem defined by (1).

P. Guillaume: Symmetric Periodic Orbits of the Restricted Problem.

An approximate analytic description is given of the families of periodic symmetric solutions of the planar circular restricted three body problem for small values of μ.

M. Hénon: Vertical Stability of Periodic Orbits in the Restricted Problem.

The stability of plane periodic orbits has generally been studied only with respect to perturbations in the plane. Here we consider their stability with respect to small perturbations perpendicular to the plane. A new index of stability a_v is defined; the orbit is stable if $|a_v| < 1$. This index is computed numerically for a number of orbits belonging to the main families of the restricted problem of three bodies. In each family, intervals of stability and instability are found. Instability is generally milder than in the plane: a_v does not take very high values.

Cases of orbits which are stable in the plane, but unstable in the perpendicular direction are found.

G. Hori: Secular Variations of the Orbital Plane of Satellites.

A. Jupp: On the Global Solution in the Resonance Problem of Poincaré.

Poincaré formulated the general problem of resonance in the case of a dynamical system which is reducible ot one degree of freedom.

He introduced the concept of the global solution; in essence, this means that the domain of the solution(s) covers the entire phase plane, comprising regions of libration and circulation.

It is the author's opinion that the technique proposed by Poincaré for the construction of the global solution is impractical. An alternative procedure, which admits secular terms into the determining function, is outlined. The latter method has been successfully applied to the Ideal Resonance Problem, which is a special case of the more general problem considered by Poincaré.

U. Kirchgraber: A Set of Elements based on Polar Coordinates in the KS-Space and its Applications.

A new set of canonical elements is introduced into the field of KS-theory. The close relationship of these elements with a set of elements proposed by G. Scheifele is analyzed. Some applications are outlined.

W.T. Kyner: Resonance.

P.J. Message: A Problem in Resonance.

Saturn's satellite Hyperion experiences large perturbations by Titan, the largest of Saturn's satellites, because of the closeness of Hyperion's orbital period to three-quarters of that of Titan. The motion of Hyperion is a superposition of periodic fluctuations (both free and forced) onto a motion which is periodic in a suitable uniformly rotating frame of reference, and in which Hyperion would be at a maximum distance from Saturn at each conjunction with Titan. Successive attempts to determine the mass of Titan from observed perturbations of Hyperion have suffered from omissions in the theory of terms subsequently found to be significant. An attempt is in progress which, it is believed, comprises all long-period changes in the osculating elements which are of second degree in the mass of Titan, and of third degree in the eccentricity of its orbit. Results so far obtained indicate that the period of the free motion of the orbit plane of Hyperion is better determined by Woltjer's reduction of the observations than by his theoretical calculations.

F. Nahon: Trajectoires rectilignes du problème des 3 corps lorsque la constante des forces vives est nulle.

H. Pollard: Recent Progress in the n-Body Problem.

E. Rabe: Parameter Distribution of small Periodic Librations about Equilateral Points of the Elliptic Restricted Problem.

As previously shown (Rabe, 1970), two classes of small periodic librations exist in the plane elliptic restricted problem, for an infinite sequence of easily specified oscillation frequencies Z_j. The present paper considers the dependence of Z on the eccentricity e of the primary motion, in addition to its dependence on the mass parameter κ, and determines the resulting relations between κ and e, for any given periodic frequency Z_j. These relationships are obtained from the condition $D(Z_j, \kappa, e) = 0$, where the basic determinant D has been expanded up to terms of order Z^{20}, κ^5, and e^4.

E.A. Roth: Fast Computation of High E centricity Orbits by the Stroboscopic Method.

In order to compute with a reasonable accuracy satellite orbits subjected to perturbations by J_2, the sun, the moon and airdrag, a first order expansion is used. The Lagrange equations are solved semi-analytically by the stroboscopic method. The intermediate , long-periodic and secular terms are obtained, but if desired the same formulism also produces the short-periodic terms. The method is well suited for use on a computer and requires only about 1% of computing time needed for numerical integration.

D. Saari: On Global Existence and Uniqueness Theorems for the Problem of n Bodies.

Recent results concerning singularities and collisions in the problem of n bodies are reviewed. It is shown how they relate to the question of finding a global existence and uniqueness theorem for this dynamical system.

G. Scheifele: Canonical Satellite Theory based on finite Fourier Series with respect to the Canonical Elements.

Transformations of the independent variable of the type $dt/ds=f(r)$ (r=distance) lead, in the problem of two bodies, to a separable Jacobian equation when the extended phase space is used. A set of eight elements is obtained. Six of these elements are related very closely to the classical elements of Delaunay.

The case $dt/ds = r^2$ is of particular interest, since the independent variable is proportional to the true anomaly, and in addition, the true anomaly is one of the canonical angular variables. The Hamiltonian due to the perturbation by a finite number of zonal harmonics of an oblate planet turns out to be a finite Fourier series with respect to the canonical angular variables. By applying the von Zeipel perturbation method, advantage from this finite representation is drawn.

J. Schubart: Some Recent Work on the Orbital Theory of Asteroids.

This is a brief report on three theoretical projects about asteroids. The results presented here were recently obtained at the Astronomisches Rechen-Institut in Heidelberg, and it is planned to publish them in "Astronomy and Astrophysics" in detail. The first project is a continuation of former work on asteroids with a mean motion commensurable to that of Jupiter (compare Schubart 1968). In the two other cases the problem consists in the best possible determination of the value of a planetary mass in units of solar mass.

A.T. Sinclair: The Formation of Commensurabilities in Satellite Systems due to the Action of Tidal Forces.

The five types of resonance possible between a pair of satellites at a 2:1 commensurability are described. By a modification of the method usually used in the restricted three-body problem, phase-plane diagrams are constructed for these resonances for the more general case where both satellite masses are non-zero. These phase-plane diagrams are used to discuss the different types of motion possible at the five resonances.

It is shown that tidal forces can drive a pair of satellites towards a commensurability, and at the 2:1 commensurability it is possible for the satellites to be captured into a libration at any of the five resonances, the probability of capture depending on the eccentricities, inclinations, and masses of the satellites. The tidal hypothesis provides a reasonable explanation of the origin of the commensurabilities between Mimas and Tethys, and between Enceladus and Dione, in the satellite system of Saturn.

V. Szebehely: Triple Close Approaches in the Problem of Three Bodies.

The gravitational problem of three bodies is treated in the case when the masses of the participating bodies are of the same order of magnitude and their distances are arbitrary. Estimates for the minimum perimeter of the triangle formed by the bodies and for the rate of the expansion of the system are obtained from Sundman's modified general inequality when the total energy of the system is negative. These estimates are used to propose and to describe an escape mechanism based on genuine three-body dynamics and to offer a method to control the accuracy of numerical integrations of the problem of three bodies. The requirements for these two applications are contradictory since an escape is the consequence of a close triple approach which phenomenon is detrimental to the accuracy of the computations. Consequently, the numerical study of escape from a triple system must treat triple close approaches with high reliability.

J. Tschauner: Das elliptische eingeschränkte Dreikörperproblem.

Eine einfache Herleitung der allgemeinen Gleichungen des elliptischen eingeschränkten Dreikörperproblems und der daraus folgenden linearen Variationsgleichungen mit periodischen Koeffizienten für die Bewegung in der Nähe der Librationszentren wird gegeben. Die in zwei früheren Arbeiten vorgelegte Aufspaltung der ebenen Variationsgleichungen 4. Ordnung in zwei Komponenten 2. Ordnung wird kurz rekapituliert. Bei Benützung dieser Aufspaltung wird die Bewegung in der Umgebung der Dreieckspunkte näher untersucht. Mit Hilfe einer Näherungslösung werden Frequenzverteilung und Stabilitätsbereiche bestimmt.

J.P. Vinti: Quadrature Solution for the General Relativistic Motion of a Satellite or a Planet.

O. Volk: Kepleriana.

J. Waldvogel: The Rectilinear Restricted Problem of Three Bodies.

In this paper we discuss some aspects of the isosceles case of the rectilinear restricted problem of three bodies, where two primaries of equal mass move on rectilinear ellipses, and the particle is confined to the symmetry axis of the system. In particular, the behaviour near a collision of the primaries and also near a collision of all three bodies is investigated. It is shown that this latter singularity is a triple collision in the sense of Siegel's theory. Furthermore, asymptotic expansions for the particle's motion during a parabolic and a hyperbolic escape are derived.

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft